$SU(3)$ symmetry breaking in decay constants and electromagnetic properties of pseudoscalar heavy mesons

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In this paper, the decay constants and mean square radii of pseudoscalar heavy mesons are studied in the $SU(3)$ symmetry breaking. Within the light-front framework, the ratios f_{D_s}/f_D and f_{B_s}/f_B are individually estimated using the hyperfine splittings in the $D^*_{(s)} - D_{(s)}$ and $B^*_{(s)} - B_{(s)}$ states and the light quark masses, $m_{s,q}$ ($q = u, d$), to extract the wave function parameter β. The values $f_{D_s}/f_D = 1.29 \pm 0.07$ and $f_{B_s}/f_B = 1.32 \pm 0.08$ are obtained, which are not only chiefly determined by the ratio of light quark masses m_s/m_q, but also insensitive to the heavy quark masses $m_{c,b}$ and the decay constants $f_{D,B}$. The dependence of f_{B_c}/f_B on $\Delta M_{B_cB_c^*}$ with the varied charm quark masses is also shown. In addition, the mean square radii are estimated as well. The values $\sqrt{\langle r^2_{D_s^+}\rangle/\langle r^2_{D^+}\rangle} = 0.740^{-0.041}_{+0.050}$ and $\sqrt{\langle r^2_{B_s^0}\rangle/\langle r^2_{B^0}\rangle} = 0.711^{-0.049}_{+0.058}$ are obtained, and the sensitivities of $\langle r^2_P\rangle$ on the heavy and light quark masses are similar to those of the decay constants.

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I. INTRODUCTION

The decay constants of pseudoscalar heavy mesons with c and b quarks play an important role for studies of CP violation and in extracting the Cabibbo-Kobayashi-Maskawa matrix elements. Experimentally, new data on the charm meson decay constants f_D and f_{D_s} have been reported [1,2]. As the calculations of the decay constants are related to the wave function overlap of the quark and antiquark which are governed by the strong interaction, they therefore provide a crucial manner to compare different theoretical methods. In addition, the determination of f_{B_s} remains beyond the reach of current experiments, thus the reliability of estimated f_{B_s} by a theoretical approach is dependent on whether the determinations of f_D and f_{D_s} by this approach are consistent with the new data. During the last decade, the decay constants of pseudoscalar heavy mesons have been studied in lattice simulations [3–9], in the QCD sum rules approach [10–13], and in the relativistic quark model [14–18].

The understanding of the electromagnetic (EM) properties of hadrons is also an important topic, and the EM form factors which are calculated using nonperturbative methods are the useful tool for this purpose. There have been numerous experimental [19–24] and theoretical studies [25] of the EM form factors of the light pseudoscalar meson (π and K). However, the EM form factors of heavy mesons (which contain one heavy quark) have much fewer studies [26,27] than those of light ones. The present paper is devoted to an analysis of the wave function and decay constant by the hyperfine mass splitting of heavy mesons and the formulas of the decay constant and mean square radius within the light-front (LF) framework. We present the $SU(3)$ symmetry breaking effect in decay constants and electromagnetic properties of pseudoscalar heavy mesons.

The light-front quark model (LFQM) is a promising analytic method for solving the nonperturbative problems of hadron physics [28], as well as offering many insights into the internal structures of bound states. The basic ingredient in LFQM is the relativistic hadron wave function which generalizes distribution amplitudes by including transverse momentum distributions and contains all the information of a hadron from its constituents. The hadronic quantities are represented by the overlap of wave functions and can be derived in principle. The light-front wave function is manifestly a Lorentz invariant, expressed in terms of internal momentum fraction variables which are independent of the total hadron momentum. Moreover, the fully relativistic treatment of quark spins and center-of-mass motion can be carried out using the so-called Melosh rotation [29]. This treatment has been successfully applied to calculate phenomenologically many important meson decay constants and hadronic form factors [30–36].

The remainder of this paper is organized as follows. In Sec. II an analysis of wave function and decay constant is presented. In Sec. III the formulism of LFQM is reviewed briefly, and the formulas of decay constant and mean square radius are derived. In Sec. IV numerical results and discussions are presented. Finally, the conclusions are given in Sec. V.

II. ANALYSES OF WAVE FUNCTION AND DECAY CONSTANT

The decay constant f_P for a pseudoscalar meson is defined by a matrix element of the axial vector current between the vacuum and the meson bound state:

$$\langle 0|\bar{q}_1\gamma_\mu\gamma_5 q_2|P(P)\rangle = if_P P_\mu. \tag{2.1}$$

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In a nonrelativistic approximation, f_P is related to the Bethe-Salpeter wave function at the origin $|\Psi(0)|$ as [37–39]

$$f_P \simeq \frac{2\sqrt{N_c}}{\sqrt{M_P}} |\Psi(0)|, \tag{2.2}$$

where N_c is the color number and M_P is the mass of the meson. If we consider the potential of a hyperfine interaction inside the meson to $O(\alpha_s)$:

$$V_{\rm hf} = \frac{4\alpha_s(3\vec{s}_1 \cdot \hat{r}\vec{s}_2 \cdot \hat{r} - \vec{s}_1 \cdot \vec{s}_2)}{3m_1 m_2 r^3} + \frac{32\pi\alpha_s \vec{s}_1 \cdot \vec{s}_2}{9m_1 m_2} \delta^3(\vec{r}), \tag{2.3}$$

where α_s is the strong coupling constant, $s_{1,2}(m_{1,2})$ are the spins (masses) of the constituent quark. For the s-wave meson, the first term of Eq. (2.3) has no contribution and the second term can distinguish the pseudoscalar and vector mesons. Therefore, the hyperfine mass splitting is obtained as

$$\Delta M_{\rm PV} = \frac{32\pi\alpha_s}{9m_1 m_2} |\Psi(0)|^2. \tag{2.4}$$

By combining Eqs. (2.2) and (2.4) and canceling $|\Psi(0)|$, we obtain

$$f_P = \left(\frac{27\Delta M_{PV} m_1 m_2}{8\pi\alpha_s M_P}\right)^{1/2}. \tag{2.5}$$

If we suppose the strong coupling constants $\alpha_s(D) \simeq \alpha_s(D_s)$ and $\alpha_s(B) \simeq \alpha_s(B_s)$, then the ratios of the decay constants can be obtained as

$$\frac{f_{D_s}}{f_D} = \left(\frac{\Delta M_{D_s D_s^*}}{\Delta M_{DD^*}} \frac{M_D}{M_{D_s}} \frac{m_s}{m_q}\right)^{1/2}, \quad \frac{f_{B_s}}{f_B} = \left(\frac{\Delta M_{B_s B_s^*}}{\Delta M_{BB^*}} \frac{M_B}{M_{B_s}} \frac{m_s}{m_q}\right)^{1/2}, \tag{2.6}$$

where $q = u, d$. From Eq. (2.6), we find that the ratios are dependent on the ratio of light quark masses m_s/m_q and are independent of heavy quark masses. Furthermore, by canceling the ratio m_s/m_q, we have a relation which does not contain any parameter in the nonrelativistic approximation:

$$\frac{f_{B_s}}{f_B} = \left(\frac{\Delta M_{DD^*} \Delta M_{B_s B_s^*}}{\Delta M_{D_s D_s^*} \Delta M_{BB^*}} \frac{M_{D_s} M_B}{M_D M_{B_s}}\right)^{1/2} \frac{f_{D_s}}{f_D}. \tag{2.7}$$

