

Extracting CP violation and strong phase in D decays by using quantum correlations in $\psi(3770) \rightarrow D^0 \bar{D}^0 \rightarrow (V_1 V_2)(V_3 V_4)$ and $\psi(3770) \rightarrow D^0 \bar{D}^0 \rightarrow (V_1 V_2)(K\pi)$

J erome Charles,² Sebastien Descotes-Genon,³ Xian-Wei Kang,^{1,4} Hai-Bo Li,¹ and Gong-Ru Lu⁴

¹*Institute of High Energy Physics, P.O. Box 918, Beijing 100049, China*

²*Centre de Physique Th eorique*, CNRS and Universit  Aix-Marseille 1 et 2*

and Sud Toulon-Var (UMR 6207), Luminy Case 907, 13288 Marseille Cedex 9, France

³*Laboratoire de Physique Th eorique, CNRS and Universit  Paris-Sud 11 (UMR 8627), 91405 Orsay Cedex, France*

⁴*Department of Physics, Henan Normal University, Xinxiang 453007, China*

(Received 13 December 2009; published 30 March 2010)

The charm quark offers interesting opportunities to cross check the mechanism of CP violation precisely tested in the strange and beauty sectors. In this paper, we exploit the angular and quantum correlations in the $D\bar{D}$ pairs produced through the decay of the $\psi(3770)$ resonance in a charm factory to investigate CP -violation in two different ways. We build CP -violating observables in $\psi(3770) \rightarrow D\bar{D} \rightarrow (V_1 V_2)(V_3 V_4)$ to isolate specific new physics effects in the charm sector. We also consider the case of $\psi(3770) \rightarrow D\bar{D} \rightarrow (V_1 V_2)(K\pi)$ decays, which provide a new way to measure the strong phase difference δ between Cabibbo-favored and doubly-Cabibbo suppressed D decays required in the determination of the Cabibbo-Kobayashi-Maskawa quark-mixing matrix angle γ . Neglecting the systematics, we give a first rough estimate of the sensitivities of these measurements at BES-III with an integrated luminosity of 20 fb^{-1} at $\psi(3770)$ peak and at a future Super τ -charm factory with a luminosity of $10^{35} \text{ cm}^{-2} \text{ s}^{-1}$.

DOI: 10.1103/PhysRevD.81.054032

PACS numbers: 13.25.Ft, 11.30.Er, 12.15.Hh, 14.40.Lb

I. INTRODUCTION

Outstanding progress has been made over the last decade thanks to the data gathered at B -factories, confirming that the Cabibbo-Kobayashi-Maskawa (CKM) mechanism embedded in the standard model (SM) is the main source of CP violation in the quark sector. The impressive agreement between results from the s -quark and the b -quark sectors [1,2] calls for further checks in less tested areas. The recent discussions concerning the leptonic decays of D and D_s mesons, about a possible disagreement between lattice results and experimental data [3–5], suggest that the charm sector has not been explored as extensively as other quarks [6]. Another illustration of this situation stems from D -meson mixing, which has only very recently provided interesting tests of the SM and its extensions [7–11].

Indeed, the D -meson sector is a remarkable place to improve our knowledge on CP violation in and beyond SM, for at least two different reasons. First, the SM predictions for CP violation in the charm sector are very small, due to the hierarchical structure of the CKM matrix and the difference of masses between the fermion generations. Any significant amount of CP violation would provide clear signals of new physics, and a contrario, the absence of observation of CP violation already sets bounds on models beyond the SM [6]. Second, D decays play a prominent role in determining γ , the least well known of the three angles from the B -meson unitarity triangle. A better understanding of the strong dynamics of related D decays would help in reducing the current uncertainty on this angle [12,13].

On the experimental side, the final results from CLEO-c and the start of BES-III provide interesting opportunities. These charm factories are known to offer the possibility to exploit the quantum entanglement of $D\bar{D}$ pairs, as explained in several references [12–17]. In addition, it is also interesting to note that $D \rightarrow VV$ (vector-vector) modes exhibit rather large branching ratios, of similar size with respect to the pseudoscalar (PP) or vector-pseudoscalar (VP) modes, and provide further angular observables to study the above issues. In this paper, we investigate this question which had not been detailed so far.

In Sec. II, we discuss production of coherent $D^0 - \bar{D}^0$ pairs from $\psi(3770)$ decay, and, in particular, the angular distribution when at least one of the D meson decays into a pair of vector mesons. In Sec. III, we apply these results to two different situations: the determination of CP -violating observables exploiting angular and quantum correlations in cases where both D decay into vector pairs, and the extraction of $D \rightarrow K\pi$ hadronic parameters in relation with the measurement of the CKM angle γ . In Sec. IV, we briefly discuss the application of these results for BES-III and Super τ -charm factory, before concluding.

II. CORRELATED D DECAYS

A. Basics

We want to describe the decay chain for $\psi(3770)$ as

$$\psi(3770) \rightarrow D^0 \bar{D}^0 \rightarrow (M_1 M_2)(M_3 M_4), \quad (1)$$

where $M_1 M_2$ and $M_3 M_4$ are mesons from two-body decays of D^0 and \bar{D}^0 , respectively, [Hereafter, ψ denotes $\psi(3770)$]. Since we do not tag the D mesons, and just

*Laboratoire affili     la FRUMAM.

observe their decay products, we can use two descriptions of D mesons—either the flavor states D^0 and \bar{D}^0 , or the CP eigenstates (neglecting, for the sake of simplicity, CP -violation in D mixing):

$$|D_1\rangle = \frac{|D^0\rangle + |\bar{D}^0\rangle}{\sqrt{2}}, \quad |D_2\rangle = \frac{|D^0\rangle - |\bar{D}^0\rangle}{\sqrt{2}} \quad (2)$$

with respective CP -parity: $\eta_{CP}(D_1) = -1$ and $\eta_{CP}(D_2) = 1$ (we take the convention $CP|D^0\rangle = -|\bar{D}^0\rangle$).

Because of the spin of ψ , the D -pair is emitted with an orbital momentum $L = 1$ corresponding to an antisymmetric coherent state:

$$|(D\bar{D})_{L=1}\rangle = \frac{-|D_1\rangle|D_2\rangle + |D_2\rangle|D_1\rangle}{\sqrt{2}}. \quad (3)$$

One can in principle consider different situations as below, where V stands for a vector and P for a pseudoscalar meson:

- (i) $(PP) + (PP)$, $(PP) + (VP)$, $(VP) + (VP)$: the only available observable is the branching ratio, since the partial waves and helicities are all fixed by angular momentum conservation.
- (ii) $(PP) + (VV)$, $(VP) + (VV)$: (VV) can have three helicity states, and thus there are new angular observables. This can be exploited for $(PP) = K\pi$ in connection with the measurement of the CKM angle γ .
- (iii) $(VV) + (VV)$: this will be studied with an interest in new observables for CP -violation.

The relevant modes for our studies can be extracted from Ref. [18] for the branching ratios and Ref. [19] for the projected efficiency at BES-III.

Some of the VV modes have not been measured yet, but some estimates combining naive factorization and models for final-state interactions are available [20–23]. It is interesting to notice that the $\rho^+\rho^-$ decay mode has not been measured yet, even though one would expect it to be larger than $\rho^0\rho^0$ (the latter being a color suppressed mode).

As it will become clear below, for the modes of definite CP -parity, one can sum over all the possible subsequent

decay channels (enhancing therefore the branching ratios). For modes without such definite CP -parity, one needs to pick up a subsequent decay channel providing a definite CP parity. For instance, in the case of the modes where K^{*0} is identified through the channel $K^{*0} \rightarrow K_S\pi^0$, we have included a factor $1/6$ in the branching ratio due to Clebsch-Gordan coefficients ($1/\sqrt{3}$ from K^{*0} to $K^0\pi^0$, then $1/\sqrt{2}$ from K^0 to K_S).

