

Grand unified theories with dimension-5 interactions: Gauge unification and intermediate scales

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Dimension-5 corrections to the gauge kinetic term of grand unified theories may capture effects of quantum gravity or string compactification. Such operators modify the usual gauge coupling unification prediction in a calculable manner. Here we examine $SU(5)$, $SO(10)$, and $E(6)$ grand unified theories in the light of all such permitted operators and calculate the impact on the intermediate scales and the unification program. We show that in many cases at least one intermediate scale can be lowered to even 1–10 TeV, where a neutral Z' and possibly other states are expected.

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I. INTRODUCTION

Grand unified theories (GUTs) [1] relate the strong and electroweak interactions of the standard model (SM) at a high energy, M_X , and embody quark-lepton unification, leading to testable predictions such as proton decay and $n - \bar{n}$ oscillations. The characteristic energy of the SM, which is based on the gauge group $\mathcal{G}_{\text{SM}} \equiv SU(2)_L \otimes U(1)_Y \otimes SU(3)_c$, is the electroweak scale M_Z . The vast difference between M_X and M_Z introduces a hierarchy problem in GUTs which is often addressed through the introduction of supersymmetry (SUSY). The rich predictions of these theories—of both the nonsupersymmetric and supersymmetric varieties—have received much attention. At the moment a clear experimental confirmation of the GUT paradigm is keenly awaited.

The fourth fundamental interaction, namely, gravity, is not a part of GUTs. It is widely expected that grand unified theories will have a setting in some larger framework, e.g., string theory, effective at higher energies close to the Planck scale, M_{Pl} , which will encompass gravitational interactions within its fold. Without going into the details of such a theory one can hope to probe some of its implications through effective operators at the GUT scale, suppressed by inverse powers of M_{Pl} , which may emerge from it and alter the grand unified theory predictions.

The particular higher dimensional operator which we consider here impacts the gauge kinetic term:

$$\mathcal{L}_{\text{kin}} = -\frac{1}{4c} \text{Tr}(F_{\mu\nu} F^{\mu\nu}), \quad (1)$$

where $F^{\mu\nu} = \sum_i \lambda_i F_i^{\mu\nu}$ is the gauge field strength tensor with λ_i being the matrix representations of the generators normalized to $\text{Tr}(\lambda_i \lambda_j) = c \delta_{ij}$. For $SU(n)$ groups the λ_i are conventionally chosen in the fundamental representation with $c = 1/2$.

The dimension-5 (dim-5) interaction which we include is [2,3]

$$\mathcal{L}_{\text{dim-5}} = -\frac{\eta}{M_{\text{Pl}}} \left[\frac{1}{4c} \text{Tr}(F_{\mu\nu} \Phi_D F^{\mu\nu}) \right], \quad (2)$$

where Φ_D denotes the D -component Higgs multiplet and η parametrizes the strength of this interaction. In order for it to be possible to form a gauge invariant of the form in Eq. (2), Φ_D can be in any representation included in the symmetric product of two adjoint representations of the group. When Φ_D develops a vacuum expectation value (VEV) v_D , which breaks the GUT symmetry and sets the scale of grand unification M_X , an effective gauge kinetic term is generated from Eq. (2). Depending on the structure of the VEV, this additional contribution usually will not be the same for the different subgroups to which the GUT group is broken, leading, after a scaling of the gauge fields, to a modification of the unification condition to

$$g_i^2(M_X)(1 + \epsilon \delta_i) = g_U^2, \quad (3)$$

wherein g_U is the unified gauge coupling, $\epsilon = \eta v_D / 2M_{\text{Pl}} \sim \mathcal{O}(M_X / M_{\text{Pl}})$, and the group-theoretic factors δ_i arise from Eq. (2). The δ_i were available in the literature for some selected choices of Φ_D and GUT groups [2]. They were exhaustively evaluated for the first time for all possible Φ_D for $SU(5)$, $SO(10)$, and $E(6)$ GUTs¹ in [3]

While normally in GUTs the gauge couplings are expected to reach a common value at M_X [6], in the presence of dim-5 terms, as in Eq. (2), the modified boundary conditions of Eq. (3) must be satisfied. It is indeed possible that this tweaking will be just enough to entail the unification program to succeed with the current low energy values of the coupling constants as a boundary condition. To check this for $SU(5)$, $SO(10)$, and $E(6)$ -based GUT models is the main goal of this work. We discuss both the nonsupersymmetric and supersymmetric alternatives.

For $SU(5)$ this analysis has appeared in our earlier short note [3] and it is briefly recapitulated here. GUTs based on $SO(10)$ and $E(6)$ provide several routes of descent to the

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¹In SUSY the δ_i also have a direct application in the non-universality of gaugino masses [3–5].

SM, with different levels of symmetry being active at the intermediate stages. This richer structure often bears new testable features. One of these is the possibility of $n - \bar{n}$ oscillations which in $SO(10)$ can be mediated via scalar fields that are not superheavy. Also, the right-handed neutrino, ν_R , which is present in both $SO(10)$ and $E(6)$ GUTs, can lead to light neutrinos through the seesaw mechanism. If the neutrino Yukawa couplings are not unnaturally small, the seesaw mechanism posits a large Majorana mass for the ν_R . This mass is fixed by the scale of $(B - L)$ symmetry breaking which is determined in our analyses below.

For $SO(10)$ we examine the breaking through the intermediate Pati-Salam [$\mathcal{G}_{224} \equiv SU(2)_L \otimes SU(2)_R \otimes SU(4)_c$] symmetry. \mathcal{G}_{224} itself can break directly to the SM or via another intermediate group $\mathcal{G}_{2131} \equiv SU(2)_L \otimes U(1)_R \otimes SU(3)_c \otimes U(1)_{(B-L)}$. We explore both routes. $E(6)$ allows an intermediate $SO(10)$ symmetry and in this case the results are to a great extent similar to that of $SO(10)$ GUTs. Here we look at $E(6)$ breaking via the intermediate gauge group $\mathcal{G}_{333} \equiv SU(3)_L \otimes SU(3)_R \otimes SU(3)_c$ with possibly also an intervening $\mathcal{G}_{21213} \equiv SU(2)_L \otimes U(1)_{Y'_L} \otimes SU(2)_R \otimes U(1)_{Y'_R} \otimes SU(3)_c$ symmetry before descending to the SM.

When there are intermediate scales in the GUT-symmetry breaking the scalar masses have been fixed using the ‘‘extended survival hypothesis’’ (ESH) [7] which is motivated along the following lines. Normally, the lack of any protection mechanism will tend to move all scalar masses to the GUT scale. The necessity of light scalars is dictated by the requirement to trigger spontaneous symmetry breaking at lower energies and this entails a fine-tuning in the scalar sector. The extended survival hypothesis, which can also be termed ‘‘minimal fine-tuning,’’ simply requires that all scalars acquire mass at the GUT scale barring those that are essential for symmetry breaking at lower scales. The latter carry masses of the order of the scales of the symmetry breakings for which they are responsible. For any such scalar, at intermediate stages of symmetry above its mass scale, out of the full GUT scalar multiplet only the submultiplet containing this scalar remains at that scale, the remainder being at M_X . As an illustrative example consider the decay chain

$$\begin{aligned} SO(10) &\xrightarrow{M_X} SU(2)_L \otimes SU(2)_R \otimes SU(4)_c \\ &\xrightarrow{M_R} SU(2)_L \otimes U(1)_Y \otimes SU(3)_c \xrightarrow{M_Z} U(1)_{em} \otimes SU(3)_c. \end{aligned} \quad (4)$$

The electroweak symmetry breaking is through the \mathcal{G}_{SM} doublets, $(2, \pm 1, 1)$, which emerge from a \mathcal{G}_{224} submultiplet $(2, 2, 1)$ which is a part of the $SO(10)$ multiplet 10. Under \mathcal{G}_{224} , $10 \equiv (1, 1, 6) + (2, 2, 1)$. According to the ESH, out of the 10 of $SO(10)$ the scalars forming the $(1, 1, 6)$ submultiplet acquire a mass M_X , while the $(2, 2, 1)$ under \mathcal{G}_{224} are at the electroweak scale M_Z . The scalar masses deter-

mine from which energy their effect on gauge coupling evolution has to be included. Whenever earlier work including the ESH contribution to gauge coupling evolution is available with which our results can be compared, we do so.

The generic renormalization group (RG) equations governing gauge coupling evolution are

$$\mu \frac{dg_i}{d\mu} = \beta_i(g_i, g_j), \quad (i, j = 1, \dots, n), \quad (5)$$

where n is the number of couplings in the theory and at two-loop order

$$\beta_i(g_i, g_j) = (16\pi^2)^{-1} b_i g_i^3 + (16\pi^2)^{-2} \sum_{j=1}^n b_{ij} g_j^2 g_i^3. \quad (6)$$

When using this two-loop formula, the matching of the coupling constant α_k below an intermediate scale M_I which goes over to α_l thereafter follows the relation [8,9]

$$\frac{1}{\alpha_k(M_I)} - \frac{C_k}{12\pi} = \frac{1}{\alpha_l(M_I)} - \frac{C_l}{12\pi}, \quad (7)$$

where C_k is the quadratic Casimir for the k th subgroup. At the unification scale, M_X , this has to be supplemented with the contributions from the dim-5 operators in Eq. (3).

