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Transverse momentum dependent quark distributions and polarized Drell-Yan processes

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We study the spin-dependent quark distributions at large transverse momentum in the collinear factorization approach. The naive-time-reversal-even and $k_\perp$-odd quark distributions are calculated in terms of a class of twist-three quark-gluon correlation functions. We further calculate the angular distribution of the Drell-Yan lepton pair production in the polarized nucleon-nucleon collisions. In the intermediate transverse momentum region, we find that the two approaches, the collinear factorization and the transverse momentum dependent factorization, are consistent in the description of the lepton pair angular distributions.

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I. INTRODUCTION

Spin-dependent semi-inclusive hadronic processes have attracted much interest from both experiment and theory sides in recent years. These processes provide more opportunities to study the quantum chromodynamics (QCD) and internal structure of hadrons, as compared to the inclusive hadronic processes or spin averaged processes. Measurements have been made in different reactions. In particular, the single transverse spin asymmetry (SSA) observed in various hadronic processes [1–7] has stimulated remarkable theoretical developments [8–26]. Two approaches in the QCD framework have been most explored: the higher twist collinear factorization approach [27–30] and the transverse momentum dependent (TMD) approach [11–26]. In these two approaches, the spin-dependent differential cross sections can be calculated in terms of the collinear twist-three quark-gluon correlation functions in the collinear factorization formalism and the TMD distributions in the TMD factorization approach. Such functions generalize the original Feynman parton picture, where the partons only carry a longitudinal momentum fraction of the parent hadron. Further study has shown that the transverse momentum dependence of the naive-time-reversal-odd TMD distributions which are responsible for the SSAs can be calculated and expressed in terms of the collinear twist-three correlation functions. By using these results, it was shown that the above two approaches are consistent in the intermediate transverse momentum region where both apply [31–33]. In this paper, we will extend these studies to more general Drell-Yan (DY) processes, in particular, the lepton angular distributions in polarized nucleon-nucleon scattering.

The single transverse spin asymmetry in the Drell-Yan lepton pair production process has been used as an example to demonstrate the consistency between these two approaches [31], where the single transverse spin asymmetry is represented as a correlation between the lepton pair transverse momentum $q_\perp$ and the transverse polarization vector $S_\perp$. For this contribution, the transverse spin-dependent differential cross section is proportional to $d\sigma(S_\perp) \propto \epsilon^{ij} S_\perp^i q_\perp^j$. The transverse momentum of the lepton pair is also the transverse momentum of the virtual photon which decays into the lepton pair in the Drell-Yan process. Therefore, this SSA is related to the quark Sivers function and is the only contribution from the quark-antiquark channel. However, if we further study the lepton angular distribution in the Drell-Yan process [34], it will open more contributions to the single spin asymmetries, as well as other spin-dependent observables [35,36]. More recently, by analyzing the general Lorentz structure of the hadronic tensor, the complete spin and transverse momentum dependent angular distribution of the lepton pair has been presented in Ref. [22], and 48 structure functions contribute. One can calculate some of them which are leading power in the context of the TMD factorization and relate them to the TMD distributions [22].

In this paper, we will study the angular distribution of the lepton pair in the polarized Drell-Yan process at large transverse momentum of the lepton pair. The relevant calculations are carried out in the collinear factorization framework. We mainly focus on the single spin asymmetry A_{UT} and double spin asymmetry A_{LT}, taking into account the contributions from the twist-three quark-gluon correlation functions from one of the incident hadrons. We will limit ourselves up to this order. For the single spin asymmetry A_{UT}, the corresponding twist-three quark-gluon correlation functions are those studied in [28–32] and the calculations will be similar. On the other hand, for the A_{LT} asymmetry, more general quark-gluon correlation functions will contribute and the calculations will be different from those in [28–32]. Following the same procedure as that in Refs. [31–33], we will compare the predictions from the two formalisms and check their consistency.

We will calculate the TMD quark distributions at large transverse momentum $k_\perp \gg \Lambda_{\rm QCD}$, and express them in terms of the collinear correlation functions. There are eight leading order TMD quark distributions [14]: three $k_\perp$-even TMD distributions $f_1(x, k_\perp)$, $g_{1L}(x, k_\perp)$, and $h_1(x, k_\perp)$; two naive-time-reversal-odd TMD quark distributions $f_{1T}^\perp(x, k_\perp)$ (Sivers function), $h_1^\perp(x, k_\perp)$ (Boer-Mulders function); and three naive-time-reversal-even but $k_\perp$-odd TMD quark distributions $g_{1T}(x, k_\perp)$, $h_{1L}(x, k_\perp)$, and $h_{1T}^\perp(x, k_\perp)$. The transverse momentum dependence of these TMD quark distributions can be calculated from perturbative QCD in the collinear factorization framework. For the $k_\perp$-even TMD quark distributions, the results are well known and can be expressed in terms of the integrated leading-twist parton distributions (see, e.g., [23]). The naive-time-reversal-odd TMD quark distributions (the Sivers function and the Boer-Mulders function) have also been calculated [31,32]. In this paper, we will extend these calculations to the naive-time-reversal-even but $k_\perp$-odd TMD quark distributions g_{1T} and h_{1L}. These results will depend on the the novel twist-three distributions $\tilde{g}(x)$, $\tilde{h}(x)$ [20,30], and the general twist-three quark-gluon correlation functions G_D, $\tilde{G}_D$, H_D, and E_D [37]. The last TMD quark distribution $h_{1T}^\perp(x, k_\perp)$ will involve twist-four quark-gluon correlation functions. We will not discuss it in this paper.

The rest of the paper is organized as follows. In Sec. II, we give a brief review on the twist-three quark-gluon collinear correlation matrix elements and discuss the relation between them and the TMD distributions. A general feature of the TMD quark distributions at large transverse momentum will be presented in Sec. III. In Sec. IV, we derive the naive-time-reversal-even TMD quark distributions $g_{1T}(x, k_\perp)$, $h_{1L}(x, k_\perp)$ in the twist-three quark-gluon correlation approach. In Sec. V, we calculate the relevant polarized Drell-Yan differential cross section using the same collinear factorization and compare to the results from the TMD factorization. We conclude the paper in Sec. VI.

II. TWIST-3 CORRELATION MATRIX ELEMENTS AND TMD DISTRIBUTIONS

In order to extract more information on hadron structure, various spin-dependent and/or transverse momentum dependent parton correlation functions have been introduced based on the QCD factorization theorem. They are universal between semi-inclusive deep inelastic scattering (SIDIS) and Drell-Yan processes (up to a sign for the naive-time-reversal-odd TMD parton distributions) and can be pinned down by a complete set of experiments. It has been shown that there exist interesting connections between the twist-three collinear functions and the naive-time-reversal-odd TMD quark distributions [20,30]. In this section, we will review the general property of the collinear correlation functions and introduce two novel twist-three functions $\tilde{g}$, $\tilde{h}$ [20]. We will further explore their relations to the naive-time-reversal-even but $k_\perp$-odd TMD quark distributions g_{1T} and h_{1L}.

