Magnetic moments of octet baryons, angular momenta of quarks, and sea antiquark polarizations

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One can determine antiquark polarizations in a proton using the information from deep inelastic scattering, β decays of baryons, orbital angular momenta of quarks, as well as their integrated magnetic distributions. The last quantities were determined previously by us performing a fit to magnetic moments of a baryon octet. However, because of the $SU(3)$ symmetry our results depend on two parameters. The quantity Γ_V, measured recently in a COMPASS experiment, gives the relation between these parameters. We can fix the last unknown parameter using the ratio of up and down quark magnetic moments which one can get from the fit to radiative vector meson decays. We calculate antiquark polarizations with the orbital momenta of valence quarks that follow from lattice calculations. The value of the difference of up and down antiquark polarizations obtained in our calculations is consistent with the result obtained in a HERMES experiment.

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I. INTRODUCTION AND FRAMEWORK

In Ref. [1] we have proposed a model for magnetic moments of $SU(3)$ octet baryons. We get an excellent fit using few parameters. In this model magnetic moments of baryons are sums of products of magnetic moments of quarks and corresponding integrated quark densities. The corrections, which take into account exchange phenomena, were also included. To determine the size of such corrections we use sum rules for magnetic moments of octet baryons (as in [1,2]). After subtraction of the exchange contributions we are left with a $SU(3)$ symmetric part of these moments, which can be expressed as independent contributions from quarks and antiquarks in considered baryon. The magnetic moment of a quark as well as its integrated magnetic density are Q^2 dependent, whereas its product is not. Other corrections coming from exchange effects of pions and gluons are incorporated in redefinition magnetic moments of quarks and their integrated densities; hence magnetic moments of quarks are not equal to their Dirac values.

So, after subtracting the pion correction to nucleon magnetic moments and taking into account $\Sigma^0 - \Lambda$ mixing, we are left with independent one particle contributions to baryon magnetic moments (sum rules for magnetic moments are satisfied) and we use for them high energy parametrization (integrated parton densities) to describe such contributions. We believe that most of all other pion exchange and gluon exchange corrections are taken into account in the high energy parametrization (see, e.g., [3]). For such parametrization, in the case of axial densities, one does not include explicitly pion and gluon corrections and we do the same for integrated magnetic densities. In contrary such corrections are present in models of bound quarks, e.g., in [4–6]. One gluon correction with gluon exchanged between different quarks can also correspond to higher twist diagrams in deep inelastic scattering.

In integrated magnetic quark densities, besides spin contributions, we have also orbital angular momentum contributions (see also [7]). Here we shall consider two models: the one in which we neglect orbital angular momentum contribution and the second with such contribution included. In the first case we have for such integrated densities

$$\delta q \equiv \Delta q_{\rm val} + \Delta q_{\rm sea} - \Delta\bar{q}. \tag{1}$$

In the case with angular momentum the formulas are

$$\delta_L q \equiv \Delta q_{\rm val} + \Delta q_{\rm sea} - \Delta\bar{q} + L_q, \tag{2}$$

where

$$L_q = \langle \hat{L}_z^q\rangle - \langle \hat{L}_z^{\bar{q}}\rangle = \langle \hat{L}_z^{q_{\rm val}}\rangle + \langle \hat{L}_z^{q_{\rm sea}}\rangle - \langle \hat{L}_z^{\bar{q}_{sea}}\rangle. \tag{3}$$

Taking into account exchange contributions (as was explained in detail in [1]), i.e., isovector contribution connected with charged pion exchange between different quarks (see also Franklin [8,9]) and $\Sigma^0 - \Lambda$ mixing, the $SU(3)$ symmetric part of baryon octet magnetic moments can be parametrized in terms of four quantities: c_0, c_3, c_8, r. From the fit we get for these parameters [1]

$$c_0 = 0.054 \pm 0.001\,\text{n.m.}, \qquad c_3 = 1.046 \pm 0.005\,\text{n.m.},$$
$$c_8 = 0.193 \pm 0.000\,\text{n.m.}, \qquad r = 1.395 \pm 0.010. \tag{4}$$