If one wants to include the relativistic correction to the ratios of the decay constants, not only the values of $m_{s,q}$, but also the form of wave function $\Psi(\vec{r})$ must be known. Let us come back to Eq. (2.2). The deviation of Eq. (2.2) uses the Fourier transform

$$\Psi(\vec{r}) = \int \frac{d^3 k}{(2\pi)^{3/2}} e^{i\vec{r}\cdot\vec{k}} \phi(\vec{k}), \tag{2.8}$$

where $\phi(\vec{k})$ is the wave function in the momentum space. If the Fourier transform is evaluated for the positive-energy projection of the Bethe-Salpeter wave function at equal "time" $z^+ = z^0 + z^3 = 0$, then [40]

$$f_P \sim \int \frac{dx d^2 k_\perp}{2(2\pi)^3} \phi(x, k_\perp), \tag{2.9}$$

where x is the longitudinal momentum fraction, $k_\perp$ are the relative transverse momenta, and $\phi(x, k_\perp)$ satisfies the normalization condition:

$$\int \frac{dx d^2 k_\perp}{2(2\pi)^3} |\phi(x, k_\perp)|^2 = 1. \tag{2.10}$$

In general, the momentum distribution amplitude $\phi(x, k_\perp)$ is obtained by solving the light-front QCD bound state equation $H_{\rm LF}|P\rangle = M|P\rangle$ which is the familiar Schrödinger equation in ordinary quantum mechanics, and $H_{\rm LF}$ is the light-front Hamiltonian. However, at the present time, how to solve the bound state equation is still unknown. Alternatively, we come back to the still-unknown wave function $\Psi(\vec{r})$ and express it as a linear combination of the arbitrary known functions which form a complete set. Of course, the complete set is not unique. Here we give two examples for comparison. One is the solution of $1/r$ potential [41]:

$$\Psi^c_{nlm}(\vec{r}) = \left(\frac{2\beta}{n}\right)^{3/2} \left[\frac{(n-l-1)!}{2n[(n+l)!]^3}\right]^{1/2} \times \left(\frac{2\beta r}{n}\right)^l L^{2l+1}_{n-l-1}(2\beta r/n) \exp\left(-\frac{\beta r}{n}\right) Y_{lm}, \tag{2.11}$$

where superscript c means "Coulomb," β is a parameter which has the energy dimension, Y_{lm} is the spherical harmonics, and $L^p_{q-p}(x)$ is an associated Laguerre polynomial which is defined as

$$L^p_{q-p}(x) \equiv (-1)^p \left(\frac{d}{dx}\right)^p e^x \left(\frac{d}{dx}\right)^q (e^{-x} x^q). \tag{2.12}$$

The other complete set is the solution of an isotropic harmonic oscillator [41]:

$$\Psi^g_{nlm}(\vec{r}) = \frac{\beta^{3/2}}{\pi^{1/4}} (\beta r)^l h_{nl}(\beta r) \exp\left(-\frac{\beta^2 r^2}{2}\right) Y_{lm}, \tag{2.13}$$

where superscript g means Gaussian and the first few $h_{nl}(\beta r)$'s are

$$h_{00} = 2, \qquad h_{11} = \sqrt{\frac{8}{3}}, \qquad h_{22} = \frac{4}{\sqrt{15}}, \qquad h_{20} = \sqrt{6}\left(1 - \frac{2}{3}\beta^2 r^2\right). \tag{2.14}$$

Then the exact solution can be expressed as

$$\Psi(\vec{r}) = \sum_{nlm}^{\infty} a^{c(g)}_{nlm} \Psi^{c(g)}_{nlm}(\vec{r}), \tag{2.15}$$

where $\sum_{nlm}^{\infty}|a_{nlm}^{c(g)}|^2 = 1$. This way seems very clumsy because, apart from β, it also introduces a series of undetermined coefficients, a_{nlm}. The following considerations, however, improve the situation. First, only the coefficients a_{n00} survive because we just studied the s-wave meson. Second, we substitute Eq. (2.15) to Eq. (2.4) and obtain

$$\Delta M_{\rm PV} = \frac{32\pi\alpha_s}{9m_1m_2}\left(\frac{\beta^3}{4\pi^{3/2}}\right)\left[\sum_{\text{even } n}^{\infty} a_{n00}^{g} h_{n0}(0)\right]^2, \quad (2.16)$$

which takes the Gaussian case, for example. It is worth noting that the square bracket in Eq. (2.16) is independent of β. Then, the ratio of hyperfine mass splittings can be reduced as

$$\frac{\Delta M_{D_sD_s^*}}{\Delta M_{DD^*}} = \frac{m_q}{m_s}\left(\frac{\beta_{cs}}{\beta_{cq}}\right)^3, \qquad \frac{\Delta M_{B_sB_s^*}}{\Delta M_{BB^*}} = \frac{m_q}{m_s}\left(\frac{\beta_{bs}}{\beta_{bq}}\right)^3, \quad (2.17)$$

which does not include any coefficient a_{nlm}. Equation (2.17) is also suitable to the Coulomb case. In fact, due to the δ function in Eq. (2.3) having the dimension of an energy cube and $\vec{r}$ is vanishing here, Eq. (2.17) holds for any wave function which contains only one hadronic parameter β. In addition, the ratios β_{cs}/β_{cq} and β_{bs}/β_{bq} in Eq. (2.17) are mainly influenced by the ratio m_s/m_q. This situation leads to the ratios f_{D_s}/f_D and f_{B_s}/f_B are sensitive to the $SU(3)$ symmetry breaking, but are insensitive to the heavy quark masses, which will be shown later. In the literature, there are some early attempts [42,43] to account for flavor symmetry breaking in pseudoscalar meson decay constants. In the next section the wave function $\Psi(\vec{r})$ and the values of $m_{s,q}$ are studied within the light-front framework.