Only the product of intrinsic parity is given, and one has to include the partial wave of the outgoing state. If PP as well as VV in S and D waves are not affected, one has to include an additional (-1) for P -wave states:

$$\begin{aligned} \eta_{CP}(PV) &= -\eta_{CP}(P)\eta_{CP}(V), \\ \eta(VV, \ell = 1) &= -\eta_{CP}(V)\eta_{CP}(V). \end{aligned} \quad (4)$$

B. Differential decay width

An adequate formalism to treat the question of decay chains is the framework of helicity amplitudes, described for instance in Refs. [24,25]. The decay chain is described by the product of amplitudes corresponding to each reaction. For a reaction $A \rightarrow BC$, we define polar angles (θ_A, ϕ_A) describing the momentum of particle B in the rest frame of A in a basis where the z -axis defined by the momentum of B in the rest frame of its mother particle. The decay amplitude depends on (θ_A, ϕ_A) and is denoted $A_{\lambda_B\lambda_C}^{A \rightarrow BC}$ where λ_B, λ_C are the helicities of the daughter mesons.

Let us start by describing the chain (we will study the other “path” later):

$$\psi \rightarrow D_1 D_2, \quad D_1 \rightarrow V_1 V_2, \quad D_2 \rightarrow V_3 V_4, \quad (5)$$

$$\begin{aligned} V_1 &\rightarrow M_1 M'_1, & V_2 &\rightarrow M_2 M'_2, \\ V_3 &\rightarrow M_3 M'_3, & V_4 &\rightarrow M_4 M'_4. \end{aligned} \quad (6)$$

The helicity formalism yields an amplitude of the form

$$\begin{aligned} M_{12}^m &= \sum_{\lambda_V} A_{00}^{\psi \rightarrow D_1 D_2} A_{00}^{V_1 \rightarrow M_1 M'_1} A_{00}^{V_3 \rightarrow M_3 M'_3} A_{00}^{V_2 \rightarrow M_2 M'_2} A_{00}^{V_4 \rightarrow M_4 M'_4} A_{\lambda_{V_1} \lambda_{V_2}}^{D_1 \rightarrow V_1 V_2} A_{\lambda_{V_3} \lambda_{V_4}}^{D_2 \rightarrow V_3 V_4} \\ &= \sqrt{\frac{3}{4\pi}} \frac{9}{(4\pi)^3} \sum_{\lambda_V} D_{m,0}^{1*}(\phi_\psi, \theta_\psi, 0) H_{D_1 D_2}^\psi \\ &\quad \times D_{0, \lambda_{V_1} - \lambda_{V_2}}^{0*}(\phi_{D_1}, \theta_{D_1}, 0) H_{V_1 V_2}^{D_1} D_{\lambda_{V_1}, 0}^{1*}(\phi_{V_1}, \theta_{V_1}, 0) H_{M_1 M'_1}^{V_1} D_{-\lambda_{V_2}, 0}^{1*}(\phi_{V_2}, \theta_{V_2}, 0) H_{M_2 M'_2}^{V_2} \\ &\quad \times D_{0, \lambda_{V_3} - \lambda_{V_4}}^{0*}(\phi_{D_2}, \theta_{D_2}, 0) H_{V_3 V_4}^{D_2} D_{\lambda_{V_3}, 0}^{1*}(\phi_{V_3}, \theta_{V_3}, 0) H_{M_3 M'_3}^{V_3} D_{-\lambda_{V_4}, 0}^{1*}(\phi_{V_4}, \theta_{V_4}, 0) H_{M_4 M'_4}^{V_4}, \end{aligned} \quad (7)$$

$$\quad (8)$$

where m is the projection of the spin of the ψ along an arbitrary axis, and λ_V denotes collectively the helicities of the 4 vector mesons. The vector mesons are emitted from a spinless D -meson, so that: $\lambda = \lambda_{V_1} = \lambda_{V_2}$ and $\kappa = \lambda_{V_3} = \lambda_{V_4}$. We used the rotation matrix $D_{m'm}^j(\alpha, \beta, \gamma) = e^{-im'\alpha} d_{m'm}^j(\beta) e^{-im\gamma}$ with the Wigner d -matrix:

$$d_{10}^1(\theta) = -\frac{1}{\sqrt{2}} \sin\theta, \quad d_{00}^1(\theta) = \cos\theta, \quad d_{-10}^1(\theta) = \frac{1}{\sqrt{2}} \sin\theta, \quad d_{00}^0(\theta) = 1. \quad (9)$$

The probability amplitude becomes

$$\begin{aligned} M_{12}^m &= \sqrt{\frac{3}{4\pi}} \frac{9}{(4\pi)^3} e^{im\phi_\psi} d_{m0}^1(\theta_\psi) H^{\psi V_1 V_2 V_3 V_4} \times \sum_\lambda e^{i\lambda\Phi_{12}} (-1)^\lambda d_{\lambda 0}^1(\theta_{V_1}) d_{\lambda 0}^1(\theta_{V_2}) H_\lambda^{D_1} \sum_\kappa e^{i\kappa\Phi_{34}} (-1)^\kappa d_{\kappa 0}^1(\theta_{V_3}) d_{\kappa 0}^1(\theta_{V_4}) H_\kappa^{D_2} \\ &= \sqrt{\frac{3}{4\pi}} \frac{9}{(4\pi)^3} e^{im\phi_\psi} d_{m0}^1(\theta_\psi) H^{\psi V_1 V_2 V_3 V_4} \left[\cos\theta_{V_1} \cos\theta_{V_2} A_0^{D_1 \rightarrow V_1 V_2} - \frac{1}{\sqrt{2}} \sin\theta_{V_1} \sin\theta_{V_2} \cos\Phi_{12} A_{\parallel}^{D_1 \rightarrow V_1 V_2} \right. \\ &\quad \left. - \frac{i}{\sqrt{2}} \sin\theta_{V_1} \sin\theta_{V_2} \sin\Phi_{12} A_{\perp}^{D_1 \rightarrow V_1 V_2} \right] \times \left[\cos\theta_{V_3} \cos\theta_{V_4} A_0^{D_2 \rightarrow V_3 V_4} - \frac{1}{\sqrt{2}} \sin\theta_{V_3} \sin\theta_{V_4} \cos\Phi_{34} A_{\parallel}^{D_2 \rightarrow V_3 V_4} \right. \\ &\quad \left. - \frac{i}{\sqrt{2}} \sin\theta_{V_3} \sin\theta_{V_4} \sin\Phi_{34} A_{\perp}^{D_2 \rightarrow V_3 V_4} \right], \end{aligned} \quad (10)$$

where we defined $\Phi_{12} = \phi_{V_1} - \phi_{V_2}$ and $\Phi_{34} = \phi_{V_3} - \phi_{V_4}$ (i.e. the angle between the two relevant vector mesons) and the combination of amplitudes

$$H^{\psi V_1 V_2 V_3 V_4} = H_{D_1 D_2}^{\psi} H_{M_1 M_1'}^{V_1} H_{M_2 M_2'}^{V_2} H_{M_3 M_3'}^{V_3} H_{M_4 M_4'}^{V_4}, \quad (11)$$

$$H_\lambda^{D_1 \rightarrow V_1 V_2} = H_{\lambda\lambda}^{D_1 \rightarrow V_1 V_2}, \quad H_\kappa^{D_2 \rightarrow V_3 V_4} = H_{\kappa\kappa}^{D_2 \rightarrow V_3 V_4}. \quad (12)$$

We introduce the transversity amplitudes

$$A_{\parallel} = \frac{1}{\sqrt{2}} (H_{+1} + H_{-1}), \quad A_0 = H_0, \quad A_{\perp} = \frac{1}{\sqrt{2}} (H_{+1} - H_{-1}). \quad (13)$$