Hence, six quantities: three quark magnetic moments and three quark densities cannot be determined using only four parameters given in Eq. (4). So as in [1], we introduce two additional parameters, ϵ and g, and our quantities become the functions of them. The parameters ϵ and g are defined in Eqs. (5) and (6):

$$\epsilon = -1 - 2\frac{\mu_d}{\mu_u}, \tag{5}$$

$$g = \delta_L u - \delta_L d. \tag{6}$$

Now we can express magnetic quark densities as

$$\begin{aligned} \delta_L u &= \frac{g}{6r}[f(\epsilon) + 1 + 3r], \\ \delta_L d &= \frac{g}{6r}[f(\epsilon) + 1 - 3r], \\ \delta_L s &= \frac{g}{6r}[f(\epsilon) - 2], \end{aligned} \tag{7}$$

where

$$f(\epsilon) = \frac{(3+\epsilon)rc_0}{c_3 - 3rc_8 - \epsilon(c_3 + rc_8)}. \tag{8}$$

One can also express magnetic moments of u, d, and s quarks in terms of our parameters ϵ and g:

$$\mu_u = \frac{8c_3}{g(3+\epsilon)}, \qquad \mu_d = -\frac{4(1+\epsilon)c_3}{g(3+\epsilon)}, \qquad \mu_s = -\frac{2[9rc_8 - c_3 + \epsilon(c_3 + 3rc_8)]}{g(3+\epsilon)}. \tag{9}$$

The parameter g sets a scale at which we have calculated our quantities. From Eqs. (7) and (9) we have that $\mu_q \delta_L q$ and quark magnetic moment ratios, e.g., μ_u/μ_d, do not depend on g.

The new quantity Γ_V, which in our notation is

$$\Gamma_V \equiv \delta u + \delta d = \delta_L u + \delta_L d - L_u - L_d, \tag{10}$$

is measured in the COMPASS experiment [10] and one gets

$$\Gamma_V = 0.41 \pm 0.07(\text{stat}) \pm 0.06(\text{syst}). \tag{11}$$

In the case when $\Delta q_{\text{sea}} \neq \Delta\bar{q}$ this quantity is not a valence one: $\Delta u_{\text{val}} + \Delta d_{\text{val}}$.

Using Eqs. (7) and (10) we can express our parameter g as

$$g = \frac{3r(\Gamma_V + L_u + L_d)}{f(\epsilon) + 1}. \tag{12}$$

Hence, the COMPASS measurement gives the relation between introduced parameters ϵ and g. So we will have only one unknown parameter however, the orbital angular momenta of quarks are present in the formulas.

We know that integrated axial densities, used in deep inelastic scattering analysis, differ from δq by a sign in an antiquark term:

$$\Delta q \equiv \Delta q_{\text{val}} + \Delta q_{\text{sea}} + \Delta\bar{q}. \tag{13}$$

From Eqs. (1), (2), and (13) we can express $\Delta\bar{q}$ as

$$\Delta\bar{q} = \tfrac{1}{2}(\Delta q - \delta q) = \tfrac{1}{2}(\Delta q - \delta_L q + L_q). \tag{14}$$

Let us express the function $f(\epsilon)$ in the form

$$f(\epsilon) = \frac{3r(\Gamma_v + L_u + L_d)}{a_3 - 2\eta + L_u - L_d} - 1, \tag{15}$$

where η is defined by

$$\eta \equiv \Delta\bar{u} - \Delta\bar{d}. \tag{16}$$

Equation (15) gives us a relation between our two basic parameters ϵ and η (which replaces parameter g). The quark integrated axial densities Δu, Δd, Δs can be determined from

$$\begin{aligned} \Delta u &= \tfrac{1}{3}a_0 + \tfrac{1}{6}a_8 + \tfrac{1}{2}a_3, \\ \Delta d &= \tfrac{1}{3}a_0 + \tfrac{1}{6}a_8 - \tfrac{1}{2}a_3, \\ \Delta s &= \tfrac{1}{3}a_0 - \tfrac{1}{3}a_8, \end{aligned} \tag{17}$$

where the values of a_3, a_8 and a_0 are obtained from neutron and hyperon β decays [11] and deep inelastic scattering spin experiments [12]. We have

$$a_0 = 0.33 \pm 0.06, \qquad a_8 = 0.585 \pm 0.025, \qquad a_3 = 1.2694 \pm 0.0028. \tag{18}$$

We can get additional information (although not very precise) using the value of η from the HERMES experiment [13]. We will take $\eta = 0.05 \pm 0.06$ which was however measured not in the whole range of x: $(0.023 \leq x \leq 0.6)$.