III. LIGHT-FRONT FRAMEWORK

A. General formulism

An s-wave meson bound state, consisting of a quark q_1 and an antiquark $\bar{q}_2$ with total momentum P and spin J, can be written as (see, for example [32])

$$|M(P, S, S_z)\rangle = \int \{d^3p_1\}\{d^3p_2\}2(2\pi)^3\delta^3(\tilde{P} - \tilde{p}_1 - \tilde{p}_2) \times \sum_{\lambda_1,\lambda_2}\Phi^{SS_z}(\tilde{p}_1, \tilde{p}_2, \lambda_1, \lambda_2)|q_1(p_1, \lambda_1) \times \bar{q}_2(p_2, \lambda_2)\rangle, \quad (3.1)$$

where p_1 and p_2 are the on-mass-shell light-front momenta,

$$\tilde{p} = (p^+, p_\perp), \qquad p_\perp = (p^1, p^2), \qquad p^- = \frac{m_q^2 + p_\perp^2}{p^+}, \quad (3.2)$$

and

$$\{d^3p\} \equiv \frac{dp^+d^2p_\perp}{2(2\pi)^3},$$
$$|q(p_1, \lambda_1)\bar{q}(p_2, \lambda_2)\rangle = b^\dagger(p_1, \lambda_1)d^\dagger(p_2, \lambda_2)|0\rangle,$$
$$\{b(p', \lambda'), b^\dagger(p, \lambda)\} = \{d(p', \lambda'), d^\dagger(p, \lambda)\} = 2(2\pi)^3\delta^3(\tilde{p}' - \tilde{p})\delta_{\lambda'\lambda}. \quad (3.3)$$

In terms of the light-front relative momentum variables $(x, k_\perp)$ defined by

$$p_1^+ = (1-x)P^+, \qquad p_2^+ = xP^+,$$
$$p_{1\perp} = (1-x)P_\perp + k_\perp, \qquad p_{2\perp} = xP_\perp - k_\perp, \quad (3.4)$$

the momentum-space wave function Ψ^{SS_z} can be expressed as

$$\Phi^{SS_z}(\tilde{p}_1, \tilde{p}_2, \lambda_1, \lambda_2) = \frac{1}{\sqrt{Nc}}R_{\lambda_1\lambda_2}^{SS_z}(u, \kappa_\perp)\phi(x, k_\perp), \quad (3.5)$$

where $R_{\lambda_1\lambda_2}^{SS_z}$ constructs a state of definite spin (S, S_z) out of light-front helicity (λ_1, λ_2) eigenstates. Explicitly,

$$R_{\lambda_1\lambda_2}^{SS_z}(x, k_\perp) = \sum_{s_1,s_2}\langle\lambda_1|\mathcal{R}_M^\dagger(1-x, k_\perp, m_1)|s_1\rangle \times \langle\lambda_2|\mathcal{R}_M^\dagger(x, -k_\perp, m_2)|s_2\rangle \times \left\langle\frac{1}{2}\frac{1}{2}; s_1s_2\Big|\frac{1}{2}\frac{1}{2}; SS_z\right\rangle, \quad (3.6)$$

where $|s_i\rangle$ are the usual Pauli spinors, and $\mathcal{R}_M$ is the Melosh transformation operator [30,31]:

$$\langle s|\mathcal{R}_M(x_i, k_\perp, m_i)|\lambda\rangle = \frac{m_i + x_iM_0 + i\vec{\sigma}_{s\lambda}\cdot\vec{k}_\perp\times\vec{n}}{\sqrt{(m_i + x_iM_0)^2 + k_\perp^2}}, \quad (3.7)$$

with $x_1 = 1 - x$, $x_2 = x$, and $\vec{n} = (0, 0, 1)$ as a unit vector in the $\hat{z}$ direction. In addition,

$$M_0^2 = (e_1 + e_2)^2 = \frac{m_1^2 + k_\perp^2}{1-x} + \frac{m_2^2 + k_\perp^2}{x},$$
$$e_i = \sqrt{m_i^2 + k_\perp^2 + k_z^2}, \quad (3.8)$$

where k_z is the relative momentum in $\hat{z}$ direction and can be written as

$$k_z = \frac{xM_0}{2} - \frac{m_2^2 + k_\perp^2}{2xM_0}. \quad (3.9)$$

M_0 is the invariant mass of $q\bar{q}$ and generally different from the mass M of the meson which satisfies $M^2 = P^2$. This is due to the fact that the meson, quark, and antiquark cannot be simultaneously on shell. We normalized the meson state as

$$\langle M(P', S', S'_z)|M(P, S, S_z)\rangle = 2(2\pi)^3 P^+ \delta^3(\tilde{P}' - \tilde{P})\delta_{S'S}\delta_{S'_z S_z}, \tag{3.10}$$

which led to Eq. (2.10).

In practice, it is more convenient to use the covariant form of $R^{SS_z}_{\lambda_1\lambda_2}$ [30,31,36,44]:

$$R^{SS_z}_{\lambda_1\lambda_2}(x, k_\perp) = \frac{\sqrt{p_1^+ p_2^+}}{\sqrt{2}\tilde{M}_0(M_0 + m_1 + m_2)} \times \bar{u}(p_1, \lambda_1)(\bar{\not{P}} + M_0)\Gamma v(p_2, \lambda_2), \tag{3.11}$$

where

$$\begin{aligned} \tilde{M}_0 &\equiv \sqrt{M_0^2 - (m_1 - m_2)^2}, \\ \bar{P} &\equiv p_1 + p_2, \ \bar{u}(p, \lambda)u(p, \lambda') = \frac{2m}{p^+}\delta_{\lambda,\lambda'}, \\ \sum_\lambda u(p, \lambda)\bar{u}(p, \lambda) &= \frac{\not{p} + m}{p^+}, \\ \bar{v}(p, \lambda)v(p, \lambda') &= -\frac{2m}{p^+}\delta_{\lambda,\lambda'}, \\ \sum_\lambda v(p, \lambda)\bar{v}(p, \lambda) &= \frac{\not{p} - m}{p^+}. \end{aligned} \tag{3.12}$$

For the pseudoscalar meson, we have $\Gamma = \gamma_5$, Eq. (3.11) can then be further reduced by the applications of equations of motion on spinors [36]:

$$R^{SS_z}_{\lambda_1\lambda_2}(x, k_\perp) = \frac{\sqrt{p_1^+ p_2^+}}{\sqrt{2}\tilde{M}_0}\bar{u}(p_1, \lambda_1)\gamma_5 v(p_2, \lambda_2). \tag{3.13}$$

Next, we derive the formulas of the decay constant and the mean square radius for the pseudoscalar meson. The former is the main subject of this work, and the latter is used to fix some parameters.

B. Formulas for decay constant and mean square radius

The decay constants of pseudoscalar mesons $P(q_1\bar{q}_2)$ are defined in Eq. (2.1). The matrix element can be calculated using the formulism in the last subsection:

$$\begin{aligned} \langle 0|\bar{q}_2\gamma_\mu\gamma_5 q_1|P(P)\rangle = &\int \{d^3p_1\}\{d^3p_2\}2(2\pi)^3 \\ &\times \delta^3(\tilde{P} - \tilde{p}_1 - \tilde{p}_2)\phi_P(x, k_\perp) \\ &\times R^{00}_{\lambda_1\lambda_2}(x, k_\perp) \\ &\times \langle 0|\bar{q}_2\gamma_\mu\gamma_5 q_1|q_1\bar{q}_2\rangle. \end{aligned} \tag{3.14}$$

Since $\tilde{M}_0\sqrt{x(1-x)} = \sqrt{A^2 + k_\perp^2}$, the decay constant can be extracted as

$$f_P = 2\sqrt{2N_c}\int\{dx\}\frac{A}{\sqrt{A^2 + k_\perp^2}}\phi_P(x, k_\perp) \tag{3.15}$$

where $\{dx\} = \frac{dx d^2k_\perp}{16\pi^3}$ and $A = m_1 x + m_2(1 - x)$.