A subtle feature [10,11], considered most recently within the context of $SO(10)$ in [12], has to do with the dynamical mixing of two $U(1)$ subgroups of an intermediate gauge symmetry even at the one-loop level. The $U(1)$ gauge currents and the $U(1)$ gauge boson fields are by themselves gauge invariant and so cross couplings between them are not forbidden by gauge symmetry. Even if the mixing is set to zero at some scale it emerges again through the RG flow. The origin of this mixing in the RG equations lies in the following fact: while the trace of the product of two different $U(1)$ generators vanishes over an entire gauge multiplet, when only a submultiplet is light [e.g., some scalars of a multiplet remaining light due to the extended survival hypothesis in $SO(10)$ or $E(6)$, or incomplete light fermion multiplets in $E(6)$] this is no longer so. This requires a more sophisticated analysis leading to a coupling of g_{1m} and g_{1n} in the one- and two-loop RG equations where m and n identify two $U(1)$ groups. These terms, not made explicit in Eq. (6), arise in the two-step breaking options for $SO(10)$ and $E(6)$ and are detailed in the discussions in the respective sections.

We consider both nonsupersymmetric as well as supersymmetric versions of the theory. In the latter case the contributions of the superpartners to the beta functions are included. (We assume that the SUSY scale is at $M_{SUSY} = 1$ TeV.) As is well known [13], unification of coupling constants is compatible with TeV-scale supersymmetry. We find that the addition of the dim-5 contributions does not spoil this.

This paper is structured as follows. In the next section we recapitulate the case of $SU(5)$ GUTs to set the *modus*

operandi for the program. In the two subsequent sections we consider $SO(10)$ - and $E(6)$ -based theories where we also explore the possibility of one or more intermediate mass scales. We require that the unification scale be above the lower bound from proton decay² and below the Planck scale and that all couplings should remain perturbative throughout the energy range. We find that in most cases there is one intermediate scale which can be as low as 1–10 TeV at which one expects a Z' neutral gauge boson and possibly other new particles. These provide a testable prediction within striking range of the LHC. The other scale(s) populating the GUT desert are usually high and $n - \bar{n}$ oscillations may not be observable.³ In the final section we summarize the results.

II. $SU(5)$

The group $SU(5)$ supports the leanest grand unified theory. It incorporates the quarks and leptons of one generation in two irreducible representations: $\bar{5}$ and 10. Unlike $SO(10)$ and $E(6)$, which are groups of rank 5 and 6, respectively, $SU(5)$ being a group of rank 4, only permits a direct breaking to the SM with no intermediate step possible. Though one of our aims in this work is to look for intermediate scales in GUT-symmetry breaking, for the sake of completeness we give a brief account of the results for $SU(5)$ [2,3]. The symmetry breaking is

$$SU(5) \xrightarrow{M_X} SU(2)_L \otimes U(1)_Y \otimes SU(3)_c. \quad (8)$$

The adjoint representation of $SU(5)$ is 24 dimensional. Since $(24 \otimes 24)_{\text{sym}} = 1 \oplus 24 \oplus 75 \oplus 200$, nontrivial contributions in Eq. (2) can arise if Φ_D transforms as the 24, 75, or 200 representation. The deviations from gauge unification due to these representations, parametrized by the δ_i in Eq. (3), are listed in Table I. The evolution of the gauge couplings⁴ are governed by the one- and two-loop beta-function coefficients:

$$b_1 = 4 + \frac{1}{10}n_H; \quad b_2 = -10/3 + \frac{1}{6}n_H; \quad b_3 = -7; \quad (9)$$

and

$$b_{ij} = \begin{pmatrix} 19/5 & 9/5 & 44/5 \\ 3/5 & 11/3 & 12 \\ 11/10 & 9/2 & -26 \end{pmatrix} + n_H \begin{pmatrix} 9/50 & 9/10 & 0 \\ 3/10 & 13/6 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (10)$$

²The current bound [14] $\tau_p(p \rightarrow e^+ \pi^0) > 1.6 \times 10^{33}$ yr translates to $M_X > 10^{15.4}$ GeV. Conservatively, we use a lower limit of 10^{16} GeV for M_X .

³For one exceptional case, see Sec. III B 1.

⁴ $g_{1,2,3}$ correspond to the $U(1)_Y$, $SU(2)_L$, and $SU(3)_c$ subgroups, respectively.

TABLE I. Effective contributions (δ_i) to gauge kinetic terms from different Higgs representations in Eq. (2) for $SU(5)$ [see Eq. (3)].

$SU(5)$ representations	δ_1	δ_2	δ_3
24	$1/\sqrt{15}$	$3/\sqrt{15}$	$-2/\sqrt{15}$
75	$4/\sqrt{3}$	$-12/5\sqrt{3}$	$-4/5\sqrt{3}$
200	$1/\sqrt{21}$	$1/5\sqrt{21}$	$1/10\sqrt{21}$

TABLE II. $SU(5)$ dimension-5 interaction strength, ϵ , and the gauge unification scale, M_X , for different Φ_D representations using the two-loop RG equations.

$SU(5)$ representations	Non-SUSY		SUSY	
	ϵ	M_X (GeV)	ϵ	M_X (GeV)
24	0.077	4.78×10^{13}	-0.009	1.64×10^{16}
75	-0.039	2.37×10^{15}	0.004	1.22×10^{16}
200	-1.27	2.59×10^{17}	0.146	9.35×10^{15}

n_H ($= 1$ for the SM) is the number of Higgs doublets. These are for the nonsupersymmetric case.

For SUSY one must also include the contributions from the superpartners to the beta-function coefficients. With three generations and two Higgs doublets one has

$$b_1 = \frac{33}{5}; \quad b_2 = 1; \quad b_3 = -3; \quad (11)$$

$$b_{ij} = \begin{pmatrix} 199/25 & 27/5 & 88/5 \\ 9/5 & 25 & 24 \\ 11/5 & 9 & 14 \end{pmatrix}.$$

Below M_{SUSY} , Eqs. (9) and (10) are operative with $n_H = 2$ while beyond M_{SUSY} Eq. (11) is employed.

The results of a two-loop RG analysis are shown in Table II. We find that for both the non-SUSY as well as the SUSY alternatives unification is possible in the $SU(5)$ GUT when additional effective interactions of dimension-5 are in play. M_X , the unification scale, and ϵ , the strength of the dim-5 interaction, are shown in Table II for the different choices of Φ_D . It is seen that for the non-SUSY case, unification, though achievable with the dim-5 interactions, is not satisfactory. For Φ_{24} and Φ_{75} the unification scale M_X is too low to be consistent with the current limits on the proton decay lifetime while for Φ_{200} , ϵ is larger than unity. The solutions for the SUSY case are satisfactory on every count.

III. $SO(10)$

$SO(10)$ [15] is the smallest GUT which accommodates all the fermions of a generation in one irreducible multiplet, the spinorial 16. The group admits a left-right symmetric subgroup [16]—the Pati-Salam $SU(2)_L \otimes SU(2)_R \otimes SU(4)_c$ which we denote by \mathcal{G}_{224} —with inter-

TABLE III. Effective contributions (δ_i) to gauge kinetic terms from different Higgs representations in Eq. (2) for $SO(10)$ [3] [see Eq. (3)].

$SO(10)$ representations	δ_{2L}	δ_{2R}	δ_{4c}
54	$3/2\sqrt{15}$	$3/2\sqrt{15}$	$-1/\sqrt{15}$
210	$1/\sqrt{2}$	$-1/\sqrt{2}$	0
770	$5/3\sqrt{5}$	$5/3\sqrt{5}$	$2/3\sqrt{5}$

esting new phenomenology including quark-lepton unification within the $SU(4)_c$. The chain of symmetry breaking that we discuss here is

$$SO(10) \xrightarrow{M_X} SU(2)_L \otimes SU(2)_R \otimes SU(4)_c \xrightarrow{M_C} SU(2)_L \otimes U(1)_R \otimes SU(3)_c \otimes U(1)_{(B-L)} \xrightarrow{M_R} \text{SM}. \quad (12)$$

Some subcases which we also look at are when (i) $M_X = M_C = M_R$ which corresponds to a breaking of $SO(10)$ to the SM with no intervening steps, and (ii) $M_C = M_R$ which is a situation where $SO(10)$ reduces to the SM through one intermediate step. We consider these cases one by one. All results presented below are based on two-loop RG analyses.

The adjoint representation of $SO(10)$ is 45 dimensional. Since $(45 \otimes 45)_{\text{sym}} = 1 \oplus 54 \oplus 210 \oplus 770$, Φ_D in Eq. (2) transforms as the 54, 210, or 770 representation. The deviations from gauge unification due to these representations, parametrized by the δ_i in Eq. (3), are listed in Table III.

A. No-step breaking in $SO(10)$

This is the most straightforward symmetry breaking for $SO(10)$ and is much like the $SU(5)$ case discussed in Sec. II.

$$SO(10) \xrightarrow{M_X} SU(2)_L \otimes U(1)_Y \otimes SU(3)_c. \quad (13)$$

When there are no intermediate scales the gauge coupling evolutions are governed by Eqs. (9) and (10) for the nonsupersymmetric case and Eq. (11) for the SUSY version.

TABLE IV. Dimension-5 interaction strength, ϵ , and the gauge unification scale, M_X , for different Φ_D representations using two-loop RG equations when $SO(10)$ descends directly to the SM.

$SO(10)$ representations	Non-SUSY		SUSY	
	ϵ	M_X (GeV)	ϵ	M_X (GeV)
54	0.170	3.99×10^{13}	-0.013	1.54×10^{16}
210	0.088	4.39×10^{14}	-0.008	1.35×10^{16}
770	0.274	4.10×10^{13}	-0.018	1.54×10^{16}

The results are shown in Table IV. As for $SU(5)$, we find that the nonsupersymmetric solutions are untenable. For all three choices of Φ_D the unification scale is $\mathcal{O}(10^{13}-10^{14})$ GeV, which is excluded by the current observational bounds on the proton decay lifetime.

