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Coupled channel approach to the structure of the $X(3872)$

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We have performed a coupled channel calculation of the 1^{++} $c\bar{c}$ sector including $q\bar{q}$ and DD^* molecular configurations. The calculation was done within a constituent quark model which successfully describes the meson spectrum, in particular, the $c\bar{c}$ 1^{--} sector. Two and four-quark configurations are coupled using the $^3P_0$ model. The elusive $X(3872)$ meson appears as a new state with a high probability for the DD^* molecular component. When the mass difference between neutral and charged states is included, a large D^0D^{*0} component is found which dominates for large distances and breaks isospin symmetry in the physical state. The original $c\bar{c}(2^3P_1)$ state acquires a sizable DD^* component and can be identified with the $X(3940)$. We study the $B \to K\pi^+\pi^-J/\psi$ and $B \to KD^0D^{*0}$ decays, finding a good agreement with Belle and *BABAR* experimental data.

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I. INTRODUCTION

In the last years, a number of exciting discoveries of new hadron states have challenged our description of the hadron spectroscopy. One of the most mysterious states is the well established $X(3872)$. It was first discovered by the Belle Collaboration in the $J/\psi\pi\pi$ invariant mass spectrum of the decay $B^+ \to K^+\pi^+\pi^-J/\psi$ [1]. Its existence was soon confirmed by the *BABAR* [2], CDF [3], and D0 [4] Collaborations. The world average mass is $M_X = 3871.2 \pm 0.5$ MeV and its width $\Gamma_X < 2.3$ MeV. The measurements of the $X(3872) \to \gamma J/\psi$ decay [5,6] imply an even C parity. Moreover, angular correlation between final state particles in the $X(3872) \to \pi^+\pi^-J/\psi$ decay measured by Belle [5] suggests that the $J^{PC} = 0^{++}$ and $J^{PC} = 0^{+-}$ may be ruled out and strongly favors the $J^{PC} = 1^{++}$ quantum numbers, although the 2^{++} combination cannot be excluded. A later analysis by the CDF Collaboration [7] of the same decay is compatible with the Belle results and concludes from the dipion mass spectrum that the most likely quantum numbers should be $J^{PC} = 1^{++}$ but cannot totally exclude the $J^{PC} = 2^{-+}$ combination. These conclusions were confirmed by a new CDF analysis of the decay $X(3872) \to \pi^+\pi^-J/\psi$ followed by $J/\psi \to \mu^+\mu^-$ excluding all the other possible quantum numbers at 99.7% confidence level [8]. However, the small phase space available for the decay $X(3872) \to D^0\bar{D}^0\pi^0$ observed by Belle [9] discards the $J = 2$, leaving the 1^{++} assignment as the most probable option.

In the 1^{++} sector, the only well established state in the Particle Data Group (PDG) [10] is the $\chi_{c_1}(1P)$ with a mass $M = 3510.66 \pm 0.07$ MeV. The first excitation is expected around 3950 MeV. In this energy region, Belle has reported the observation of three resonant structures denoted by $X(3940)$, $Y(3940)$, and $Z(3930)$. The last one was observed by Belle in the $\gamma\gamma \to D\bar{D}$ reaction [11] and is already included in the PDG as the $\chi_{c_2}(2P)$. The $X(3940)$ has been seen as a peak in the recoiling mass spectrum of J/ψ produced in e^+e^- collision. Its main decay channel is DD^* [12]. The $Y(3940)$ appears as a threshold enhancement in the $J/\psi\omega$ invariant mass distribution of the $B \to J/\psi\omega K$ decay [13].

The relative decay rates outlines a puzzling structure for the $X(3872)$. The $\gamma J/\psi$ and $\gamma\psi'$ decay rates [14]

$$\frac{X(3872) \to \gamma J/\psi}{X(3872) \to \pi^+\pi^-J/\psi} = 0.33 \pm 0.12, \qquad \frac{X(3872) \to \gamma\psi'}{X(3872) \to \pi^+\pi^-J/\psi} = 1.1 \pm 0.4 \tag{1}$$

suggest a $c\bar{c}$ structure; whereas, the $X(3872) \to \pi^+\pi^-\pi^0J/\psi$ decay mode

$$\frac{X(3872) \to \pi^+\pi^-\pi^0J/\psi}{X(3872) \to \pi^+\pi^-J/\psi} = 1.0 \pm 0.4 \pm 0.3 \tag{2}$$

indicates a very different one [15]. The dipion mass spectrum in the $\pi^+\pi^-J/\psi$ channel shows that the pions come from the ρ^0 resonance. On the other hand, the $\pi^+\pi^-\pi^0$ mass spectrum has a strong peak around 750 MeV, suggesting that the process is dominated by a ω meson. Thus, the ratio $R \sim 1$ indicates that there should be an isospin violation incompatible with a traditional charmonium assumption.

Concerning the mass value, in 2006 Belle measured [9] an enhancement in the $D^0D^0\pi^0$ channel just above the D^0D^{*0} threshold using the $B^+ \to K^+D^0D^0\pi^0$ decay. The amazing aspect of this enhancement is that it appears at $M_X = 3875.2 \pm 0.7^{+0.3}_{-1.6} \pm 0.8$ MeV just 3 MeV above the M_X world average mass value. This fact triggered a new discussion about the possibility of two different charmonium-like states. The Belle mass value was confirmed later by the *BABAR* Collaboration [16]. Last year, the Belle Collaboration announced a new measurement of the $B \to KD^0D^0\pi^0$ decay [17] with a lower position of the

$X(3872)$ peak in $M_X = 3872.6^{+0.5}_{-0.4} \pm 0.4$ MeV. New data of the $\pi^+\pi^- J/\Psi$ decay has been also recently reported by the Belle [18], *BABAR* [19], and CDF [20] Collaborations, confirming a mass value in agreement with the world average.