Next, the EM form factor of a meson P is determined by the scattering of one virtual photon and one meson. It describes the deviation from the pointlike structure of the meson, and is a function of Q^2. Here, we considered the momentum of the virtual photon in a spacelike region, so it was always possible to orient the axes in such a manner that $Q^+ = (P' - P)^+ = 0$. Thus, the EM form factor was determined by the matrix element:

$$\langle P(P')|J^+|P(P)\rangle = eF_P(Q^2)(P + P')^+, \tag{3.16}$$

where $J^\mu = \bar{q}e_q e\gamma^\mu q$, e_q is the charge of quark q in e unit, and $Q^2 = (P' - P)^2 < 0$. With the light-front framework, F_P can be extracted by Eq. (3.16):

$$\begin{aligned} F_P(Q^2) = &\, e_{q_1}\int\{dx\}\frac{A^2 + k_\perp\cdot k'_\perp}{\sqrt{A^2 + k_\perp^2}\sqrt{A^2 + k'^2_\perp}} \\ &\times \phi_P(x, k_\perp)\phi_{P'}(x, k'_\perp) \\ &+ e_{\bar{q}_2}\int\{dx\}\frac{A^2 + k_\perp\cdot k''_\perp}{\sqrt{A^2 + k_\perp^2}\sqrt{A^2 + k''^2_\perp}} \\ &\times \phi_P(x, k_\perp)\phi_{P'}(x, k''_\perp), \end{aligned} \tag{3.17}$$

where $k'_\perp = k_\perp + xQ_\perp$, $k''_\perp = k_\perp - (1 - x)Q_\perp$. For applying this to Eq. (3.22), it is convenient to consider the term $\tilde{\phi}_P \equiv \phi_P(x, k_\perp)/\sqrt{A^2 + k_\perp^2}$ and take the Taylor expansion around $k_\perp^2$

$$\tilde{\phi}_{P'}(k'^2_\perp) = \tilde{\phi}_{P'}(k_\perp^2) + \frac{d\tilde{\phi}_{P'}}{dk_\perp^2}\bigg|_{Q_\perp=0}(k'^2_\perp - k_\perp^2) + \frac{d^2\tilde{\phi}_{P'}}{2(dk_\perp^2)^2}\bigg|_{Q_\perp=0}(k'^2_\perp - k_\perp^2)^2 + \ldots \tag{3.18}$$

Then, by using the identity

$$\int d^2k_\perp(k_\perp\cdot A_\perp)(k_\perp\cdot B_\perp) = \frac{1}{2}\int d^2k_\perp k_\perp^2 A_\perp\cdot B_\perp, \tag{3.19}$$

we can rewrite (3.17) to

$$\begin{aligned} F_P(Q^2) = &\, (e_{q_1} + e_{\bar{q}_2}) - Q^2\int\{dx\}\phi_P^2(x, k_\perp) \\ &\times [x^2 e_{q_1} + (1 - x)^2 e_{\bar{q}_2}] \\ &\times \left(\Theta_P\frac{A^2 + 2k_\perp^2}{A^2 + k_\perp^2} + \tilde{\Theta}_P k_\perp^2\right) + \mathcal{O}(Q^4), \end{aligned} \tag{3.20}$$

where

$$\Theta_M = \frac{1}{\tilde{\phi}_M}\left(\frac{d\tilde{\phi}_M}{dk_\perp^2}\right), \qquad \tilde{\Theta}_M = \frac{1}{\tilde{\phi}_M}\left(\frac{d^2\tilde{\phi}_M}{(dk_\perp^2)^2}\right). \tag{3.21}$$

It should be realized that the size and the density of a hadron depend on the probe. For an EM probe, it is the electric charge radius $\langle r^2\rangle^{1/2}$ that is obtained. In the experimental view, $\langle r_P^2\rangle$ cannot be measured directly and is obtained by fitting the slope of $F_P(Q^2)$ at $Q^2 = 0$, i.e.,

$$\langle r_P^2\rangle = 6\frac{dF_P(Q^2)}{dQ^2}\bigg|_{Q^2=0}. \tag{3.22}$$

Here the mean square radius is easily obtained:

$$\begin{aligned}\langle r_P^2\rangle &= \langle r_{q_1}^2\rangle + \langle r_{\bar{q}_2}^2\rangle \\ &= e_{q_1}\left\{-6\int\{dx\}x^2\tilde{\phi}_P\left[(A^2+2k_\perp^2)\frac{d}{dk_\perp^2} + (A^2+k_\perp^2)k_\perp^2\left(\frac{d}{dk_\perp^2}\right)^2\right]\tilde{\phi}_P\right\} \\ &\quad + e_{\bar{q}_2}\left\{-6\int\{dx\}(1-x)^2\tilde{\phi}_P\left[(A^2+2k_\perp^2)\frac{d}{dk_\perp^2} + (A^2+k_\perp^2)k_\perp^2\left(\frac{d}{dk_\perp^2}\right)^2\right]\tilde{\phi}_P\right\}.\end{aligned} \tag{3.23}$$

It is worth mentioning that, first, the static property $F_P(0) = e_P$ is quite easily checked in Eq. (3.20). Second, from Eq. (3.23), we find that the mean square radius is related to the first and second longitudinal momentum square derivatives of $\tilde{\phi}$ which contain the Melosh transformation effect.

If we take the heavy quark limit $m_1 = m_Q \to \infty$, $m_Q(M_P)$ is unimportant for the low energy properties of the meson state, so it is more natural to use velocity v instead of momentum variable P. The normalization of the meson state is rewritten as [45]

$$\langle P(v')|P(v)\rangle = 2(2\pi)^3 v^+ \delta^3(\bar{\Lambda}v - \bar{\Lambda}v'), \tag{3.24}$$

where $\bar{\Lambda} = M_P - m_Q$ is the residual center mass of the heavy meson and the meson states have a relation $|P(v)\rangle = (M_P)^{-1/2}|P(P)\rangle$. In addition, since x is the longitudinal momentum fraction carried by the light antiquark, the meson wave function should be sharply peaked near $x \sim \Lambda_{\rm QCD}/m_Q$. It is thus clear that $x \to 0$ and only terms of the form $X \equiv x m_Q$ survive in the wave function as $m_Q \to \infty$; that is, X is independent of m_Q in the heavy quark limit. Therefore, the normalization of the wave function Eq. (2.10) is rewritten as

$$\int \frac{dXd^2k_\perp}{2(2\pi)^3}|\varphi(X, k_\perp)|^2 = 1, \tag{3.25}$$

where $\varphi(X, k_\perp) = (m_Q)^{-1/2}\phi(x, k_\perp)$. Other replacements are $A \to \tilde{A} = X + m_{q_2}$ and $\tilde{\phi}(x, k_\perp) \to \tilde{\varphi}(X, k_\perp)$. Thus we can rewrite (3.23) as

$$\begin{aligned}\langle r_{Qq_2}^2\rangle &= \langle r_Q^2\rangle + \langle r_{\bar{q}_2}^2\rangle \\ &= \frac{e_Q}{m_Q^2}\left\{-6\int\{dX\}X^2\tilde{\varphi}\left[(\tilde{A}^2+2k_\perp^2)\frac{d}{dk_\perp^2} + (\tilde{A}^2+k_\perp^2)k_\perp^2\left(\frac{d}{dk_\perp^2}\right)^2\right]\tilde{\varphi}\right\} \\ &\quad + e_{\bar{q}_2}\left\{-6\int\{dX\}\tilde{\varphi}\left[(\tilde{A}^2+2k_\perp^2)\frac{d}{dk_\perp^2} + (\tilde{A}^2+k_\perp^2)k_\perp^2\left(\frac{d}{dk_\perp^2}\right)^2\right]\tilde{\varphi}\right\}.\end{aligned} \tag{3.26}$$