B. One-step breaking in $SO(10)$

Here we have to consider the following breaking chain of $SO(10)$:

$$SO(10) \xrightarrow{M_X} SU(2)_L \otimes SU(2)_R \otimes SU(4)_c \xrightarrow{M_C} \text{SM}. \quad (14)$$

The \mathcal{G}_{224} intermediate group offers a new discrete symmetry— D parity [9,17]. This symmetry relates the gauged $SU(2)_L$ and $SU(2)_R$ subgroups of $SO(10)$ much the same way that ordinary parity relates the $SU(2)_L$ and $SU(2)_R$ subgroups of the Lorentz group $SO(3,1)$. Alternative routes of $SO(10)$ symmetry breaking are admissible which either preserve or violate D parity at the intermediate stages. We will consider both in the following. The first step of symmetry breaking from $SO(10)$ to \mathcal{G}_{224} is accomplished by assigning an appropriate VEV to a 54, 210, or 770-dimensional Higgs. $\langle \Phi_{54} \rangle$ or $\langle \Phi_{770} \rangle$ ensure that D parity is conserved while $\langle \Phi_{210} \rangle$ breaks D parity. This is reflected in Table III in that $\delta_{2L} = -\delta_{2R}$ in this case whereas in the other cases they are equal.

The next step breaking of \mathcal{G}_{224} to the SM is achieved through the VEV of a 126-dimensional Higgs. The submultiplet of 126_H that develops a VEV for this purpose at the scale M_C transforms as $(1, 3, \bar{10})$ under \mathcal{G}_{224} . According to the extended survival hypothesis the entire submultiplet acquires a mass $\mathcal{O}(M_C)$ while the other members of 126_H are at M_X . This is true if D parity is not conserved. When D parity remains unbroken then it relates the $(1, 3, \bar{10})$ submultiplet to the $(3, 1, 10) \subset 126_H$ and it too has a mass of $\mathcal{O}(M_C)$.

One must also consider the Higgs scalars ϕ_{SM} responsible for the breaking of SM at $\sim M_Z$. They transform under \mathcal{G}_{SM} , \mathcal{G}_{224} , and $SO(10)$ as $\{(2, 1, 1) + (2, -1, 1)\}$, $(2, 2, 1)$, and 10, respectively. Notice that the extended survival hypothesis mandates that the $(1, 1, 6)$ under \mathcal{G}_{224} contained in the $SO(10)$ 10-dimensional representation has a mass at M_X while the $(2, 2, 1)$ is at M_Z .

The scalars contributing to the RG evolution in different stages are summarized in Table V.

When the couplings are evolved from their low energy inputs the key matching formula at M_C is⁵

$$\frac{1}{\alpha_{1Y}(M_C)} = \frac{3}{5} \left[\frac{1}{\alpha_{2R}(M_C)} - \frac{1}{6\pi} \right] + \frac{2}{5} \left[\frac{1}{\alpha_{4c}(M_C)} - \frac{1}{3\pi} \right]. \quad (15)$$

This is a consequence of the relation $Y/2 = T_{3R} + (B - L)/2$. On the right-hand side (rhs) T_3 resides within the

⁵ α_{1Y} is the GUT-normalized $U(1)_Y$ coupling.

TABLE V. Higgs scalars for the one-step symmetry breaking of $SO(10)$ and the submultiplets contributing to RG evolution according to the ESH. The submultiplet in the braces also contributes if D parity is conserved.

$SO(10)$ representation	Symmetry breaking	Scalars contributing to RG	
		$M_Z \rightarrow M_C$ under \mathcal{G}_{SM}	$M_C \rightarrow M_X$ under \mathcal{G}_{224}
10	$\mathcal{G}_{SM} \rightarrow EM$	$(2, \pm 1, 1)$	$(2, 2, 1)$
126	$\mathcal{G}_{224} \rightarrow \mathcal{G}_{SM}$	\dots	$(1, 3, \overline{10})$ {(3,1,10)}

$SU(2)_R$ while $(B - L)$ is included in $SU(4)_c$ and Eq. (7) has been used. Similarly, $\alpha_{4c}(M_C) = \alpha_{3c}(M_C) + 1/12\pi$ and is fixed from the RG evolution of α_{3c} from M_Z . The two cases that we discuss here are as follows:

(a) If D parity is not conserved then for every choice of M_C , Eq. (15) determines $\alpha_{2R}(M_C)$. The three couplings have to be further evolved to determine M_X and ϵ .

$$\text{non-SUSY: } b_{2L} = -3; \quad b_{2R} = 11/3; \quad b_{4c} = -23/3; \quad b_{ij} = \begin{pmatrix} 8 & 3 & 45/2 \\ 3 & 584/3 & 765/2 \\ 9/2 & 153/2 & 643/6 \end{pmatrix}. \quad (16)$$

$$\text{SUSY: } b_{2L} = 1; \quad b_{2R} = 21; \quad b_{4c} = 3; \quad b_{ij} = \begin{pmatrix} 25 & 3 & 45 \\ 3 & 265 & 405 \\ 9 & 81 & 231 \end{pmatrix}. \quad (17)$$

The one- and two-loop beta-function coefficients we have calculated are in agreement⁶ with those obtained in [9,18]. Both papers deal only with the non-SUSY case.

Results: For this chain, the low energy measured gauge couplings allow a range of values for M_C . The results for this case are shown in the left (non-SUSY) and middle (SUSY) panels of Fig. 1. As shown, for every allowed M_C one can determine M_X (dark solid, red curve) and ϵ (pale broken, green curve) from the unification of coupling constants satisfying Eq. (3). As a general observation, lower values of M_C correspond to increased M_X and larger ϵ . Notice that in the non-SUSY case, M_C can be as low as 10^3 GeV and therefore within the range of detectability for the Large Hadron Collider. Further, the $(1, 3, \overline{10})$ scalars which have mass $\sim M_C$ can mediate $n - \bar{n}$ oscillations⁷ and it is known that current experimental limits place a lower bound on M_C around 10 TeV depending on hadronic factors not precisely known [19]. The mass of the ν_R is also

(b) If D parity is conserved at M_C then in Eq. (15) we must further impose $\alpha_{2R}(M_C) = \alpha_{2L}(M_C)$, with the latter fixed by the RG evolution of α_{2L} from its low energy value. This identifies a unique M_C . M_X can then be determined in terms of ϵ .

We discuss these options in detail below.

From M_Z to M_C : For the RG running of the coupling constants in this range Eqs. (9)–(11) are applicable irrespective of whether D parity is conserved or not.

1. D parity not conserved

This is the case when Φ_{210} is responsible for the $SO(10)$ GUT-symmetry breaking.

From M_C to M_X : The beta-function coefficients receive contributions from $(1, 3, \overline{10}) \subset 126_H$ along with the $(2, 2, 1) \subset 10_H$ scalars and the three generations of fermions: $(2, 1, 4) + (1, 2, \overline{4}) = 16_F$. These are

$\mathcal{O}(M_C)$. While a low M_C is desirable for detectability of $n - \bar{n}$ oscillations it is not the preferred choice for a seesaw mechanism for generating light neutrino masses. In the SUSY case M_X and M_C are restricted to a very limited range, a reflection of the large beta functions beyond M_C . Here M_C (10^{14} – 10^{16} GeV) is too high for observable $n - \bar{n}$ oscillations but quite appropriate for light neutrino seesaw masses.

2. D parity conserved

This is the situation which arises when either Φ_{54} or Φ_{770} is responsible for the $SO(10)$ breaking.

From M_C to M_X : According to the extended survival hypothesis the only change from the previous section is that one must include contributions from both $(1, 3, \overline{10})$ and $(3, 1, 10)$ within the 126_H . This gives

$$\text{non-SUSY: } b_{2L} = b_{2R} = 11/3; \quad b_{4c} = -14/3; \quad b_{ij} = \begin{pmatrix} 584/3 & 3 & 765/2 \\ 3 & 584/3 & 765/2 \\ 153/2 & 153/2 & 1759/6 \end{pmatrix}. \quad (18)$$

⁶There are minor differences in b_{2L2R} and b_{2L4c} between our results and those in [9].

⁷The oscillation period $\tau_{n-\bar{n}} \sim (M_{(1,3,\overline{10})})^5$.

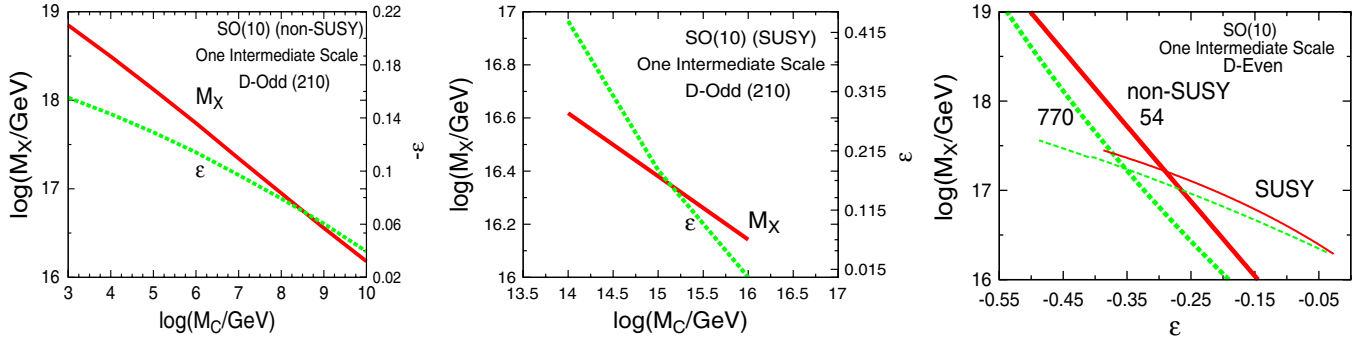


FIG. 1 (color online). $SO(10)$ one-step breaking results: The unification scale, M_X (dark solid, red lines) and the strength of the dim-5 interaction, ϵ (pale broken, green lines) as a function of M_C for the D -parity nonconserving (Φ_{210}) case for non-SUSY (left panel) and SUSY (center panel). M_X vs ϵ for the D -parity conserving case (right panel). Thick (thin) lines correspond to non-SUSY (SUSY). The results for both Φ_{54} (dark solid, red lines) and Φ_{770} (pale broken, green lines) are shown.

$$\text{SUSY: } b_{2L} = b_{2R} = 21; \quad b_{4c} = 12; \quad b_{ij} = \begin{pmatrix} 265 & 3 & 405 \\ 3 & 265 & 405 \\ 81 & 81 & 465 \end{pmatrix}. \quad (19)$$

The beta-function coefficients for the non-SUSY case agree with those in [18].

