Invisible Υ decays into light dark matter

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Invisible ψ and Υ decays into light neutralinos, within the minimal or next-to-minimal supersymmetric standard model, are smaller than for $\nu\bar\nu$ production, even if light spin-0 particles are coupled to quarks and neutralinos. In a more general way, light dark matter particles are normally forbidden, unless they can annihilate sufficiently through a new interaction stronger than weak interactions (at lower energies), as induced by a light spin-1 U boson, or heavy-fermion exchanges in the case of scalar dark matter. We discuss the possible contributions of U-boson, heavy-fermion, or spin-0 exchanges to invisible ψ and Υ decays. U exchanges could lead, but not necessarily, to significant branching fractions for invisible decays into light dark matter. We show how one can get the correct relic density together with sufficiently small invisible branching fractions, and the resulting constraints on the U couplings to ordinary particles and dark matter, in particular $|c_\chi f_{bV}| < 5 \times 10^{-3}$ from Υ decays, for $2m_\chi$ smaller than a few GeV. We also explain why there is no model-independent way to predict ψ and Υ branching fractions into light dark matter, from dark matter annihilation cross sections at freeze-out time.

I. NEW PARTICLES IN INVISIBLE ψ AND Υ DECAYS

The nature of dark matter is one of the most challenging issues facing physics. Observation of standard model (SM) particles coupling to invisible final states, as searched for recently in Υ decays [1], might provide information on new neutral particles such as photinos or neutralinos and very light gravitinos, and candidate dark matter constituents [2–4]. In the standard model, invisible decays of the Υ(1S) proceed by $b\bar b$ annihilation into a $\nu\bar\nu$ pair, with a branching fraction [3]

$$B(\Upsilon(1S) \to \nu\bar\nu) \simeq 10^{-5}, \tag{1}$$

well below the current experimental sensitivity [1]. However, low-mass dark matter candidates could couple through *stronger-than-weak* interactions to SM particles, and possibly enhance the invisible branching fraction of the Υ(1S) to the level of $\approx 10^{-5}$ to 10^{-2} [3,4], in contrast with weakly-interacting particles, as indicated by (1). A new light boson U associated with the gauging of an extra-$U(1)$ symmetry, as considered long ago in [5], may play a crucial role as a mediator of such a new interaction.

Upper limits on the invisible Υ branching fraction have been obtained long ago by CLEO [6] and ARGUS [7], already having in mind the search for new weakly-interacting particles such as photinos and very light gravitinos, as in invisible ψ decays [2]. These limits (7×10^{-3} for ψ, 5×10^{-2} then 2.3×10^{-2} for Υ) are obtained by looking for

$$\psi(2S) \to \pi^+ \pi^- \psi(1S)_{\hookrightarrow\text{invisible}},$$

$$\Upsilon(3S), \quad \text{or} \quad \Upsilon(2S) \to \pi^+ \pi^- \Upsilon(1S)_{\hookrightarrow\text{invisible}}, \tag{2}$$

which provide signatures for the production and invisible decays of ψ and Υ.

The Υ bounds have been improved by Belle and CLEO [8,9], and recently *BABAR* [1], down to

$$B(\Upsilon(1S) \to \text{invisible}) < 3 \times 10^{-4}, \tag{3}$$

at the 90% C.L. We also have, from BES II [10],

$$B(\psi(1S) \to \text{invisible}) < 7.2 \times 10^{-4}. \tag{4}$$

Although the present paper is in general formulated with the Υ, the analysis applies to invisible ψ decays as well.

What can we learn about the light neutral particles that could be produced? We shall discuss possible invisible decays of the Υ, and at first into *light neutralinos* within the minimal (MSSM) or next-to-minimal [N(n)MSSM] supersymmetric standard model (cf. Sec. II). They are significantly smaller than for Υ(1S) → $\nu\bar\nu$, even in the presence of light spin-0 particles coupling directly quarks to neutralinos.

We shall also discuss, in a more general way, invisible decays of ψ and Υ into *light dark matter particles* (of mass $< m_\psi/2$ or $m_\Upsilon/2$) [3,4], which could be much *more strongly coupled to ordinary particles than through ordinary weak interactions*, possibly leading to significant invisible ψ and Υ branching fractions.

Indeed, light dark matter (LDM) particles [11,12] are normally required to annihilate through a new interaction stronger than weak interactions (at least at lower energies), otherwise their relic density would be too large. Is this compatible with the new experimental bounds [1,10] on invisible ψ and Υ decays? This is part of the more general question [4,12,13]: how can we have a new interaction stronger than weak interactions, responsible for sufficient annihilations of LDM particles in the early Universe, and at the same time how could it remain unnoticed if it is stronger than weak interactions? Indeed this stronger-than-weak feature cannot persist up to high energies, es-

pecially with (production, annihilation, or interaction) cross sections increasing like s, without getting in conflict with experimental results.

The apparent contradiction is solved for an interaction mediated by *a new light neutral boson* with small couplings to quarks and leptons, such as the light spin-1 U boson [5] introduced and discussed long ago, associated with the gauging of an extra-$U(1)$ symmetry. Another possibility, for spin-0 dark matter particles, is obtained for interactions mediated by *heavy-fermion exchanges* [11], which may also allow for sufficient annihilations of light dark matter in the early Universe. We shall also discuss (in Sec. II) light spin-0 exchanges, which do not contribute to invisible ψ and Υ decays, both in the N(n) MSSM and in a more general way.

The choice is thus, for the production of light dark matter particles in invisible ψ and Υ decays, between

$$\text{a new neutral current} \quad \text{and} \quad \text{new heavy fermions,} \quad (5)$$

or both (reminding us of the early days of gauge theories, before the discovery of the weak neutral current coupled to the Z). We shall discuss in Sec. III the possible production of scalar dark matter through heavy-fermion exchanges, according to

$$\Upsilon \xrightarrow{b_M} \varphi \bar{\varphi}. \quad (6)$$

We shall then concentrate, in Secs. IV and V, on U-induced reactions, discussing the implications of the experimental limit (3) on the couplings of the light U boson that may be responsible for Υ annihilations into light spin-$\frac{1}{2}$ (χ) or spin-0 (φ) dark matter particles:

$$\Upsilon \xrightarrow{U} \chi\chi \quad (\text{or } \varphi\bar{\varphi} \text{ or } \chi\bar{\chi}), \quad (7)$$

at $\sqrt{s} = m_\Upsilon$ [4]. This U allows for the correct relic density of light dark matter particles [11,12], by inducing sufficient annihilations in the early Universe,

$$\chi\chi \quad (\text{or } \varphi\bar{\varphi} \text{ or } \chi\bar{\chi}) \xrightarrow{U} f\bar{f}, \quad (8)$$

at lower values of the energy, $\sqrt{s} \simeq 2m_\chi$ (or $2m_\varphi$).

Finally, we shall discuss in Sec. VI whether it makes sense to attempt predicting, in a model-independent way, ψ and Υ invisible branching fractions into dark matter, from dark matter annihilation cross sections at freeze-out time; and consider briefly, in Sec. VII, possible nonperturbative effects associated with light U exchanges.