Let us start by introducing the following collinear quark-antiquark correlation matrix:

$$\hat{M}_{\alpha\beta}(x) \equiv \int \frac{dy^-}{2\pi} e^{-ixP^+y^-} \langle P, S|\bar{\psi}_\beta(y^-)\psi_\alpha(0)|P, S\rangle, \tag{1}$$

where P, S are the hadron momentum and spin, respectively, and we have suppressed the light-cone gauge links between different fields. The hadron momentum P^μ is proportional to the light-cone vector $p^\mu = (1^+, 0^-, 0_\perp)$, whose conjugate light-cone vector is $n = (0^+, 1^-, 0_\perp)$. x is the momentum fraction of the hadron carried by the quark. Up to the twist-three level, the above matrix can be expanded as [37]

$$\hat{M}(x) = \frac{1}{2}[f_1(x)\not{p} + g_1(x)\lambda\gamma_5\not{p} + h_1(x)\gamma_5\not{S}_\perp\not{p}] + \frac{M}{2P^+} \times\left[e(x)1 + g_T(x)\gamma_5\not{S}_\perp + h_L(x)\frac{\lambda}{2}\gamma_5[\not{p}, \not{n}]\right], \tag{2}$$

where λ represents the helicity for the nucleon for the longitudinal polarized nucleon, $S_\perp$ is the transverse polarization vector, and M is the hadron mass. The first three are the leading-twist quark distributions: spin average f_1, longitudinal spin g_1, and quark transversity h_1 distributions. The twist-three quark distributions $e(x)$, $g_T(x)$, and $h_L(x)$ do not have simple interpretations and belong to more general quark-gluon correlation functions [37]. These correlation functions can be defined through the following matrix [37–39]:

$$\hat{M}^\mu_{D\alpha\beta}(x, x_1) \equiv \int \frac{dy^-}{2\pi}\frac{dy_1^-}{2\pi} e^{-ixP^+y^-} e^{i(x_1-x)P^+y_1^-} \times \langle P, S|\bar{\psi}_\beta(y^-)iD_\perp^\mu(y_1^-)\psi_\alpha(0)|P, S\rangle, \tag{3}$$

where we have adopted the covariant derivative as $iD_\perp^\rho = i\partial^\rho + gA_\perp^\rho$. The expansion of the above matrix contains the following four twist-three quark-gluon correlation functions:

$$\hat{M}_D^\mu(x, x_1) = \frac{M}{2P^+}[G_D(x, x_1)i\epsilon_\perp^{\mu\nu}S_{\perp\nu}\not{p} + \tilde{G}_D(x, x_1)S_\perp^\mu\gamma_5\not{p}] + \frac{M}{2P^+} \times[H_D(x, x_1)\lambda\gamma_5\gamma_\perp^\mu\not{p} + E_D(x, x_1)\gamma_\perp^\mu\not{p}]. \tag{4}$$

By imposing the Hermiticity, parity, and time-reversal invariance, we have the following constrains:

$$G_D(x, x_1) = -G_D(x_1, x), \tilde{G}_D(x, x_1) = \tilde{G}_D(x_1, x),$$
$$H_D(x, x_1) = H_D(x_1, x), E_D(x, x_1) = -E_D(x_1, x), \tag{5}$$

and these functions are real. As mentioned, the twist-three quark distributions $g_T(x)$, $e(x)$, and $h_L(x)$ can be expressed in terms of the above quark-gluon correlation functions [37,40],

$$g_T(x) = \frac{1}{x}\int dx_1[G_D(x, x_1) + \tilde{G}_D(x, x_1)], \tag{6}$$

$$h_L(x) = \frac{2}{x}\int dx_1 H_D(x, x_1), \tag{7}$$

$$e(x) = \frac{2}{x}\int dx_1 E_D(x, x_1). \tag{8}$$

Therefore, G_D, $\tilde{G}_D$, H_D, and E_D functions are more fundamental, which becomes evident when we study the scale evolution for the twist-three quark distributions, and the next-to-leading order perturbative corrections to the relevant cross sections [40,41].

In terms of the twist expansion, of course, D-type correlations are not the only ones at the twist-three level. One can also define a set of the F-type twist-3 correlation matrix elements,

$$\hat{M}^{\mu}_{F\alpha\beta}(x, x_1) \equiv \int \frac{dy^-}{2\pi}\frac{dy_1^-}{2\pi} e^{-ixP^+y^-} e^{i(x_1-x)P^+y_1^-} \times \langle P, S|\bar{\psi}_\beta(y^-) g F^{\mu}_{+\perp}(y_1^-)\psi_\alpha(0)|P, S\rangle. \tag{9}$$

Again, the expansion of the above matrix defines the following F-type quark-gluon correlation functions:

$$\hat{M}^{\mu}_{F}(x, x_1) = \frac{M}{2}[T_F(x, x_1)\epsilon_\perp^{\nu\mu} S_{\perp\nu}\not{p} + \tilde{T}_F(x, x_1) i S_\perp^\mu \gamma_5 \not{p}] + \frac{M}{2}[\tilde{T}_F^{(\sigma)}(x, x_1) i\lambda\gamma_5\gamma_\perp^\mu \not{p} + T_F^{(\sigma)}(x, x_1) i\gamma_\perp^\mu \not{p}], \tag{10}$$

where, for convenience, we have used different normalization factors for $T_F(x, x_1)$ and $T_F^{(\sigma)}(x, x_1)$ as compared to Refs. [31,32], with a relative factor $2\pi M$. Similarly, the parity and time-reversal invariance implies

$$T_F(x, x_1) = T_F(x_1, x), \qquad \tilde{T}_F(x, x_1) = -\tilde{T}_F(x_1, x),$$
$$\tilde{T}_F^{(\sigma)}(x, x_1) = -\tilde{T}_F^{(\sigma)}(x_1, x), \qquad T_F^{(\sigma)}(x, x_1) = T_F^{(\sigma)}(x_1, x). \tag{11}$$

The F-type correlation functions are usually regarded as an alternative but not independent functions in the calculations of the inclusive DIS structure functions, such as the g_T structure function [41]. On the other hand, it has been found that the F-type correlation functions are more relevant for the single transverse spin asymmetry and have been intensively studied [28–30]. By using the equation of motion, these two types of the correlation functions can be related to each other [20,30],

$$G_D(x, x_1) = P\frac{1}{x - x_1} T_F(x, x_1), \tag{12}$$

$$\tilde{G}_D(x, x_1) = P\frac{1}{x - x_1}\tilde{T}_F(x, x_1) + \delta(x - x_1)\tilde{g}(x), \tag{13}$$

$$E_D(x, x_1) = P\frac{1}{x - x_1} T_F^{(\sigma)}(x, x_1), \tag{14}$$

$$H_D(x, x_1) = P\frac{1}{x - x_1}\tilde{T}_F^{(\sigma)}(x, x_1) + \delta(x - x_1)\tilde{h}(x), \tag{15}$$

where P stands for the principal value, and $\tilde{g}$, $\tilde{h}$ are given by [20,30]

$$\int \frac{dy^-}{2\pi} e^{-ixP^+y^-}\langle P, S|\bar{\psi}(y^-)\Big(iD_\perp^\mu - ig\int_0^\infty d\zeta^- F^{+\perp}(\zeta^-)\Big)\psi(0)|P, S\rangle = \frac{M}{2}[\tilde{g}(x) S_\perp^\mu \gamma_5 \not{p} + \tilde{h}(x)\lambda\gamma_5\gamma_\perp^\mu \not{p}]. \tag{16}$$

From the above results, we find that indeed the F-type and D-type correlation functions are not completely independent, and they form an overcomplete set of functions. However, we still need $\tilde{g}$ and $\tilde{h}$ to completely describe the associated physics at this order, in particular, for the calculation performed in this paper. In practice, we can either use D-type or F-type plus $\tilde{g}$ and $\tilde{h}$ as a complete set of twist-three functions.