Results: In this case, the relationship between the $SU(2)_L$ and $SU(2)_R$ couplings uniquely fixes the intermediate scale M_C . We find that for the non-SUSY case $M_C = 5.37 \times 10^{13}$ GeV, while in the SUSY case it is higher and is around 1.9×10^{16} GeV. This fixed intermediate scale, M_C , is the same for Φ_{54} and Φ_{770} . The $(1, 3, \bar{10})$ and $(3, 1, 10)$ scalars at $\sim M_C$ are thus too heavy for observable $n - \bar{n}$ oscillations. Depending on whether the non-SUSY or the SUSY theory is under consideration, a range of allowed M_X can be obtained as a function of ϵ for either choice of Φ_D . The results for the non-SUSY (thick lines) and SUSY (thin lines) cases are shown in the right panel of Fig. 1. The dark solid (red) lines correspond to Φ_{54} while the pale broken (green) lines are for Φ_{770} .

C. Two-step breaking in $SO(10)$

Here we consider the breaking of $SO(10)$ to SM via two intermediate steps:

$$\begin{aligned} SO(10) &\xrightarrow{M_X} SU(2)_L \otimes SU(2)_R \otimes SU(4)_c \\ &\xrightarrow{M_C} SU(2)_L \otimes U(1)_R \otimes SU(3)_c \otimes U(1)_{(B-L)} \xrightarrow{M_R} \text{SM}. \end{aligned} \quad (20)$$

The symmetry breaking at different stages is arranged as follows. The breaking of the Pati-Salam \mathcal{G}_{224} to \mathcal{G}_{2131} is through the VEV of a $(1, 3, 15)$ component of 210_H . The subsequent descent to the SM is through the VEV to a $(1, 3, 1, -2) \subset (1, 3, \bar{10}) \subset 126_H$. The Higgs scalars responsible for the SM symmetry breaking, ϕ_{SM} , transform as $(2, \pm 1, 1)$ under the SM group and as $(2, \pm \frac{1}{2}, 1, 0) \subset (2, 2, 1) \subset 10$ under \mathcal{G}_{2131} , \mathcal{G}_{224} , and $SO(10)$, respectively. The contributing scalars at different stages of RG evolution, as determined by the ESH, are summarized in Table VI.

If D parity is conserved, and it can be conserved only till M_C in this chain, then one must include the contribution from a $(3, 1, 10)$ and a $(3, 1, 15)$ in the final stage of evolution (see Table VI).

TABLE VI. Higgs scalars for the two-step symmetry breaking of $SO(10)$ and the submultiplets contributing to RG evolution according to the ESH. The submultiplets in the braces also contribute if D parity is conserved.

$SO(10)$ representation	Symmetry breaking	Scalars contributing to RG		
		$M_Z \rightarrow M_R$ under \mathcal{G}_{SM}	$M_R \rightarrow M_C$ under \mathcal{G}_{2131}	$M_C \rightarrow M_X$ under \mathcal{G}_{224}
10	$\mathcal{G}_{\text{SM}} \rightarrow EM$	$(2, \pm 1, 1)$	$(2, \pm \frac{1}{2}, 1, 0)$	$(2, 2, 1)$
126	$\mathcal{G}_{2131} \rightarrow \mathcal{G}_{\text{SM}}$	\dots	$(1, 3, 1, -2)$	$(1, 3, \bar{10})$
210	$\mathcal{G}_{224} \rightarrow \mathcal{G}_{2131}$	\dots	\dots	$\{(3, 1, 10)\}$ $(1, 3, 15)$ $\{(3, 1, 15)\}$

A point worth noting in Table VI is that in the range M_C to M_X there are contributions from (1,3,15) [and possibly (3,1,15)] scalar fields over and above those in the one-step breaking case (see Table V). Because of these large-dimensional multiplets the RG evolutions are quite different and the naïve expectation of the two-step results going over to the one-step one in the limit $M_R = M_C$ is invalid.

In the energy range M_R to M_C there are two $U(1)$ gauge groups. As observed in [10,11] and stressed most recently in [12], due to incomplete scalar multiplets remaining light according to the extended survival hypothesis there is a dynamical mixing between these two $U(1)$ subgroups which is manifested in the RG evolution equations. In particular, below the M_R threshold there is one $U(1)$ coupling corresponding to hypercharge, Y , while above one must consider the possibility of a 2×2 matrix of $U(1)$ couplings, G :

$$G = \begin{pmatrix} g_{RR} & g_{RX} \\ g_{XR} & g_{XX} \end{pmatrix}, \quad (21)$$

where $X \equiv (B - L)$. This is the most general form permitted for the coupling of the gauge currents to gauge bosons which for the $U(1)$ groups are both by themselves gauge invariant. Here, g_{ij} is the strength of the coupling of the i th current to the j th gauge boson. In the range M_R to M_C the evolution of all elements of G will occur.⁸ The RG equations for g_{RX} and g_{XR} at the one-loop level involve one additional beta-function coefficient, $\tilde{b}_{XR} = \tilde{b}_{RX} \propto$

$\sum_i Q_R^i Q_X^i$. At the two-loop level, besides the usual ones, one requires the following independent coefficients:

- (1) $\tilde{b}_{RX,RR}, \tilde{b}_{XR,XX}$;
- (2) $\tilde{b}_{RX,p}, \tilde{b}_{XR,p}$;
- (3) $\tilde{b}_{p,RX}$.

The first beta coefficient in point 1 appears in, among others, the evolution equation of g_{RX} as the coefficient of $g_{RR}^4 g_{XX}$ while the second is readily obtainable from the above through $R \leftrightarrow X$. For points 2 and 3, p represents a non-Abelian subgroup of the gauge symmetry. The coefficient of $g_{RX}^3 g_p^2$ ($g_{XR}^3 g_p^2$) in the RG equation of g_{RX} (g_{XR}) is listed under point 2. Similarly, in point 3, $\tilde{b}_{p,RX}$ is the coefficient of $g_p^3 (g_{RR} g_{XR} + g_{XX} g_{RX})$. For the $SO(10)$ model we are considering, the entries in points 2 and 3 turn out to be zero.

At the boundary M_R there is freedom to choose G to be upper triangular. On RG evolution all elements will, however, become nonzero. The matching of the elements of G with the coupling below M_R and those above M_C is made through projection operators which relate the basis of evolution with the $U(1)$ gauge basis defining the groups at the boundary.

Taking all this into account, the gauge couplings evolve as follows:

(i-a) From M_C to M_X (D parity not conserved):

$$\text{non-SUSY: } b_{2L} = -3; \quad b_{2R} = 41/3; \quad b_{4c} = -11/3; \quad b_{ij} = \begin{pmatrix} 8 & 3 & 45/2 \\ 3 & 1424/3 & 1725/2 \\ 9/2 & 345/2 & 1987/6 \end{pmatrix}. \quad (22)$$

$$\text{SUSY: } b_{2L} = 1; \quad b_{2R} = 51; \quad b_{4c} = 15; \quad b_{ij} = \begin{pmatrix} 25 & 3 & 45 \\ 3 & 625 & 885 \\ 9 & 177 & 519 \end{pmatrix}. \quad (23)$$

(i-b) From M_C to M_X (D parity conserved):

$$\text{non-SUSY: } b_{2L} = b_{2R} = 41/3; \quad b_{4c} = 10/3; \quad b_{ij} = \begin{pmatrix} 1424/3 & 3 & 1725/2 \\ 3 & 1424/3 & 1725/2 \\ 345/2 & 345/2 & 4447/6 \end{pmatrix}. \quad (24)$$

$$\text{SUSY: } b_{2L} = b_{2R} = 51; \quad b_{4c} = 36; \quad b_{ij} = \begin{pmatrix} 625 & 3 & 885 \\ 3 & 625 & 885 \\ 177 & 177 & 1041 \end{pmatrix}. \quad (25)$$

(ii) From M_R to M_C : Below M_C , where the gauge group is $SU(2)_L \otimes U(1)_R \otimes SU(3)_c \otimes U(1)_{(B-L)}$, there is no $L \leftrightarrow R$ symmetry and hence there can be no D parity. Thus for the two cases just discussed the evolution will be identical. Here we are giving the decompositions of the contributing fields under the gauge symmetry at this level:

⁸Because of the mixing of the two $U(1)$ groups, the RG equations will be somewhat more involved and are not presented. They can be found in [11,12].