II. INVISIBLE DECAYS INTO NEUTRINOS, AND NEUTRALINOS (OR GRAVITINOS, …)

A. Standard decays into neutrinos

The expected Υ decay rate into *neutrinos* (1), which involves the vector part in the weak neutral current $J_Z = J_3 - \sin^2\theta J_{em}$ of the b quark, is obtained from

$$\frac{B(\Upsilon \to \nu\bar{\nu})}{B(\Upsilon \to e^+e^-)} \simeq \frac{3}{2}\left(\frac{\frac{1}{2}(-\frac{1}{4} + \frac{1}{3}\sin^2\theta)(g^2 + g'^2)/m_Z^2}{-\frac{e^2}{3}/m_\Upsilon^2}\right)^2$$
$$\simeq \frac{27G_F^2 m_\Upsilon^4}{64\pi^2\alpha^2}\left(-1 + \frac{4}{3}\sin^2\theta\right)^2 \simeq 4 \times 10^{-4}, \quad (9)$$

with $B(\Upsilon \to e^+e^-) \simeq 2.4\%$ or possibly 2.5%. The smallness of the resulting $B(\Upsilon \to \nu\bar{\nu}) \simeq 10^{-5}$ reflects the smallness of $(m_\Upsilon/m_Z)^4 \approx 10^{-4}$ [3,14].

In a similar way,

$$\frac{B(\psi \to \nu\bar{\nu})}{B(\psi \to e^+e^-)} \simeq \frac{3}{2}\left(\frac{\frac{1}{2}(\frac{1}{4} - \frac{2}{3}\sin^2\theta)(g^2 + g'^2)/m_Z^2}{\frac{2e^2}{3}/m_\Upsilon^2}\right)^2$$
$$\simeq \frac{27G_F^2 m_\psi^4}{256\pi^2\alpha^2}\left(1 - \frac{8}{3}\sin^2\theta\right)^2 \simeq 4 \times 10^{-7}, \quad (10)$$

the precise value depending on how renormalization effects are taken into account. With $B(\psi \to e^+e^-) \simeq 6\%$ this leads to a very small $B(\psi \to \nu\bar{\nu}) \simeq (2 \text{ to } 3) \times 10^{-8}$ [2,14], well below present experimental sensitivity [10].

B. Decays into light neutralinos, in the MSSM or N(n) MSSM

The branching fraction for the pair-production of light *neutralinos* by squark or Z exchanges (Fig. 1) is also expected to be small. The amplitudes involving the exchanges of the two spin-0 squarks $\tilde{b}$ (Fig. 1) are induced in the local limit approximation by

$$\bar{b}\gamma^\mu b \; \bar{\chi}\gamma_\mu\gamma_5\chi \quad (11)$$

4-fermion effective interactions proportional to $1/m_{\tilde{b}}^2$ [15], in a way which depends on the composition (gaugino/Higgsino) of the neutralino χ considered, and of the mass matrix of the two squark fields $\tilde{b}_L$ and $\tilde{b}_R$. Indeed as Υ has the same quantum numbers 1^{--} as the photon, the $\bar{b}\gamma^\mu\gamma_5 b$, $\bar{b}b$, and $\bar{b}\gamma_5 b$ operators, which have $C = +$, cannot contribute to the decay, nor $\bar{b}\sigma^{\mu\nu}b$ (which has $C = -$) as $\bar{\chi}\sigma_{\mu\nu}\chi$ vanishes identically for a Majorana χ, as for $\bar{\chi}\gamma_\mu\chi$.
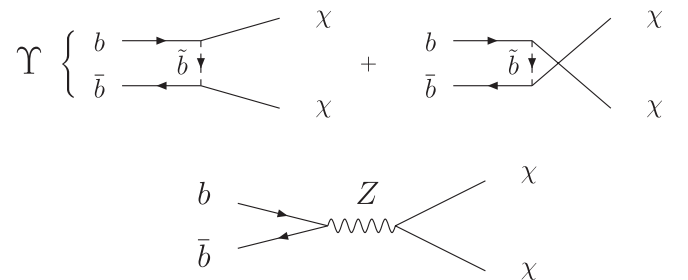

FIG. 1. Υ decay into light neutralinos, induced by squark or Z exchanges (ignoring a possible U contribution as in Fig. 6).

These amplitudes may be easily compared with the Z amplitudes for $\Upsilon \to \nu\bar{\nu}$. They are very small for the two $\tilde{b} \gtrsim 100$ GeV, corresponding to an invisible branching fraction into $\chi\chi$ of less than 5×10^{-8} [16,17]. This leaves us with the Z amplitude, to be discussed soon.

But what about possible contributions from the exchanges of neutral spin-0 particles? The question arises especially as neutral particles such as a pseudoscalar a or scalar h (with $\bar{b}\gamma_5 b$ or $\bar{b}b$ couplings to the b quark) could be light when the MSSM is extended to include a singlet S with a $\lambda H_1 H_2 S$ trilinear superpotential term [19], in connection with associated global $U(1)$ symmetries of 2-Higgs-doublet models, which may be almost-spontaneously broken [20,21]. Such light spin-0 particles might conceivably induce relatively large $\Upsilon \to \chi\chi$ invisible decay amplitudes.

But $\Upsilon \to \chi\chi$ cannot proceed through the virtual production of a spin-0 boson with pseudoscalar or scalar couplings to $\bar{b}\gamma_5 b$ or $\bar{b}b$. The $b\bar{b}$ annihilation of the Υ, a $C = -$ state, can only occur (independently of the fact that C may be conserved or not in the decay) through a $C = -$ hadronic operator such as the quark vector current $\bar{b}\gamma^\mu b$, but not the axial current $\bar{b}\gamma^\mu \gamma_5 b$, nor $\bar{b}\gamma_5 b$ or $\bar{b}b$, which all have $C = +$. We thus always have

$$\mathcal{A}\left(\Upsilon \overset{\underset{(a,h,\ldots)}{\text{spin-0}}}{\to} \chi\chi\right) = 0, \tag{12}$$

as represented in Fig. 2.

One remains, in the MSSM as well as in the N(n)MSSM, with the virtual-Z amplitude for $\Upsilon \to \chi\chi$ (Fig. 1). The Z does not couple to neutral gauginos, only to Higgsinos $\tilde{h}_1{}^0$ and $\tilde{h}_2{}^0$, with opposite signs. With $\chi = \alpha_1 \tilde{h}_1{}^0 + \alpha_2 \tilde{h}_2{}^0 + \ldots$, the Z coupling (written as a coupling to a chiral χ_L) is $|\alpha_1|^2 - |\alpha_2|^2$ times the Z coupling to an ordinary neutrino ν_L. This leads to a contribution to the invisible width of the Υ fixed by $(|\alpha_1|^2 - |\alpha_2|^2)^2$. Its size is experimentally limited as the neutralino χ should not contribute too much to the invisible decay width of the Z. That is, conservatively, considering "N_ν" $= 2.92 \pm 0.05$ from the direct measurement of the Z invisible width, at most $\simeq 10\%$ of the contribution of a single neutrino flavor. As a result we get the estimate

$$B(\Upsilon \to \chi\chi) \simeq (|\alpha_1|^2 - |\alpha_2|^2)^2 \beta^3 B(\Upsilon \to \nu_e \bar{\nu}_e)$$
$$\lesssim 0.1 \beta^3 B(\Upsilon \to \nu_e \bar{\nu}_e) \lesssim 3 \times 10^{-7}. \tag{13}$$

(The β^3 factor, with $\beta = v_\chi/c$, is associated with the Majorana character of the neutralino χ, coupled to the Z through its axial current $\bar{\chi}\gamma^\mu \gamma_5 \chi$.)