In the following, we will further reveal the physical meaning of $\tilde{g}$ and $\tilde{h}$ and build the connection between them and the transverse momentum dependent quark distributions. The TMD parton distributions are an important generalization of the conventional Feynman parton distributions. Because of additional dependence on the transverse momentum of partons, these distributions open more opportunities to study the partonic structure in the nucleon. The nontrivial correlations between the parton transverse momentum and the polarization vectors of the parent nucleon or the quark itself provide novel consequences in the transverse component in the hadronic processes, for example, the single transverse spin asymmetry. Of course, upon the integral over transverse momentum, these TMD parton distributions will naturally connect to the leading-twist and higher-twist parton distributions. In this paper, we will focus on the TMD quark distributions, which are relevant to the Drell-Yan lepton pair production. The TMD quark distributions can be defined through the following matrix [18,19,42]:

$$\hat{\mathcal{M}}_{\alpha\beta}(x, k_\perp) = \int \frac{dy^- d^2y_\perp}{(2\pi)^3} e^{-ixP^+\cdot y^- + i\vec{k}_\perp\cdot\vec{y}_\perp}\langle PS|\bar{\psi}_\beta(y^-, y_\perp) \times \mathcal{L}_v^\dagger(y^-, y_\perp)\mathcal{L}_v(0)\psi_\alpha(0)|PS\rangle, \tag{17}$$

where x is the longitudinal momentum fraction and $k_\perp$ is the transverse momentum carried by the quark. The gauge link $\mathcal{L}_v$ is along the direction represented by v which is conjugated to p. In the case that we need to regulate the light-cone singularities, we will use an off-light-cone vector, $v^- \gg v^+$ and $v_\perp = 0$, and further define $\zeta^2 = (v \cdot P)^2/v^2$. Compared to the integrated parton distributions definition in the above, we find that the two quark fields are not only separated by light-cone distance ξ^-, but also by the transverse distance $\xi_\perp$, which is conjugate to the transverse momentum of the quark $k_\perp$ [42]. Because of this difference, the additional transverse gauge link for the TMD parton distributions has to be contained in order to make the above quark matrix gauge invariant [19], and the gauge link direction depends on the process [18,19]. Since we will study the Drell-Yan lepton pair production in this paper, in the following we will adopt the TMD definition for this process and the gauge link will go to $-\infty$ [18,19]. The gauge link plays an essential role in the naive-time-reversal-odd TMD parton distributions.

The leading order expansion of the quark distribution matrix $\mathcal{M}$ contains eight quark distributions [14,15,21],

$$\hat{\mathcal{M}} = \frac{1}{2}\bigg[f_1(x,k_\perp)\not{p} + \frac{1}{M}h_1^\perp(x,k_\perp)\sigma^{\mu\nu}k_\mu p_\nu + g_{1L}(x,k_\perp)\lambda\gamma_5\not{p} + \frac{1}{M}g_{1T}(x,k_\perp)\gamma_5\not{p}(\vec{k}_\perp\cdot\vec{S}_\perp) + \frac{1}{M}h_{1L}\lambda i\sigma_{\mu\nu}\gamma_5 p^\mu k_\perp^\nu + h_1(x,k_\perp)i\sigma_{\mu\nu}\gamma_5 p^\mu S_\perp^\nu + \frac{1}{M^2}h_{1T}^\perp(x,k_\perp)i\sigma_{\mu\nu}\gamma_5 p^\mu\left(\vec{k}_\perp\cdot\vec{S}_\perp k_\perp^\nu - \frac{1}{2}\vec{k}_\perp^2 S_\perp^\nu\right) + \frac{1}{M}f_{1T}^\perp(x,k_\perp)\epsilon^{\mu\nu\alpha\beta}\gamma_\mu p_\nu k_\alpha S_\beta\bigg]. \tag{18}$$

Out of the eight TMD distributions, three of them are associated with the $k_\perp$-even structure: $f_1(x,k_\perp)$, $g_{1L}(x,k_\perp)$, and $h_1(x,k_\perp)$. They are a simple extension of the above integrated quark distributions in Eq. (2). The other five distributions are associated with the $k_\perp$-odd structures, and hence vanish when $k_\perp$ are integrated out for $\mathcal{M}_{\alpha\beta}$. For an unpolarized nucleon target, one can introduce the unpolarized quark distribution $f_1(x,k_\perp)$ and naive-time-reversal-odd transversely polarized quark distribution $h_1^\perp(x,k_\perp)$—the Boer-Mulders function. For a longitudinally polarized nucleon, one introduces a longitudinally polarized quark distribution $g_{1L}(x,k_\perp)$ and a transversely polarized distribution $h_{1L}^\perp(x,k_\perp)$. Finally, for a transversely polarized nucleon, one introduces a quark spin-independent distribution $f_{1T}^\perp(x,k_\perp)$, the Sivers function, and a longitudinally polarized quark polarization $g_{1T}(x,k_\perp)$, a symmetrical transversely polarized quark distribution $h_1(x,k_\perp)$, and an asymmetric transversely polarized quark distribution $h_{1T}^\perp(x,k_\perp)$.

If we weight the integral of $\mathcal{M}_{\alpha\beta}$ with linear dependent transverse momentum, the $k_\perp$-odd quark distributions will lead to the higher-twist quark-gluon correlation functions [20]. Four of them will correspond to the four quark-gluon correlation functions introduced above, including $f_{1T}^\perp$, $h_1^\perp$, g_{1T}, and h_{1L}. The last one $h_{1T}^\perp$, as we mentioned, will correspond to a twist-four correlation function. First, the two naive-time-reversal-odd quark distributions $f_{1T}^\perp$ and $h_1^\perp$ lead to the following quark-gluon correlations[1] [20,43]:

$$T_F(x,x) = \int\frac{d^2\vec{k}_\perp}{2\pi}\frac{\vec{k}_\perp^2}{M^2}f_{1T}^\perp|_{\rm DY}(x,k_\perp), \tag{19}$$

$$T_F^{(\sigma)}(x,x) = \int\frac{d^2\vec{k}_\perp}{2\pi}\frac{\vec{k}_\perp^2}{M^2}h_1^\perp|_{\rm DY}(x,k_\perp), \tag{20}$$

where the TMD quark distributions follow their definitions in the Drell-Yan process. Similarly, the two naive-time-reversal-even but $k_\perp$-odd quark distributions g_{1T} and h_{1L} can be related to the following twist-three matrix element:

$$\tilde{g}(x) = \int d^2\vec{k}_\perp\frac{\vec{k}_\perp^2}{2M^2}g_{1T}(x,k_\perp), \tag{21}$$

$$\tilde{h}(x) = \int d^2\vec{k}_\perp\frac{\vec{k}_\perp^2}{2M^2}h_{1L}(x,k_\perp). \tag{22}$$

Here, because they are naive-time-reversal-even distributions, they will not change sign between DIS and Drell-Yan processes. In summary, the four $k_\perp$-odd TMD quark distributions correspond to the four twist-three quark-gluon correlation functions introduced above Eqs. (12)–(15). From these relations, we can further study the scale evolutions for the above twist-three correlations [44–46].

The above relations are only one aspect of the connections between the TMD quark distributions and higher-twist quark-gluon correlation functions. In the following section, we will explore another aspect, i.e., the large transverse momentum behavior of the TMD quark distributions in terms of the collinear leading-twist or twist-three quark-gluon correlation functions.

III. QUARK DISTRIBUTIONS AT LARGE TRANSVERSE MOMENTUM

When the transverse momentum $k_\perp$ is the order of $\Lambda_{\rm QCD}$, the TMD parton distribution functions are entirely nonperturbative objects. However, the transverse momentum dependence can be calculated in perturbative QCD and related to the collinear matrix elements when $k_\perp$ is much larger than $\Lambda_{\rm QCD}$. The collinear matrix elements are the relevant integrated leading-twist parton distributions or higher-twist quark-gluon correlation functions. For the $k_\perp$-even quark distributions, they will depend on the integrated leading-twist parton distributions. However, for the $k_\perp$-odd quark distributions, they depend on the twist-three

[1] These relations Eqs. (19)–(22) are valid at the leading order in perturbative expansion which we will use in this paper. They may differ from these forms at higher orders.