$$16_F = [2, 0, 3, -1/3] + [2, 0, 1, 1] + [1, 1/2, \bar{3}, 1/3] + [1, 1/2, 1, -1] + [1, -1/2, \bar{3}, 1/3] + [1, -1/2, 1, -1],$$

$$10_H \supset [2, 1/2, 1, 0] + [2, -1/2, 1, 0], 126_H \supset [1, -1, 1, 2], \quad (26)$$

whence⁹ [$X \equiv (B - L)$]

$$\text{non-SUSY: } b_{2L} = -3; \quad b_{RR} = 14/3; \quad b_{3c} = -7; \quad b_{XX} = 9/2; \quad \tilde{b}_{RX} = \tilde{b}_{XR} = -1/\sqrt{6}; \quad (27)$$

$$b_{ij} = \begin{pmatrix} 8 & 1 & 12 & 3/2 \\ 3 & 8 & 12 & 15/2 \\ 9/2 & 3/2 & -26 & 1/2 \\ 9/2 & 15/2 & 4 & 25/2 \end{pmatrix}; \quad \tilde{b}_{XR,RR} = -2\sqrt{6}; \quad \tilde{b}_{RX,XX} = -3\sqrt{6}, \quad \tilde{b}_{RX,p} = \tilde{b}_{XR,p} = \tilde{b}_{p,RX} = 0. \quad (28)$$

$$\text{SUSY: } b_{2L} = 1; \quad b_{RR} = 8; \quad b_{3c} = -3; \quad b_{XX} = 15/2; \quad \tilde{b}_{RX} = \tilde{b}_{XR} = -\sqrt{6}/2; \quad (29)$$

$$b_{ij} = \begin{pmatrix} 25 & 1 & 24 & 3 \\ 3 & 11 & 24 & 9 \\ 9 & 3 & 14 & 1 \\ 9 & 9 & 8 & 16 \end{pmatrix}; \quad \tilde{b}_{XR,RR} = -2\sqrt{6}; \quad \tilde{b}_{RX,XX} = -3\sqrt{6}; \quad \tilde{b}_{RX,p} = \tilde{b}_{XR,p} = \tilde{b}_{p,RX} = 0. \quad (30)$$

(iii) *From M_Z to M_R* : In this range Eqs. (9)–(11) are applicable.

The one- and two-loop beta-function coefficients in the D -parity conserving case agree with those obtained in [12,18] with the proviso that in [18] only one Higgs doublet is assumed to contribute in the range M_Z to M_R . In addition, the $U(1)$ mixing contribution at the one-loop level has been included only in [12].

Results: At M_R one must now use the matching relation:

$$\frac{1}{\alpha_{1Y}(M_R)} = 4\pi P(GG^T)^{-1}P^T, \quad (31)$$

where $P = (\sqrt{\frac{5}{3}} \sqrt{\frac{5}{3}})$. At the M_C boundary, the $U(1)_R$ and $U(1)_{(B-L)}$ couplings are obtained from the RG evolved G using a similar formula while choosing $P = (1 \ 0)$ and $(0 \ 1)$, respectively.

When D parity is not conserved, i.e., the first stage of symmetry breaking is due to Φ_{210} , Eq. (31) fixes the couplings at M_R . The meeting of the $U(1)_{(B-L)}$ and $SU(3)_c$ couplings determines M_C and at that scale α_{1R} goes over to α_{2R} . At M_R , the ratios $g_{RR}/g_{(B-L)(B-L)}$ and $g_{R(B-L)}/g_{(B-L)(B-L)}$ can be varied to first determine M_C via Eq. (31) and subsequently M_X . In Fig. 2 are shown the ranges of M_C and M_X , consistent with all constraints, as a function of M_R for the non-SUSY (left panel) and SUSY

(right panel) cases. Notice that in both cases M_R can be as low as 10 TeV. This is the scale for a new neutral Z' boson which could be seen at the LHC. On the other hand, in both cases, M_R , which is also the $(B - L)$ -violating scale relevant for seesaw neutrino masses, can be 10^{14-16} GeV, which is of the desirable size for $\mathcal{O}(1)$ Yukawa couplings. For the non-SUSY case, M_C is around $10^{11.5}$ GeV or above which is too high for the detectability of $n - \bar{n}$ oscillations. For SUSY M_C is even higher, $\sim 10^{15}$ GeV or more. This is also the mass scale for the right-handed charged gauge bosons. For the solutions discussed above the parameter $|\epsilon|$ lies in the range (0.004–0.160) for non-SUSY and (0.04–1.0) for SUSY.

When D parity is conserved, i.e., the GUT symmetry breaking is due to Φ_{54} or Φ_{770} , M_R must be such that the α_{1R} and $\alpha_{1(B-L)}$ matches with α_{2L} and α_{3c} , respectively [as per Eq. (7)] at precisely the same energy scale M_C . This is quite constraining. Though for both non-SUSY and SUSY M_R can range from 10^4 – 10^{16} GeV, M_C and M_X are very close to each other¹⁰ and around 10^{16} GeV always. Thus, barring the Z' neutral gauge boson there will be no other observable signatures in this scenario. The high values of M_C preclude the possibility of detectable $n - \bar{n}$ oscillations. On the other hand, such a high M_{ν_R} will be able to accommodate the light neutrino masses through a

⁹The coefficients superscribed with a tilde arise due to $U(1)$ mixing.

¹⁰This is a consequence of the large beta functions due to the contributions from big submultiplets introduced to maintain D -parity symmetry (see Table VI).

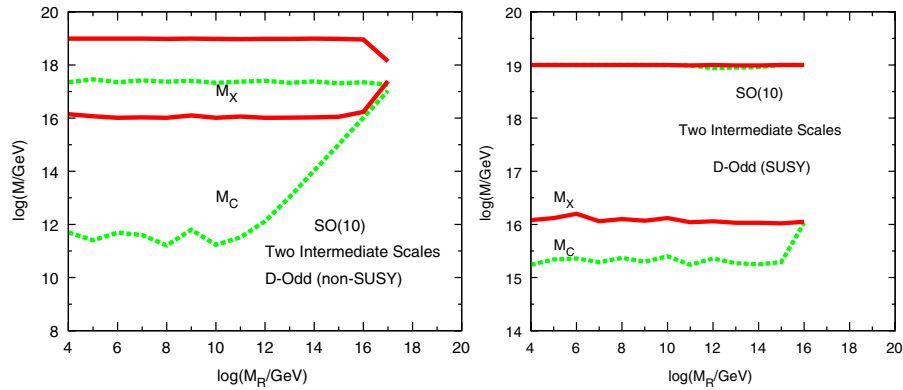


FIG. 2 (color online). The allowed ranges of M_X (dark solid, red lines) and M_C (pale broken, green lines) vs M_R for the non-SUSY (left panel) and SUSY (right panel) cases for $SO(10)$ breaking through two intermediate steps when D parity is not conserved. Note that the upper limits for M_X and M_C are almost identical for SUSY.

type I seesaw. For the strength of the dim-5 interaction, ϵ , it is found $0 \leq |\epsilon| \leq 0.18$ for non-SUSY and $0 \leq |\epsilon| \leq 0.25$ for SUSY. For both non-SUSY as well as SUSY, the results for Φ_{54} and Φ_{770} are practically identical excepting for small differences in the values of ϵ .

IV. $E(6)$

The exceptional group $E(6)$ has also been discussed in the literature as a possible GUT symmetry [20]. The breaking scheme of $E(6)$ that we consider here is

$$\begin{aligned}
 E(6) &\xrightarrow{M_X} SU(3)_L \otimes SU(3)_R \otimes SU(3)_c \\
 &\xrightarrow{M_I} SU(2)_L \otimes U(1)_{Y_L} \otimes SU(2)_R \otimes U(1)_{Y_R} \otimes SU(3)_c \\
 &\xrightarrow{M_R} \text{SM}.
 \end{aligned} \tag{32}$$

In $E(6)$ the fermions of one generation are accommodated in the 27-dimensional fundamental representation which under \mathcal{G}_{333} consists of $(\bar{3}, 3, 1) + (1, \bar{3}, 3) + (3, 1, \bar{3})$. At the stage where the \mathcal{G}_{333} symmetry is broken, all fermions other than those in the SM become massive.

In contrast to the $SO(10)$ cases discussed in the previous section, here the quark-lepton symmetry is lost at M_X and $n - \bar{n}$ oscillations will be highly suppressed in this class of $E(6)$ models.

The adjoint representation of $E(6)$ is 78 dimensional. Since $(78 \otimes 78)_{\text{sym}} = 1 \oplus 650 \oplus 2430$, nontrivial contributions in Eq. (2) can arise if Φ_D transforms as the 650 or 2430 representation. Of these, Φ_{650} has two distinct directions for the VEV which can accomplish the symmetry breaking to \mathcal{G}_{333} , one of which protects D parity while the other does not. We denote these by 650 and 650', respectively. In Table VII we collect the dimension-5 contributions for the different representations of $E(6)$.

A. No-step breaking in $E(6)$

This corresponds to the situation when $M_X = M_I = M_R$ and the symmetry breaking is simply

$$E(6) \xrightarrow{M_X} SU(2)_L \otimes U(1)_Y \otimes SU(3)_c. \tag{33}$$

Here, Eqs. (9)–(11) determine the gauge coupling evolution in the entire range. The results obtained including the dimension-5 operators in Eq. (2) are shown in Table VIII.

As for the other GUT groups, though gauge unification is possible in the non-SUSY case, the scale of unification is too low and is ruled out by the proton decay limits. The SUSY solutions are acceptable for Φ_{650} . For Φ_{2430} the scale M_X is too low (note that all the δ_i are equal), but

TABLE VII. Effective contributions (δ_i) to gauge kinetic terms from different Higgs representations in Eq. (2) for $E(6)$ [3] [see Eq. (3)]. Note that there are two $SU(3)_L \otimes SU(3)_R \otimes SU(3)_c$ singlet directions in 650 of which the first conserves D parity while the second does not.