The first term of Eq. (3.26) vanished when $m_Q \to \infty$. This means that not only $\langle r_{Qq_2}^2\rangle$ is blind to the flavor of Q, but also $\langle r_P^2\rangle$ is insensitive to m_1 for the heavy meson. The former is the so-called flavor symmetry and the latter will be proven in the numerical calculation.

IV. NUMERICAL RESULTS AND DISCUSSIONS

In the nonrelativistic (NR) approximation, we substituted the experimental data [1] to Eq. (2.7), and obtained $f_{B_s}/f_B = (1.03 \pm 0.02)f_{D_s}/f_D$. If one wanted to evaluate f_{B_s}/f_B and f_{D_s}/f_D individually, then $m_s = 483$ MeV and $m_q = 310$ MeV were the "best-fit" values for the pseudoscalar and vector light meson masses [46]. The ratios were

$$\frac{f_{D_s}}{f_D}\bigg|_{\rm NR} = 1.226 \pm 0.002, \qquad \frac{f_{B_s}}{f_B}\bigg|_{\rm NR} = 1.24 \pm 0.02. \tag{4.1}$$

The former was a little smaller than the data [1,2] $f_{D_s}/f_D|_{\rm exp} = 1.27 \pm 0.06$, and the latter was almost larger than the other theoretical calculations (see Tables II and III).

In the light-front framework, the momentum distribution amplitude $\phi(x, k_\perp)$ or the wave function $\Psi(\vec{r})$ in principle is unknown unless all the coefficients a_{n00} are obtained. However, we may suppose $a_{100}^c = 1$ or $a_{000}^g = 1$, that is, $\Psi(\vec{r}) = \Psi_{100}^c(\vec{r})$ or $\Psi(\vec{r}) = \Psi_{000}^g(\vec{r})$ as a trial wave function to fit the relevant data. The other a_{n00}'s will be subsumed if the parameters appearing in the momentum distribution amplitude cannot satisfy all experimental results. In other words, the coefficients a_{nlm} are taken as another kind of parameter. Of course, based on the principle of quantum mechanics, the physical meanings of these new parameters are clear. Here we list the first $\phi^{c(g)}_{n00}$:

$$\phi^c_{100}(x, k_\perp) = 8\left(\frac{2\pi}{\beta^3}\right)^{1/2}\sqrt{\frac{e_1 e_2}{x(1-x)M_0}}\left[\frac{\beta^2}{k_\perp^2 + k_z^2 + \beta^2}\right]^2, \tag{4.2}$$

$$\phi^g_{000}(x, k_\perp) = 4\left(\frac{\pi}{\beta^2}\right)^{3/4}\sqrt{\frac{e_1 e_2}{x(1-x)M_0}}\exp\left[-\frac{k_\perp^2 + k_z^2}{2\beta^2}\right], \tag{4.3}$$

and use the experimental data of $f_{\pi^+} = 130.4 \pm 0.2$ MeV and $\langle r^2_{\pi^+}\rangle^{1/2} = 0.672 \pm 0.008$ fm to fit the parameters m_q and β_{qq}. The results are $m_q = 0.172(0.251)$ GeV and $\beta_{qq} = 0.555 \mp 0.011(0.317 \mp 0.007)$ GeV for $\phi^c_{100}(\phi^g_{000})$. As for the strange quark mass, in Ref. [47] $m_s - m_u = 0.23$ GeV was obtained with some interaction potentials, while in Ref. [31] $m_s - m_u = 0.12$ GeV in the invariant meson mass scheme. So here we use the values $m_s - m_u = 0.180 \pm 0.050$ GeV and $f_{K^+} = 155.5 \pm 0.8$ MeV to fix β_{sq}. The results are $\beta_{sq} = 0.463^{-0.032}_{+0.054}(0.354^{-0.009}_{+0.015})$ GeV for $\phi^c_{100}(\phi^g_{000})$. The charge radius $\langle r^2_{K^+}\rangle^{1/2}$ and the mean square radius $\langle r^2_{K^0}\rangle$ were calculated by these parameters and were listed in Table I. We found that, on one hand, the value of $\langle r^2_{K^+}\rangle^{1/2}$ ($\langle r^2_{K^0}\rangle$) for ϕ^c_{100} was too large (small) than that obtained in the experiment. Then, the coefficients a^c_{n00} for $n > 1$ may be taken as nonzero to correct the fitting of $\langle r^2_{K^+}\rangle^{1/2}$ and $\langle r^2_{K^0}\rangle$. However, the mean square radii of ϕ^c_{n00} is greater when n is larger, or $\langle r^2\rangle_{\phi^c_{100}} < \langle r^2\rangle_{\phi^c_{200}} < \langle r^2\rangle_{\phi^c_{300}} < \ldots$ This means, for decreasing the value of $\langle r^2_{K^+}\rangle^{1/2}$, the values of a^c_{n00} must be artificially arranged in order to cancel out the contributions of ϕ^c_{n00} ($n \geq 2$) mutually. It is too hard to achieve now. On the other hand, the results for ϕ^g_{000} were consistent with the experimental data. Therefore we only use the Gaussian-type wave function, ϕ^g_{000}, to the following calculation. By combining Eq. (2.17), the experimental data [1], and the light quark mass in above, we obtained the ratios as

$$\left.\frac{\beta_{cs}}{\beta_{cq}}\right|^g = 1.20 \pm 0.04, \qquad \left.\frac{\beta_{bs}}{\beta_{bq}}\right|^g = 1.20 \pm 0.05. \tag{4.4}$$

TABLE I. Charge radius $\langle r^2_{K^+}\rangle^{1/2}$ and the mean square radius $\langle r^2_{K^0}\rangle$ of the experiment, this work, and other theoretical estimations. DS is Dyson-Schwinger equations; VMDχ is vector meson dominance plus an effective chiral theory; BS is Bethe-Salpeter equation.