The $X(3872)$ mass is difficult to reproduce by the standard quark models (see Ref. [21] for a review). The state appears to be too heavy for a $1D$ charmonium state and too light for a $2P$ charmonium one. Moreover, no four-quark bound state configurations have been found in this mass region which rules out the possibility that this particle was a compact tetraquark system [22,23].

An important property of the $X(3872)$ is that its mass is extremely close to the D^0D^{*0} threshold with a difference using PDG values given by -0.6 ± 0.6 MeV. The proximity of the D^0D^{*0} threshold made the $X(3872)$ a natural candidate to a $C = +$ D^0D^{*0} molecule. The hypothesis of a DD^* molecule mainly bound by pion exchange has been suggested by several authors [24]. In particular, in Ref. [25] it is argued that the $X(3872)$ is a $J^{PC} = 1^{++}$ D^0D^{*0} molecule stabilized by admixture of $\rho J/\psi$ and $\omega J/\psi$ states. The author shows that pion exchange alone can not bind the molecule being the combined effect of pion exchange and coupled channels responsible for that. The D^0D^{*0} component dominates the wave function at the experimental binding, all other contributions becoming small.

The molecular interpretation runs into trouble when it tries to explain the high $\gamma\psi'$ decay rate. For a molecular state, this can only proceed through annihilation diagrams and hence is very small.

This puzzling situation suggests for the $X(3872)$ state a combination of a $2P$ $c\bar{c}$ state and a weakly-bound D^0D^{*0} molecule [13,14]. The experimental assignment $J^{PC} = 1^{++}$ favors this conclusion, because it allows the molecule to be in a relative S-wave state; whereas, the corresponding $c\bar{c}$ should be in a relative P-wave state. Then, the masses of the additional light quarks are compensated by the angular momentum excitation, and both configurations may be almost in the same mass region. Similar behavior has been already observed in the open charmed sector [26]. Recently, Zhang *et al.* [27] have analyzed, using the coupled channel Flatté formula, the $B \to KD^0D^0\pi^0$ [17], and $B \to K\pi^+\pi^- J/\Psi$ [18] Belle data. They found that a third sheet pole close, but below, the D^0D^{*0} threshold is needed to describe the data, which supports the idea of the $X(3872)$ as a mixed state of χ'_{c1} and D^0D^{*0} components. An updated Flatté analysis of the same data together with the new *BABAR* data of the same reactions [16,19] has been performed in Ref. [28] assuming a mechanism for the $X(3872)$ production via the charmonium components. The authors conclude that the data clearly indicates a sizable $c\bar{c}$ 2^3P_1 component in the $X(3872)$ wave function. Finally, Dong *et al.* [29] show in their analysis of the $J/\psi\gamma$ and $\psi(2S)\gamma$ decay modes of the $X(3872)$ that the large value of the ratio $\mathrm{BR}(X(3872) \to J/\psi\gamma)/\mathrm{BR}(X(3872) \to \psi(2S)\gamma)$ measured by the *BABAR* Collaboration provides a constraint on the value of the $c\bar{c}$ component in the $X(3872)$. From the experimental values, they deduce a small admixture of the $c\bar{c}$ component.

Having in mind these evidences, in this paper we perform a microscopic coupled channel calculation of the 1^{++} sector including both $c\bar{c}$ and DD^* states. The calculation is done in the framework of a constituent quark model widely used in hadronic spectroscopy. The paper is organized as follows. In the next section, we review the main ingredients of our model. Section III is devoted to discuss the numerical procedures and the results. Finally, we summarize the main findings of our work in the last section.

II. THE MODEL

A. The constituent quark model

The constituent quark model used in this work has been extensively described elsewhere [30,31], and therefore we will only summarize here its most relevant aspects. The model is based on the assumption that the light constituent quark mass appears as a consequence of the spontaneous breaking of the chiral symmetry at some momentum scale. As a consequence, the quark propagator gets modified and quarks acquire a dynamical momentum dependent mass. The simplest Lagrangian must therefore contain chiral fields to compensate the mass term and can be expressed as [32]

$$\mathcal{L} = \bar{\psi}(i\not{\partial} - M(q^2)U^{\gamma_5})\psi, \tag{3}$$

where $U^{\gamma_5} = \exp(i\pi^a\lambda^a\gamma_5/f_\pi)$, π^a denotes nine pseudoscalar fields $(\eta_0, \vec{\pi}, K_i, \eta_8)$ with $i = 1, \ldots, 4$, and $M(q^2)$ is the constituent mass. This constituent quark mass, which vanishes at large momenta and is frozen at low momenta at a value around 300 MeV, can be explicitly obtained from the theory, but its theoretical behavior can be simulated by parametrizing $M(q^2) = m_q F(q^2)$ where $m_q \simeq 300$ MeV, and

$$F(q^2) = \left[\frac{\Lambda^2}{\Lambda^2 + q^2}\right]^{1/2}. \tag{4}$$

The cutoff Λ fixes the chiral symmetry breaking scale.

The Goldstone boson field matrix U^{γ_5} can be expanded in terms of boson fields,

$$U^{\gamma_5} = 1 + \frac{i}{f_\pi}\gamma^5\lambda^a\pi^a - \frac{1}{2f_\pi^2}\pi^a\pi^a + \cdots. \tag{5}$$

The first term of the expansion generates the constituent quark mass, while the second gives rise to a one-boson exchange interaction between quarks. The main contribution of the third term comes from the two-pion exchange which has been simulated by means of a scalar exchange potential.

In the heavy quark sector, chiral symmetry is explicitly broken and this type of interaction does not act. However, it constrains the model parameters through the light meson phenomenology and provides a natural way to incorporate the pion exchange interaction in the DD^* dynamics.