Hence we can express $\Delta\bar{u}$, $\Delta\bar{d}$, and $\Delta\bar{s}$ as a function of parameter η using Eqs. (7), (12), (15), and (17), getting

$$\begin{aligned} \Delta\bar{u} &= \frac{1}{6}a_0 + \frac{1}{12}a_8 - \frac{1}{4}\Gamma_V + \frac{1}{2}\eta, \\ \Delta\bar{d} &= \frac{1}{6}a_0 + \frac{1}{12}a_8 - \frac{1}{4}\Gamma_V - \frac{1}{2}\eta, \\ \Delta\bar{s} &= \frac{1}{6}a_0 - \frac{1}{6}a_8 - \frac{1}{4}\Gamma_V + \frac{a_3 - 2\eta}{4r} + \frac{L_u - L_d}{4r} \\ &\quad - \frac{L_u + L_d - 2L_s}{4}. \end{aligned} \tag{19}$$

One sees that $\Delta\bar{u}$, and $\Delta\bar{d}$ do not depend directly on orbital angular momenta. Knowing the precise value of η in the whole range $(0 \leq x \leq 1)$ one is able to determine $\Delta\bar{u}$ and $\Delta\bar{d}$ in our model.

II. NUMERICAL RESULTS AND DISCUSSION

A. Angular momenta of quarks neglected

Let us start with an assumption that all angular momenta of quarks are negligible (i.e., we put them equal to zero). In Fig. 1 we present antiquark polarizations $\Delta\bar{q}$ for u, d, and s quarks as a functions of parameter η.

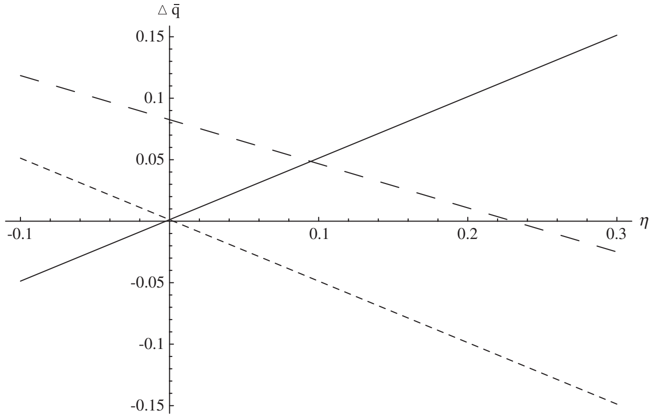

FIG. 1. The antiquark polarizations for $\bar{u}$ (solid line), $\bar{d}$ (short-dashed line), and $\bar{s}$ (long-dashed line) versus η in the model where angular momenta of quarks are neglected.

The fact that $\Delta\bar{u} + \Delta\bar{d} \approx 0$ is connected with the value of Γ_V measured by the COMPASS experiment. We can also see how the values of $\Delta\bar{u}$, $\Delta\bar{d}$, and $\Delta\bar{u}$ change when we change η between -0.01 and 0.11, i.e., within 1 standard deviation off central value. When we take the number from the HERMES experiment ($\eta = 0.05$), we can determine polarizations of all sea antiquarks:

$$\Delta\bar{u} = 0.03 \pm 0.04, \qquad \Delta\bar{d} = -0.02 \pm 0.04, \qquad \Delta\bar{s} = 0.06 \pm 0.03. \tag{20}$$

The values are a little bit different from the antiquark values quoted by HERMES [13]. Using Eqs. (4), (7), and (15) we can calculate the corresponding value of parameter ϵ. We get $\epsilon = -0.17$ for $\eta = 0.05$. In general we can use Eq. (15) to eliminate η and express $\Delta\bar{q}$ as a function of ϵ in the case when we neglect dependence on orbital angular momenta. In Fig. 2 we show such dependence, i.e., $\Delta\bar{q}(\epsilon)$.

One can try to determine the value of parameter ϵ using the experimental data for radiative vector meson decays.