	Experiment [1]	ϕ^c_{100}	ϕ^g_{000}	DS [48]	VMDχ [49]	pQCD [50]	BS [51]
$\langle r^2_{K^+}\rangle^{1/2}$ (fm)	0.560 ± 0.031	$0.710^{+0.033}_{-0.044}$	$0.607^{+0.010}_{-0.012}$	0.49	0.616	0.570	0.62
$\langle r^2_{K^0}\rangle$ (fm^2)	-0.077 ± 0.010	$-0.121^{-0.036}_{+0.039}$	$-0.072^{-0.017}_{+0.019}$	-0.020	0.057	-0.0736	-0.085

TABLE II. Theoretical calculations of the decay constants f_D, f_{D_s} (MeV), and the ratio f_{D_s}/f_D. QL is quenched lattice calculations, BS is Bethe-Salpeter equation, Linear and HO are the different potentials within LFQM. We have quoted only the value with $m_c = 1.5$ GeV in this work (LF).

	f_D	f_{D_s}	f_{D_s}/f_D
Experiment	205.8 ± 8.9 [1]	261.2 ± 6.9 [2]	1.27 ± 0.06[a]
This work (LF)	$\underline{205.8 \pm 8.9}$	264.5 ± 17.5	1.29 ± 0.07
This work (NR)			1.226 ± 0.002
Lattice (HPQCD + UKQCD) [3]	208 ± 4	241 ± 3	1.162 ± 0.009
QL (QCDSF) [4]	$206 \pm 6 \pm 3 \pm 22$	$220 \pm 6 \pm 5 \pm 11$	$1.068 \pm 0.018 \pm 0.020$
QL (Taiwan) [5]	$235 \pm 8 \pm 14$	$266 \pm 10 \pm 18$	$1.13 \pm 0.03 \pm 0.05$
Lattice (FNAL + MILC + HPQCD) [6]	$201 \pm 3 \pm 17$	$249 \pm 3 \pm 16$	$1.24 \pm 0.01 \pm 0.07$
QL (UKQCD) [8]	$210 \pm 10^{+17}_{-16}$	$238 \pm 8^{+17}_{-14}$	$1.13 \pm 0.02^{+0.04}_{-0.02}$
QL [9]	$211 \pm 14^{+0}_{-12}$	$231 \pm 12^{+6}_{-1}$	1.10 ± 0.02
QCD Sum Rules [10]	177 ± 21	205 ± 22	$1.16 \pm 0.01 \pm 0.02$
QCD Sum Rules [12]	203 ± 23	235 ± 24	1.15 ± 0.04
Field Correlators [14]	210 ± 10	260 ± 10	1.24 ± 0.04
Potential Model [15]	234	268	1.15
BS [16]	230 ± 25	248 ± 27	1.08 ± 0.01
BS [17]	238	241	1.01
Linear{HO} [18]	211{194}	248{233}	1.18{1.20}

[a]This value is obtained by combining $f_D = 205.8 \pm 8.9$ MeV [1] and $f_{D_s} = 261.2 \pm 6.9$ MeV [2].

TABLE III. Theoretical calculations of the decay constants f_B, f_{B_s} (MeV), and the ratio f_{B_s}/f_B. Only the value with $m_b = 4.7$ GeV has been quoted in this work (LF).

	f_B	f_{B_s}	f_{B_s}/f_B
Experiment	204 ± 31[a]		
This work (LF)	$\underline{204 \pm 31}$	270.0 ± 42.8	1.32 ± 0.08
This work (NR)			1.24 ± 0.02
QL (QCDSF) [4]	$190 \pm 8 \pm 23 \pm 25$	$205 \pm 7 \pm 26 \pm 17$	$1.080 \pm 0.028 \pm 0.031$
Lattice (HPQCD) [7]	$216 \pm 9 \pm 19 \pm 4 \pm 6$	259 ± 32	$1.20 \pm 0.03 \pm 0.01$
QL (UKQCD) [8]	$177 \pm 17 \pm 22$	$204 \pm 12^{+24}_{-23}$	$1.15 \pm 0.02^{+0.04}_{-0.02}$
QL [9]	$179 \pm 18^{+26}_{-9}$	$204 \pm 16^{+28}_{-0}$	$1.14 \pm 0.03^{+0.00}_{-0.01}$
QCD Sum Rules [11]	178 ± 14	200 ± 14	$1.12 \pm 0.01 \pm 0.03$
QCD Sum Rules [12]	203 ± 23	236 ± 30	1.16 ± 0.05
QCD Sum Rules [13]	210 ± 19	244 ± 21	1.16
Field Correlators [14]	182 ± 8	216 ± 8	1.19 ± 0.03
Potential Model [15]	189	218	1.15
BS [16]	196 ± 29	216 ± 32	1.10 ± 0.01
BS [17]	193	195	1.01
Linear{HO} [18]	189{180}	234{237}	1.24{1.32}

[a]This value is extracted by the branching ratio: $\mathcal{B}(B^- \to \tau^- \bar{\nu}) = (1.42 \pm 0.43) \times 10^{-4}$ [52].

Obviously, the $SU(3)$ symmetry breaking is the major contribution to the ratios in Eq. (4.4).

For the heavy quark masses, the quite different values were also used in the model calculations. For example, $m_c = 1.38$ GeV and $m_b = 4.76$ GeV which were fitted for the spectrum of the p-wave charmonium and bottomonium states [53]; and $m_c = 1.8$ GeV and $m_b = 5.2$ GeV which were obtained from the potential models and the variational principle [54]. Here the values $m_c = 1.2$, 1.5, 1.8 GeV and $m_b = 4.2$, 4.7, 5.2 GeV were taken into account. By combining Eqs. (3.15), (4.3), and (4.4), and the quark masses $m_{q(s)} = 0.251(0.431)$ GeV, the dependences of f_{D_s}/f_D on f_D with three different m_c's and f_{B_s}/f_B on f_B with three different m_b's were shown in Fig. 1 and 2, respectively. It was easily found that the ratios f_{D_s}/f_D and f_{B_s}/f_B were not only insensitive to the heavy quark masses m_c and m_b, but also insensitive to the decay constants f_D and f_B, respectively.

Recently, the CLEO collaboration updated their data about the branching fraction for the purely leptonic decay $D^+ \to \mu^+ \nu$ and reported [1] $f^{\rm exp}_{D^+} = 205.8 \pm 8.9$ MeV. By using this value, we determined the ratio $f_{D_s}/f_D = 1.29 \pm 0.07$ and the decay constant $f_{D_s} = 264.5 \pm 17.5$ MeV with $m_c = 1.5$ GeV. We found these results were consistent with the data [2]: $f^{\rm exp}_{D_s^+} = 261.2 \pm 6.9$ MeV and $f^{\rm exp}_{D_s^+}/f^{\rm exp}_{D^+} = 1.27 \pm 0.06$, which were the average of the CLEO and Belle results (which included the radiative corrections). In addition, our values were generally larger than the other theoretical calculations. Table II compares

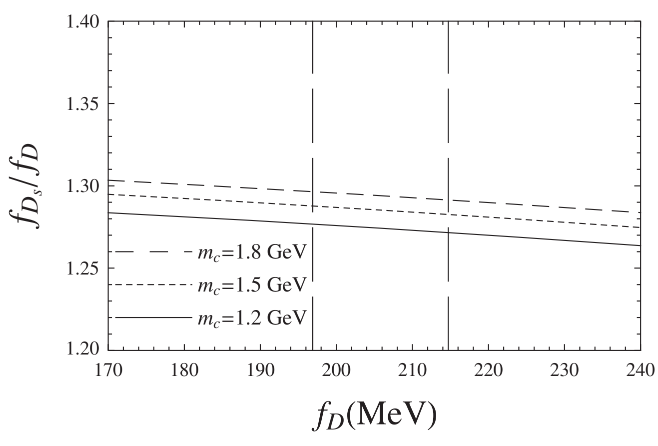

FIG. 1. Dependence of f_{D_s}/f_D on f_D with $m_c = 1.2$, 1.5, 1.8 GeV. The left and right vertical dash lines correspond to the lower and upper limits of the data $f_D = 205.8 \pm 8.9$ MeV, respectively.