Beyond the chiral symmetry breaking scale, one expects the dynamics to be governed by QCD perturbative effects. They are taken into account through the one gluon-exchange interaction [33] derived from the Lagrangian

$$\mathcal{L}_{gqq} = i\sqrt{4\pi\alpha_s}\bar{\psi}\gamma_\mu G_c^\mu \lambda_c \psi, \tag{6}$$

where λ_c are the SU(3) color generators, and G_c^μ is the gluon field.

The other QCD nonperturbative effect corresponds to confinement, which prevents from having colored hadrons. Such a term can be physically interpreted in a picture in which the quark and the antiquark are linked by a one-dimensional color flux tube. The spontaneous creation of light-quark pairs may give rise at the same scale to a breakup of the color flux tube [34]. This can be translated into a screened potential [35] in such a way that the potential saturates at the same interquark distance:

$$V_{\mathrm{CON}}(\vec{r}_{ij}) = \{-a_c(1 - e^{-\mu_c r_{ij}}) + \Delta\}(\vec{\lambda}^c{}_i \cdot \vec{\lambda}^c{}_j), \tag{7}$$

where Δ is a global constant to fit the origin of energies. At short distances, this potential presents a linear behavior with an effective confinement strength $a = -a_c\mu_c(\vec{\lambda}^c{}_i \cdot \vec{\lambda}^c{}_j)$ while it becomes constant at large distances. It has been shown that this form of the potential is important to explain the huge degeneracy observed in the high excited light meson spectrum [36] and turns out to be very important for the correct assignment of $J^{PC} = 1^{--}$ charmonium states [37]. Explicit expressions for all these interactions are given in [37].

All of these ingredients are needed to explain the hadronic phenomenology. Apart from the obvious confinement potential, gluon exchange is demanded from the hyperfine splitting in charmonium. Moreover, pion exchange is one of the best established interactions in nature, since its parameters are constrained by a huge number of experiments. When Goldstone boson exchanges are considered at the quark level together with the one gluon exchange, the possibility of double counting emerges. This problem has been studied in the literature concluding that the pion can be safely exchanged together with the gluon [38].

Constituent quark models are also criticized, because they only incorporate a limited sector of the Fock space. In particular, its applicability to high excited states may be questionable as more thresholds open up. In our case, the parameters of the model have been fixed in the low lying part of the spectrum where these effects are more easily incorporated into them. Furthermore, the main contribution of the open channels are taken into account by the screened confinement potential.

B. The coupled channel approach

To model the 1^{++} $c\bar{c}$ system, we assume that the hadronic state is

$$|\Psi\rangle = \sum_\alpha c_\alpha|\psi_\alpha\rangle + \sum_\beta \chi_\beta(P)|\phi_{M_1}\phi_{M_2}\beta\rangle, \tag{8}$$

where $|\psi_\alpha\rangle$ are $c\bar{c}$ eigenstates of the two body Hamiltonian, ϕ_{M_i} are $c\bar{n}$ ($\bar{c}n$) eigenstates describing the D ($\bar{D}$) mesons, $|\phi_{M_1}\phi_{M_2}\beta\rangle$ is the two meson state with β quantum numbers coupled to total J^{PC} quantum numbers, and $\chi_\beta(P)$ is the relative wave function between the two mesons in the molecule. As we always work with eigenstates of the C-parity operator, we use the usual notation in which DD^* is the right combination of $D\bar{D}^*$ and $D^*\bar{D}$.

The coupling between the two sectors requires the creation of a light-quark pair $n\bar{n}$. Similar to the strong decay process, this coupling should be in principle driven by the same interquark Hamiltonian which determines the spectrum. However, Ackleh *et al.* [39] have shown that the quark pair creation $^3P_0$ model [40] gives similar results to the microscopic calculation. The model assumes that the pair creation Hamiltonian is

$$\mathcal{H} = g\int d^3x\bar{\psi}(x)\psi(x) \tag{9}$$

which, in the nonrelativistic reduction, is equivalent to the transition operator [41]

$$T = -3\sqrt{2}\gamma'\sum_\mu \int d^3p d^3p' \delta^{(3)}(p + p') \times \left[\mathcal{Y}_1\left(\frac{p - p'}{2}\right)b_\mu^\dagger(p)d_\nu^\dagger(p')\right]^{C=1,I=0,S=1,J=0}, \tag{10}$$

where μ ($\nu = \bar{\mu}$) are the quark (antiquark) quantum numbers, and $\gamma' = 2^{5/2}\pi^{1/2}\gamma$ with $\gamma = \frac{g}{2m}$ is a dimensionless constant that gives the strength of the $q\bar{q}$ pair creation from the vacuum. From this operator, we define the transition potential $V_{\beta\alpha}(P)$ within the $^3P_0$ model as [42]

$$\langle\phi_{M_1}\phi_{M_2}\beta|T|\psi_\alpha\rangle = PV_{\beta\alpha}(P)\delta^{(3)}(\vec{P}_{\mathrm{cm}}), \tag{11}$$

where P is the relative momentum of the two meson state.

Using the wave function from Eq. (8) and the coupling Eq. (11), we arrive to the coupled equations

$$\begin{aligned} & M_\alpha c_\alpha + \sum_\beta \int V_{\alpha\beta}(P)\chi_\beta(P)P^2dP = Ec_\alpha, \\ & \sum_\beta \int H_{\beta'\beta}^{M_1M_2}(P', P)\chi_\beta(P)P^2dP + \sum_\alpha V_{\beta'\alpha}(P')c_\alpha \\ & \quad = E\chi_{\beta'}(P'), \end{aligned} \tag{12}$$

where M_α are the masses of the bare $c\bar{c}$ mesons and $H_{\beta'\beta}^{M_1M_2}$ is the resonating group method Hamiltonian for the two meson states obtained from the $q\bar{q}$ interaction.