This analysis applies as well to invisible ψ decays, leading to

$$B(\psi \to \chi\chi) \lesssim 0.1 \beta^3 B(\psi \to \nu_e \bar{\nu}_e) \lesssim 10^{-9}, \tag{14}$$

assuming the two squarks $\tilde{c}$ to be somewhat heavier than $\simeq 200$ GeV so that their contribution is negligible [16].

C. Spin-0 induced decays?

More generally, could exchanges of light neutral spin-0 particles, as represented in Fig. 2 for neutralinos χ, lead to significant invisible ψ or Υ branching fractions, independently of the nature of the dark matter particle? As we saw invisible ψ or Υ decays cannot be directly induced by spin-0 bosons coupled to $\bar{b}\gamma_5 b$ or $\bar{b}b$, since the $b\bar{b}$ annihilation can only occur through a $C = -$ hadronic operator, so that

$$\mathcal{A}\left(\psi/\Upsilon \overset{\text{spin-0 part.}}{\to} \text{invisible}\right) = 0. \tag{15}$$

This applies independently of the spin of the final particles, for Majorana or Dirac ($\Upsilon \to \chi\chi$ or $\chi\bar{\chi}$) as well as spin-0 ones ($\varphi\bar{\varphi}$). The decay $\Upsilon \to \varphi\varphi$ into a pair of identical spin-0 particles, in a $L = 1$ state, is in any case forbidden by angular momentum conservation and Bose statistics, independently of its possible mediator(s). In particular,

$$B(\psi/\Upsilon \to \text{pair of self-conj. spin-0 part.}) \equiv 0. \tag{16}$$

D. Decays into gravitinos + neutralinos

A spin-$\frac{3}{2}$ gravitino, although coupled with gravitational strength ($\propto \kappa = \sqrt{8\pi G_N} \simeq 4 \times 10^{-19}$ GeV^{-1}), could still interact "weakly" or even relatively "strongly" with standard model particles, if it is sufficiently light, much less than 1 eV [15,22]. ψ or Υ could then decay into a gravitino + a photino or neutralino (the latter remaining also unobserved if it is light enough). These decays are primarily induced by a virtual photon coupled to the c or b quark (Fig. 3), having with the gravitino and photino a nondiagonal q^2-dependent chargelike effective coupling $\kappa q^2/(m_{3/2}\sqrt{6}) = q^2/d$. It leads to an effective pointlike 4-fermion interaction, which could be large if the gravitino mass $m_{3/2}$ is really very small. The branching fractions are obtained in the photino case from $\left(\frac{\kappa}{m_{3/2}\sqrt{6}} / \frac{e}{m_{\psi/\Upsilon}^2}\right)^2$, indepen-
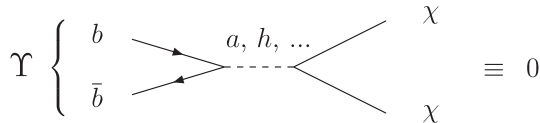

FIG. 2. Invisible decay $\Upsilon \to \chi\chi$ that might be induced by a or h, $\ldots$ These amplitudes vanish identically, as for Dirac ($\Upsilon \to \chi\bar{\chi}$) or spin-0 ($\Upsilon \to \varphi\bar{\varphi}$) dark matter particles. The production of identical spin-0 particles, $\Upsilon \to \varphi\varphi$, is forbidden by Bose statistics.
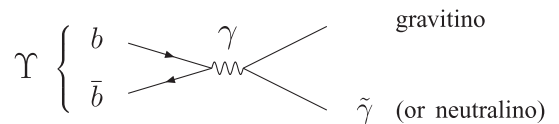

FIG. 3. Decay $\Upsilon \to$ gravitino + photino (or neutralino) [2,4]. The amplitude is $\propto \kappa e/m_{3/2}$. The photino or neutralino is unobserved if sufficiently light, otherwise it can decay into γ + gravitino.

dently of the charge 2/3 or −1/3 of the annihilating c or b quark, leading to [2,4]:

$$B(\psi/\Upsilon \rightarrow \text{gravitino} + \tilde{\gamma}) \simeq \frac{G_N m_{\psi/\Upsilon}^4}{3 m_{3/2}^2 \alpha} B(\psi/\Upsilon \rightarrow e^+ e^-).$$

(17)

If the photino $\tilde{\gamma}$ is replaced by a neutralino χ, this expression should be multiplied by $\cos^2$ of the neutralino/photino mixing angle.

The resulting lower limit on the gravitino mass, now

$$m_{3/2} > 4.4 \times 10^{-7} \text{ eV (with light quasistable } \tilde{\gamma}), \quad (18)$$

from Υ decays, is improved by $\sqrt{5 \times 10^{-2}/(3 \times 10^{-4})} \simeq$ 13 over the 3×10^{-8} eV obtained from a 5% CLEO limit in 1984 [4,6]. We also get, from invisible ψ decays [10], $m_{3/2} > 4.8 \times 10^{-8}$ eV, improved by $\simeq \sqrt{7 \times 10^{-3}/(7.2 \times 10^{-4})} \simeq 3.1$ over the first 1.5×10^{-8} eV limit of [2]. These limits, however, are largely superseded by those already obtained long ago at higher energies, e.g., in $e^+ e^-$ annihilations [23], as the associated gravitino-neutralino production cross sections, increasing with energy proportionally to s, are easier to detect at higher energies.

III. DECAYS INTO SCALAR DARK MATTER THROUGH HEAVY-FERMION EXCHANGES

A. Invisible decays $\Upsilon \rightarrow \varphi \bar{\varphi}$

Annihilations of spin-0 light dark matter particles may also occur through exchanges of new heavy fermions such as mirror fermions [11]. The dark matter particle φ is a mixing of a singlet and neutral component of an electroweak triplet, so that it is not directly coupled to the Z boson. The decay

$$\Upsilon \xrightarrow{b_M} \underbrace{\varphi \bar{\varphi}}_{\text{invisible}}$$

(19)

could then be induced by the exchange of a new heavy quark b_M (Fig. 4). The nondiagonal couplings of the spin-0 dark matter field φ to the b quark and its heavy (mirror) partner b_M are given, in terms of the two chiral Yukawa coupling constants λ_{bL} and λ_{bR}, by
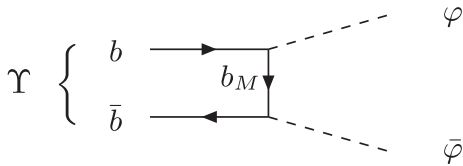

FIG. 4. Invisible decay into a pair of (non-self-conjugate) spin-0 dark matter particles, induced by a heavy quark b_M [11]. The amplitude, $\propto 1/m_{b_M}^2$, is expected to be small.

$$\mathcal{L} = -m_{b_M} \bar{b}_M b_M + [\varphi^\dagger \bar{b}_M (\lambda_{bL} b_L + \lambda_{bR} b_R) + \text{H.c.}] + \dots$$

(20)

The exchange of a heavy b_M leads in the local limit approximation $m_{b_M} \gg m_b$, in which

$$b_M = \frac{1}{m_{b_M}} \varphi^\dagger (\lambda_{bL} b_L + \lambda_{bR} b_R) + \dots,$$

(21)

to a dimension-5 effective interaction between b quarks and dark matter particles, given by

$$\mathcal{L}_{\text{eff}}^5 = m_{b_M} \bar{b}_M b_M = \frac{\lambda_{bR}^* \lambda_{bL}}{m_{b_M}} \bar{b}_R b_L \varphi^\dagger \varphi + \text{H.c.}$$

(22)

This operator, however, cannot contribute to invisible Υ decays, $\bar{b}b$ and $\bar{b}\gamma_5 b$ having $C = +$, unlike Υ [4]. We are then interested, for next-order contributions to the amplitude, in the ($C = -$) quark current $\bar{b}\gamma^\mu b$ (or $\bar{c}\gamma^\mu c$), effectively coupled to the spin-0 dark matter current $\varphi^\dagger i \overleftrightarrow{\partial}_\mu \varphi$, proportionally to $1/m_{b_M}^2$.