$E(6)$ representations	δ_{3L}	δ_{3R}	δ_{3c}
650	$1/2\sqrt{2}$	$1/2\sqrt{2}$	$-1/\sqrt{2}$
650'	$3/2\sqrt{6}$	$-3/2\sqrt{6}$	0
2430	$-3/\sqrt{26}$	$-3/\sqrt{26}$	$-3/\sqrt{26}$

TABLE VIII. Dimension-5 interaction strength, ϵ , and the gauge unification scale, M_X , for different Φ_D representations using two-loop RG equations when $E(6)$ descends directly to the SM.

$E(6)$ representations	Non-SUSY		SUSY	
	ϵ	M_X (GeV)	ϵ	M_X (GeV)
650	0.126	8.04×10^{12}	-0.012	1.72×10^{16}
650'	0.101	4.15×10^{14}	-0.011	1.30×10^{16}
2430	0.000	3.76×10^{12}	0.000	1.25×10^{15}

this can be addressed easily by changing the SUSY scale, M_{SUSY} .

B. One-step breaking in $E(6)$

This situation corresponds to $M_I = M_R$ in Eq. (32), i.e.,

$$E(6) \xrightarrow{M_X} SU(3)_L \otimes SU(3)_R \otimes SU(3)_c \xrightarrow{M_R} \text{SM}. \quad (34)$$

For this case, the symmetry breaking at M_R and subsequently the one at M_Z is through the VEVs to components within the $(\bar{3}, 3, 1)$ submultiplet under $SU(3)_L \otimes SU(3)_R \otimes$

$SU(3)_c \equiv \mathcal{G}_{333}$ which is present in a 27 of $E(6)$. According to the extended survival hypothesis this entire $(\bar{3}, 3, 1)$ submultiplet, but for the ϕ_{SM} fields which are at M_Z , has a mass M_R . Since it is symmetric under $SU(2)_L \leftrightarrow SU(2)_R$, the evolution of the couplings from M_R to M_X is controlled by the same RG equations for both the D -parity violating and D -parity conserving cases (see Table IX). The beta-function coefficients in this case are as follows:

From M_R to M_X :

$$\text{non-SUSY: } b_{3L} = b_{3R} = -9/2; \quad b_{3c} = -5; \quad b_{ij} = \begin{pmatrix} 23 & 20 & 12 \\ 20 & 23 & 12 \\ 12 & 12 & 12 \end{pmatrix}. \quad (35)$$

$$\text{SUSY: } b_{3L} = b_{3R} = 3/2; \quad b_{3c} = 0; \quad b_{ij} = \begin{pmatrix} 65 & 32 & 24 \\ 32 & 65 & 24 \\ 24 & 24 & 48 \end{pmatrix}. \quad (36)$$

From M_Z to M_R : For the RG running of the coupling constants below M_R Eqs. (9)–(11) are applicable irrespective of whether D parity is conserved or not.

Results: The chain of $E(6)$ breaking considered in this section is rather constrained. The matching formula at M_R is now

$$\frac{1}{\alpha_{1Y}(M_R)} = \frac{4}{5} \left[\frac{1}{\alpha_{3R}(M_R)} - \frac{1}{4\pi} \right] + \frac{1}{5} \left[\frac{1}{\alpha_{3L}(M_R)} - \frac{1}{4\pi} \right]. \quad (37)$$

This is a consequence of the relation $Y/2 = T_{3R} + (Y'_L + Y'_R)/2$. On the rhs T_{3R} and Y'_R reside within the $SU(3)_R$ while Y'_L is included in $SU(3)_L$. The two cases are as follows:

(a) If D parity is not conserved then for any chosen M_R , through Eq. (37) $\alpha_{3R}(M_R)$ is fixed since $\alpha_{2L}(M_R)$ is determined from its low energy value through RG evolution and $1/\alpha_{3L}(M_R) = 1/\alpha_{2L}(M_R) + 1/(12\pi)$. The three couplings have to be further evolved to determine M_X and ϵ .

(b) If D parity is conserved at M_R then in Eq. (37) $\alpha_{3R}(M_R) = \alpha_{3L}(M_R)$, with the latter fixed by the RG evolution of α_{2L} from its low energy value. This identifies a unique M_R . M_X can then be determined in terms of ϵ .

We discuss these options in detail next.

When D parity is not conserved, i.e., for $\Phi_{650'}$, we find that the intermediate scale at M_R is rather tightly restricted from the twin requirements that M_X satisfies the proton decay bound and is within the upper limit set by the Planck mass as well as all couplings remain perturbative. It is in the ballpark of 10^{14} (10^{16}) GeV for the non-SUSY (SUSY) case. The unification scale is 7.0×10^{18} (3.5×10^{16}) GeV for the respective cases with ϵ almost fixed at $= -0.04$ (0.02).

When D parity is conserved, which corresponds to Φ_{650} and Φ_{2430} , the intermediate scale M_R is uniquely fixed in both cases at the value 1.5×10^{13} (1.7×10^{16}) GeV for non-SUSY (SUSY). A plot of the unification scale M_X vs ϵ is shown in the left panel of Fig. 3 for Φ_{650} . For Φ_{2430} we have $\delta_{3L} = \delta_{3R} = \delta_{3c}$ and so the dim-5 operator does not affect the unification. We find that for non-SUSY as well as SUSY with $M_{\text{SUSY}} = 1$ TeV the couplings unify at an energy beyond the Planck scale.

For both Φ_{650} and $\Phi_{650'}$ the scale M_R is in the right range for the mass of the right-handed neutrinos to drive a type I seesaw.

C. Two-step breaking in $E(6)$

The symmetry breaking steps are

$$\begin{aligned} E(6) &\xrightarrow{M_X} SU(3)_L \otimes SU(3)_R \otimes SU(3)_c \\ &\xrightarrow{M_I} SU(2)_L \otimes U(1)_{Y'_L} \otimes SU(2)_R \otimes U(1)_{Y'_R} \otimes SU(3)_c \\ &\xrightarrow{M_R} \text{SM}. \end{aligned} \quad (38)$$

Here, $\langle \Phi_{650} \rangle$ or $\langle \Phi_{2430} \rangle$ breaks $E(6)$ to \mathcal{G}_{333} which reduces to $SU(2)_L \otimes U(1)_{Y'_L} \otimes SU(2)_R \otimes U(1)_{Y'_R} \otimes SU(3)_c \equiv \mathcal{G}_{21213}$ when the $(8, 8, 1)$ submultiplet of a 650_H acquires a VEV. The SM is reached by assigning a VEV to the $(\bar{3}, 3, 1)$ component of 27_H . The final step of SM symmetry breaking is accomplished through a different component of $(\bar{3}, 3, 1)$ (see Table X). It is seen that there is room for D parity to be conserved or broken during the running in the M_R to M_I range. But the Higgs submultiplets which acquire masses at M_I according to the extended survival hypothesis, namely, $(\bar{3}, 3, 1)$ and $(8, 8, 1)$, are $SU(2)_L \leftrightarrow SU(2)_R$ symmetric and so the running from M_I to M_X will be identical in both cases.

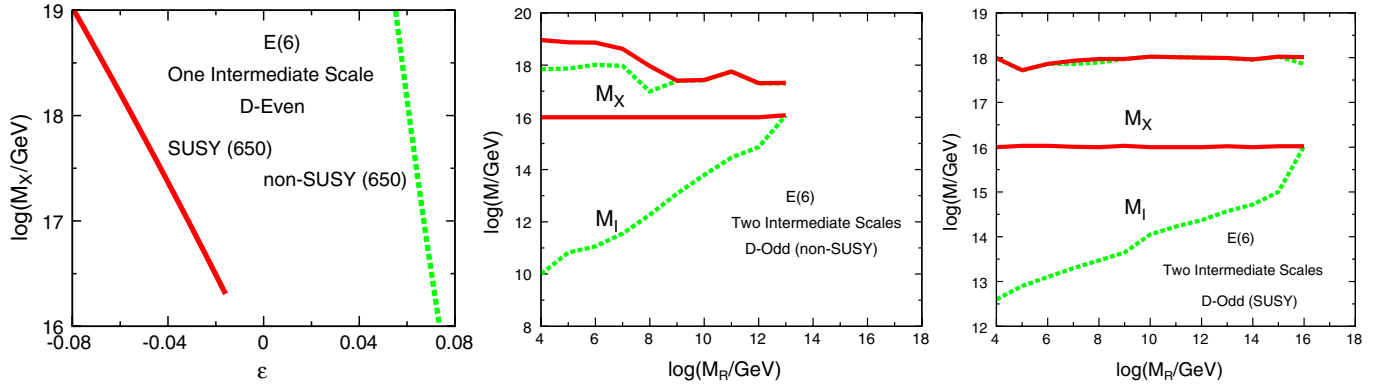


FIG. 3 (color online). Left panel: M_X as a function of ϵ for one-step breaking of $E(6)$ in the D -parity conserving case for Φ_{650} . The pale broken green (dark solid red) line corresponds to non-SUSY (SUSY). Center and right panels: The allowed ranges of M_X (dark solid, red line) and M_I (pale broken, green line) vs M_R for the non-SUSY (center panel) and SUSY (right panel) cases for $E(6)$ breaking through two intermediate steps when D parity is not conserved. Note that the upper limits for M_X and M_I are almost identical.

TABLE IX. Higgs scalars for the one-step symmetry breaking of $E(6)$ and the submultiplets contributing to RG evolution according to the ESH.