Solving the coupling with $c\bar{c}$ states, we finally end up with a Schrödinger-type equation for the relative wave function of the two meson states

$$\sum_\beta \int (H^{M_1M_2}_{\beta'\beta}(P', P) + V^{\rm eff}_{\beta'\beta}(P', P))\chi_\beta(P)P^2 dP = E\chi_{\beta'}(P'), \tag{13}$$

where

$$V^{\rm eff}_{\beta'\beta}(P', P) = \sum_\alpha \frac{V_{\beta'\alpha}(P')V_{\alpha\beta}(P)}{E - M_\alpha} \tag{14}$$

is an effective interaction between the two mesons due to the coupling with intermediate $c\bar{c}$ states.

In this way, we study the influence of the $c\bar{c}$ states on the dynamics of the two meson states. This is a different point of view from that usually found in the literature where the influence of two meson states (in general without meson-meson interaction) in the mass and width of $c\bar{c}$ states is studied [42]. Our approach allows us to generate new states through the meson-meson interaction due to the coupling with $c\bar{c}$ states and to the underlying $q\bar{q}$ interaction. As we will see, the renormalization effects of the $c\bar{c}$ mass due to this channel is small.

The $c\bar{c}$ probabilities are given by

$$c_\alpha = \frac{1}{E - M_\alpha}\sum_\beta \int V_{\alpha\beta}(P)\chi_\beta(P)P^2 dP \tag{15}$$

with the normalization condition $1 = \sum_\alpha |c_\alpha|^2 + \sum_\beta \langle \chi_\beta|\chi_\beta\rangle$.

C. Flatté parametrization

In order to compare the predictions of our model with the recent Belle and *BABAR* experimental data, we obtain from Eq. (12) a Flatté-like parametrization of the DD^* near threshold amplitude following Ref. [43]. We recall the main ideas here.

From Eq. (12), and neglecting the DD^* interaction, one can easily derive the DD^* scattering amplitude

$$F^\beta_{DD^*}(P, P; E) = -\pi\mu \sum_\alpha \frac{V^2_{\beta\alpha}(P)}{E - M_\alpha + g^\alpha_{DD^*}(E)}, \tag{16}$$

where the function $g^\alpha_{DD^*}(E)$ is given by

$$g^\alpha_{DD^*}(E) = \sum_\beta \int \frac{V^2_{\beta\alpha}(P)}{\frac{P^2}{2\mu} + M_D + M_{D^*} - E - i0^+} P^2 dP. \tag{17}$$

For small binding energies $\epsilon = M_D + M_{D^*} - E$, it can be expanded as

$$g^\alpha_{DD^*}(E) = \bar{E}^\alpha_{DD^*} + \frac{i}{2}\Gamma^\alpha_{DD^*} + \mathcal{O}(4\mu^2\epsilon/\Lambda^2), \tag{18}$$

where

$$\bar{E}^\alpha_{DD^*} = 2\mu \sum_\beta \int_0^\infty V^2_{\beta\alpha}(P)dP, \tag{19}$$

$$\Gamma^\alpha_{DD^*} = 2\pi\mu \sum_\beta V^2_{\beta\alpha}(0)P, \tag{20}$$

and $\Lambda \gg \epsilon$ is the characteristic scale of the $V_{\alpha\beta}$ production amplitude which may correspond to the scale of the quark wave function and it is assumed to be much bigger than the binding energy of the physical state.

A straightforward generalization to include the DD^* charged states and other channels gives the expression for the near threshold DD^* scattering amplitude

$$F_{DD^*} = -\frac{1}{2P}\frac{\Gamma_{DD^*}}{E - E_f + \frac{i}{2}(\Gamma_{D^0D^{*0}} + \Gamma_{D^+D^{*-}} + \Gamma(E)) + \mathcal{O}(4\mu^2\epsilon/\Lambda^2)}, \tag{21}$$

where $\Gamma(E)$ accounts for the width due to other processes different from the opening of the near DD^* threshold. Equation (21) corresponds to a Flatté parametrization with

$$D(E) = E - E_f + \frac{i}{2}(\Gamma_{D^0D^{*0}} + \Gamma_{D^+D^{*-}} + \Gamma(E)) + \mathcal{O}(4\mu^2\epsilon/\Lambda^2). \tag{22}$$

Now assuming, as in Ref. [28], that the short range dynamics of the weak $B \to KX(3872)$ transition can be absorbed into a coefficient $\mathcal{B}$, we are able to write the differential rates in the Flatté approximation as

$$\frac{d\mathrm{Br}(B \to KD^0D^{*0})}{dE} = \mathcal{B}\frac{1}{2\pi}\frac{\Gamma_{D^0D^{*0}}(E)}{|D(E)|^2}. \tag{23}$$

The analysis of the $B \to KX(3872) \to K\pi^+\pi^-J/\psi$ data is more involved, because we have to calculate the $DD^* \to \pi^+\pi^-J/\psi$ transition amplitude.

This can consistently be done in our formalism assuming that the process takes place through the DD^* components of the $X(3872)$ which decays in $\rho J/\psi$ and then into the final $\pi^+\pi^-J/\psi$ states. The decay width of the process is given by

$$\Gamma_{\pi^+\pi^-J/\psi} = \sum_{JL}\int_0^{k_{\max}} dk \frac{\Gamma_\rho}{(M_X - E_\rho - E_{J/\psi})^2 + \frac{\Gamma^2_\rho}{4}} \times |\mathcal{M}^{JL}_{X\to\rho J/\psi}(k)|^2. \tag{24}$$

The amplitude $\mathcal{M}^{JL}_{X\to\rho J/\psi}$ is calculated in our model by the