The invisible decay amplitude, vanishing at order $1/m_{b_M}$, is proportional to $1/m_{b_M}^2$. This contrasts with $\varphi \bar{\varphi} \rightarrow f \bar{f}$ dark matter annihilation amplitudes (and heavy-fermion contributions to lepton anomalous magnetic moments). These are proportional to $1/m_{f_M}$ (f_M being the exchanged fermion) in the nonchiral case where λ_{fL} and λ_{fR} couplings are both present, as seen from the formulas involving the fermion f, analogous to (20) and (22).

The invisible decay amplitude is thus usually expected to be very small, at most of the order of weak-interaction amplitudes for $m_{b_M} \gtrsim 100$ GeV and Yukawa couplings $\lesssim$ electroweak gauge couplings g and g'.

Even for m_{q_M} as low as ≈ 100 GeV, we would need, to get from heavy b_M exchanges a significant $\Upsilon \rightarrow \varphi \bar{\varphi}$ branching fraction possibly approaching the experimental limit (3), rather large Yukawa couplings λ_b of φ to b and b_M, as compared to g and g', corresponding to a stronger-than-weak interaction at $\sqrt{s} \simeq m_\Upsilon$. But analogous Yukawa couplings to charged leptons (λ_l) or light quarks (λ_q) of similar size would tend to lead to excessively large effects in $e^+ e^-$ scatterings ($e^+ e^- \rightarrow \gamma \varphi \bar{\varphi}$), anomalous magnetic moments of charged leptons, with large contributions [11]

$$\delta a_l \simeq \frac{\Re(\lambda_{lR}^* \lambda_{lL})}{16 \pi^2} \frac{m_l}{m_{lM}},$$

(23)

$K^+ \rightarrow \pi^+ \varphi \bar{\varphi}, \dots$

The residual invisible Υ decay amplitude, proportional to $1/m_{bM}^2$, may be estimated from the relevant part in the heavy b_M propagator, $\slashed{q}/(q^2 - m_{b_M}^2)$, replacing the virtual momentum $q = \frac{1}{2}(p_1 - k_1) - \frac{1}{2}(p_2 - k_2)$ by $\frac{1}{2}(k_2 - k_1)$ [as the terms proportional to $\slashed{p}_1(b)$ and $\slashed{p}_2(\bar{b})$ lead again to C-even operators which do not contribute]. This leads to the dimension-6 operator,

$$(|\lambda_{bL}|^2 \bar{b}_L \gamma^\mu b_L + |\lambda_{bR}|^2 \bar{b}_R \gamma^\mu b_R) \; \frac{\varphi^\dagger i \overleftrightarrow{\partial}_\mu \varphi}{2 m_{b_M}^2}, \quad (24)$$

in which we retain the (C-odd) $\bar{b}\gamma^\mu b$ contribution

$$\frac{|\lambda_{bL}|^2 + |\lambda_{bR}|^2}{2} \; \frac{1}{2} \bar{b}\gamma^\mu b \; \frac{\varphi^\dagger i \overleftrightarrow{\partial}_\mu \varphi}{m_{b_M}^2}. \quad (25)$$

Comparing with $\Upsilon \to e^+ e^-$, given by

$$\frac{e^2}{3} \bar{b} \gamma^\mu b \; \frac{\bar{e} \gamma_\mu e}{m_\Upsilon^2}, \quad (26)$$

we get

$$\frac{B(\Upsilon \to \varphi \bar{\varphi})}{B(\Upsilon \to e^+ e^-)} \simeq \left(\frac{|\lambda_{bL}|^2 + |\lambda_{bR}|^2}{2} \; \frac{3}{2e^2} \right)^2 \frac{m_\Upsilon^4}{m_{b_M}^4} \frac{1}{4} \beta^3$$

$$\simeq 0.536 \times 10^{-2} \left(\frac{|\lambda_{bL}|^2 + |\lambda_{bR}|^2}{2} \right)^2$$

$$\times \frac{(100 \text{ GeV})^4}{m_{b_M}^4} \beta^3. \quad (27)$$

$\frac{1}{4} \beta^3$ is associated with the pair production of spin-0 particles, as compared to massless spin-$\frac{1}{2}$ Dirac ones, with $\beta = \frac{v_\varphi}{c} = (1 - \frac{4m_\varphi^2}{m_\Upsilon^2})^{1/2}$.

With leptonic branching fractions of Υ of about $(2.38 \pm 0.11)\%$, $(2.48 \pm 0.05)\%$ and $(2.60 \pm 0.10)\%$ for $e^+ e^-$, $\mu^+ \mu^-$ and $\tau^+ \tau^-$, respectively, and choosing to use the value $\simeq 2.48\%$ to evaluate the invisible branching fraction (27), we get

$$B(\Upsilon \to \varphi \bar{\varphi}) \simeq 1.33 \times 10^{-4} \left(\frac{|\lambda_{bL}|^2 + |\lambda_{bR}|^2}{2} \right)^2$$

$$\times \frac{(100 \text{ GeV})^4}{m_{b_M}^4} \beta^3. \quad (28)$$

To get an upper bound on the Yukawa couplings, however, we may use $\simeq 2.38\%$ but as a lower bound on the branching fraction into $e^+ e^-$, the 1.33 in (28) getting replaced by about 1.275. This leads to

$$\sqrt{\frac{|\lambda_{bL}|^2 + |\lambda_{bR}|^2}{2}} < 9.4 [\lim(B_{\text{inv}}(\Upsilon))]^{1/4}$$

$$\times \frac{m_{b_M}}{100 \text{ GeV}} \left(1 - \frac{4m_\varphi^2}{m_\Upsilon^2} \right)^{-3/8}, \quad (29)$$

i.e., for $B_{\text{inv}}(\Upsilon) < 3 \times 10^{-4}$ [1],

$$\sqrt{\frac{|\lambda_{bL}|^2 + |\lambda_{bR}|^2}{2}} < \underbrace{1.24 \left(1 - \frac{4m_\varphi^2}{m_\Upsilon^2} \right)^{-3/8}}_{<1.5 \text{ for } m_\varphi < 3 \text{ GeV}} \frac{m_{b_M}}{100 \text{ GeV}}, \quad (30)$$

assuming it makes sense to consider such large values of the Yukawa couplings. This upper bound is in agreement

with the one obtained in [24] from a similar analysis of Υ decays into scalar dark matter particles.