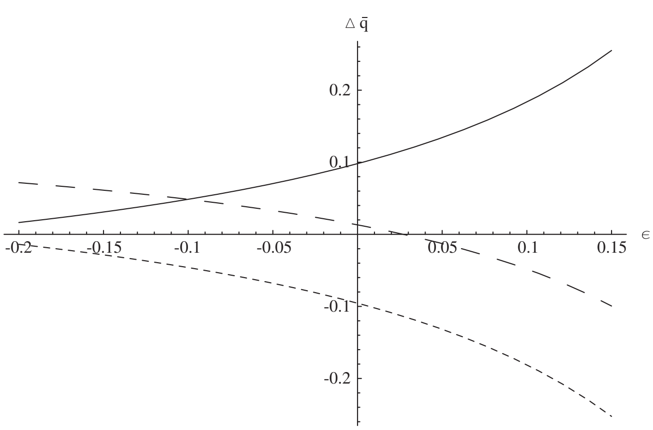

FIG. 2. The antiquark polarizations for $\bar{u}$ (solid line), $\bar{d}$ (short-dashed line), and $\bar{s}$ (long-dashed line) versus ϵ in the model where angular momenta of quarks are neglected.

The model which is used to determine $\frac{\mu_u}{\mu_d}$ is not as sophisticated as is the one for baryon magnetic moments; we have used similar formulas as in [14]. One does not include the contribution from orbital momenta of quarks in such a model; however, in [15] it was shown that such contributions may be small. Performing the fit one gets $\frac{\mu_u}{\mu_d} = -1.87 \pm 0.07$ which gives, with the help of Eq. (5), $\epsilon = 0.06 \pm 0.04$. If we use this value of parameter ϵ we will get for $\Delta\bar{q}$

$$\Delta\bar{u} = 0.14 \pm 0.07, \qquad \Delta\bar{d} = -0.14 \pm 0.04, \qquad \Delta\bar{s} = -0.02 \pm 0.03. \tag{21}$$

In this case the corresponding value of parameter η is 0.28 ± 0.08. It looks as if $\Delta\bar{q}$ calculated from the HERMES value of η and $\Delta\bar{q}$ calculated using ϵ gotten from vector meson decays are not consistent.

B. Angular momenta of quarks taken into account

Now we shall consider the model with nonzero orbital angular momenta of quarks. We will use the values of such momenta calculated numerically on the lattice. From [16] we have

$$L_u = L_u^{\rm val}(\text{lattice}) = -0.195 \pm 0.044, \qquad L_d = L_d^{\rm val}(\text{lattice}) = 0.200 \pm 0.044. \tag{22}$$

These values are determined at $Q^2 = 4\ \text{GeV}^2$. Let us make some comments. From Eq. (3) angular momentum of quarks consists of angular momentum of valence quarks, sea quarks, and antiquarks. From [16] we have only information on valence quark contribution. We neglect the rest because of lack of knowledge; it actually means that we assume that orbital angular momenta of sea quarks and antiquarks are equal [see Eq. (3)]. The existence of a small correction to this hypothesis cannot be excluded. The orbital angular momentum of quarks is scale dependent [17]. There are two possibilities: First one can take into account evolution equations for angular momenta and start with initial conditions at low energies taking as is suggested by A. W. Thomas values that follow from the cloudy bag model [18,19], which take into account the relativistic motion of quarks, chiral pion cloud, and one gluon exchange corrections. Second, one can take initial conditions at high energy as was done in [20]. In our case we use high energy parameters so it is natural to use high energy initial conditions as in [20]. From our procedure it seems that we cannot go in Q^2 scale below 1 GeV^2. From [20] it follows that the angular momenta of quarks for $Q^2 > 1\ \text{GeV}^2$ are only weakly scale dependent. The angular momenta of valence quarks and Γ_V determine the scale used in our equations. Using Eq. (19) and eliminating η [using Eq. (15)] we can get formulas for $\Delta\bar{q}$ (for u, d, and s quarks) with orbital angular momenta taken into account:

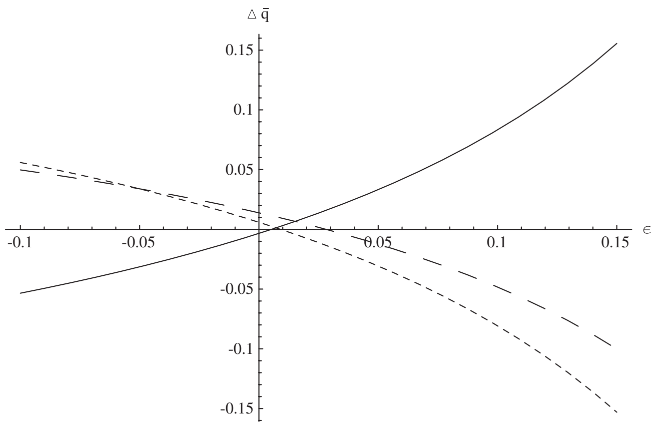

FIG. 3. The antiquark polarizations for $\bar{u}$ (solid line), $\bar{d}$ (short-dashed line), and $\bar{s}$ (long-dashed line) versus ϵ in the model where angular momenta of quarks are taken into account.

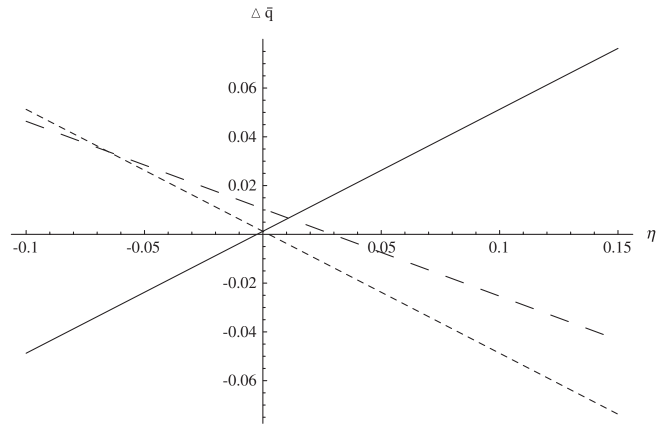

FIG. 4. The antiquark polarizations for $\bar{u}$ (solid line), $\bar{d}$ (short-dashed line), and $\bar{s}$ (long-dashed line) versus η in the model where angular momenta of quarks are taken into account.

$$\Delta\bar{u} = \frac{1}{6}a_0 + \frac{1}{12}a_8 + \frac{1}{4}a_3 - \frac{1}{4}\Gamma_V - \frac{3r}{4}\frac{\Gamma_V + L_u + L_d}{f(\epsilon)+1} + \frac{1}{4}(L_u - L_d),$$
$$\Delta\bar{d} = \frac{1}{6}a_0 + \frac{1}{12}a_8 - \frac{1}{4}a_3 - \frac{1}{4}\Gamma_V + \frac{3r}{4}\frac{\Gamma_V + L_u + L_d}{f(\epsilon)+1} - \frac{1}{4}(L_u - L_d), \tag{23}$$
$$\Delta\bar{s} = \frac{1}{6}a_0 - \frac{1}{6}a_8 - \frac{1}{4}\Gamma_V + \frac{3r}{4}\frac{\Gamma_V + L_u + L_d}{f(\epsilon)+1} - \frac{1}{4}(L_u + L_d - 2L_s).$$

The dependence of $\Delta\bar{q}$ on ϵ, calculated from Eq. (23), is shown in Fig. 3.

When we use the result for ϵ from the fit to radiative decays of vector mesons we get for $\Delta\bar{u}$, $\Delta\bar{d}$, and $\Delta\bar{s}$

$$\Delta\bar{u} = 0.04 \pm 0.08, \qquad \Delta\bar{d} = -0.04 \pm 0.05, \qquad \Delta\bar{s} = -0.02 \pm 0.03. \tag{24}$$

The errors are quite big so the determination is not very conclusive. For $\eta = \Delta\bar{u} - \Delta\bar{d}$ we get the value 0.08 ± 0.09 which has to be compared with 0.05 ± 0.06. The agreement is reasonable despite the fact that all errors are relatively big. Let us stress that we have used the value of ϵ calculated from the fit to experimental data on radiative vector meson decays and have taken into account orbital angular momenta of quarks from lattice calculations.