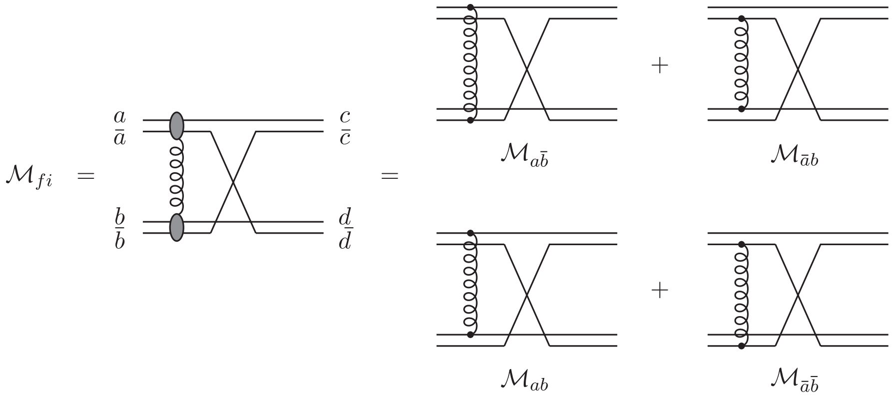

FIG. 1. Diagrams included in the quark rearrangement process $DD^* \to \rho J/\psi$.

rearrangement diagrams of Fig. 1, averaged with the DD^* component of the $X(3872)$ wave function. The rearrangement diagrams are calculated following Ref. [44]. The amplitude is given by

$$\mathcal{M}_{fi} = \sum_{i=a,\bar{a};j=b,\bar{b}} \mathcal{M}_{ij}, \tag{25}$$

where

$$\mathcal{M}_{ij}(\vec{P}', \vec{P}) = \langle \phi_{M_1'}\phi_{M_2'}|H_{ij}^{O}|\phi_{M_1}\phi_{M_2}\rangle \times \langle \xi^{SFC}_{M_1'M_2'}|\mathcal{O}^{SFC}_{ij}|\xi^{SFC}_{M_1M_2}\rangle \tag{26}$$

and the orbital part can be written as [e.g. for the case $(ij) = (a\bar{b})$]

$$\begin{aligned}\langle \phi_{M_1'}\phi_{M_2'}|H_{ij}^{O}|\phi_{M_1}\phi_{M_2}\rangle = \int & d^3P_{M_1'}d^3P_{M_2'}d^3P_{M_1}d^3P_{M_2}\phi^*_{M_1'} \\ & \times (P_{M_1'})\phi^*_{M_2'}(P_{M_2'})\delta(\vec{P}_{M_2'} - \vec{P}_{M_1}) \\ & \times \delta(\vec{P}_{M_2'} - \vec{P}_{M_2} - (\vec{P}' - \vec{P})) \\ & \times H\Big(-\frac{1}{2}(\vec{P}_{M_1} + \vec{P}_{M_2}) + \vec{P}_{M_1'} \\ & + \frac{1}{2}(\vec{P}' - \vec{P})\Big)\phi_{M_1}(P_{M_1}) \\ & \times \phi_{M_2}(P_{M_2}).\end{aligned} \tag{27}$$

The spin-flavor-color matrix elements are taken from Ref. [44].

Once the decay width $\Gamma_{\pi^+\pi^-J/\psi}$ is calculated, the differential rate is given by

$$\frac{d\mathrm{Br}(B \to K\pi^+\pi^-J/\psi)}{dE} = \mathcal{B}\frac{1}{2\pi}\frac{\Gamma_{\pi^+\pi^-J/\psi}(E)}{|D(E)|^2}. \tag{28}$$

In order to compare with the experimental data, we determine the number of event distributions from the differential cross section

$$N^{\pi\pi J/\psi}_{\mathrm{Belle}}(E) = 2.5\ [\mathrm{MeV}]\left(\frac{131}{8.310^{-6}}\right) \times \frac{d\mathrm{Br}(B \to K\pi^+\pi^-J/\psi)}{dE}, \tag{29}$$

$$N^{D^0\bar{D}^0\pi^0}_{\mathrm{Belle}}(E) = 2.0\ [\mathrm{MeV}]\left(\frac{48.3}{0.7310^{-4}}\right) \times \frac{d\mathrm{Br}(B \to KD^0\bar{D}^0\pi^0)}{dE}, \tag{30}$$

$$N^{\pi\pi J/\psi}_{BABAR}(E) = 5\ [\mathrm{MeV}]\left(\frac{93.4}{8.410^{-6}}\right) \times \frac{d\mathrm{Br}(B \to K\pi^+\pi^-J/\psi)}{dE}, \tag{31}$$

$$N^{D^0D^{*0}}_{BABAR}(E) = 2.0\ [\mathrm{MeV}]\left(\frac{33.1}{1.6710^{-4}}\right)\frac{d\mathrm{Br}(B \to KD^0\bar{D}^{*0})}{dE}. \tag{32}$$

In all reactions, a background is taken into account modeled as in Ref. [28]. For the $B \to KD^0\bar{D}^0\pi^0$, the D^0D^{*0} signal interferes with the background and so a phase $\phi^{\mathrm{Belle}} = 0^0$ and $\phi^{BABAR} = 324^0$ have been introduced. Also, the experimental branching ratio $B(D^{*0} \to D^0\pi^0) = 0.62$ is introduced. We use a value for $\mathcal{B} = 3.510^{-4}$ which is in the order of the one used in Ref. [28].

III. RESULTS

A. Numerical methods

To find the quark-antiquark bound states, we solve the Schrödinger equation using the Gaussian expansion method [45]. In this method, the radial wave functions solution of the Schrödinger equation are expanded in terms of basis functions

$$R_\alpha(r) = \sum_{n=1}^{n_{\max}} b_n^\alpha \phi^G_{nl}(r), \tag{33}$$

where α refers to the channel quantum numbers. The coefficients b_n^α and the eigenenergy E are determined from the Rayleigh-Ritz variational principle

$$\sum_{n=1}^{n_{\max}}\left[(T^\alpha_{n'n} - EN^\alpha_{n'n})b^\alpha_n + \sum_{\alpha'} V^{\alpha\alpha'}_{n'n} b^{\alpha'}_n = 0\right], \qquad (34)$$

where the operators $T^\alpha_{n'n}$ and $N^\alpha_{n'n}$ are diagonal, and the only operator that mixes the different channels is the potential $V^{\alpha\alpha'}_{n'n}$.