These invisible decays into scalar dark matter induced by heavy-fermion exchanges are likely to remain unaccessible, due to the smaller expected values of the Yukawa couplings. We may then turn to the radiative decays [4]

$$\Upsilon \xrightarrow{b_M} \gamma \underbrace{\varphi \bar{\varphi}}_{\text{invisible}}, \quad (31)$$

which may be induced by the effective dimension-5 operator (22), with a branching fraction $\propto \alpha |\lambda_{bR}^* \lambda_{bL}|^2 m_\Upsilon^2 / m_{b_M}^2$ (to be compared, for example, with a branching fraction into $\gamma \pi^+ \pi^-$ of about 6×10^{-5}). We would need, again, rather large values of the Yukawa couplings for this branching fraction into γ + invisible to be significant, as also discussed in [24].

B. Invisible decays $\psi \to \varphi \bar{\varphi}$

For invisible ψ decays we have, in a similar way,

$$\frac{B(\psi \to \varphi \bar{\varphi})}{B(\psi \to e^+ e^-)} \simeq \left(\frac{|\lambda_{cL}|^2 + |\lambda_{cR}|^2}{2} \; \frac{3}{4e^2} \right)^2 \frac{m_\psi^4}{m_{c_M}^4} \frac{1}{4} \beta^3$$

$$\simeq 1.54 \times 10^{-5} \left(\frac{|\lambda_{cL}|^2 + |\lambda_{cR}|^2}{2} \right)^2$$

$$\times \frac{(100 \text{ GeV})^4}{m_{c_M}^4} \beta^3. \quad (32)$$

With $B(\psi \to e^+ e^-) \simeq 5.94\%$ we obtain

$$B(\psi \to \varphi \bar{\varphi}) \simeq 0.91 \times 10^{-6} \left(\frac{|\lambda_{cL}|^2 + |\lambda_{cR}|^2}{2} \right)^2$$

$$\times \frac{(100 \text{ GeV})^4}{m_{c_M}^4} \beta^3, \quad (33)$$

leading to

$$\sqrt{\frac{|\lambda_{cL}|^2 + |\lambda_{cR}|^2}{2}} < 32 [\lim(B_{\text{inv}}(\psi))]^{1/4}$$

$$\times \left(1 - \frac{4m_\varphi^2}{m_\psi^2} \right)^{-3/8} \frac{m_{c_M}}{100 \text{ GeV}}. \quad (34)$$

From the experimental limit 7.2×10^{-4} [10] we get

$$\sqrt{\frac{|\lambda_{cL}|^2 + |\lambda_{cR}|^2}{2}} < 5.3 \left(1 - \frac{4m_\varphi^2}{m_\Upsilon^2} \right)^{-3/8} \frac{m_{c_M}}{100 \text{ GeV}}. \quad (35)$$

This mainly indicates that quite large Yukawa couplings would be required to get from heavy-fermion exchanges a significant branching fraction of ψ into scalar dark matter.
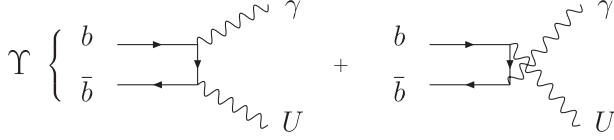
FIG. 5. The decay $\Upsilon \rightarrow \gamma U$ [5,21] induced by the axial coupling f_{bA} of the b quark.

IV. U-INDUCED DECAYS OF ψ AND Υ

A. The U as a mediator of a new interaction

We now turn again to the situation, that we consider more promising, of production and annihilation reactions induced by the light spin-1 U boson, as represented later in Figs. 6 and 7.

Meanwhile we recall that the U may be directly produced in radiative ψ and Υ decays, $\psi \rightarrow \gamma U$ and $\Upsilon \rightarrow \gamma U$, through its *axial* couplings to quarks f_{cA} and f_{bA}, as shown in Fig. 5. The corresponding rates may be sufficiently small, a light U behaving very much as an equivalent pseudoscalar, linear combination of doublet (interacting) and singlet (essentially "inert") components [5,25]. Upper limits on the production of a monochromatic photon + invisible particles (or a $\mu^+\mu^-$ pair, etc.), lead to strong upper limits on these axial couplings [13,21], e.g., from the radiative production of invisibly decaying light U bosons,

$$|f_{bA}| < 4 \times 10^{-7} m_U \ (\text{MeV})/\sqrt{B_{\text{inv}}(U)},$$
$$|f_{cA}| < 1.5 \times 10^{-6} m_U \ (\text{MeV})/\sqrt{B_{\text{inv}}(U)}. \tag{36}$$

The *vector* couplings f_{cV} and f_{bV}, on the other hand, may induce invisible ψ and Υ decays into light dark matter particles (Fig. 6), possibly at a significant rate, as we now discuss.

B. Invisible branching fractions

If we take aside the possibility of large Yukawa couplings of scalar dark matter to the b and c quarks, discussed in Sec. III, invisible Υ and ψ decays only give significant new information on possible decays into light dark matter particles induced by a new (light) spin-1 U boson with *vector* couplings to quarks, as illustrated in Fig. 6 [3,4,11,12]. Decays into $\nu\bar{\nu}$ and $\chi\chi$ are induced by the vector parts in the Z and U currents, involving the same quark current $\bar{b}\gamma^\mu b$, or $\bar{c}\gamma^\mu c$. The Υ branching fraction into light dark matter particles is given [4], for a spin-$\frac{1}{2}$ Majorana χ with an axial coupling $c_\chi/2$ to the U boson (corresponding for a nearly massless χ to a coupling c_χ to χ_L), by

$$\frac{B(\Upsilon \rightarrow \chi\chi)}{B(\Upsilon \rightarrow e^+e^-)} \simeq \frac{1}{2}\beta^3 \left(\frac{c_\chi f_{bV}}{e^2/3}\right)^2 \frac{1}{(1 - \frac{m_U^2}{m_\Upsilon^2})^2}. \tag{37}$$

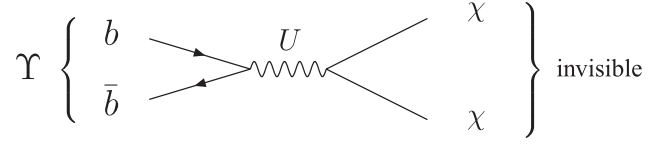FIG. 6. Υ decay into light dark matter particles, induced by a U boson [5] with vector couplings to b [3,4]. This applies to spin-0 or spin-$\frac{1}{2}$ Majorana or Dirac particles [11,12], $\Upsilon \rightarrow \varphi\bar{\varphi}$, $\chi\chi$, or $\chi\bar{\chi}$. The amplitude is $\propto c_\chi f_{bV}/(m_\Upsilon^2 - m_U^2)$, compared to $e^2/(3m_\Upsilon^2)$ for $\Upsilon \rightarrow e^+e^-$.

With $\beta = \frac{v_\chi}{c} = (1 - \frac{4m_\chi^2}{m_\Upsilon^2})^{1/2}$, and using as before $B(\Upsilon \rightarrow e^+e^-) \simeq 2.48\%$, we get in the Majorana case

$$B(\Upsilon \rightarrow \chi\chi) \simeq 13.3(c_\chi f_{bV})^2 \frac{(1 - \frac{4m_\chi^2}{m_\Upsilon^2})^{3/2}}{(1 - \frac{m_U^2}{m_\Upsilon^2})^2}, \tag{38}$$

i.e.,

$$B(\Upsilon \rightarrow \chi\chi) \simeq 13.3(c_\chi f_{bV})^2[\beta^3] \tag{39}$$

for a light U compared to m_Υ. This allows in principle for relatively large (or on the other hand very small) values of the invisible branching fraction into LDM particles. This is also compatible with an appropriate value of the annihilation cross section of these light dark matter particles [12], as discussed more in Secs. V and VI. We get, for m_U and $2m_\chi$ small compared to m_Υ,

$$B(\Upsilon \rightarrow \chi\chi) \simeq 1.33 \times 10^{-3} \left(\frac{c_\chi f_{cV}}{10^{-2}}\right)^2, \tag{40}$$

$|c_\chi f_{bV}| \simeq 10^{-2}$, in particular, being now excluded by (3).