(or twist-four) quark-gluon correlation functions. In general, we will have the following expression for the quark distributions at large transverse momentum [23][2]:

$$q(x, k_\perp)|_{k_\perp \gg \Lambda_{\rm QCD}} = \frac{1}{(k_\perp^2)^n} \int \frac{dx'}{x'} f_i(x') \mathcal{H}_{q/i}(x; x'), \quad (23)$$

where $q(x, k_\perp)$ represents the TMD quark distribution we are interested in, f_i represents the integrated quark distribution for the $k_\perp$-even TMDs, and the higher-twist quark-gluon correlation function for the $k_\perp$-odd TMDs. For the latter case, x' should be understood as two variables for the twist-three quark-gluon correlation functions as we discussed in the last section. The overall power behavior $1/(k_\perp^2)^n$ can be analyzed by the power counting rule [48]. The hard coefficient $\mathcal{H}_{q/i}(x; x')$ is calculated from perturbative QCD. In this paper, we will show the one-gluon radiation contribution to this hard coefficient.

The $k_\perp$-even TMD quark distribution functions, $f_1(x, k_\perp)$, $g_{1L}(x, k_\perp)$, and $h_1(x, k_\perp)$, can be calculated from the associated integrated quark distributions [23].[3] For the nonsinglet contributions, they are expressed as [23]

$$f_1(x_B, k_\perp) = \frac{\alpha_s}{2\pi^2} \frac{1}{\vec{k}_\perp^2} C_F \int \frac{dx}{x} f_1(x) \times \left[\frac{1+\xi^2}{(1-\xi)_+} + \delta(1-\xi)\left(\ln\frac{x_B^2\zeta^2}{\vec{k}_\perp^2} - 1\right)\right], \quad (24)$$

$$g_{1L}(x_B, k_\perp) = \frac{\alpha_s}{2\pi^2} \frac{1}{\vec{k}_\perp^2} C_F \int \frac{dx}{x} g_{1L}(x) \times \left[\frac{1+\xi^2}{(1-\xi)_+} + \delta(1-\xi)\left(\ln\frac{x_B^2\zeta^2}{\vec{k}_\perp^2} - 1\right)\right], \quad (25)$$

$$h_1(x_B, k_\perp) = \frac{\alpha_s}{2\pi^2} \frac{1}{\vec{k}_\perp^2} C_F \int \frac{dx}{x} f_1(x) \times \left[\frac{2\xi}{(1-\xi)_+} + \delta(1-\xi)\left(\ln\frac{x_B^2\zeta^2}{\vec{k}_\perp^2} - 1\right)\right], \quad (26)$$

where the color factor $C_F = (N_c^2 - 1)/2N_c$ with $N_c = 3$, $\xi = x_B/x$, and $\zeta^2 = (2v \cdot P)^2/v^2$. Here, we have adopted an off-light-cone vector v to regulate the light-cone singularity associated with the above calculations [23].

In the same spirit, the naive-time-reversal-odd TMD distributions, the quark Sivers function $f_{1T}^\perp$, and the Boer-Mulders function $h_1^\perp$ at large $k_\perp$ can be calculated perturbatively. The contributions come from the twist-three correlation matrix elements T_F, $\tilde{T}_F$, and $T_F^{(\sigma)}$. Furthermore, it is known that the time-reversal-odd TMD distributions are process dependent because the difference on the gauge link directions will lead to a sign difference between the SIDIS and the Drell-Yan processes [17–19],

$$f_{1T}^\perp|_{\rm DY} = -f_{1T}^\perp|_{\rm DIS}, \qquad h_1^\perp|_{\rm DY} = -h_1^\perp|_{\rm DIS}. \quad (27)$$

The quark Sivers function and the Boer-Mulders function have been calculated [31,32][4] as

$$f_{1T}^\perp|_{\rm DY}(x_B, k_\perp) = \frac{\alpha_s}{\pi} \frac{M^2}{(\vec{k}_\perp^2)^2} \int \frac{dx}{x} \left[A_{f_{1T}^\perp} + C_F T_F(x, x) \times \delta(1-\xi)\left(\ln\frac{x_B^2\zeta^2}{\vec{k}_\perp^2} - 1\right)\right], \quad (28)$$

$$h_1^\perp|_{\rm DY}(x_B, k_\perp) = \frac{\alpha_s}{\pi} \frac{M^2}{(\vec{k}_\perp^2)^2} \int \frac{dx}{x} \left[A_{h_1^\perp} + C_F T_F^{(\sigma)}(x, x) \times \delta(1-\xi)\left(\ln\frac{x_B^2\zeta^2}{\vec{k}_\perp^2} - 1\right)\right], \quad (29)$$

where A factors are defined as

$$A_{f_{1T}^\perp} = C_F T_F(x, x) \frac{1+\xi^2}{(1-\xi)_+} + \frac{C_A}{2}\left[\frac{1+\xi}{1-\xi} T_F(x, x_B) - \frac{1+\xi^2}{1-\xi} T_F(x, x)\right] + \frac{C_A}{2} \tilde{T}_F(x_B, x), \quad (30)$$

$$A_{h_1^\perp} = C_F T_F^{(\sigma)}(x, x) \frac{2\xi}{(1-\xi)_+} + \frac{C_A}{2}\left[\frac{2}{1-\xi} T_F^{(\sigma)}(x, x_B) - \frac{2\xi}{1-\xi} T_F^{(\sigma)}(x, x)\right], \quad (31)$$

where the color factor $C_A = N_c$.

From the above results, we can see that the large transverse momentum TMD quark distributions have a generic structure. They contain two parts: one part is similar to the splitting kernel for the relevant collinear functions, and one part is a delta function associated with a large logarithm $\ln\zeta^2/k_\perp^2$ which comes from the light-cone singularity regulated by an off-light-cone gauge link. The splitting kernel may be different for different TMD quark distributions. However, the logarithmic term is the same for all of them. This is because this term is related to the soft-gluon radiation and is spin independent. We can also use this as an important consistent check for all the calculations. In the next section, we will calculate the $k_\perp$-odd but time-reversal even quark distributions g_{1T} and h_{1L}, and we will find that they have the same structure.

The energy dependence of the TMD quark distributions, the derivative with respect to ζ^2, is controlled by the so-

[2]This is not a rigorous factorization formula. However, we shall be able to construct a QCD factorization formalism in the impact parameter b space for the TMD distributions [23,42,47].

[3]Mixing with the gluonic contributions will have to be taken into account for f_1 and g_1 distributions. In this paper, we will not discuss these contributions.

[4]The derivative terms in these results [31] have been transformed into the nonderivative terms by partial integrals. The associated boundary terms were canceled out by the same boundary terms from the derivative terms [45].

called Collins-Soper (CS) evolution equation [42,49]. These evolution equations can be used to perform soft-gluon resummation for the final $k_\perp$ distribution in the cross section and the TMD quark distributions. It is more convenient to study this resummation in the impact parameter b space [42,49]. We will address this issue in the future.

IV. TRANSVERSE MOMENTUM DEPENDENT QUARK DISTRIBUTIONS g_{1T} AND h_{1L}

In this section, we will calculate the large transverse momentum behavior for the naive-time-reversal-even but $k_\perp$-odd quark distributions g_{1T} and h_{1L}. These calculations will follow the previous calculations on the naive-time-reversal-odd quark distributions $f_{1T}^\perp$ and $h_1^\perp$. However, they differ in a significant way. As shown in [17–19], for the naive-time-reversal-odd distributions, the gauge link contributions play very important roles. For example, these time-reversal-odd distributions will have opposite signs between SIDIS and Drell-Yan processes, because the gauge links go in different directions. In the practical calculations, we have to take pole contributions from the gauge links in these TMD quark distributions [31,32]. The calculations for g_{1T} and h_{1L} are different. Because they are naive time reversal even, we do not take the pole contributions from the gauge links. That makes the calculations a little more involved, as the two-variable dependent correlation functions will enter explicitly. As we mentioned above, these two TMD quark distributions will depend on the correlation functions, G_D (T_F), $\tilde{G}_D$ ($\tilde{T}_F$), and H_D ($\tilde{T}_F^{(\sigma)}$). Moreover, they will have contributions from the twist-three functions $\tilde{g}$ and $\tilde{h}$.