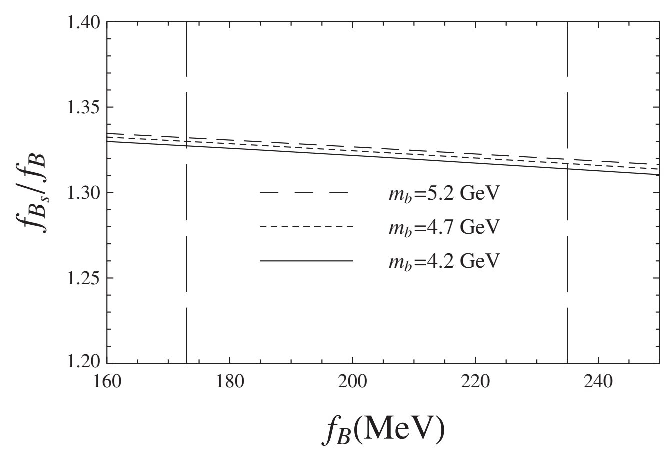

FIG. 2. Dependence of f_{B_s}/f_B on f_B with $m_c = 4.2$, 4.7, 5.2 GeV. The left and right vertical dash lines correspond to the lower and upper limits of the data $f_B = 204 \pm 31$ MeV, respectively.

the theoretical calculations with experimental value. For the bottom sector, the Belle [55] and *BABAR* [56,57] collaborations found evidence for $B^- \to \tau^- \bar{\nu}$ decay which was not helicity suppressed. However, the Belle and *BABAR* values had 3.5 and 2.6 standard-deviation significances, respectively; thus the average was provisional [52]: $\mathcal{B}(B^- \to \tau^- \bar{\nu}) = (1.42 \pm 0.43) \times 10^{-4}$. We extracted the decay constant $f_B^{\rm exp} = 204 \pm 31$ MeV. By using this value, the ratio $f_{B_s}/f_B = 1.32 \pm 0.08$ and the decay constant $f_{B_s} = 270.0 \pm 42.8$ MeV with $m_b = 4.7$ GeV were obtained. Table III compares the theoretical calculations with the experimental value. Similar to the charm sector, our ratio f_{B_s}/f_B was almost larger than all other calculations. It is worth mentioning that the decay constants of both pseudoscalar and vector heavy mesons have already been investigated by the author of Ref. [18] with the analysis of magnetic dipole decays of various heavy flavored mesons in the light-front quark model. The parameters in Ref. [18] were constrained by the variational principle for the QCD-motivated effective Hamiltonian. Roughly speaking, our above results showed the flavor $SU(3)$ symmetry breaking $m_s/m_q = 1.72 \pm 0.20$ leads into the ratios

$$\frac{f_{D_s}}{f_D} = 1.29 \pm 0.07, \qquad \frac{f_{B_s}}{f_B} = 1.32 \pm 0.08.$$

For the B_c meson, however, both the decay constant and the hyperfine splitting have not been measured yet. We considered the ratio of hyperfine mass differences:

$$\frac{\Delta M_{B_c B_c^*}}{\Delta M_{BB^*}} = \frac{m_q}{m_c}\left(\frac{\beta_{bc}}{\beta_{bq}}\right)^3. \tag{4.5}$$

Similar to the above cases, the ratio f_{B_c}/f_B was insensitive to the value of m_b and sensitive to that of m_c/m_q. The dependences of f_{B_c}/f_B on $\Delta M_{B_c B_c^*}$ with $m_c = 1.2$, 1.5, 1.8 GeV and $m_b = 4.7$ GeV were shown in Fig. 3. Some model predictions were made for f_{B_c} [15,54,58–62], and the range of these values was $f_{B_c} = 360 \sim 517$ MeV or $f_{B_c}/f_B = 1.76 \sim 2.53$. As shown in Fig. 3, this range corresponded to $\Delta M_{B_c B_c^*} = 16 \sim 64$ MeV. We found that this result was consistent with a calculation using the nonrelativistic renormalization group [63] $\Delta M_{B_c B_c^*} = 48 \pm 15^{+14}_{-11}$ MeV.

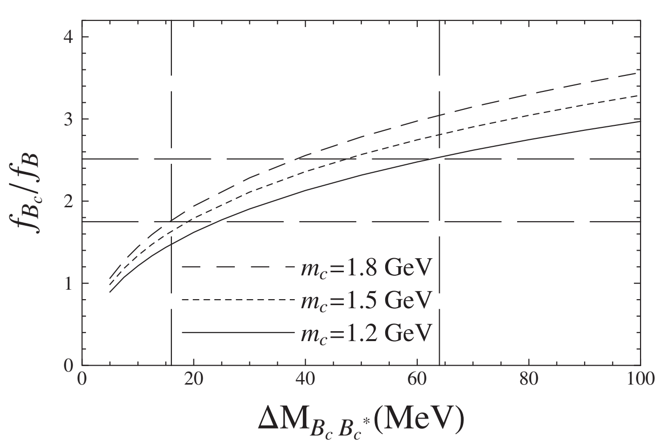

FIG. 3. Dependences of f_{B_c}/f_B on $\Delta M_{B_c B_c^*}$ with $m_c = 1.2$, 1.5, 1.8 GeV and $m_b = 4.7$ GeV. The low and high horizontal dash lines correspond to $f_{B_c} = 360$ MeV and $f_{B_c} = 517$ MeV, respectively. The left and right vertical dash lines correspond to $\Delta M_{B_c B_c^*} = 16$ MeV and $\Delta M_{B_c B_c^*} = 64$ MeV, respectively.

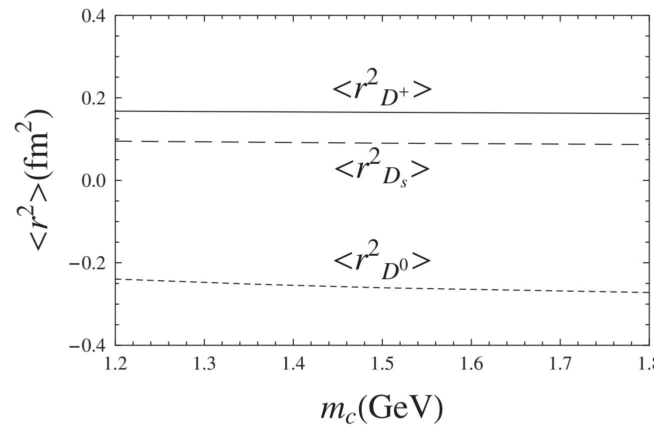

FIG. 4. Dependences of $\langle r^2_{D^+,D^0,D_s}\rangle$ on $m_c = 1.2 \sim 1.8$ GeV.