If we do not want to use the information about ϵ from the fit to meson decays, we can use Eq. (19) where this parameter is eliminated and $\Delta\bar{q}$ are functions of η. The dependence of $\Delta\bar{q}$ on η for u, d, and s antiquarks is shown in Fig. 4.

If we knew precisely the value of η we could predict $\Delta\bar{u}$, $\Delta\bar{d}$, and $\Delta\bar{s}$ values. From Eq. (19) we see that $\Delta\bar{u}$ and $\Delta\bar{d}$ do not depend on orbital angular momenta of quarks; only $\Delta\bar{s}$ does. It means that precise determination of $\Delta\bar{s}$ could be the additional test of the importance of orbital angular momenta of quarks. When we use the η value obtained in the HERMES experiment we will get $\Delta\bar{u}$ and $\Delta\bar{d}$ as in Eq. (20) and $\Delta\bar{s} = -0.007 \pm 0.04$. We cannot expect additional experimental information about magnetic moments of quarks. In the future more precise measurements of antiquark polarizations could be a real verification of our model.

We also want to give a comparison of two models, i.e., without and with orbital angular momenta of quarks taken into account. The relation that follows from the COMPASS measurement of Γ_V, Eq. (15), could be rewritten in the form

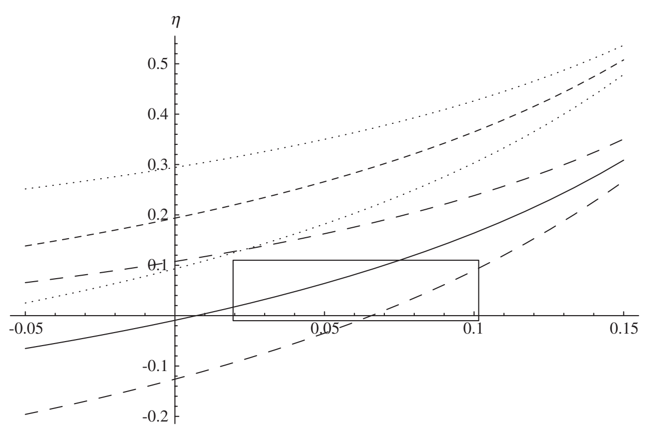

FIG. 5. The parameter η versus ϵ in the model where angular momenta of quarks are taken into account (solid line) and corresponding errors (long-dashed lines) compared to curves in the model where angular momenta are neglected (short-dashed lines) and corresponding errors (dotted lines). The size of the rectangle is determined by the error of ϵ from radiative vector meson decays and the error in the measurement of η in the HERMES experiment.

$$\eta = \frac{1}{2}a_3 - \frac{3r}{2}\frac{\Gamma_V + L_u + L_d}{f(\epsilon) + 1} + \frac{1}{2}(L_u - L_d). \qquad (25)$$

In Fig. 5 we show the function $\eta(\epsilon)$ for both models with the corresponding errors. The rectangle is given by the errors (1 standard deviation) of ϵ and η. It seems that it is an indication in favor of including nonzero orbital angular momenta of quarks.

III. CONCLUSIONS

From the COMPASS measurement of Γ_V we have a relation between two parameters not determined in our previous fit to magnetic moments of baryons. Hence, we have only one independent parameter and it could be either ϵ or η. We have discussed two possibilities: the first with inclusion of the orbital angular momenta of quarks suggested by calculations on the lattice and the second with such angular momenta neglected. We have presented in both cases the dependence of antiquark polarizations on a single independent parameter (being ϵ or η). The results are plotted in Figs. 1 to 4. In order to find the antiquark polarizations we can use the result for η from the HERMES experiment or the value of ϵ obtained from the fit to radiative vector meson decays. Unfortunately the errors are big and the results are not very conclusive.

The relation between η and ϵ plotted with errors in Fig. 5 shows that the solution with orbital angular momenta of valence quarks taken into account is preferred.

With orbital angular momenta taken into account and the value of ϵ taken from the fit to radiative vector meson decays, we obtain values of antiquark polarizations. Such a procedure gives the prediction $\eta = 0.08 \pm 0.09$ that seems to be consistent with the value 0.05 ± 0.06 from the HERMES experiment. Because it is difficult to get additional information on magnetic moments of quarks, it seems that more precise values of antiquark polarizations (maybe from a Jefferson Lab experiment) could be a real verification of our model.

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