The twist expansion will be the key step in the calculations. The technique used to calculate these contributions has been well developed in the last few decades [28–33,37,38,40,41]. In the following, we will sketch the main points of our calculations for the TMD quark distributions g_{1T} and h_{1L} and the Drell-Yan lepton pair production cross section in the next section.

In the twist expansion, a set of nonperturbative matrix elements of the hadron state will be analyzed according to the power counting of the associated contributions. At the twist-three order, from a generic power counting we have contributions from the following matrix elements [38,39,41]:

$$\langle \bar{\psi} \partial_\perp \psi \rangle, \qquad \langle \bar{\psi} A_\perp \psi \rangle, \qquad \langle \bar{\psi} \partial_\perp A^+ \psi \rangle, \tag{32}$$

where the quark field spin indices have been suppressed for simplicity. The relevant Feynman diagrams can be drawn accordingly. We illustrate the typical diagrams for the associated contributions from the above matrix elements in Fig. 1. For comparison, we have also shown the diagram corresponding to the leading-twist contribution from the matrix element $\langle \bar{\psi} \psi \rangle$ in Fig. 1(a). Figures 1(b)–1(d) represent the contributions up to twist-three quark-gluon correlation matrix elements. Figure 1(b) corresponds to the

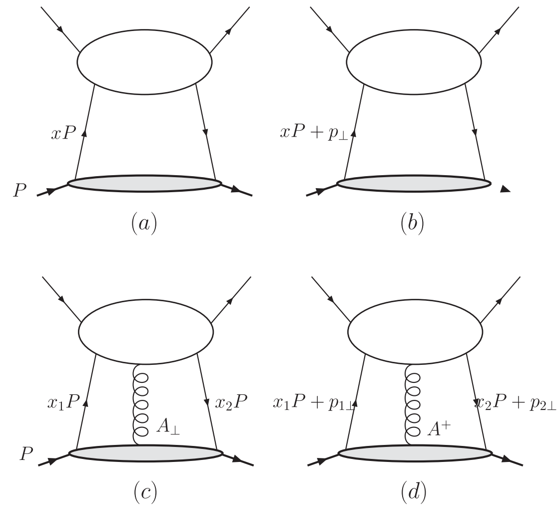

FIG. 1. Generic diagram interpretations for the twist expansions in the high energy scattering amplitudes up to the twist-three level: (a) corresponds to a leading-twist matrix element $\langle \bar{\psi} \psi \rangle$; (b)–(d) for twist-three contributions, (b) for $\langle \bar{\psi} \partial_\perp \psi \rangle$, (c) for $\langle \bar{\psi} A_\perp \psi \rangle$, and (d) for $\langle \bar{\psi} \partial_\perp A^+ \psi \rangle$. An additional A^+ gluon connection between the hard partonic part and the nonperturbative nucleon structure part can be added to these diagrams. This is because they do not change the power counting in these diagrams. The contributions from these diagrams (b)–(d) are not gauge invariant individually. However, they will combine into the gauge invariant results in terms of the correlation functions introduced in Sec. II.

contributions from the matrix element $\langle \bar{\psi} \partial_\perp \psi \rangle$, Fig. 1(c) from $\langle \bar{\psi} A_\perp \psi \rangle$, and Fig. 1(d) from $\langle \bar{\psi} \partial_\perp A^+ \psi \rangle$. Because of additional gluon components in the matrix elements for Figs. 1(c) and 1(d), there will be gluon attachment from the nonperturbative part to the perturbative part as shown in these diagrams. To calculate the contributions from Figs. 1(b) and 1(d), we have to do a collinear expansion of the partonic scattering amplitudes in terms of $p_\perp^\alpha$ and $k_{g\perp}^\alpha = p_{2\perp}^\alpha - p_{1\perp}^\alpha$, respectively. These expansions, combining with the quark field and gluon field, will lead to the contributions in terms of the matrix elements $\langle \bar{\psi} \partial_\perp \psi \rangle$ and $\langle \bar{\psi} \partial_\perp A^+ \psi \rangle$. The calculation of Fig. 1(b) is straightforward, without expansion in terms of the transverse momenta of the quarks and gluon. Furthermore, all these calculations have to be combined into the gauge invariant matrix elements, such as G_D, $\tilde{G}_D$, H_D, E_D, T_F, $\tilde{T}_F$, $T_F^{(\sigma)}$, $\tilde{T}_F^{(\sigma)}$, $\tilde{g}$, and $\tilde{h}$.

However, these functions form an overcomplete set of correlation functions at this order (twist-three) as we discussed in Sec. II. Therefore, we can express the results in terms of either D-type or F-type correlation functions. For example, in the calculations of the twist-three g_T structure function [40,41], one has to combine the contributions from $\langle \bar{\psi} \partial_\perp \psi \rangle$ and $\langle \bar{\psi} A_\perp \psi \rangle$ into the gauge invariant form $\langle \bar{\psi} D_\perp \psi \rangle$ which is associated with G_D and $\tilde{G}_D$. Meanwhile, for the single spin asymmetry observables (or the naive-time-reversal-odd TMD quark distributions) [28–32], it is more convenient to calculate the contributions in terms of $\langle\bar{\psi}\partial_\perp A^+\psi\rangle$ matrix element. Such matrix element is part of the gauge invariant matrix element $\langle\bar{\psi}F^{\perp+}\psi\rangle$ which is associated with the twist-three correlation functions T_F, $\tilde{T}_F$, and $T_F^{(\sigma)}$. The contributions from the diagrams associated with $\langle\bar{\psi}\partial^+A_\perp\psi\rangle$ have also been shown to exactly coincide with those from $\langle\bar{\psi}\partial_\perp A^+\psi\rangle$ to form a complete result into the form in terms of $\langle\bar{\psi}F^{\perp+}\psi\rangle$ [30].

In the above two examples, it seems that the $\tilde{g}$ and $\tilde{h}$ functions are not necessary in these calculations, because they do not appear in the final results. This is only because in these calculations one has chosen a particular set of correlation functions for the final results. Otherwise, $\tilde{g}$ and $\tilde{h}$ functions will show up if we choose a different set of correlation functions. For example, the structure function g_T can be solely expressed in terms of G_D and $\tilde{G}_D$. However, if we rewrite the g_T structure function in terms of T_F and $\tilde{T}_F$, we will have to introduce the $\tilde{g}$ function, because of the relation of Eq. (13). Similar arguments apply to the single spin asymmetry calculations. Furthermore, the necessity of $\tilde{g}$ and $\tilde{h}$ will be manifest in the following calculations of the TMD quark distributions g_{1T} and h_{1L}. As shown below, their roles become so essential that we have to include them in the first place. This can also be seen from the relations between g_{1T} (h_{1L}) and $\tilde{g}$ ($\tilde{h}$) discussed in Sec. II.