M_{12}^m is actually only one of the two ‘‘paths’’ that can be chosen. The total amplitude for a given projection m of the spin of ψ along an arbitrary z -axis is: $M^m = (-M_{12}^m + M_{21}^m)/\sqrt{2}$. The differential decay width is obtained by averaging the squared modulus of the amplitude over the three possible values of $m = +1, 0, -1$. The three squared Wigner functions $d_{m0}^1(\theta_\psi)$ add up to 1, so that the differential width is

$$\begin{aligned} d\Gamma_{4V} &= \frac{81}{32\pi^2} d(\cos\theta_{V_1}) d(\cos\theta_{V_2}) d\Phi_{12} d(\cos\theta_{V_3}) d(\cos\theta_{V_4}) d\Phi_{34} \times |A^{\psi V_1 V_2 V_3 V_4}|^2 \\ &\times \left[\left[\cos\theta_{V_1} \cos\theta_{V_2} A_0^{D_0 \rightarrow V_1 V_2} - \frac{1}{\sqrt{2}} \sin\theta_{V_1} \sin\theta_{V_2} \cos\Phi_{12} A_{\parallel}^{D_0 \rightarrow V_1 V_2} - \frac{i}{\sqrt{2}} \sin\theta_{V_1} \sin\theta_{V_2} \sin\Phi_{12} A_{\perp}^{D_0 \rightarrow V_1 V_2} \right] \right. \\ &\times \left[\cos\theta_{V_3} \cos\theta_{V_4} A_0^{\bar{D}^0 \rightarrow V_3 V_4} - \frac{1}{\sqrt{2}} \sin\theta_{V_3} \sin\theta_{V_4} \cos\Phi_{34} A_{\parallel}^{\bar{D}^0 \rightarrow V_3 V_4} - \frac{i}{\sqrt{2}} \sin\theta_{V_3} \sin\theta_{V_4} \sin\Phi_{34} A_{\perp}^{\bar{D}^0 \rightarrow V_3 V_4} \right] \\ &- \left[\cos\theta_{V_1} \cos\theta_{V_2} A_0^{\bar{D}^0 \rightarrow V_1 V_2} - \frac{1}{\sqrt{2}} \sin\theta_{V_1} \sin\theta_{V_2} \cos\Phi_{12} A_{\parallel}^{\bar{D}^0 \rightarrow V_1 V_2} - \frac{i}{\sqrt{2}} \sin\theta_{V_1} \sin\theta_{V_2} \sin\Phi_{12} A_{\perp}^{\bar{D}^0 \rightarrow V_1 V_2} \right] \\ &\times \left. \left[\cos\theta_{V_3} \cos\theta_{V_4} A_0^{D^0 \rightarrow V_3 V_4} - \frac{1}{\sqrt{2}} \sin\theta_{V_3} \sin\theta_{V_4} \cos\Phi_{34} A_{\parallel}^{D^0 \rightarrow V_3 V_4} - \frac{i}{\sqrt{2}} \sin\theta_{V_3} \sin\theta_{V_4} \sin\Phi_{34} A_{\perp}^{D^0 \rightarrow V_3 V_4} \right] \right|^2. \end{aligned} \quad (14)$$

We have an integration over $[0, \pi]$ for θ 's and $[0, 2\pi]$ for Φ_{12} and Φ_{34} . The amplitudes A are normalized so that: $\Gamma(X \rightarrow YZ) = |A(X \rightarrow YZ)|^2$.

The above formalism can be adapted easily to describe the situation where one D meson decays into PP' rather than VV' . Indeed, it amounts to considering only the longitudinal decay amplitude for $D \rightarrow V_3 V_4$ and to remove the angular phase space related to the decay products of V_3 and V_4 .

III. OBSERVABLES FROM CORRELATED D DECAYS

A. Observables from $\psi \rightarrow 2D \rightarrow 4V$ for CP violation

If we take the decay chain [14,15]

$$e^+ e^- \rightarrow \psi \rightarrow D^0 \bar{D}^0 \rightarrow f_a f_b \quad (15)$$

with f_a and f_b CP eigenstates of same CP -parity, we have

$$\begin{aligned} CP|\psi\rangle &= |\psi\rangle, \\ CP|f_a f_b\rangle &= \eta_a \eta_b (-1)^\ell |f_a f_b\rangle = -|f_a f_b\rangle \end{aligned} \quad (16)$$

since f_a and f_b are in a P wave. Therefore, the decay of ψ into states of identical CP parity is, by itself, a CP -violating observable [14,15].

One obtains, neglecting CP -violation in $D\bar{D}$ mixing, the following result for the combined branching ratio, which can be recovered from [16]

$$\begin{aligned} \mathcal{BR}((D^0\bar{D}^0)_{C=-1} \rightarrow f_a f_b) &= 2\mathcal{BR}(D^0 \rightarrow f_a) \\ &\times \mathcal{BR}(D^0 \rightarrow f_b) \\ &\times (|\rho_a - \rho_b|^2 \\ &+ r_D |1 - \rho_a \rho_b|^2), \end{aligned} \quad (17)$$

with the ratio of CP -conjugate amplitudes and the combination of D -mixing parameters

$$\rho_f = \frac{A(\bar{D}^0 \rightarrow f)}{A(D^0 \rightarrow f)}, \quad r_D = (x^2 + y^2)/2 < 10^{-4}. \quad (18)$$

where $x = \Delta m/\Gamma$ and $y = \Delta\Gamma/(2\Gamma)$ are the difference of masses and widths of the mass eigenstates in the $D\bar{D}$ system, normalized by their average width [6].

If we assume that CP is conserved in decay, we have $\rho_f = \eta_f$, and thus $\mathcal{BR} = 0$ for a, b with same CP -parity. Therefore, we have indeed that the observation of $(D^0\bar{D}^0)_{C=-1} \rightarrow f_a f_b$ with a, b of same CP -parity is an indication of CP -violation. Let us notice that a and b must be different eigenstates (either different mesons, or for VV , different partial waves), and that this branching ratio is sensitive to different aspects of CP -violation compared to uncorrelated decays of $D \rightarrow f_a$ and $D \rightarrow f_b$, since the latter would be sensitive to $1 - |\rho_a|^2$ or $1 - |\rho_b|^2$. We can thus construct observables for CP violation in VV decays by considering states with the same CP parity, which depends on the relative angular momentum between the two mesons. The favorite channels among the measured ones are K^+K^- , $\pi^+\pi^-$, $K_S\pi^0$, $\rho^0\pi^0$, $K_S\rho^0$, $\bar{K}^{*0}\rho^0 \rightarrow (K_S\pi^0)(\pi^+\pi^-)$ and $\rho^0\phi$.

The transversity amplitudes A for $D_{1,2} \rightarrow VV'$ have simple transformation laws under CP :

$$A_0^{D \rightarrow VV'} \rightarrow +\eta_{CP}(V)\eta_{CP}(V')\eta_{CP}(D)A_0^{D \rightarrow \bar{V}\bar{V}'}, \quad (19)$$

$$A_{\parallel}^{D \rightarrow VV'} \rightarrow +\eta_{CP}(V)\eta_{CP}(V')\eta_{CP}(D)A_{\parallel}^{D \rightarrow \bar{V}\bar{V}'}, \quad (20)$$

$$A_{\perp}^{D \rightarrow VV'} \rightarrow -\eta_{CP}(V)\eta_{CP}(V')\eta_{CP}(D)A_{\perp}^{D \rightarrow \bar{V}\bar{V}'}. \quad (21)$$

Following Eq. (17), CP conservation at the level of the amplitude would require that only two combinations of transversity amplitudes are allowed: $(0, \perp)$ or (\parallel, \perp) . In

terms of partial waves, 0 and \parallel are combinations of S and D waves, whereas \perp is P wave, which means that CP conservation at the level of the amplitude would impose the vector mesons to be emitted in (S, D) waves on one side and P wave on the other.