$E(6)$ representation	Symmetry breaking	Scalars contributing to RG	
		$M_Z \rightarrow M_R$ under \mathcal{G}_{SM}	$M_R \rightarrow M_X$ under \mathcal{G}_{333}
27	$\mathcal{G}_{SM} \rightarrow EM$	$(2, \pm 1, 1)$	$(\bar{3}, 3, 1)$
27	$\mathcal{G}_{333} \rightarrow \mathcal{G}_{SM}$	\dots	$(\bar{3}, 3, 1)$

It is seen from Table X that in the range M_I to M_X there are additional contributions from the $(8, 8, 1)$ scalar fields besides those in the one-step breaking case (Table IX). The RG evolution in the two cases is therefore different and, as in the case of $SO(10)$, the naive expectation of the two-step

results going over to the one-step one in the limit $M_R = M_I$ does not hold.

Below we list the one- and two-loop beta-function coefficients for gauge coupling evolution in the different stages. Notice that in the range M_R to M_I there are two $U(1)$ components and the RG evolution here has to take into account mixing and follows the same procedure as discussed in detail for $SO(10)$ in the previous section.

(i) From M_I to M_X : The fermion and scalar fields which contribute in the RG equations are

$$\begin{aligned}
 27_F &= [\bar{3}, 3, 1] + [3, 1, 3] + [1, \bar{3}, \bar{3}], \\
 650_H &\supset [8, 8, 1], \quad 27_H \supset [\bar{3}, 3, 1].
 \end{aligned}
 \tag{39}$$

Thus,

$$\text{non-SUSY: } b_{3L} = 7/2; \quad b_{3R} = 7/2; \quad b_{3c} = -5; \quad b_{ij} = \begin{pmatrix} 359 & 308 & 12 \\ 308 & 359 & 12 \\ 12 & 12 & 12 \end{pmatrix}.
 \tag{40}$$

$$\text{SUSY: } b_{3L} = 51/2; \quad b_{3R} = 51/2; \quad b_{3c} = 0; \quad b_{ij} = \begin{pmatrix} 497 & 320 & 24 \\ 320 & 497 & 24 \\ 24 & 24 & 48 \end{pmatrix}.
 \tag{41}$$

TABLE X. Higgs scalars for the two-step symmetry breaking of $E(6)$ and the submultiplets contributing to RG evolution according to the ESH. The submultiplet in the braces also contributes if D parity is conserved.

$E(6)$ representation	Symmetry breaking	Scalars contributing to RG		
		$M_Z \rightarrow M_R$ under \mathcal{G}_{SM}	$M_R \rightarrow M_I$ under \mathcal{G}_{21213}	$M_I \rightarrow M_X$ under \mathcal{G}_{333}
27	$\mathcal{G}_{SM} \rightarrow EM$	$(2, \pm 1, 1)$	$(2, -\frac{1}{2\sqrt{3}}, 2, \frac{1}{2\sqrt{3}}, 1)$	$(\bar{3}, 3, 1)$
27	$\mathcal{G}_{21213} \rightarrow \mathcal{G}_{SM}$	\dots	$(1, \frac{1}{\sqrt{3}}, 2, \frac{1}{2\sqrt{3}}, 1)$	$(\bar{3}, 3, 1)$
		\dots	$\{(2, \frac{1}{2\sqrt{3}}, 1, \frac{1}{\sqrt{3}}, 1)\}$	
650	$\mathcal{G}_{333} \rightarrow \mathcal{G}_{21213}$	\dots	\dots	$(8, 8, 1)$

(ii-a) From M_R to M_I (D parity not conserved): At this stage the non-SM fermions have acquired mass and decoupled. Taking the extended survival hypothesis into consideration, the fields that contribute to the RG equations are

$$\begin{aligned} 27_F &\supset [2, -1/2\sqrt{3}, 1, -1/\sqrt{3}, 1] + [2, 1/2\sqrt{3}, 1, 0, 3] + [1, 1/\sqrt{3}, 2, 1/2\sqrt{3}, 1] + [1, 0, 2, -1/2\sqrt{3}, \bar{3}], \\ 27_H &\supset [1, 1/\sqrt{3}, 2, 1/2\sqrt{3}, 1] + [2, -1/2\sqrt{3}, 2, 1/2\sqrt{3}, 1]. \end{aligned} \quad (42)$$

This gives¹¹

$$\text{non-SUSY: } b_{2L} = -3; \quad b_{LL} = 3; \quad b_{2R} = -17/6; \quad b_{RR} = 17/6; \quad b_{3c} = -7; \quad \tilde{b}_{LR} = \tilde{b}_{RL} = 4/3; \quad (43)$$

$$\begin{aligned} b_{ij} &= \begin{pmatrix} 8 & 4/3 & 3 & 4/3 & 12 \\ 4 & 8/3 & 6 & 1 & 4 \\ 3 & 2 & 61/6 & 3/2 & 12 \\ 4 & 1 & 9/2 & 11/6 & 4 \\ 9/2 & 1/2 & 9/2 & 1/2 & -26 \end{pmatrix}; \\ \tilde{b}_{LR,RR} &= 5/6; \quad \tilde{b}_{RL,LL} = 7/6; \quad \tilde{b}_{2R,RL} = 1/2; \quad \tilde{b}_{2L,RL} = 1/6; \quad \tilde{b}_{3c,RL} = 0; \\ \tilde{b}_{RL,2R} &= 3/2; \quad \tilde{b}_{RL,2L} = 1/2; \quad \tilde{b}_{RL,3c} = 0; \quad \tilde{b}_{RL,p} = \tilde{b}_{LR,p}; \quad \tilde{b}_{p,LR} = \tilde{b}_{p,RL}. \end{aligned} \quad (44)$$

$$\text{SUSY: } b_{2L} = 1; \quad b_{LL} = 5; \quad b_{2R} = 3/2; \quad b_{RR} = 9/2; \quad b_{3c} = -3; \quad \tilde{b}_{LR} = \tilde{b}_{RL} = 2; \quad (45)$$

$$\begin{aligned} b_{ij} &= \begin{pmatrix} 25 & 7/3 & 3 & 7/3 & 24 \\ 7 & 13/3 & 9 & 5/3 & 8 \\ 3 & 3 & 57/2 & 5/2 & 24 \\ 7 & 5/3 & 15/2 & 7/2 & 8 \\ 9 & 1 & 9 & 1 & 14 \end{pmatrix}; \\ \tilde{b}_{LR,RR} &= 5/3; \quad \tilde{b}_{RL,LL} = 2; \quad \tilde{b}_{2R,RL} = 1; \quad \tilde{b}_{2L,RL} = 2/3; \quad \tilde{b}_{3c,RL} = 0; \\ \tilde{b}_{RL,2R} &= 3; \quad \tilde{b}_{RL,2L} = 2; \quad \tilde{b}_{RL,3c} = 0; \quad \tilde{b}_{RL,p} = \tilde{b}_{LR,p}; \quad \tilde{b}_{p,LR} = \tilde{b}_{p,RL}. \end{aligned} \quad (46)$$

(ii-b) From M_R to M_I (D parity conserved): Because of D -parity conservation the scalar sector is slightly enlarged and the fields contributing to the RG equations are

$$\begin{aligned} 27_F &\supset [2, -1/2\sqrt{3}, 1, -1/\sqrt{3}, 1] + [2, 1/2\sqrt{3}, 1, 0, 3] + [1, 1/\sqrt{3}, 2, 1/2\sqrt{3}, 1] + [1, 0, 2, -1/2\sqrt{3}, \bar{3}], \\ 27_H &\supset [1, 1/\sqrt{3}, 2, 1/2\sqrt{3}, 1] + [2, 1/2\sqrt{3}, 1, 1/\sqrt{3}, 1] + [2, -1/2\sqrt{3}, 2, 1/2\sqrt{3}, 1]. \end{aligned} \quad (47)$$

We find

$$\begin{aligned} \text{non-SUSY: } b_{2L} &= -17/6; \quad b_{LL} = 55/18; \quad b_{2R} = -17/6; \quad b_{RR} = 55/18; \quad b_{3c} = -7; \\ \tilde{b}_{LR} &= \tilde{b}_{RL} = 13/9; \end{aligned} \quad (48)$$

¹¹The coefficients superscribed with a tilde arise due to $U(1)$ mixing.

$$b_{ij} = \begin{pmatrix} 61/6 & 3/2 & 3 & 2 & 12 \\ 9/2 & 49/18 & 6 & 11/9 & 4 \\ 3 & 2 & 61/6 & 3/2 & 12 \\ 6 & 11/9 & 9/2 & 49/18 & 4 \\ 9/2 & 1/2 & 9/2 & 1/2 & -26 \end{pmatrix};$$

$$\begin{aligned} \tilde{b}_{LR,RR} &= 23/18; & \tilde{b}_{RL,LL} &= 23/18; & \tilde{b}_{2R,RL} &= 1/2; & \tilde{b}_{2L,LR} &= 1/2; & \tilde{b}_{3c,RL} &= 0; \\ \tilde{b}_{RL,2R} &= 3/2; & \tilde{b}_{RL,2L} &= 3/2; & \tilde{b}_{RL,3c} &= 0; & \tilde{b}_{RL,p} &= \tilde{b}_{LR,p}; & \tilde{b}_{p,LR} &= \tilde{b}_{p,RL}. \end{aligned} \quad (49)$$

$$\text{SUSY: } b_{2L} = 3/2; \quad b_{LL} = 31/6; \quad b_{2R} = 3/2; \quad b_{RR} = 31/6; \quad b_{3c} = -3; \quad \tilde{b}_{LR} = \tilde{b}_{RL} = 7/3; \quad (50)$$

$$b_{ij} = \begin{pmatrix} 57/2 & 5/2 & 3 & 3 & 24 \\ 15/2 & 79/18 & 9 & 17/9 & 8 \\ 3 & 3 & 57/2 & 5/2 & 24 \\ 9 & 17/9 & 15/2 & 79/18 & 8 \\ 9 & 1 & 9 & 1 & 14 \end{pmatrix};$$

$$\begin{aligned} \tilde{b}_{LR,RR} &= 19/9; & \tilde{b}_{RL,LL} &= 19/9; & \tilde{b}_{2R,RL} &= 1; & \tilde{b}_{2L,LR} &= 1; & \tilde{b}_{3c,RL} &= 0; \\ \tilde{b}_{RL,2R} &= 3; & \tilde{b}_{RL,2L} &= 3; & \tilde{b}_{RL,3c} &= 0; & \tilde{b}_{RL,p} &= \tilde{b}_{LR,p}; & \tilde{b}_{p,LR} &= \tilde{b}_{p,RL}. \end{aligned} \quad (51)$$

From M_Z to M_R : For the RG running of the coupling constants below M_R Eqs. (9)–(11) are applicable irrespective of whether D parity is conserved or not.