Besides, the mean square radii of the heavy meson are calculated by the above parameters and Eq. (3.23). The dependences of $\langle r^2_{D^+,D^0,D_s}\rangle$ on m_c and $\langle r^2_{B^+,B^0,B_s}\rangle$ on m_b were shown in Figs. 4 and 5, respectively. It was easily found that, as mentioned at the end of Sec. III, the mean square radii $\langle r^2_{D^+,D^0,D_s}\rangle$ and $\langle r^2_{B^+,B^0,B_s}\rangle$ were insensitive to the heavy quark masses m_c and m_b, respectively. We used $m_c = 1.5$ GeV and $m_b = 4.7$ GeV to estimate the mean square radii of the heavy meson and the results are listed in Table IV. It is interesting to note that the value $\langle r^2_{B^+}\rangle$ is slightly lower but comparable to what one would obtain from the lattice calculation of Ref. [27], and it is considerably smaller than the results obtained by applying the

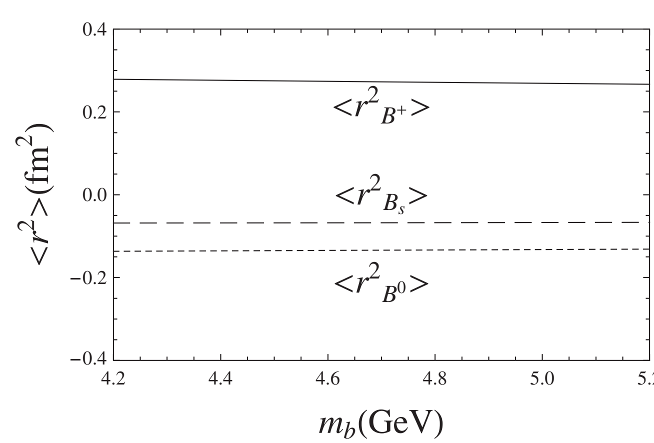

FIG. 5. Dependences of $\langle r^2_{B^+,B^0,B_s}\rangle$ on $m_c = 4.2 \sim 5.2$ GeV.

TABLE IV. Mean square radius $\langle r_P^2\rangle$ (fm^2) of this work and the other theoretical calculations.

	This work	Lattice [27]	VMD
D^+	$0.165^{-0.010}_{+0.011}$		
D^0	$-0.261^{+0.018}_{-0.019}$		
D_s^+	$0.0902^{-0.011}_{+0.014}$		
B^+	$0.273^{-0.043}_{+0.059}$	0.334 ± 0.003	0.393[a]
B^0	$-0.134^{+0.022}_{-0.029}$		
B_s^0	$-0.0676^{+0.0141}_{-0.0189}$		
B_c^+	$0.0277 \sim 0.0451$[b]		

[a]This value is obtained by $\langle r^2\rangle_{\rm VMD} = 6/M_\rho^2$.
[b]This value is obtained by $f_{B_c} = 360 \sim 517$ MeV.

simple vector meson dominance. The $SU(3)$ symmetry breaking in $\langle r_P^2\rangle^{1/2}$, which mainly come from the mass difference $m_s - m_q = 180 \pm 50$ MeV, are obtained as

$$\sqrt{\frac{\langle r^2_{D_s^+}\rangle}{\langle r^2_{D^+}\rangle}} = 0.740^{-0.041}_{+0.050}, \qquad \sqrt{\frac{\langle r^2_{B_s^0}\rangle}{\langle r^2_{B^0}\rangle}} = 0.711^{-0.049}_{+0.058}.$$

The radius ratio of B_c^+ and B^+ are also obtained as

$$\sqrt{\frac{\langle r^2_{B_c^+}\rangle}{\langle r^2_{B^+}\rangle}} = 0.407^{+0.037}_{-0.038} \sim 0.319^{+0.029}_{-0.030},$$

which corresponds to the range $f_{B_c} = 360 \sim 517$ MeV.

V. CONCLUSIONS

In this study, we discussed the ratios of decay constants and mean square radii for pseudoscalar heavy mesons. By considering the hyperfine interaction inside the meson, we found that the ratio of light quark masses m_s/m_q was the important factor for determining the ratio of the decay constants. First, in the nonrelativistic approximation, we obtained the relation $f_{B_s}/f_B = (1.05 \pm 0.02) f_{D_s}/f_D$ which did not use any parameters. These two ratios were individually evaluated by including the "best-fit" light quark masses $m_s/m_q = 483/310 = 1.558$ and the values $f_{D_s}/f_D = 1.226 \pm 0.002$ and $f_{B_s}/f_B = 1.24 \pm 0.02$ were obtained. Second, in the light-front framework, we utilized the mass difference of light quark masses $m_s - m_q = 180 \pm 50$ MeV and the fittings of the decay constants for light mesons to compare the mean square radii of $K^{+,0}$ mesons in the power-law and Gaussian momentum distribution amplitudes. The latter was consistent with the data and it extracted the light quark masses ratio $m_s/m_q = 1.72 \pm 0.20$. This mass ratio led to $f_{D_s}/f_D = 1.29 \pm 0.07$ and $f_{B_s}/f_B = 1.32 \pm 0.08$. The former was in agreement with the experimental data and the latter was almost larger than all other theoretical calculations. Both these ratios were not only insensitive to the heavy quark masses $m_{c,b}$, but also insensitive to the decay constants $f_{D,B}$. Similar to the above, the ratio f_{B_c}/f_B was mainly determined by the mass ratio m_c/m_q and the mass splitting $\Delta M_{B_c B_c^*}$. The dependences of f_{B_c}/f_B on $\Delta M_{B_c B_c^*}$ with the varied charm quark masses have been shown. We found that $f_{B_c}/f_B = 1.76 \sim 2.53$ corresponded to $\Delta M_{B_c B_c^*} = 16 \sim 64$ MeV. In addition, the mean square radii of heavy meson were estimated. We found the mean square radii $\langle r^2_{D^+,D^0,D_s}\rangle$ and $\langle r^2_{B^+,B^0,B_s}\rangle$ were insensitive to the heavy quark masses m_c and m_b, respectively, which was consistent with the behavior when the heavy quark limit was taken. Our $\langle r^2_{B^+}\rangle$ was slightly lower but comparable to that of lattice calculation, and was considerably smaller than that of vector meson dominance (VMD). The light quark mass ratio and the range of f_{B_c} given above also led the radius ratios $\sqrt{\langle r^2_{D_s^+}\rangle/\langle r^2_{D^+}\rangle} = 0.740^{-0.041}_{+0.050}$, $\sqrt{\langle r^2_{B_s^0}\rangle/\langle r^2_{B^0}\rangle} = 0.711^{-0.049}_{+0.058}$, and $\sqrt{\langle r^2_{B_c^+}\rangle/\langle r^2_{B^+}\rangle} = 0.407^{+0.037}_{-0.038} \sim 0.319^{+0.029}_{-0.030}$, respectively.

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