Therefore, the following combinations of transversity amplitudes in the partial differential decay rate can be in principle CP violating observables:

$$(0, 0), \quad (0, \parallel), \quad (\parallel, 0), \quad (\parallel, \parallel), \quad (\perp, \perp). \quad (22)$$

Let us notice that in the case of identical meson pairs in the final state, there is only one CP -violating configuration that is available: $(0, \parallel)$, due to the Bose-Einstein statistics. It seems more interesting to consider two different meson pairs, both with longitudinal polarization, to get a larger BR. From the above table, the most interesting modes are $\bar{K}^{*0}\rho^0 \rightarrow (K_S\pi^0)(\pi^+\pi^-)$ and $\rho^0\rho^0$. It is straightforward to construct the corresponding CP -violating observable:

$$\begin{aligned} &\int d\Gamma_{4V} \frac{1}{128} (5\cos^2\theta_{V_1} - 1)(5\cos^2\theta_{V_2} - 1) \\ &\times (5\cos^2\theta_{V_3} - 1)(5\cos^2\theta_{V_4} - 1) \\ &= |A^{\psi V_1 V_2 V_3 V_4}|^2 |A_0^{D^0 \rightarrow V_1 V_2}|^2 |A_0^{D^0 \rightarrow V_3 V_4}|^2 \\ &\times |\rho_{V_1, V_2}^0 - \rho_{V_3, V_4}^0|^2. \end{aligned} \quad (23)$$

Similar weights can be obtained for the other CP -violating combinations, exploiting orthogonality relationships for Legendre and Chebyshev polynomials to select specific angular dependences as in the previous case. For instance, we have:

$$\begin{aligned} &\int d\Gamma_{4V} \frac{1}{32} (5\cos^2\theta_{V_1} - 3)(5\cos^2\theta_{V_2} - 3)(5\cos^2\theta_{V_3} - 3) \\ &\times (5\cos^2\theta_{V_4} - 3) \cdot (4\cos^2\Phi_{12} - 1)(4\cos^2\Phi_{34} - 1) \\ &= |A^{\psi V_1 V_2 V_3 V_4}|^2 |A_{\parallel}^{D^0 \rightarrow V_1 V_2}|^2 |A_{\parallel}^{D^0 \rightarrow V_3 V_4}|^2 \\ &\times |\rho_{V_1, V_2}^{\parallel} - \rho_{V_3, V_4}^{\parallel}|^2 \end{aligned} \quad (24)$$

and

$$\begin{aligned} &\int d\Gamma_{4V} \frac{1}{32} (5\cos^2\theta_{V_1} - 3)(5\cos^2\theta_{V_2} - 3)(5\cos^2\theta_{V_3} - 3) \\ &\times (5\cos^2\theta_{V_4} - 3) \cdot (4\cos^2\Phi_{12} - 3)(4\cos^2\Phi_{34} - 3) \\ &= |A^{\psi V_1 V_2 V_3 V_4}|^2 |A_{\perp}^{D^0 \rightarrow V_1 V_2}|^2 |A_{\perp}^{D^0 \rightarrow V_3 V_4}|^2 \\ &\times |\rho_{V_1, V_2}^{\perp} - \rho_{V_3, V_4}^{\perp}|^2. \end{aligned} \quad (25)$$

In the case of the same VV final state for both D decays, one can obtain the appropriate observable corresponding to $(0, \parallel)$ using for instance

$$\int d\Gamma_{4V}[(5\cos^2\theta_{V_1} - 1)(5\cos^2\theta_{V_2} - 1)(5\cos^2\theta_{V_3} - 3)(5\cos^2\theta_{V_4} - 3)(4\cos^2\Phi_{12} - 1) \\ \cdot [(5\cos^2\theta_{V_1} - 3)(5\cos^2\theta_{V_2} - 3)(5\cos^2\theta_{V_3} - 1)(5\cos^2\theta_{V_4} - 1)(4\cos^2\Phi_{34} - 1)] \\ = |A^{\psi V_1 V_2 V_3 V_4}|^2 |A_0^{D^0 \rightarrow V_1 V_2}|^2 |A_{\parallel}^{D^0 \rightarrow V_3 V_4}|^2 \times |\rho_{V_1, V_2}^0 - \rho_{V_3, V_4}^{\parallel}|^2. \quad (26)$$

B. $\psi \rightarrow 2D \rightarrow (VV)(K\pi)$ for the extraction of γ

The measurement of γ from the Atwood-Dunietz-Soni (ADS) method [26] requires the determination of the hadronic parameters r and δ . At BES-III, we can also take advantage of the coherence of the D^0 mesons produced at the $\psi(3770)$ peak to extract the strong phase difference δ between doubly-Cabibbo-suppressed and Cabibbo-favored decay amplitudes that appears in the γ measurements [12,13]. Here we introduce, in the standard phase

convention where δ vanishes in the SU(3) limit,

$$r \cdot e^{i\delta} = \frac{\langle K^- \pi^+ | \bar{D}^0 \rangle}{\langle K^- \pi^+ | D^0 \rangle}. \quad (27)$$

The process of one D^0 decaying to $K^- \pi^+$, while the other D^0 decaying to a VV CP eigenstate can be described as (for our purposes, it will prove more convenient to express this decay rate in terms of D^0 and \bar{D}^0 amplitudes):

$$d\Gamma_{2V} = \frac{9}{4\pi} d(\cos\theta_{V_1}) d(\cos\theta_{V_2}) d\Phi \times |A^{\psi V_1 V_2}|^2 |A^{D^0 \rightarrow K\pi}|^2 \times \left| \cos\theta_{V_1} \cos\theta_{V_2} (A_0^{\bar{D}^0 \rightarrow V_1 V_2} - r e^{i\delta} A_0^{D^0 \rightarrow V_1 V_2}) \right. \\ \left. - \frac{1}{\sqrt{2}} \sin\theta_{V_1} \sin\theta_{V_2} \cos\Phi (A_{\parallel}^{\bar{D}^0 \rightarrow V_1 V_2} - r e^{i\delta} A_{\parallel}^{D^0 \rightarrow V_1 V_2}) - \frac{i}{\sqrt{2}} \sin\theta_{V_1} \sin\theta_{V_2} \sin\Phi (A_{\perp}^{\bar{D}^0 \rightarrow V_1 V_2} - r e^{i\delta} A_{\perp}^{D^0 \rightarrow V_1 V_2}) \right|^2. \quad (28)$$

We can introduce:

$$A_{0,\parallel,\perp}(\bar{D}^0 \rightarrow V_a V_b) = A_{0,\parallel,\perp}(D^0 \rightarrow V_a V_b) \rho_{V_a, V_b}^{0,\parallel,\perp}. \quad (29)$$

In the absence of CP violation, which we will assume in this section, we have:

$$\rho_{V_a, V_b}^{0,\parallel} = -\eta_{CP}(V_a) \eta_{CP}(V_b) = -\rho_{V_a, V_b}^{\perp}. \quad (30)$$