Results: When $E(6)$ breaks to the SM through two intermediate steps, at M_R one must set

$$\frac{1}{\alpha_{1Y}(M_R)} = \frac{3}{5} \left[\frac{1}{\alpha_{2R}(M_R)} - \frac{1}{6\pi} \right] + 4\pi P(GG^T)^{-1} P^T, \quad (52)$$

where $P = (\sqrt{\frac{1}{5}} \sqrt{\frac{1}{5}})$, which follows from $Y/2 = T_{3R} + (Y'_L + Y'_R)/2$.

When the initial symmetry breaking of $E(6)$ is through the $\Phi_{650'}$, D parity is not conserved. It might seem that there is some flexibility here and at M_R one can choose $g_{Y'_R Y'_R}$, $g_{Y'_R Y'_L}$, and g_{2R} independently, determining $g_{Y'_L Y'_L}$ from Eq. (52). In fact, there is a rather severe constraint that $\alpha_{Y'_R}$ and α_{2R} must meet at M_I and at precisely the same scale $\alpha_{Y'_L}$ must equal α_{2L} . In Fig. 3 we show the allowed range of the intermediate scale M_I and the unification scale M_X as a function of M_R . Note that for both cases these scales are on the high side. The scale of the second stage of symmetry breaking, M_R , is permitted to be as low as 10^4 GeV for the non-SUSY as well as the SUSY case. It determines the mass scale of a Z' boson and may offer room for experimental probing at the LHC. The right-handed charged weak bosons are at M_C and hence beyond reach. ϵ is bounded in the range $0 \leq |\epsilon| \leq 0.16$.

When the first stage of symmetry breaking is driven through the Φ_{650} , D parity is preserved. This implies that $\alpha_{2R}(M_R) = \alpha_{2L}(M_R)$ and is fixed by the RG evolution of

g_{2L} from M_Z . Also at M_R , $g_{Y'_L Y'_L} = g_{Y'_R Y'_R}$ and one can choose $g_{Y'_L Y'_R} = g_{Y'_R Y'_L} = 0$, so all couplings are determined once M_R is chosen. Requiring that the constraint on M_X from proton decay be satisfied along with perturbativity, we find that there is a very limited range of allowed solutions with 10^{11} GeV $\leq M_R \leq 10^{13}$ GeV (non-SUSY case) and 10^{15} GeV $\leq M_R \leq 10^{16}$ GeV (SUSY case). M_I and M_X are always close together around 10^{16-17} GeV. For these solutions $0 \leq |\epsilon| \leq 0.14$.

The case of Φ_{2430} is not distinguishable from the situation of no dim-5 operators at all since here $\delta_1 = \delta_2 = \delta_3$.

V. SUMMARY AND DISCUSSIONS

In this paper we have examined the GUT-symmetry breaking consequences of dim-5 operators which can arise from quantum gravity or string compactification leading to a correction to the gauge kinetic term. When the GUT symmetry is broken their effect is to modify gauge coupling unification to the relation $g_i^2(M_X)(1 + \epsilon\delta_i) = g_U^2$ [Eq. (3)]. The relevant group-theoretic factors δ_i were exhaustively calculated in [3]. Here we have focused on the implications for grand unification and intermediate energy scales, both for single and multistep breaking and also for nonsupersymmetric as well as supersymmetric theories. We have required all coupling constants to remain perturbative in the entire energy range and that the bound on the GUT scale from nonobservation of proton decay be respected. We have remarked on $n - \bar{n}$ oscillations and seesaw light neutrino mass implications in passing.

For multistep symmetry breaking cases we have utilized the extended survival hypothesis to decide which scalar

submultiplet gets mass at which scale. When there are two $U(1)$ factors at some intermediate stage, we consider the effect of their mixing.

For $SU(5)$ we show that even after the inclusion of the effect of dim-5 operators the non-SUSY version cannot be rescued from the proton decay limit impasse while the SUSY version works fine not just when the initial GUT breaking is through the usual Φ_{24} but also by Φ_{75} and Φ_{200} .

For $SO(10)$ we consider the direct breaking to the SM as well as multistep breaking via the Pati-Salam \mathcal{G}_{224} route. For the former case, the conclusions are pretty much the same as that for $SU(5)$. For the latter alternative, the spontaneous symmetry breaking can be achieved through Φ_{54} , Φ_{210} , and Φ_{770} . We classify the solutions according to whether (a) they conserve D parity (Φ_{54} and Φ_{770}) or (b) not (Φ_{210}). (b) turns out to be phenomenologically more interesting. If there is one intermediate scale then in (b) this can be as low as 10^3 GeV with a plethora of observable consequences including charged and neutral gauge bosons and a possibility of observable $n - \bar{n}$ oscillations. For (a) this scale is very high: 10^{13} GeV or more. This is also the energy at which ν_R develops a mass and so it could conveniently generate light neutrino masses with $\mathcal{O}(1)$ Yukawa couplings. In the case of two intermediate scales, for both (a) and (b) one can have one of them as low as 1 TeV where a neutral gauge boson is expected. The other scale can be $10^{6.5}$ GeV or higher for (b) and 10^{13} GeV or more for (a).

For $E(6)$ the GUT-symmetry breaking can be achieved through two possible VEVs for the 650-dimensional Higgs scalar multiplet, which we call Φ_{650} and $\Phi_{650'}$ as well as through a Φ_{2430} . For the direct breaking to the SM the results are again as in the case of other GUT groups, namely, the non-SUSY case is disfavored and the SUSY option is consistent with all requirements. For multistep breaking we consider the \mathcal{G}_{333} route. Here the solutions that we obtain with Φ_{650} and $\Phi_{650'}$ all have options with one intermediate scale as low as 10^4 GeV or higher. For Φ_{2430} , $\delta_{3L} = \delta_{3R} = \delta_{3c}$ and the situation remains identical to the usual case but for a scaling of the unified coupling.

A general remark about two-step and one-step breaking is that the additional scalar fields which drive the symmetry breaking at M_C for $SO(10)$ [M_I for $E(6)$] in the former case contribute in the RG evolution in the stage $M_C \rightarrow M_X$ ($M_I \rightarrow M_X$) over and above whatever is present in the one-step breaking case. Because of this, the simple-minded expectation of the two-step case going over to the one-step

case in the limit of $M_R = M_C$ for $SO(10)$ and $M_R = M_I$ for $E(6)$ is not valid.

Finally, we would like to compare our results with some of the earlier analyses of GUT-symmetry breaking with intermediate scales, albeit without dim-5 operators. Multistep symmetry breaking of $SO(10)$ has been looked at, for example, in [9,12]. For the chain of Eq. (20), i.e., via \mathcal{G}_{224} and \mathcal{G}_{2131} no acceptable solutions were found in the non-SUSY case in [12] while in [9] solutions with M_R in the range 10^5 – 10^7 GeV were presented.¹² Here we obtained a wider span of 10^4 – 10^{16} GeV. For the one-step symmetry breaking of non-SUSY $SO(10)$ the scale M_C was found in [9] to be in the 10^5 – 10^7 GeV range whereas with the inclusion of dim-5 operators we have shown that this scale is in the phenomenologically attractive 10^3 – 10^{10} GeV region.

$SU(5)$ GUT with the inclusion of the dim-5 operator from Φ_{24} has been examined in [21]. The results for non-SUSY as well as the SUSY cases are in agreement with the ones in Sec. II. Our analysis also covers Φ_{75} and Φ_{200} of $SU(5)$. For $SO(10)$ the effect of Φ_{54} and Φ_{210} has been considered in [22] using the one-loop RG equations. They also noted, like us, that for Φ_{54} , when D parity is conserved, the scale M_C is uniquely fixed by the measured $\sin^2\theta_W$ and the M_X they obtain is in agreement with our results. For the D -parity nonconserving case of Φ_{210} they find a range of M_C and M_X similar to what is depicted in Fig. 1.

With regards to $E(6)$, we could not trace any earlier published analysis in the descent to the SM through the \mathcal{G}_{333} chain [23]. The attention has invariably focused on $E(6)$ breaking through an intermediate $SO(10) \times U(1)$ [24].

It can be hoped that further refinement in the determination of the low energy gauge couplings, proton decay tests, and explorations of $n - \bar{n}$ oscillation will enable us to extract signals of physics that lies beyond the grand unification scale.

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¹²In [9], the $U(1)$ mixing contributions to the RG equations at one loop were not included. We also have differences with a few beta-function coefficients calculated in this paper.

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