We will take the g_{1T} calculation as an example to show how we perform the computation at the twist-three level with the quark-gluon correlation functions G_D (T_F), $\tilde{G}_D$ ($\tilde{T}_F$), and $\tilde{g}$. The calculations for the TMD quark distribution h_{1L} and the Drell-Yan cross sections in the next section will follow accordingly. As outlined above, we first draw the associated Feynman diagrams for the large transverse momentum g_{1T} quark distribution. In order to calculate the large transverse momentum behavior for the g_{1T} function, we have to radiate a hard gluon. The relevant diagrams are plotted in Fig. 2, where the probing quark carries the momentum $k = x_B P + k_\perp$ and the nucleon momentum is denoted by P. The double lines in these diagrams represent the gauge link expansion from the quark distribution definition in Eq. (17). Again, these diagrams include the contributions from the matrix element $\langle\bar{\psi}\partial_\perp\psi\rangle$, Figs. 2(a1)–(a4); from $\langle\bar{\psi}A^\mu\psi\rangle$, Figs. 2(b1)–(e4). To obtain a complete result, we have to attach the gluon to all possible places as shown in the diagrams of Figs. 2(b1)–(e4). This also guarantees that we will obtain the gauge invariant result. The mirror diagrams of (b1)–(e4) where the gluon attaches to the right of the cutting line of these diagrams were not shown in Fig. 2, but included in the final results. Part of the diagrams of Figs. 2(b1)–(e4) correspond to the contributions from $\langle\bar{\psi}A_\perp\psi\rangle$, whereas all of them contribute to that from $\langle\bar{\psi}\partial_\perp A^+\psi\rangle$.

To calculate the TMD quark distribution g_{1T} at large transverse momentum, we first compute the individual contributions from the matrix elements shown in Eq. (32). Then we will combine the individual results into the gauge invariant twist-three quark-gluon correlation functions defined in Sec. II. We can parametrize the associated matrix elements in Eq. (32) which correspond to our calculations. For example, the relevant $\langle\bar{\psi}\partial_\perp\psi\rangle$ matrix elements are parametrized as

$$M^\mu_{\partial_\perp}(x) \equiv \int\frac{dy^-}{2\pi}e^{-ixP^+y^-}\langle P,S|\bar{\psi}(y^-)i\partial^\mu_\perp\psi(0)|P,S\rangle = \frac{M}{2}[T_{\partial_\perp}(x)i\epsilon^{\mu\nu}_\perp S_{\perp\nu}\not{p} + \tilde{T}_{\partial_\perp}(x)S^\mu_\perp\gamma_5\not{p}], \quad (33)$$

where $T_{\partial_\perp}$ and $\tilde{T}_{\partial_\perp}$ correspond to the relevant parts in the gauge invariant functions G_D and $\tilde{G}_D$, respectively. Similarly, we can define the associated $\langle\bar{\psi}A_\perp\psi\rangle$ matrix elements,

$$M^\mu_{A_\perp}(x,x_1) \equiv \int\frac{dy^-}{2\pi}\frac{dy_1^-}{2\pi}e^{-ixP^+y^-}e^{i(x_1-x)P^+y_1^-} \times\langle P,S|\bar{\psi}(y^-)gA^\mu_\perp(y_1^-)\psi(0)|P,S\rangle = \frac{M}{2P^+}[T_{A_\perp}(x,x_1)i\epsilon^{\mu\nu}_\perp S_{\perp\nu}\not{p} + \tilde{T}_{A_\perp}(x,x_1)S^\mu_\perp\gamma_5\not{p}]. \quad (34)$$

Notice that because of additional gluon attachment to the nucleon state, the above matrix elements will depend on two variables (x, x_1) which represent the momentum fractions carried by the quarks from the left and right sides of the cut line in the diagrams. In the case where there is no gluon attachment like that in Eq. (33), they are equal and become one variable. From the above matrix elements we can easily define those of $\langle\bar{\psi}\partial^+A_\perp\psi\rangle$,

$$M^\mu_{\partial^+A_\perp}(x,x_1) \equiv \int\frac{dy^-}{2\pi}\frac{dy_1^-}{2\pi}e^{-ixP^+y^-}e^{i(x_1-x)P^+y_1^-} \times\langle P,S|\bar{\psi}(y^-)g\partial^+A^\mu_\perp(y_1^-)\psi(0)|P,S\rangle = \frac{M}{2}[T_{\partial^+A_\perp}(x,x_1)\epsilon^{\nu\mu}_\perp S_{\perp\nu}\not{p} + \tilde{T}_{\partial^+A_\perp}(x,x_1)iS^\mu_\perp\gamma_5\not{p}]. \quad (35)$$

At the same order, we shall also have the following matrix elements:

$$M^\mu_{\partial_\perp A^+}(x,x_1) \equiv -\int\frac{dy^-}{2\pi}\frac{dy_1^-}{2\pi}e^{-ixP^+y^-}e^{i(x_1-x)P^+y_1^-} \times\langle P,S|\bar{\psi}(y^-)g\partial^\mu_\perp A^+(y_1^-)\psi(0)|P,S\rangle = \frac{M}{2}[T_{\partial_\perp A^+}(x,x_1)\epsilon^{\nu\mu}_\perp S_{\perp\nu}\not{p} + \tilde{T}_{\partial_\perp A^+}(x,x_1)iS^\mu_\perp\gamma_5\not{p}]. \quad (36)$$

From the definitions in Sec. II, we find that the above matrix elements can form the following gauge invariant correlation functions:

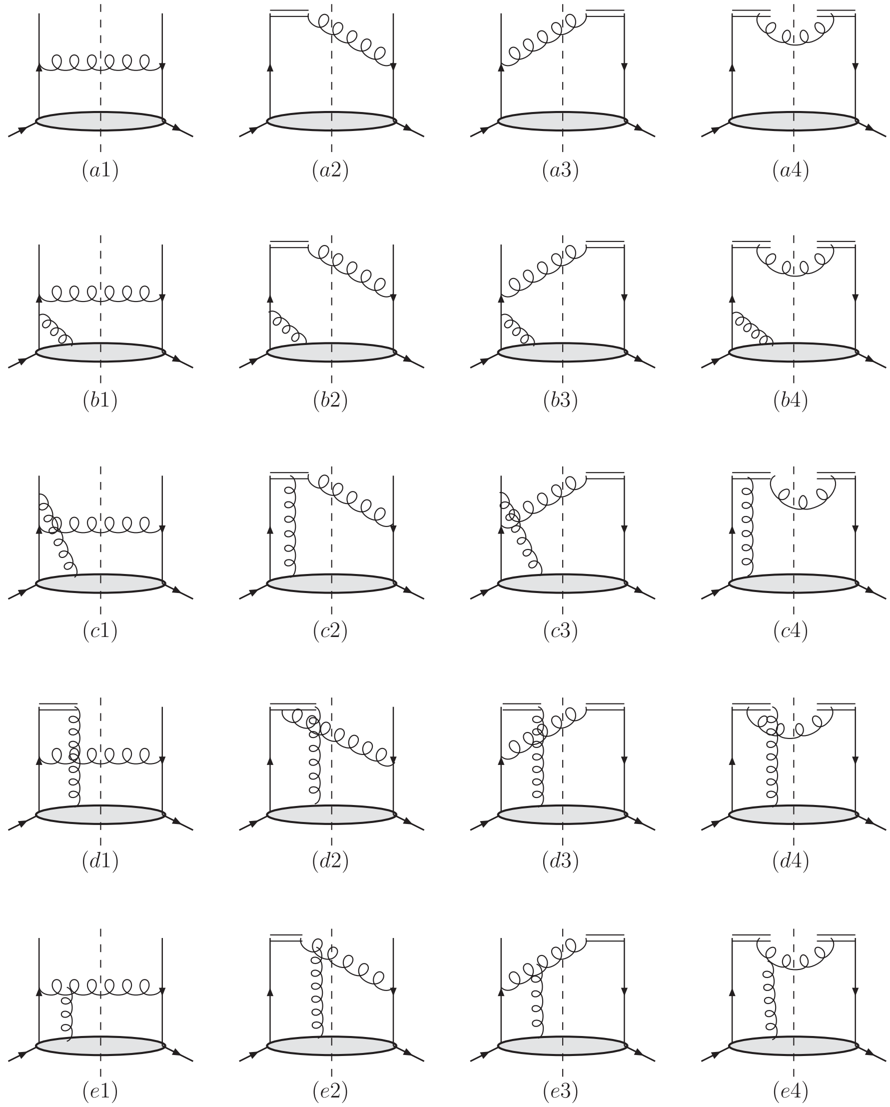

FIG. 2. Feynman diagrams for the TMD quark distributions at large transverse momentum calculated from the twist-three quark-gluon correlation functions. The mirror diagrams of (b1)–(e4) are not shown, but included in the final results. (a1)–(a4) correspond to the contributions from the matrix elements of $\langle \bar{\psi} \partial_\perp \psi \rangle$; (b1)–(b4), (e1)–(e4), (c1), and (c3) correspond to the diagrams contributions from $\langle \bar{\psi} A_\perp \psi \rangle$; and (b1)–(e4) for $\langle \bar{\psi} \partial_\perp A^+ \psi \rangle$.

$$\tilde{T}_F(x, x_1) = \tilde{T}_{\partial_\perp A^+}(x, x_1) + \tilde{T}_{\partial^+ A_\perp}(x, x_1), \tag{37}$$

$$T_F(x, x_1) = T_{\partial_\perp A^+}(x, x_1) + T_{\partial^+ A_\perp}(x, x_1), \tag{38}$$

$$\tilde{g}(x) = \tilde{T}_{\partial_\perp}(x) + \int dx_1 P \frac{1}{x - x_1} \tilde{T}_{\partial_\perp A^+}(x, x_1), \tag{39}$$

where we have used the time-reversal invariance to derive the last equation. There is no similar relation for $T_{\partial_\perp}$, which, on the other hand, can be related to $T_F(x, x)$ depending on the choice of the boundary conditions for the gauge potential [50],

$$T_{\partial_\perp}(x) = \begin{cases} \text{Adv:} \ -T_F(x, x) & A_\perp(+\infty) = 0, \\ \text{Ret:} \ T_F(x, x) & A_\perp(-\infty) = 0, \\ \text{PV:} \ 0 & A_\perp(+\infty) + A_\perp(-\infty) = 0. \end{cases} \tag{40}$$

It has been shown that the final results on the single spin asymmetries do not depend on the boundary conditions for the gauge potential, although they correspond to different relations between the matrix element $T_{\partial_\perp}(x)$ and $T_F(x, x)$, and different contributions from individual diagrams [50].

Having sorted out the above relations, it is relatively straightforward to perform the calculations. As outlined above, we will calculate the Feynman diagram contributions in terms of the matrix elements at the right-hand sides of Eqs. (37)–(39). These results will be combined into the gauge invariant correlation functions at the left sides of Eqs. (37)–(39). In the following, we will calculate these contributions separately and show how we will combine them into the gauge invariant results.

A. Contributions from $\tilde{T}_{\partial_\perp}$ and $T_{\partial_\perp}$

The contributions from $\tilde{T}_{\partial_\perp}$ and $T_{\partial_\perp}$ come from the diagrams of Figs. 2(a1)–(a4). As mentioned above, we will perform the collinear expansion to calculate their contributions. That is, the hard partonic part illustrated in the upper parts of these diagrams can be expanded in terms of the transverse momentum of the quark connecting to the nucleon state in these diagrams [see also Fig. 1(b)]. This momentum can be parametrized as

$$p^\mu = xP^\mu + p_\perp^\mu, \tag{41}$$

where x is the longitudinal momentum fraction of the nucleon and $p_\perp$ is the transverse momentum. In the collinear expansion, we take the approximation that $p_\perp \ll k_\perp$, and only keep the leading nontrivial terms which are relevant for our calculations. For example, the hard partonic part H can be expanded as

$$H(k,p) = H(k,p)|_{p=xP} + p_\perp^\alpha \frac{\partial H(k,p)}{\partial p_\perp^\alpha}\bigg|_{p=xP} + \cdots, \tag{42}$$

where the ellipsis stands for higher order expansion terms, and α is a transverse index $\alpha = 1, 2$. The first term in the above expansion does not contribute to the TMD quark distribution $g_{1T}(x_B, k_\perp)$ at large transverse momentum. The second term will lead to the contribution from the matrix elements $\tilde{T}_{\partial_\perp}$ and $T_{\partial_\perp}$.

In the calculations, we substitute Eq. (42) into the hard partonic part $H(k, p)$ in the Feynman diagrams of Figs. 2(a1)–(a4), and take the linear term in the expansion. One particular contribution is the so-called derivative term, which comes from the expansion of the on-shell condition for the radiated gluon $k_1 = p - k$. To calculate this contribution, we only keep the $p_\perp$ dependence in the delta function of the on-shell condition, and set $p_\perp = 0$ for all other factors in the hard partonic scattering amplitude. The final result will be proportional to the corresponding Born diagram in the collinear limit [28],

$$g_{1T}(x_B,k_\perp)|^D_{\tilde{T}_{\partial_\perp}} = \frac{\alpha_s}{\pi^2}\frac{1}{k_\perp^4} C_F \int \frac{dx}{x}\left(-x\frac{\partial}{\partial x}\tilde{T}_{\partial_\perp}(x)\right)(1+\xi^2), \tag{43}$$

where $\xi = x_B/x$ and $1/k_\perp^4$ behavior comes from the power counting for the $k_\perp$-odd TMD quark distributions.

For the nonderivative terms, we keep all $p_\perp$ dependence in the hard partonic scattering amplitude and expand to the linear term in $p_\perp$. Although it is tedious, the calculation is straightforward, and we obtain

$$\frac{\alpha_s}{\pi^2}\frac{1}{k_\perp^4} C_F \int \frac{dx}{x}\tilde{T}_{\partial_\perp}(x)\left[\frac{\xi(1-\xi^2+2\xi)}{(1-\xi)_+} + \delta(1-\xi)\left(\ln\frac{x_B^2\zeta^2}{k_\perp^2}-1\right)\right], \tag{44}$$

where ζ^2 has been introduced in Sec. III. After partial integrating for the derivative term, we can add these two terms Eqs. (43) and (44) together,[5]

$$g_{1T}(x_B,k_\perp)|_{\tilde{T}_{\partial_\perp}} = \frac{\alpha_s}{\pi^2}\frac{1}{k_\perp^4} C_F \int_{x_B} \frac{dx}{x}\tilde{T}_{\partial_\perp}(x)\left[\frac{\xi(1+\xi^2)}{(1-\xi)_+} + \delta(1-\xi)\left(\ln\frac{x_B^2\zeta^2}{k_\perp^2}-1\right)\right]. \tag{45}$$

Similar calculations can be performed for the contributions from $T_{\partial_\perp}$, and we find that it does not contribute to the TMD quark distribution $g_{1T}(x_B, k_\perp)$.