Moreover, we notice that all the decays presented in Sec. II A have CP parities such that $\rho^0 = -1$, which yields the further expression of the differential decay width in Eq. (28)

$$d\Gamma_{2V} = \frac{9}{4\pi} d(\cos\theta_{V_1}) d(\cos\theta_{V_2}) d\Phi \times |A^{\psi V_1 V_2}|^2 |A^{D^0 \rightarrow K\pi}|^2 \times \left[\cos^2\theta_{V_1} \cos^2\theta_{V_2} |A_0^{D^0 \rightarrow V_1 V_2}|^2 (1 + 2r \cos\delta + r^2) \right. \\ \left. + \frac{1}{2} \sin^2\theta_{V_1} \sin^2\theta_{V_2} \cos^2\Phi |A_{\parallel}^{D^0 \rightarrow V_1 V_2}|^2 (1 + 2r \cos\delta + r^2) - \sqrt{2} \cos\theta_{V_1} \sin\theta_{V_1} \cos\theta_{V_2} \sin\theta_{V_2} \right. \\ \left. \times \cos\Phi \operatorname{Re}[A_0^{D^0 \rightarrow V_1 V_2} (A_{\parallel}^{D^0 \rightarrow V_1 V_2})^*] (1 + 2r \cos\delta + r^2) + \frac{1}{2} \sin^2\theta_{V_1} \sin^2\theta_{V_2} \sin^2\Phi |A_{\perp}^{D^0 \rightarrow V_1 V_2}|^2 (1 - 2r \cos\delta + r^2) \right. \\ \left. + \sqrt{2} \cos\theta_{V_1} \sin\theta_{V_1} \cos\theta_{V_2} \sin\theta_{V_2} \sin\Phi \{\operatorname{Re}[A_0^{D^0 \rightarrow V_1 V_2} (A_{\perp}^{D^0 \rightarrow V_1 V_2})^*] (2r \sin\delta) + \operatorname{Im}[A_0^{D^0 \rightarrow V_1 V_2} (A_{\perp}^{D^0 \rightarrow V_1 V_2})^*] (1 - r^2)\} \right. \\ \left. - \sin^2\theta_{V_1} \sin^2\theta_{V_2} \cos\Phi \sin\Phi \{\operatorname{Re}[A_{\parallel}^{D^0 \rightarrow V_1 V_2} (A_{\perp}^{D^0 \rightarrow V_1 V_2})^*] (2r \sin\delta) + \operatorname{Im}[A_{\parallel}^{D^0 \rightarrow V_1 V_2} (A_{\perp}^{D^0 \rightarrow V_1 V_2})^*] (1 - r^2)\} \right]. \quad (31)$$

We see that the differential decay width provides six different angular observables depending on the following (real) quantities:

- (i) three products of moduli for VV decays: $|A^{\psi V_1 V_2} A^{D^0 \rightarrow K\pi} A_{0,\parallel,\perp}^{D^0 \rightarrow V_1 V_2}|^2$
- (ii) two relative phases between the three amplitudes $A_{0,\parallel,\perp}^{D^0 \rightarrow V_1 V_2}$
- (iii) two strong parameters describing the $K\pi$ decay: r and δ

Whereas the full angular integration yields the sum of the three transversity amplitudes:

$$\int d\Gamma_{2V} = \frac{9}{4\pi} |A^{\psi V_1 V_2}|^2 |A^{D^0 \rightarrow K\pi}|^2 \\ \times \left[\frac{8\pi}{9} |A_0^{D^0 \rightarrow V_1 V_2}|^2 (1 + 2r \cos\delta + r^2) \right. \\ \left. + \frac{8\pi}{9} |A_{\parallel}^{D^0 \rightarrow V_1 V_2}|^2 (1 + 2r \cos\delta + r^2) \right. \\ \left. + \frac{8\pi}{9} |A_{\perp}^{D^0 \rightarrow V_1 V_2}|^2 (1 - 2r \cos\delta + r^2) \right], \quad (32)$$

one can easily separate the different contributions by choosing suitable weights for the angular integration

(they can be obtained easily by exploiting orthogonality relations among Legendre polynomials). In practice the best way to perform the experimental analysis is usually to do a maximum likelihood fit on Eq. (31).

The branching ratio only depends on the three amplitude combinations:

$$\begin{aligned} M_0 &= A_0^{D^0 \rightarrow V_1 V_2} (1 + r e^{i\delta}), \\ M_{\parallel} &= A_{\parallel}^{D^0 \rightarrow V_1 V_2} (1 + r e^{i\delta}), \\ M_{\perp} &= A_{\perp}^{D^0 \rightarrow V_1 V_2} (1 - r e^{i\delta}). \end{aligned} \quad (33)$$

Table IV shows that $P_{1,2,3}$ yield the relative size and phase of M_0 and M_{\parallel} , whereas $P_{4,5,6}$ yield the relative size and phase of M_{\perp} . Therefore, without further knowledge, one can extract a combined constraint on r , δ and A_{\perp} from the differential decay width, and one can also determine the ratio A_{\parallel}/A_0 .

Note the invariance under the simultaneous transformation $A_0 A_{\parallel}^* \rightarrow A_0^* A_{\parallel}$, $A_{\parallel} A_{\perp}^* \rightarrow -A_{\parallel}^* A_{\perp}$, $\delta \rightarrow -\delta$, which implies that for fixed value of r and A_{\perp} there is a twofold ambiguity on δ (in other words there is no information on the sign of $\sin\delta$). It is worth noting here that the decay rate is sensitive to $|\sin\delta|$ terms (thanks to $\text{Re}[M_{0,\parallel} M_{\perp}^*]$), while in the standard analysis with PP modes one is only sensitive to $\cos\delta$ (neglecting the small mixing contributions which lift the ambiguity [12]). Since δ is small, the sensitivity on the sine in addition to the cosine is expected to improve the final result.

The above constraint can be improved by exploiting our current or expected knowledge of the polarization of $D \rightarrow VV$. If we extract the relative size and phase of the three amplitudes $A_{0,\parallel,\perp}^{D^0 \rightarrow V_1 V_2}$ from independent single $D \rightarrow V_1 V_2$ decay (single-tag-ST) measurements, and if the three amplitudes are not too different in size (as seems to be the case for $\rho^0 \rho^0$), the measurement of the M_i amplitudes in the correlated (double-tag-DT) $DD \rightarrow (V_1 V_2)(K\pi)$ decay leads to the determination of both r and δ (more precisely, r , $\cos\delta$ and $|\sin\delta|$). Since the ratio r is already well known, $r = 0.055 \pm 0.002$ [27], our method may lead to a good measurement of δ .

Note that for relatively low statistics, a simplified transversity analysis can be performed. Instead of considering the full angular distribution in both single and double D decays, one can perform a one-parameter fit to the distribution of the transversity¹ angle θ_{tr} , which yields the perpendicular polarization fraction in single-tag and double-tag decays:

¹ $(\theta_{V_1}, \theta_{\text{tr}}, \Phi_{\text{tr}})$ transversity angles are related to $(\theta_{V_1}, \theta_{V_2}, \Phi)$ helicity angles by

$$\begin{aligned} \cos\theta_{V_2} &= \sin\theta_{\text{tr}} \cos\theta_{\text{tr}} & \sin\theta_{V_2} \sin\Phi &= \cos\theta_{\text{tr}} \\ \sin\theta_{V_2} \cos\Phi &= \sin\theta_{\text{tr}} \sin\Phi_{\text{tr}} \end{aligned} \quad (34)$$

$$\begin{aligned} f_{\perp}^{\text{ST}} &= \frac{|A_{\perp}|^2}{|A_0|^2 + |A_{\parallel}|^2 + |A_{\perp}|^2}, \\ f_{\perp}^{\text{DT}} &= \frac{|M_{\perp}|^2}{|M_0|^2 + |M_{\parallel}|^2 + |M_{\perp}|^2}. \end{aligned} \quad (35)$$

The above observables lead to:

$$\begin{aligned} \frac{|A_{\perp}|^2}{|A_0|^2 + |A_{\parallel}|^2} &= \frac{f_{\perp}^{\text{ST}}}{1 - f_{\perp}^{\text{ST}}}, \\ \left| \frac{1 + r e^{i\delta}}{1 - r e^{i\delta}} \right|^2 &= \frac{f_{\perp}^{\text{ST}}}{1 - f_{\perp}^{\text{ST}}} \frac{1 - f_{\perp}^{\text{DT}}}{f_{\perp}^{\text{DT}}}, \end{aligned} \quad (36)$$

which implies a one-dimensional parabolic constraint in the $(r, \cos\delta)$ plane. An independent constraint comes from the ratio of double-tag to single-tag widths proportional to $(|A_0|^2 + |A_{\parallel}|^2 + |A_{\perp}|^2)/(|M_0|^2 + |M_{\parallel}|^2 + |M_{\perp}|^2)$, that can be expressed in terms of r , $\cos\delta$, and f_{\perp}^{ST} . This simplified transversity analysis allows one to determine r and $\cos\delta$. However, the main novelty of our proposal comes from the sensitivity of the complete correlated decay rate to $|\sin\delta|$ terms, which needs the study of the full angular dependence.