For invisible ψ decays we get

$$\frac{B(\psi \rightarrow \chi\chi)}{B(\psi \rightarrow e^+e^-)} \simeq \frac{1}{2}\beta^3 \left(\frac{c_\chi f_{bV}}{2e^2/3}\right)^2 \frac{1}{(1 - \frac{m_U^2}{m_\psi^2})^2}, \tag{41}$$

leading for a light U, with a Υ branching fraction into e^+e^- of $(5.94 \pm 0.06)\%$, to

$$B(\psi \rightarrow \chi\chi) \simeq 8(c_\chi f_{bV})^2[\beta]^3, \tag{42}$$

i.e.,

$$B(\psi \rightarrow \chi\chi) \simeq 8 \times 10^{-4} \left(\frac{c_\chi f_{cV}}{10^{-2}}\right)^2, \tag{43}$$

for m_U and $2m_\chi$ small compared to m_ψ.

This also applies to the pair-production of non-self-conjugate spin-0 or spin-$\frac{1}{2}$ particles, with the replacements in (37) [4]

$$\frac{1}{2}\beta^3_{\text{Majorana}} \rightarrow \begin{cases} \frac{1}{4}\beta^3: & \text{spin-0,} \\ \beta^3: & \text{Dirac, axial,} \\ \frac{3}{2}\beta - \frac{1}{2}\beta^3: & \text{Dirac, vector,} \end{cases} \tag{44}$$

so that (40) and (43) are replaced, for light spin-0 particles,

by

$$B(\Upsilon \to \varphi\bar\varphi) \simeq 6.6 \times 10^{-4}\left(\frac{c_\chi f_{bV}}{10^{-2}}\right)^2,$$

$$B(\psi \to \varphi\bar\varphi) \simeq 4 \times 10^{-4}\left(\frac{c_\chi f_{bV}}{10^{-2}}\right)^2, \tag{45}$$

and 4 times as much, for light spin-$\frac{1}{2}$ Dirac particles.

C. Limits on U couplings

The Υ branching fraction (38) (but now estimated with an electronic branching fraction of 2.38% to get an upper limit, which leads to replace 13.3 by 12.7) leads to

$$|c_\chi f_{bV}| < 0.28\sqrt{\lim(B_{\text{inv}})}\left|1 - \frac{m_U^2}{m_\Upsilon^2}\right|\left(1 - \frac{4m_\chi^2}{m_\Upsilon^2}\right)^{-3/4}, \tag{46}$$

which simplifies into

$$|c_\chi f_{bV}| < \frac{e^2\sqrt{2}}{3}\left(\frac{\lim(B_{\text{inv}})}{B_{ee}}\right)^{1/2} \simeq 0.28\sqrt{\lim(B_{\text{inv}})} \tag{47}$$

for χ and U light compared to Υ. From the *BABAR* limit 3×10^{-4} [1] we get

$$|c_\chi f_{bV}| < 5 \times 10^{-3} \quad (\text{for } m_U, 2m_\chi \lesssim 2 \text{ GeV}), \tag{48}$$

still approximately valid for $2m_\chi$ and m_U smaller than $\simeq m_\Upsilon/2$ (and which may even be used up to $m_U \simeq 13$ GeV although it overestimates the limit for m_U in the vicinity of m_Υ). As in the very light gravitino case this bound is improved by $\sqrt{5 \times 10^{-2}/(3 \times 10^{-4})} \simeq 13$, as compared to $|c_\chi f_{bV}| \lesssim 6 \times 10^{-2}$ [4] derived from the 5% CLEO limit on B_{inv}.

It may be compared with the similar limit for the vector coupling of c,

$$|c_\chi f_{cV}| < 0.95 \times 10^{-2}, \tag{49}$$

still approximately valid for m_U and $2m_\chi$ smaller than $\simeq m_\psi/2$, deduced [cf. (43)] from the recent BES II limit on invisible ψ decays, $B(\psi \to \chi\chi) < 7.2 \times 10^{-4}$ [4,10], which should be improved soon at BES III [26].

With the kinematic factors reestablished as in (46), these limits read, in the Majorana case,

$$|c_\chi f_{bV}| < 5 \times 10^{-3}\left|1 - \frac{m_U^2}{m_\Upsilon^2}\right|\left(1 - \frac{4m_\chi^2}{m_\Upsilon^2}\right)^{-3/4}, \tag{50}$$

and similarly for (49). They disappear if m_χ approaches $m_\Upsilon/2$ (or $m_\psi/2$), and get stronger if m_U approaches m_Υ (or m_ψ) as the cross sections get enhanced from the U propagator effect.

The limits (48) and (49), multiplied or divided by $\sqrt{2}$ owing to (44), apply to non-self-conjugate spin-0 or $\frac{1}{2}$ dark matter particles:

$$|c_\varphi f_{bV}| < 7 \times 10^{-3}: \text{spin-0},$$
$$|c_\chi f_{bV}| < 3.5 \times 10^{-3}: \text{spin-}\tfrac{1}{2} \text{ Dirac};$$
$$|c_\varphi f_{cV}| < 1.4 \times 10^{-2}: \text{spin-0},$$
$$|c_\chi f_{cV}| < 7 \times 10^{-3}: \text{spin-}\tfrac{1}{2} \text{ Dirac}. \tag{51}$$

The effects of m_U, m_χ are taken into account by reintroducing $|1 - \frac{m_U^2}{m_\Upsilon^2}|\beta^{-3/2}$ as in (50), or $|1 - \frac{m_U^2}{m_\Upsilon^2}|(\frac{3}{2}\beta - \frac{1}{2}\beta^3)^{-1/2}$ for a vectorially-coupled Dirac particle.

V. COMPATIBILITY WITH RELIC ABUNDANCE REQUIREMENTS

How do the new limits (48)–(51) on $|c_\chi f_{bV}|$, $|c_\chi f_{cV}|$, etc., compare with the requirement that LDM particles must have sufficiently large annihilation cross sections at freeze-out time, for the correct relic abundance? The cross section for (P-wave) annihilation of Majorana particles, $\chi\chi \to e^+e^-$, may be written as [12]

$$\sigma_{\text{ann}}v_{\text{rel}} \simeq \frac{v_\chi^2}{0.16}\left(\frac{c_\chi f_e}{10^{-6}}\right)^2\left(\frac{m_\chi \times 1.8 \text{ MeV}}{m_U^2 - 4m_\chi^2}\right)^2(4 \text{ pb}) \tag{52}$$

disregarding m_e for simplicity. $c_\chi/2$ is the axial U coupling to χ, f_{eV} and f_{eA} the vector and axial couplings to the electron, with $f_e^2 = f_{eV}^2 + f_{eA}^2$. Similar expressions apply to the P-wave annihilations of non-self-conjugate spin-0 particles, or spin-$\frac{1}{2}$ Dirac ones axially coupled to U. The annihilation cross section of spin-$\frac{1}{2}$ Dirac particles with a vector coupling to U, however, can proceed through S-wave and no longer involves the v_χ^2 factor.