To solve the four body problem, we also use the Gaussian expansion of the two body wave functions obtained from the solution of the Schrödinger equation. This procedure allows us to introduce in variational way possible distortions of the two body wave function within the molecule. Using these wave functions Eq. (13) reduces to a matrix equation by Gauss integration.

A crucial problem of the variational methods is how to choose the radial functions $\phi^G_{nl}(r)$ in order to have a minimal, but enough, number of basis functions. Following [45] we employ Gaussian's trial functions whose ranges are in geometric progression. The geometric progression is useful in optimizing the ranges with a small number of free parameters. Moreover the distribution of the Gaussian ranges in geometric progression is dense at small ranges, which is well suited for making the wave function correlate with short range potentials. The fast damping of the Gaussian tail is not a real problem since we can choose the maximal range much longer than the hadronic size.

B. Results

The calculation is parameter free since all the parameters are taken from the previous calculation [31,37] including the $\gamma = 0.26$ parameter in Eq. (10). This value was fitted to the reaction $\psi(3770) \to DD$, which is the only well established charmonium strong decay. This way to determine the value of γ might overestimate it, since the $\psi(3770)$ is very close to the DD threshold and FSI effects, which were not included, might be relevant [46].

We first perform an isospin symmetric calculation including ${}^3S_1$ and ${}^3D_1$ DD^* partial waves and taking the D and D^* masses as average of the experimental values between charged states. If we neglect the coupling to $c\bar{c}$ states, we do not get a bound state for the DD^* molecule in the 1^{++} channel, neither in the $I = 0$ nor in the $I = 1$ channels. The interaction coming from one pion exchange is attractive in the $I = 0$ channel but not enough to bind the system, even allowing for distortion in the meson states.

Now, we include in the $I = 0$ channel the coupling to $c\bar{c}$ states. The most relevant are the 1^{++} ground and first excited states with bare masses within the model given by

$$\begin{aligned} c\bar{c}(1^3P_1) &\to M = 3503.9\ \text{MeV}, \\ c\bar{c}(2^3P_1) &\to M = 3947.4\ \text{MeV}. \end{aligned} \qquad (35)$$

TABLE I. Masses and channel probabilities for the three states in three different calculations. The first three states are found when we perform and isospin symmetric calculation with a value of γ fit to the decay $\psi(3770) \to DD$. The second three states shows the effect of isospin breaking in the DD^* masses. The last three states correspond to a value of $\gamma = 0.19$ that fits the experimental mass of the $X(3872)$. The probability is shown as zero when it is less than 0.5%.

	M (MeV)	$c\bar{c}(1^3P_1)$	$c\bar{c}(2^3P_1)$	D^0D^{*0}	$D^\pm D^{*\mp}$
	3936	0%	79%	10.5%	10.5%
A	3865	1%	32%	33.5%	33.5%
	3467	95%	0%	2.5%	2.5%
	3937	0%	79%	7%	14%
B	3863	1%	30%	46%	23%
	3467	95%	0%	2.5%	2.5%
	3942	0%	88%	4%	8%
C	3871	0%	7%	83%	10%
	3484	97%	0%	1.5%	1.5%

The results of this calculation are shown in part A of Table I. We find an almost pure $c\bar{c}(1^3P_1)$ state with mass 3467 MeV which we identify with the $\chi_{c_1}(1P)$ and two states with significant molecular admixture. One of them with mass 3865 MeV is almost a DD^* molecule bound by the coupling to the $c\bar{c}$ states. The second one, with mass 3936 MeV, is a $c\bar{c}(2^3P_1)$ with sizable DD^* component. We assign the first state to the $X(3872)$, as the second one is a candidate to the $X(3940)$. We have also analyzed the effect of higher bare $c\bar{c}$ states finding a negligible effect on the mass and probabilities that will not change the above numbers.

Coexistence of the $\omega J/\psi(I = 0)$ and $\rho J/\psi(I = 1)$ decay modes strongly suggest a large isospin mixing. However, the relative branching fraction of both modes can be misleading with respect to the absolute magnitude of the isospin mixing in $X(3872)$ due to the phase space suppression of the $\omega J/\psi$ channel against the $\rho J/\psi$ one. In fact, if we assume that $X(3872)$ is a D^0D^{*0} molecule, the ratio $\frac{B(X(3872)\to\pi^+\pi^-\pi^0 J/\psi)}{B(X(3872)\to\pi^+\pi^- J/\psi)}$ would be a factor 20 smaller than the experiment due to the different phase space.

It is clear that we need charged components in the wave function but with a different weight with respect to the neutral component. This rules out the intuitive idea of the dominance of the loosely bound neutral component. The clarification of this puzzle has been nicely done in Ref [47].

To introduce the isospin breaking in our calculation, we turn to the charge basis instead of the isospin symmetric basis with the transformation

$$|D^\pm D^{*\mp}\rangle = \frac{1}{\sqrt{2}}(|DD^* I = 0\rangle - |DD^* I = 1\rangle), \qquad (36)$$

$$|D^0 D^{*0}\rangle = \frac{1}{\sqrt{2}}(|DD^* I = 0\rangle + |DD^* I = 1\rangle), \qquad (37)$$

writing our isospin symmetric interaction on the charged basis. We now explicitly break isospin symmetry taking the experimental threshold difference into account in our equations and solving for the charged and neutral components. Of course, if we do not break it explicitly, we recover our previous result as a bound state in the $I = 0$ sector. Now, we get again three states being the main difference in the DD^* molecular component. The masses and channel probabilities are shown in part B of Table I. We now get a higher probability for the D^0D^{*0} component, although the isospin 0 component still dominates with a 66% probability and a 3% for isospin 1.