C. CP -violation in $D^0 \bar{D}^0$ mixing

In the previous discussions, we have neglected the tiny CP -violation in $D^0 \bar{D}^0$ mixing in order to simplify the study of correlated $D \rightarrow VV$ decays. The inclusion of this effect would impact our results in the following way:

- (i) If CP violation is indeed measured through $\psi(3770) \rightarrow D^0 \bar{D}^0 \rightarrow (V_1 V_2)(V_3 V_4)$, we cannot *a priori* disentangle CP violation in mixing from CP violation in decay. Therefore, if we want to convert this result into a bound on fundamental parameters of a new physics model, we will have to exploit external inputs on CP -violating parameters of the mixing (from other observables). On the other hand, such an input is not necessary if we only aim at setting a constraint on CP -violation itself.
- (ii) In the determination of the strong phase in $D \rightarrow K\pi$, the amplitudes exhibit in principle a small dependence on mixing effects. However, this dependence is very weak with respect to the dependence on the hadronic parameters (r, δ) , and as a first approximation, it can be neglected. As in the previous case, we can use external information on the CP -violating parameters in mixing to include their impact when required by more accurate measurements of the partial decay rate.

IV. POTENTIAL FOR BES-III AND A SUPER τ -CHARM FACTORY

In this section we give a first rough estimate of the expected sensitivity of the two different measurements

discussed above, either at the BES-III experiment or at a Super τ -charm factory.

A. CP violation

As discussed in Sec. III A, the decay chain of $e^+e^- \rightarrow \psi \rightarrow D^0\bar{D}^0 \rightarrow f_a f_b$ can be described by Eq. (17), in which both CP conserving and violating processes can occur. We parameterize the ratio of amplitudes ρ_f in Eq. (18) as $\rho_f = \eta_f(1 + \delta_f)e^{i\alpha_f}$, where the δ_f is term from CP violation in decay, and α_f is the phase difference between D^0 and \bar{D}^0 decay into the same final state f .

The D^0 decay channels in Table I can be directly used to search for CP violation by fully considering the correlation of $D^0\bar{D}^0$ production at BES-III. The background is small, and the main dilution is due to the misidentification of charged particles, which is suppressed by about 10^{-4} . The sensitivity of measurement of CP violation can reach about 10^{-3} with a 20 fb^{-1} luminosity on the $\psi(3770)$ peak at BES-III. As in the previous case, one must take care of the background due to the dilution from non- CP eigenstates that impact the quasi two-body decays of D^0 meson listed in Table II (for $D \rightarrow PV$) and III (for $D \rightarrow VV$).

Final states consisting of two vector meson pairs are particularly interesting, since one can use information on transversity amplitudes to extract different combinations of CP -violating observables, as discussed in Sec. III A: (0, 0), (0, \parallel), (\parallel , 0), (\parallel , \parallel), (\perp , \perp). For example, a back-of-the-envelope computation yields the most promising channel $\rho^0\rho^0/\bar{K}^{*0}\rho^0$:

$$\begin{aligned} & \mathcal{BR}((D^0\bar{D}^0)_{C=-1} \rightarrow \rho^0\rho^0, \bar{K}^{*0}\rho^0)|_{(0,\parallel)}^{CPV} \\ & \simeq 8 \times \mathcal{BR}^0(D^0 \rightarrow \rho^0\rho^0) \cdot \mathcal{BR}^\parallel(D^0 \rightarrow \bar{K}^{*0}\rho^0) \\ & \quad \times \sin^2 \frac{\alpha_a - \alpha_b}{2}, \end{aligned} \quad (37)$$

where \mathcal{BR}^0 means the branching fraction for longitudinal polarized $D^0 \rightarrow \rho^0\rho^0$ decay, \mathcal{BR}^\parallel means the parallel hel-

TABLE I. Branching ratios for D decays into CP -eigenstates composed of two pseudoscalar mesons. In each case, the product of intrinsic CP parities and the estimated reconstruction efficiency at BES-III are indicated. Note that the efficiency is for both D^0 decaying into a PP final state; for single D decay, the efficiency is $\sqrt{\epsilon}$.

PP	$\eta_{CP}(P)\eta_{CP}(P)$	Br (%)	Eff. (ϵ)
K^+K^-	+1	0.39	0.50
$\pi^+\pi^-$	+1	0.14	0.60
$K_S K_S$	+1	0.038	0.30
$\pi^0\pi^0$	+1	0.08	0.24
$K_S\pi^0$	-1	1.22	0.33
$K_S\eta$	-1	0.40	0.26
$K_S a_0(980) \rightarrow K_S(\eta\pi^0)$	+1	0.67	0.18
$K_S a_0(980) \rightarrow K_S(K^+K^-)$	+1	0.31	0.10

TABLE II. Branching ratios for D decays into CP -eigenstates composed of one pseudoscalar and one vector mesons. In each case, the product of intrinsic CP parities and the estimated reconstruction efficiency at BES-III are indicated. Note that the efficiency is for both D^0 decaying into a PV final state; for single D decay, the efficiency is $\sqrt{\epsilon}$.

PV	$\eta_{CP}(P)\eta_{CP}(V)$	Br (%)	Eff. (ϵ)
$\rho^0\pi^0$	-1	0.37	0.29
$\phi\pi^0 \rightarrow (K^+K^-)\pi^0$	-1	0.06	0.10
$K_S\rho^0$	+1	0.77	0.27
$K_S\phi \rightarrow K_S(K^+K^-)$	+1	0.22	0.08
$K_S\omega \rightarrow K_S(\pi^+\pi^-\pi^0)$	+1	0.98	0.20
$\bar{K}^{*0}\eta \rightarrow (K_S\pi^0)(\pi^+\pi^-\pi^0)$	+1	0.03	0.17
$\bar{K}^{*0}\eta \rightarrow (K_S\pi^0)(\gamma\gamma)$	+1	0.06	0.17
$\bar{K}^{*0}\pi^0 \rightarrow (K_S\pi^0)\pi^0$	+1	0.67	0.15

icities fraction of $D^0 \rightarrow \bar{K}^{*0}\rho^0$ decay, and where we have assumed that the CP -violating parameters δ_f vanish.

Assuming that no CP -violating signal events in $D^0\bar{D}^0$ coherent decays are observed with 20 fb^{-1} data at BES-III, we can provide an upper limit on the CP -violating branching fraction at 90% confidence level (C.L.), as indicated in Table V. A Super τ -charm factory with 2 ab^{-1} data yields naturally stronger constraints. If each polarized fraction is measured independently, an upper limit on the phase difference $|\alpha_a - \alpha_b|$ can be set. For example, the current values for the polarized fractions in $\rho\rho$ and ρK^* yields the upper limit $|\alpha_a - \alpha_b| < 4.4^\circ$ at 90% confidence level from the channel $(D^0\bar{D}^0)_{C=-1} \rightarrow \rho^0\rho^0, \bar{K}^{*0}\rho^0|_{(0,\parallel)}$. At a future Super τ -charm factory, with a data set of 2 ab^{-1} , the constraint would be more severe, $|\alpha_a - \alpha_b| < 0.5^\circ$ at 90% confidence level.

A more realistic analysis requires a likelihood fit to the full angular dependence of the VV modes. Systematics will arise from the misreconstruction as VV CP -eigenstates of the events that actually come from other resonances or

TABLE III. Branching ratios for D decays into CP -eigenstates composed of two vector mesons. In each case, the product of intrinsic CP parities and the estimated reconstruction efficiency at BES-III are indicated. The rates in brackets are not measured yet, but were predicted in Ref. [20]. Note that the efficiency is for both D^0 decaying into a VV final state; for single D decay, the efficiency is $\sqrt{\epsilon}$.