The thermally-averaged total annihilation cross section at freeze-out time should be $\langle\sigma_{\text{ann}}v_{\text{rel}}(\chi\chi \to f\bar f)\rangle \approx$ 4–5 pb, for a Majorana χ lighter than ≈ 1 GeV with P-wave annihilation. It corresponds to a cross section into $e^+e^- \approx$ 4–5 pb times the annihilation branching fraction B_{ann}^{ee}. This requires [12,27]:

$$|c_\chi f_e| \simeq (B_{\text{ann}}^{ee})^{1/2}10^{-3}\frac{|m_U^2 - 4m_\chi^2|}{m_\chi(1.8 \text{ GeV})}. \tag{53}$$

This is compatible with the constraint (48) and (50) from invisible Υ decays, for similar values of f_{bV} and f_e, provided m_U is not too large compared to $\sqrt{2m_\chi \times 1 \text{ GeV}}$. This also shows that a light U is espe-
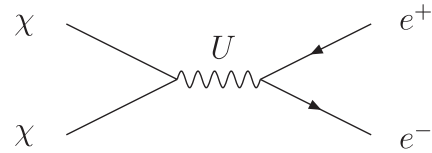

FIG. 7. Dark matter annihilation into e^+e^- induced by the U boson [11–13]. This process should be stronger than for weak interactions, for a correct relic density of light dark matter.

cially required for the smaller values of m_χ, down to MeV scale.

For example with $m_U = 10$ MeV and $m_\chi \simeq 4$ (or 6) MeV as considered in [12,13], (53) would give

$$|c_\chi f_e| \approx 3 \times 10^{-6}, \tag{54}$$

depending on $B_{\rm ann}^{ee}$ (here taken $\approx 40\%$). For a heavier U we may get larger couplings, e.g.,

$$\text{up to } |c_\chi f_e| \simeq \frac{10^{-2}}{2m_\chi \, (\text{MeV})} \quad \text{for } m_U = 100 \text{ MeV} \tag{55}$$

(with $B_{\rm ann}^{ee} \simeq 1$). These values [13] are generally smaller than the new 5×10^{-3} upper limit (48) on $|c_\chi f_{bV}|$.

VI. ESTIMATING ψ AND Υ INVISIBLE DECAYS FROM DARK MATTER ANNIHILATION CROSS SECTIONS ?

What about attempting to estimate the Υ invisible branching fraction into dark matter particles in a model-independent way from the dark matter annihilation cross section, with approximate expectations predicted $\simeq 0.41\%$ or 0.6%, or $\sim 1.8 \times 10^{-3}$ for P-wave annihilation [8,9,28,29]? As we saw in the previous sections, one cannot establish such predictions without taking into account, more specifically, essential features of these processes. In fact we are dealing, not simply with $b\bar{b} \to \chi\chi$ and the inverse reaction $\chi\chi \to b\bar{b}$, but with *different reactions*, and *at different energies*:

$$b\bar{b} \to \chi\chi \qquad \text{at } \sqrt{s} = m_\Upsilon,$$
$$\chi\chi \to l\bar{l}, \quad q\bar{q}(q \neq b) \quad \text{at a lower } \sqrt{s} \simeq 2m_\chi. \tag{56}$$

Furthermore a U axially coupled to quarks would not contribute to invisible ψ and Υ decays,

$$B(\psi \to \chi\chi) = B(\Upsilon \to \chi\chi) = 0, \tag{57}$$

while still inducing the desired dark matter annihilations into lighter quarks or leptons, in the early Universe. Direct spin-0 exchanges also do not contribute to these invisible ψ and Υ decays. For spin-0 LDM particles φ interacting nonchirally though heavy-fermion exchanges, annihilation cross sections are $\propto 1/m_{b_M}^2$ [11,12] but $B(\Upsilon \to \varphi\bar{\varphi})$ vanishes at this order [4]. This branching fraction, $\propto (|\lambda_{bL}|^2 + |\lambda_{bR}|^2)^2/m_{b_M}^4$ as we saw in Sec. III, cannot be expressed proportionally to annihilation cross sections for $\chi\chi \to f\bar{f}$, $\propto |\lambda_{fR}^* \lambda_{fL}|^2/m_{b_M}^2 \ldots$

It is thus essential to take into account the nature of the underlying process responsible for invisible decays and LDM annihilations, U exchanges or possible heavy-fermion exchanges, …, and treat correctly the fact that they occur *at different energies*, with in general *energy-dependent cross sections*.

Let us now concentrate on U-induced reactions. Relations between $B_{\rm inv}(\Upsilon)$ (or ψ) and annihilation cross

sections are implicit from the comparison between the upper limit (50) on $|c_\chi f_{bV}|$ from $B_{\rm inv}(\Upsilon)$, and the value (53) of $|c_\chi f_e|$ required for the correct relic density. Taking $|c_\chi f_{bV}| \approx |c_\chi f_e|$ as seems natural in the absence of more specific informations on the nature of the couplings, we can write the invisible branching fraction (38) proportionally to the annihilation cross section at freeze-out time,

$$B(\Upsilon \to \chi\chi) \propto (c_\chi f_{bV})^2 \left(\frac{m_\Upsilon^2}{m_\Upsilon{}^2 - m_U^2} \right)^2$$

$$\propto \frac{m_\Upsilon{}^2}{4m_\chi^2} \left(\frac{4m_\chi^2 - m_U^2}{m_\Upsilon{}^2 - m_U^2} \right)^2 \ldots \langle \sigma v \rangle_{FO}$$

$$\propto \begin{cases} \dfrac{m_\Upsilon{}^2}{4m_\chi^2}(\ldots) & \text{for large } m_U \gtrsim 2m_\Upsilon, \\[2mm] \dfrac{m_U^4}{4m_\chi^2 m_\Upsilon{}^2}(\ldots) & \text{for } 4m_\chi \lesssim m_U \lesssim m_\Upsilon/2, \end{cases} \tag{58}$$

where the … include the extra factor $(f_{bV}/f_e)^2 B_{\rm ann}^{ee}$.

The factor $m_\Upsilon^2/(4m_\chi^2)$ for a heavy U originates from production and annihilation cross sections growing with energy $\propto s$, from $\sqrt{s} \simeq 2m_\chi$ to m_Υ, just as for ordinary weak interactions. A stronger-than-weak annihilation cross section at $2m_\chi$, into e^+e^- for example, corresponds to an effective Fermi-like coupling G' larger than G_F, that would then lead to stronger-than-weak processes at higher energies, e.g., in $e^+e^- \to \gamma\chi\chi$, which have not been observed [3]. Additional constraints are obtained from lepton anomalous magnetic moments, neutrino scatterings, parity-violation effects in atomic physics, … [13].

The amplitudes corresponding to the exchanges of a *light U*, on the other hand, change behavior and start decreasing at energies (or momentum transfer) larger than $\approx m_U$. Above this value the Fermi-like couplings G', relevant in the local limit approximation corresponding to a heavy U, have to be replaced by

$$G' \to G' \frac{m_U^2}{m_U^2 - q^2} \simeq G' \frac{m_U^2}{-q^2} \ll G' \text{ for a light } U. \tag{59}$$

This mechanism was proposed to make the neutral current effects associated with the exchanges of a new neutral gauge boson sufficiently small, when this one is light compared to the energies (or momentum transfer) considered [5].