Having in mind that the $^3P_0$ model is probably too naive and we might be overestimating the value of γ, we show in Fig. 2 the variation of the $X(3872)$ mass with it. We can see that it is possible to get the experimental binding energy with a fine tune of this parameter. Using 0.6 MeV as the binding energy, we get a value of $\gamma = 0.19$, 25% smaller than the original. The results are shown in part C of Table I. Now, the D^0D^{*0} clearly dominates with a 83% probability giving a 70% for the isospin 0 component and 23% for isospin 1. Of course, as the isospin breaking is a threshold effect [25], it grows as we get closer to it as can be seen in Fig. 3 where we show the probabilities of the different components for the state $X(3872)$.

In Fig. 4, we compare our results with the $B \to KD^0\bar{D}^0\pi^0$ data from Belle (a) and $B \to KD^0\bar{D}^{*0}$ data from *BABAR* (b). The same comparison is done in Fig. 5 for the $B \to K\pi^+\pi^- J/\Psi$ data from Belle (a) and *BABAR* (b). In all figures, the dashed lines shows the results without resolution functions. The solid line gives the result using the resolution functions as in Ref. [28]. All the resolution functions are those given by Belle [17] and *BABAR* [19] Collaborations with the exception of the *BABAR* DD^* resolution where we use the prescription from Ref. [28].

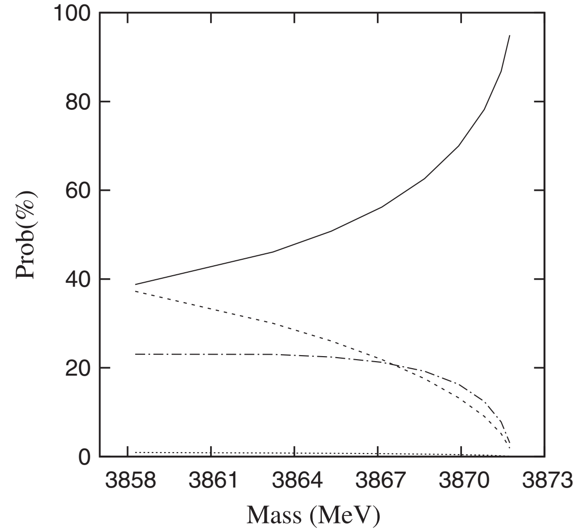

FIG. 3. Probability (in %) of different components as a function of the binding energy when we vary the γ parameter of the $^3P_0$ model. The solid line gives the D^0D^{*0} probability, the dash-dotted line is the $D^\pm D^{*\mp}$, the dashed line is the $c\bar{c}(2^3P_1)$, and the dotted line is the $c\bar{c}(1^3P_1)$.

We find a good description of the Belle $B \to KD^0D^0\pi^0$ data; whereas, the agreement is poor in the case of the *BABAR* data. It is important to notice that in the Belle analysis, the mass of the X appears as 3872 MeV, while in the *BABAR* data, the resonance is located 3 MeV above. The *BABAR* mass value does not coincide with the mass of the X obtained in our calculation which may be the reason for the disagreement.

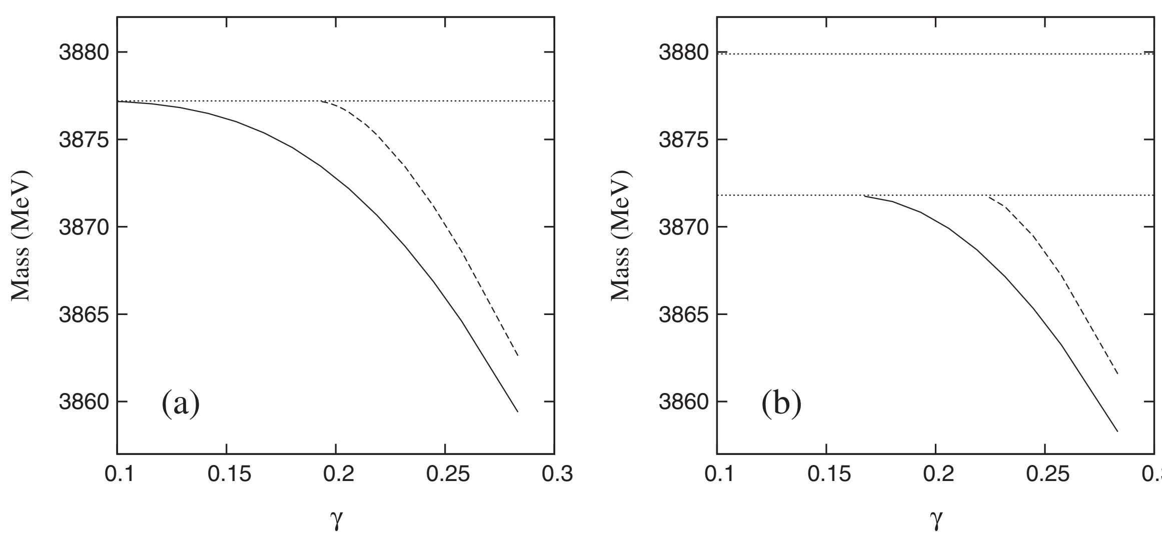

FIG. 2. Mass of the $X(3872)$ as a function of the strength γ of the $^3P_0$ model. The isospin symmetric calculation is shown in figure (a) and the isospin breaking in figure (b). Dotted lines show the threshold positions for the DD^* average in figure (a) and D^0D^{*0} and $D^\pm D^{*\mp}$ in (b). The solid lines show the full result, and the dashed lines turn off the DD^* interaction.