VV	$\eta_{CP}(V)\eta_{CP}(V)$	Br (%)	Eff. (ϵ)
$\rho^0\rho^0$	1	0.18	0.24
$\bar{K}^{*0}\rho^0 \rightarrow (K_S\pi^0)(\pi^+\pi^-)$	1	0.27	0.12
$\rho^0\phi \rightarrow (\pi^+\pi^-)(K^+K^-)$	1	0.14	0.07
$\bar{K}^{*0}\omega \rightarrow (K_S\pi^0)(\pi^+\pi^-\pi^0)$	1	0.33	0.09
$\rho^+\rho^-$	1	[0.6]	0.18
$\rho^0\omega \rightarrow (\pi^+\pi^-)(\pi^+\pi^-\pi^0)$	1	[$\simeq 0$]	0.18
$K^{*+}K^{*-} \rightarrow (K_S\pi^+)(K_S\pi^-)$	1	[0.08]	0.07
$K^{*0}\bar{K}^{*0} \rightarrow (K_S\pi^0)(K_S\pi^0)$	1	0.003	0.09

TABLE IV. Weights used to select contributions from the transversity amplitudes for $\psi \rightarrow (K\pi)(VV)$. M amplitudes are defined by Eq. (33).

i	$P_i(\theta_{V_1}, \theta_{V_2}, \Phi)$	$\int d\Gamma_{2V} P_i / (A^{\psi V_1 V_2} ^2 A^{D^0 \rightarrow K\pi} ^2)$
1	$\frac{1}{8}(5 \cos^2 \theta_{V_1}^2 - 1)(5 \cos^2 \theta_{V_2}^2 - 1)$	$ M_0 ^2$
2	$\frac{1}{16}(5 \cos^2 \theta_{V_1}^2 - 3)(5 \cos^2 \theta_{V_2}^2 - 3)(4 \cos^2 \Phi - 1)$	$ M_{\parallel} ^2$
3	$-\frac{25}{4\sqrt{2}} \cos \theta_{V_1} \cos \theta_{V_2} \sin \theta_{V_1} \sin \theta_{V_2} \cos \Phi$	$\text{Re}[M_0 M_{\parallel}^*]$
4	$-\frac{1}{16}(5 \cos^2 \theta_{V_1}^2 - 3)(5 \cos^2 \theta_{V_2}^2 - 3)(4 \cos^2 \Phi - 3)$	$ M_{\perp} ^2$
5	$\frac{25}{4\sqrt{2}} \cos \theta_{V_1} \sin \theta_{V_1} \cos \theta_{V_2} \sin \theta_{V_2} \sin \Phi$	$-\text{Re}[M_0 M_{\perp}^*]$
6	$\frac{1}{4}(5 \cos^2 \theta_{V_1}^2 - 3)(5 \cos^2 \theta_{V_2}^2 - 3) \cos \Phi \sin \Phi$	$\text{Re}[M_{\parallel} M_{\perp}^*]$

background contributions. In view of the sizable width of the vector resonances, we expect that these systematics will dominate the final result. Their precise estimate in the framework of each experiment is however beyond the scope of this paper.

B. Strong phase in $D^0 \rightarrow K\pi$

The joint decay of D^0 into $K^- \pi^+$ and of D^0 into a CP eigenstate f_{η} can be described as

$$\begin{aligned} \Gamma_{K\pi;f_{\eta}} &\equiv \Gamma[(K^- \pi^+)(f_{\eta})] \approx A^2 A_{f_{\eta}}^2 |1 + \eta r e^{-i\delta}|^2 \\ &\approx A^2 A_{f_{\eta}}^2 (1 + 2\eta r \cos\delta), \end{aligned} \quad (38)$$

where $A = |\langle K^- \pi^+ | \mathcal{H} | D^0 \rangle|$ and $A_{f_{\eta}} = |\langle f_{\eta} | \mathcal{H} | D^0 \rangle|$ are the real-valued decay amplitudes, $\eta = \pm 1$ is CP eigenvalue of the eigenstate f_{η} , $r e^{-i\delta}$ is defined in Eq. (27) and we have taken f_{η} to be a PP or VP CP -eigenstate, without any non trivial phase-space dependence. We also have neglected the subdominant r^2 term in Eq. (38). The following asymmetry can be used to determine δ [10]

$$\mathcal{A} \equiv \frac{\Gamma_{K\pi;f_+} - \Gamma_{K\pi;f_-}}{\Gamma_{K\pi;f_+} + \Gamma_{K\pi;f_-}}, \quad (39)$$

where $\Gamma_{K\pi;f_{\pm}}$ is defined in Eq. (38), which is the rate for the $\psi(3770) \rightarrow D^0 \bar{D}^0$ configuration to decay into flavor eigenstates and a CP -eigenstates f_{\pm} . Equation (38) implies a small asymmetry, $\mathcal{A} = 2r \cos\delta$. In such a case, the error

$\Delta \mathcal{A}$ is approximately $1/\sqrt{N_{K^- \pi^+}}$, where $N_{K^- \pi^+}$ is the total number of events tagged with CP -even and CP -odd eigenstates, leading to:

$$\Delta(\cos\delta) \approx \frac{1}{2r\sqrt{N_{K^- \pi^+}}}. \quad (40)$$

The expected number $N_{K^- \pi^+}$ of CP -tagged events depends on the total number of $D^0 \bar{D}^0$ pairs $N(D^0 \bar{D}^0)$, the branching ratio to the CP -eigenstate f_{η} and the tagging efficiency. Considering all decay modes listed in Tables I, II, and III we find

$$\Delta(\cos\delta) \approx \frac{300}{\sqrt{N(D^0 \bar{D}^0)}}. \quad (41)$$

At BES-III, about 72×10^6 $D^0 \bar{D}^0$ pairs can be collected with 4 yr running [19,28], which implies an accuracy of about 0.03 for $\cos\delta$, when considering both $K^- \pi^+$ and $K^+ \pi^-$ final states.

As in the previous section a more realistic analysis requires a likelihood fit to the full angular dependence of the VV modes, which in turn provides independent information on $|\sin\delta|$ as explained above. On the other hand the imperfect reconstruction of the VV events as pure CP -eigenstates will presumably introduce sizable systematics in this discussion.

At a Super τ -charm factory [29,30] with a 2 ab^{-1} data set, we can expect a factor of 10 improvement, but again

TABLE V. The projected 90%-C.L. upper limits on CP violating branching fraction of some most interesting $(VV)(VV)$ modes from correlated $D^0 \bar{D}^0$ pairs with 20 fb^{-1} data taken at $\psi(3770)$ peak at BES-III.

Reaction	Efficiency	Upper limits at BES-III ($\times 10^{-7}$)
$D^0 \bar{D}^0 \rightarrow (\rho^+ \rho^-)(\bar{K}^{*0} \omega)$	0.13	2.46
$D^0 \bar{D}^0 \rightarrow (\rho^0 \rho^0)(\bar{K}^{*0} \rho^0)$	0.17	1.88
$D^0 \bar{D}^0 \rightarrow (\bar{K}^{*0} \rho^0)(K^{*0} \omega)$	0.10	3.19
$D^0 \bar{D}^0 \rightarrow (\bar{K}^{*0} \rho^0)(\rho^0 \phi)$	0.09	3.55
$D^0 \bar{D}^0 \rightarrow (\bar{K}^{*0} \omega)(\rho^0 \phi)$	0.08	3.99
$D^0 \bar{D}^0 \rightarrow (\rho^0 \rho^0)(\bar{K}^{*0} \omega)$	0.15	2.13
$D^0 \bar{D}^0 \rightarrow (\rho^0 \rho^0)(\rho^0 \phi)$	0.13	2.46
$D^0 \bar{D}^0 \rightarrow (\rho^+ \rho^-)(\rho^0 \phi)$	0.11	2.90
$D^0 \bar{D}^0 \rightarrow (\rho^+ \rho^-)(K^{*+} K^{*-})$	0.11	2.90

the precise impact of the modeling of the vector resonances requires more studies.