For a U somewhat lighter than Υ, the Υ invisible fraction gets indeed inhibited as shown by (58) and (59), so that

$$B(\Upsilon \to \chi\chi) \propto (c_\chi f_{bV})^2 \approx (c_\chi f_e)^2 \ldots$$

$$\propto \frac{(m_U^2 - 4m_\chi^2)^2}{4m_\chi^2 m_\Upsilon^2} \ldots, \tag{60}$$

where the last … includes the factor $(f_{bV}/f_e)^2 B_{\rm ann}^{ee}$. This reflects again that U-induced cross sections grow like s from $\simeq 4m_\chi^2$ but now only up to $\approx m_U^2$, then decrease like

$1/s$ between m_U^2 and $m_Υ^2$, leading to a smaller result for the invisible branching fraction. The lighter the U (within the limits indicated as compared to $2m_χ$), the easier it is to satisfy strong bounds from invisible Υ and $ψ$ decays, in particular.

This illustrates that there is no general way to predict the invisible $Υ → χχ$ branching fraction simply from the annihilation cross section of light dark matter particles at freeze-out time.

VII. NONPERTURBATIVE EFFECTS

These results could be affected by nonperturbative exchanges of light U bosons, although this is not the situation we generally have in mind, as we would prefer the theory to remain perturbative [13]. For relatively large values of the U coupling to dark matter particles (naively for $α_χ = c_χ^2/4π ≳ 1$, or similarly for $α_φ$), potentially large nonperturbative effects may have to be considered. One could get an enhancement of the $Υ → χχ$ branching fraction from U-mediated $χχ$ interactions in the final state, especially if $2m_χ$ or $2m_φ$ is relatively close to $m_Υ$, or one should consider U-radiative decays like $Υ → χχU$ (cf. Figs. 8 and 9).

For a spin-$\frac{1}{2}$ dark matter particle one has to take into account that a U lighter than $χ$ could effectively behave as an almost equivalent pseudoscalar a, with an effective coupling to $χ$

$$c_{χp} = c_χ \frac{2m_χ}{m_U}, \qquad (61)$$

leading to an enhanced effective coupling $α_{χp} = α_χ$ $(4m_χ^2/m_U^2)$ [5,21,30]. The relative correction to the $Υ → χχ$ amplitude (Fig. 8) or relative branching ratio into $Uχχ$ (Fig. 9) are then proportional to

$$\frac{1}{π}α_{χp} = \frac{1}{π}α_χ \frac{4m_χ^2}{m_U^2} = \frac{c_χ^2}{π^2} \frac{m_χ^2}{m_U^2}, \qquad (62)$$

which tells us if we are in the nonperturbative regime or not. But $m_U < m_χ$, or $c_χ m_χ$, does not correspond to the situations we usually have in mind [13], and $m_U < m_χ$ would also lead to significant dark matter annihilations into UU rather than e^+e^-, which may be disfavored [31].

Nonperturbative effects could also affect annihilation cross sections, possibly leading to a Sommerfeld-type factor [32] enhancing annihilation cross sections of low velocity dark matter particles. Exchanges of light U's between spin-$\frac{1}{2}$ dark matter particles would lead in the
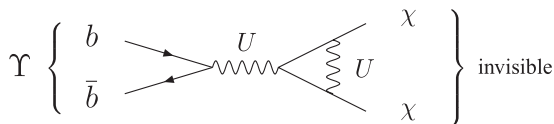

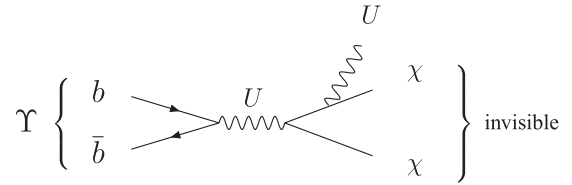FIG. 8. Radiative correction to $Υ → χχ$, from U-mediated $χχ$ interactions in the final state.

FIG. 9. Radiative correction to $Υ → χχ$, with emission of a light U in the final state.

low-mass limit, in place of the usual Coulomb or Yukawa interactions, to a long-ranged spin-spin interaction

$$c_χ^2 \frac{\vec{σ}_1 \cdot \vec{σ}_2 - 3\,\vec{σ}_1 \cdot \hat{r}\,\vec{σ}_2 \cdot \hat{r}}{4πm_U^2 r^3}, \qquad (63)$$

as for ordinary fermions with axial couplings to the U [25,33]. The potential is the same as for the exchange of a quasimassless pseudoscalar a with an effective coupling given by (61) so that $c_χ^2/(4πm_U^2) = c_{χp}^2/(16πm_χ^2)$. But such effects are not expected to be essential at freeze-out time, at least for moderate values of the coupling, as light dark matter velocities, $v ≈ 0.4c$, are not small.

Should such an enhancement factor be present for the annihilation cross section, (53) would be turned into an inequality, $|c_χ f_e| < \ldots$, and similarly for the expression of $B(Υ → χχ)$ as written in (60). The discussion further illustrates that there is no general way to predict the invisible $Υ → χχ$ branching fraction simply from the annihilation cross section of light dark matter at freeze-out time.

VIII. CONCLUSION

Searching for invisible $ψ$ and Υ decays has long been identified and used as a way to search for new light particles, especially if they are more strongly coupled to standard model ones than through weak interactions. Invisible branching fractions into neutrinos in the SM and neutralinos in the (N/n)MSSM, however, are well below present experimental limits, even if light spin-0 particles are directly coupled to both quarks and neutralinos.

Exchanges of a new heavy quark b_M or c_M also lead to small invisible branching fractions into scalar dark matter, unless the b_M or c_M quarks are only moderately heavy, and the corresponding Yukawa couplings to scalar dark matter rather large.

Exchanges of a light spin-1 U boson associated with a new gauge interaction, with a vector coupling to quarks, may induce invisible decays of $ψ$ and Υ into light dark matter particles at a significant rate, or, conversely, at a very small rate, with U exchanges responsible for sufficient dark matter annihilations in the early Universe.

The new limit on invisible Υ decays constrains the U couplings to dark matter and b quarks to satisfy $|c_χ f_{bV}| < 5 × 10^{-3}$, for $2m_χ$ smaller than a few GeV's. This may be compared with $|c_χ f_{cV}| < 9.5 × 10^{-3}$ from BES II (ex-

pected to get significantly improved at BES III). Small values of m_χ of a few MeV's, and m_U of less than a few hundred MeV's, in fact, may well be preferred.

The radiative decays ψ or $\Upsilon \to \gamma \chi \chi$, or $\gamma \varphi \bar{\varphi}$, may be induced by the axial couplings of the U to the c and b quarks, with amplitudes proportional to $c_\chi f_{cA} e$ or $c_\chi f_{bA} e$. If they can be sufficiently well constrained although the emitted photon is not monochromatic, they may lead to interesting limits on $c_\chi f_{cA}$ or $c_\chi f_{bA}$, especially for larger

c_χ, as f_{cA} and f_{bA} are already constrained from ψ or $\Upsilon \to \gamma U$. These decays may also be sensitive to heavy-quark exchanges, for scalar dark matter.

Even if standard decay modes into neutrinos are not reachable yet, searches for invisible meson decays give useful information on the U boson and its couplings, contributing to shed light on the nature of dark matter and on the dark force through which it interacts with ordinary particles.

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