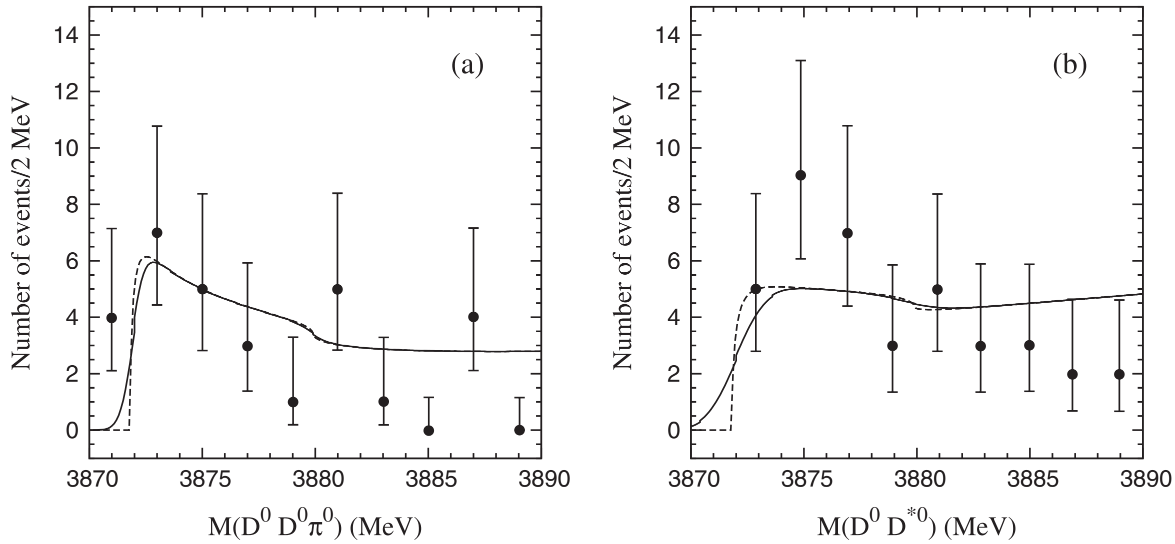

FIG. 4. Number of events for the decay $B \to KD^0D^0\pi^0$ measured by Belle (a) and for the decay $B \to KD^0D^{*0}$ measured by *BABAR* (b). The solid and dashed lines show the results from our model with and without the resolution functions as explained in the text.

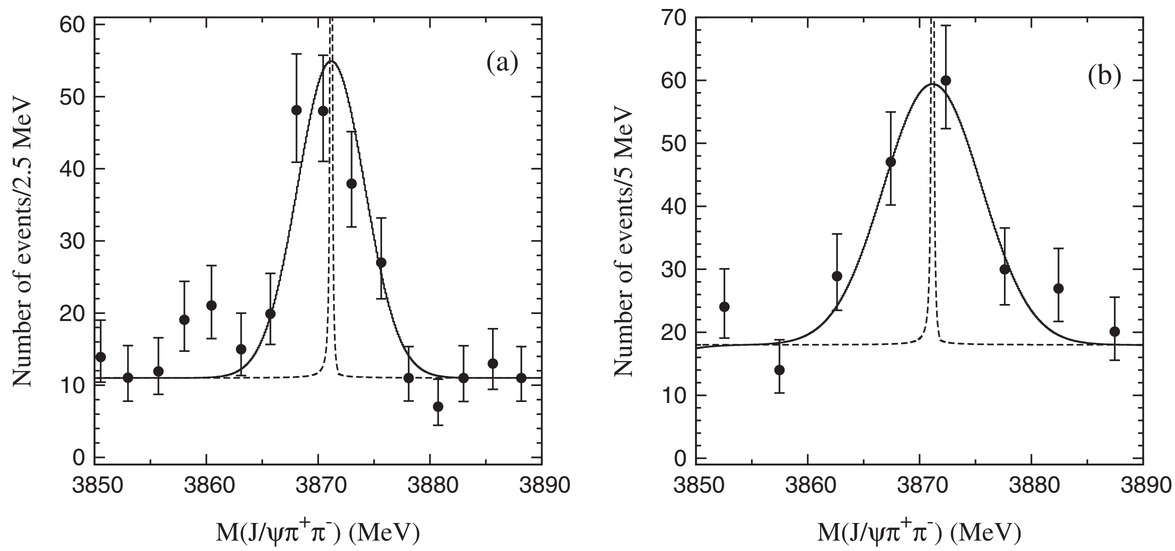

FIG. 5. Number of events for the decay $B \to K\pi^+\pi^-J/\Psi$ measured by Belle (a) and by *BABAR* (b). The solid and dashed lines show the results from our model with and without the resolution functions as explained in the text.

The $B \to K\pi^+\pi^-J/\Psi$ data are equally well described for the Belle and *BABAR* experiments. In this case, both collaborations give similar values for the mass of the resonance, namely, 3871.4 MeV, which are in much better agreement with our result.

IV. SUMMARY

As a summary, we have shown that the $X(3872)$ emerges in a constituent quark model calculation as a dynamically generated mixed state of a DD^* molecule and $\chi_{c_1}(2P)$. Although the $c\bar{c}$ mixture is less than the 10%, it is important to bind the molecular state. This result is in agreement with the analysis of Ref. [29]. The proposed structure allows to understand simultaneously the isospin violation showed by the experimental data and the radiative decay rates. Furthermore, we have demonstrated that this solution explain the new Belle data in the $D^0D^0\pi^0$ and $\pi^+\pi^-J/\Psi$ decay modes and the $\pi^+\pi^-J/\Psi$ *BABAR* data. The original $\chi_{c_1}(2P)$ state acquires a significant DD^* component and can be identified with the $X(3940)$.

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