V. CONCLUSION

The charm quark offers interesting opportunities to cross check the mechanism of CP violation precisely tested in the strange and beauty sectors. The start of BES-III will allow for extensive measurements of charm properties. Among the various tests that can be considered, one may think of exploiting the quantum correlations in the $D\bar{D}$ pairs produced at $\psi(3770)$ resonance. In this paper, we exploit these correlations in $\psi(3770) \rightarrow D^0\bar{D}^0 \rightarrow (V_1V_2)(V_3V_4)$ in connection with CP violation, and $\psi(3770) \rightarrow D^0\bar{D}^0 \rightarrow (V_1V_2)(K\pi)$ for CKM angle γ measurements, where all VV pairs are reconstructed as CP -eigenstates.

In the case of $\psi(3770) \rightarrow D^0\bar{D}^0 \rightarrow (V_1V_2)(V_3V_4)$, the existence of correlations hinders some helicity configurations for the outgoing vector mesons in the absence of CP violation. This is mirrored by the angular distribution of the differential decay width, out of which CP -violating observables can be constructed. Such observables should be interesting to isolate significant new physics effects in the charm sector. Assuming that there would be no CP -violating signal events observed in $D^0\bar{D}^0$ coherent decays with 20 fb^{-1} data taken at $\psi(3770)$ peak at BES-III, we estimated an order of magnitude of the corresponding upper limit on CP -violating parameters, in particular, for the channel $(D^0\bar{D}^0)_{C=-1} \rightarrow \rho^0\rho^0, \bar{K}^{*0}\rho^0|_{(0,\parallel)}$. Since the obtained bounds do not follow from a full angular fit and do not include the systematics corresponding to the

separation of the wanted vector resonances from the background, further studies are needed.

CP -tagged $D \rightarrow K\pi$ decays give access to the strong phase difference δ between Cabibbo-favored and doubly-Cabibbo suppressed decays, and thus improve the uncertainty on the γ measurement of the unitary triangle from $B^\pm \rightarrow D/\bar{D}K^\pm$ decays. At BES-III, with 20 fb^{-1} data at $\psi(3770)$ peak, we estimate the error of $\cos\delta$ to be of a few percents, corresponding to an error on the δ of a few degrees. We expect this estimate can be improved by taking into account the dependence of the full angular decay width to the sine of the strong phase. At the Super τ -charm factory, the expected statistical error on δ could then fall below 1° . On the other hand a further study of experimental systematics related to the background identification is required since they will presumably dominate over the uncertainty quoted here.

Since our numerical estimates are quite promising we hope that the potential of such coherent D -decays into vector mesons at charm factories will be assessed more precisely in the future.

ACKNOWLEDGMENTS

One of the author (H. B. Li) would like to thank M. Z. Yang and Z. Z. Xing for useful discussions. This work is supported in part by the ANR Contract No. ANR-06-JCJC-0056, the EU Contract No. MRTN-CT-2006-035482, "FLAVIANet", the National Natural Science Foundation of China under Contract Nos. 10521003, 10821063, 10835001, 10979008, the 100 Talents program of CAS, and the Knowledge Innovation Project of CAS under Contract Nos. U-612 and U-530 (IHEP).

-
- [1] J. Charles *et al.* (CKMfitter Group), Eur. Phys. J. C **41**, 1 (2005), updated results and plots available at: <http://ckmfitter.in2p3.fr>.
 - [2] M. Bona *et al.* (UTfit Collaboration), J. High Energy Phys. **10** (2006) 081, updated results and plots available at: <http://www.utfit.org>.
 - [3] B. A. Dobrescu and A. S. Kronfeld, Phys. Rev. Lett. **100**, 241802 (2008).
 - [4] J. P. Alexander *et al.* (CLEO Collaboration), Phys. Rev. D **79**, 052001 (2009).
 - [5] K. Abe *et al.* (Belle Collaboration), Phys. Rev. Lett. **100**, 241801 (2008).
 - [6] For a recent review see, e.g., M. Artuso, B. Meadows, and A. A. Petrov, Annu. Rev. Nucl. Part. Sci. **58**, 249 (2008).
 - [7] B. Aubert *et al.* (BABAR Collaboration), Phys. Rev. Lett. **98**, 211802 (2007).
 - [8] M. Staric *et al.* (Belle Collaboration), Phys. Rev. Lett. **98**, 211803 (2007).
 - [9] E. Golowich, J. Hewett, S. Pakvasa, and A. A. Petrov, Phys. Rev. D **76**, 095009 (2007).
 - [10] X. D. Cheng, K. L. He, H. B. Li, Y. F. Wang, and M. Z. Yang, Phys. Rev. D **75**, 094019 (2007).
 - [11] H. B. Li and M. Z. Yang, Phys. Rev. D **74**, 094016 (2006).
 - [12] M. Gronau, Y. Grossman, and J. L. Rosner, Phys. Lett. B **508**, 37 (2001).
 - [13] D. M. Asner and W. M. Sun, Phys. Rev. D **73**, 034024 (2006); **77**, 019901(E) (2008); D. Atwood and A. Soni, Phys. Rev. D **68**, 033003 (2003).
 - [14] I. I. Y. Bigi and A. I. Sanda, Phys. Lett. B **171**, 320 (1986).
 - [15] I. I. Y. Bigi, $D^0\bar{D}^0$ Mixing and CP Violation in D Decays: *Can There Be High Impact Physics in Charm Decays?* (Stanford Tau Charm, Stanford, CA, 1989), pp. 0169–195.
 - [16] Z. z. Xing, Phys. Rev. D **55**, 196 (1997).
 - [17] J. L. Rosner *et al.* (CLEO Collaboration), Phys. Rev. Lett. **100**, 221801 (2008); D. M. Asner *et al.* (CLEO Collaboration), Int. J. Mod. Phys. A **21**, 5456 (2006); Phys. Rev. D **78**, 012001 (2008).
 - [18] C. Amsler *et al.* (Particle Data Group), Phys. Lett. B **667**, 1

- (2008).
- [19] D.M. Asner *et al.*, Physics at BES-III, edited by K. T. Chao and Y.F. Wang, *Int. J. Mod. Phys. A* **24**, Supp. 1 (2009).
- [20] T. Uppal and R. C. Verma, effects due *Z*. *Phys. C* **56**, 273 (1992).
- [21] A. N. Kamal, R. C. Verma, and N. Sinha, *Phys. Rev. D* **43**, 843 (1991).
- [22] P. Bedaque, A. Das, and V. S. Mathur, *Phys. Rev. D* **49**, 269 (1994).
- [23] I. Hinchliffe and T. A. Kaeding, *Phys. Rev. D* **54**, 914 (1996).
- [24] J. D. Jackson, *Les Houches*, edited by C. de Witt and M. Jacob (Gordon and Breach, New York, 1965).
- [25] M. Jacob and G. C. Wick, *Ann. Phys. (N.Y.)* **7**, 404 (1959); **281**, 774 (2000).
- [26] D. Atwood, I. Dunietz, and A. Soni, *Phys. Rev. Lett.* **78**, 3257 (1997).
- [27] The Heavy Flavor Averaging Group (HFAG), <http://www.slac.stanford.edu/xorg/hfag/>.
- [28] (BES-III Collaboration), Report No. IHEP-BEPCII-SB-13.
- [29] D.M. Asner, in *On the Case for a Super Tau-Charm Factory*, Frascati Physics Series Vol. 41 (Frascati, Italy, 2006), p. 377.
- [30] M. Bona *et al.*, arXiv:0709.0451.