B. Contributions from $\tilde{T}_{A_\perp}$ ($\tilde{T}_{\partial^+ A_\perp}$) and $T_{A_\perp}$ ($T_{\partial^+ A_\perp}$)

Because the attaching gluon is transversely polarized ($A_\perp$), the contributions from the matrix elements $\tilde{T}_{A_\perp}$ and $T_{A_\perp}$ come from the diagrams of Figs. 2(b1)–(b4), (e1)–(e4), (c1), and (c3). To calculate the contributions from these diagrams, we take the kinematics illustrated in Fig. 1(c), where the quark and gluon lines connecting the hard partonic part and the nucleon state only contain collinear momenta,

$$p_1^\mu = x_1 P^\mu, \qquad p_2^\mu = x_2 P^\mu, \qquad k_g^\mu = (x_2 - x_1)P^\mu, \tag{46}$$

where k_g is the attaching gluon momentum. These calculations are straightforward, and we obtain the contributions from $\tilde{T}_{A_\perp}$,

$$g_{1T}(x_B,k_\perp)|_{\tilde{T}_{A_\perp}} = \frac{\alpha_s}{\pi^2}\frac{1}{k_\perp^4}\int\frac{dxdx_1}{x}\tilde{T}_{A_\perp}(x,x_1) \times\left\{C_F\left(\frac{x_B^2}{x^2}+\frac{x_B}{x_1}-\frac{2x_B^2}{x_1 x}-\frac{x_B}{x}-1\right) + \frac{C_A}{2}\frac{(x_B^2+xx_1)(2x_B-x-x_1)}{(x_B-x_1)(x-x_1)x_1}\right\}, \tag{47}$$

where we have used the symmetric property for $\tilde{T}_{A_\perp}(x, x_1)$ to combine the results from Fig. 2 and their mirrors. Because of additional gluon attachment in these diagrams, we will have two contributions from two different color factors, C_F and C_A. Similarly, we have the contribution from $T_{A_\perp}$,

[5]Note that the boundary term when we partially integrate the derivative contribution was canceled out by the boundary term when we compute the derivative term in Eq. (43) [45].

$$g_{1T}(x_B,k_\perp)|_{T_{A_\perp}} = \frac{\alpha_s}{\pi^2}\frac{1}{k_\perp^4}\int dx dx_1 T_{A_\perp}(x,x_1)\Big\{C_F\Big(\frac{x_B^2}{x^2}+\frac{x_B}{x_1} - \frac{x_B}{x}-1\Big)+\frac{C_A}{2}\frac{x_B^2-xx_1}{(x_1-x_B)x_1}\Big\}. \quad (48)$$

Moreover, the above results can be translated into the contributions from $\tilde{T}_{\partial^+ A_\perp}$ and $T_{\partial^+ A_\perp}$. This is because we have the following relations between the above matrix elements:

$$\tilde{T}_{A_\perp}(x,x_1) = P\frac{1}{x-x_1}\tilde{T}_{\partial^+ A_\perp}(x,x_1),$$
$$T_{A_\perp}(x,x_1) = P\frac{1}{x-x_1}T_{\partial^+ A_\perp}(x,x_1), \quad (49)$$

where the imaginary parts in the right-hand sides of the above equations have been dropped, because they do not contribute to the g_{1T} and h_{1L} calculations here. However, when we calculate the single spin asymmetry observables (such as the time-reversal-odd Sivers and Boer-Mulders functions), we have to keep these imaginary parts in the above equations [50]. Substituting the above results into Eqs. (47) and (48), we shall obtain the contributions from the matrix elements $\tilde{T}_{\partial^+ A_\perp}$ and $T_{\partial^+ A_\perp}$.

C. Contributions from $\tilde{T}_{\partial_\perp A^+}$ and $T_{\partial_\perp A^+}$

For these contributions, it is the A^+ component connecting from the nucleon state to the hard partonic part, and the gluon can attach to the gauge links in the Feynman diagrams. Therefore, we will have all diagrams in Figs. 2(b1)–(e4) contributing to the final results. Moreover, since these matrix elements involve $\partial_\perp A^+$, we have to perform the collinear expansion of the hard partonic parts in terms of the gluon transverse momentum. In doing so, we keep both transverse momenta for the two quark lines connecting the hard part and the nucleon state,

$$p_1^\mu = x_1 P^\mu + p_{1\perp}^\mu, \qquad p_2^\mu = x_2 P^\mu + p_{2\perp}^\mu. \quad (50)$$

Clearly, the kinematics tell us that $k_g^\mu = (x_2 - x_1)P^\mu + k_{g\perp}^\mu$ and $k_{g\perp}^\mu = p_{2\perp}^\mu - p_{1\perp}^\mu$. The corresponding collinear expansion of the hard partonic part takes the following form:

$$H(k;p_1,p_2) = H(k,p_1,p_2)|_{p_{1\perp}=p_{2\perp}=0} + p_{1\perp}^\alpha \frac{\partial H(k;p_1,p_2)}{\partial p_{1\perp}^\alpha}\Big|_{p_{1\perp}=p_{2\perp}=0} + p_{2\perp}^\alpha \frac{\partial H(k;p_1,p_2)}{\partial p_{2\perp}^\alpha}\Big|_{p_{1\perp}=p_{2\perp}=0} + \cdots. \quad (51)$$

Again, the expansion coefficients in the above equation can be calculated following the same method as we discussed in the above for that of the $\tilde{T}_{\partial_\perp}$ contribution. For example, there is also derivative terms associated with $\tilde{T}_{\partial_\perp A^+}$ matrix element. This contribution also comes from the expansion of the delta function for the on-shell condition for the radiated gluon k_1. The derivation for this part is similar, and we obtain the following result:

$$g_{1T}(x_B,k_\perp)|^D_{\tilde{T}_{\partial_\perp A^+}} = \frac{\alpha_s}{\pi^2}\frac{1}{k_\perp^4}C_F\int\frac{dx}{x}(1+\xi^2)\Big(-x\frac{\partial}{\partial x} \times \int dx_1 P\frac{1}{x-x_1}\tilde{T}_{\partial_\perp A^+}(x,x_1)\Big), \quad (52)$$

where the same hard coefficient as that for $\tilde{T}_\partial$ appears as it should due to the gauge invariance. As we mentioned above, this derivative term comes from the delta function expansion for the on-shell condition of k_1. To calculate this contribution, we only keep the $p_{i\perp}$ dependence in this delta function, and set $p_{i\perp}=0$ for all other factors in the hard partonic amplitude. Because of this and the fact that it is the A^+ insertion in the hard part, we can use the Ward identity argument to summarize all diagrams into a simple factorization form: a product of the hard partonic part in the Born diagram without the gluon insertion and the factor $1/(x-x_1)$ representing the gluon insertion.

Because they have the same hard coefficient, we can combine the derivative contributions from $\tilde{T}_\partial$ and $\tilde{T}_{\partial_\perp A^+}$ together. In particular, by using Eq. (39), we can add the results from Eqs. (43) and (52),

$$g_{1T}(x_B,k_\perp)|^D_{\tilde{g}} = \frac{\alpha_s}{\pi^2}\frac{1}{k_\perp^4}C_F\int\frac{dx}{x}(1+\xi^2)\Big(-x\frac{\partial}{\partial x}\tilde{g}(x)\Big), \quad (53)$$

in terms of the gauge invariant correlation function $\tilde{g}$. This shows how we obtain the gauge invariant results from the individual contributions. Moreover, it also demonstrates that $\tilde{g}$ is an independent contribution to the large transverse momentum g_{1T} quark distribution.

The nonderivative contributions from $\tilde{T}_{\partial_\perp A^+}$ can be calculated accordingly, by keeping all the $p_{i\perp}$ dependence in the partonic amplitude. The final result is

$$g_{1T}(x_B,k_\perp)|_{\tilde{T}_{\partial_\perp A^+}} = \frac{\alpha_s}{\pi^2}\frac{1}{k_\perp^4}\int\frac{dxdx_1}{x}\frac{1}{x-x_1}\tilde{T}_{\partial_\perp A^+}(x,x_1) \times\Big\{C_F\Big[\frac{(1-\xi^2+2\xi)\xi}{(1-\xi)_+}+\delta(\xi-1)\times\Big(\ln\frac{x_B^2\zeta^2}{k_\perp^2}-1\Big)\Big] + C_F\Big(\frac{x_B^2}{x^2}+\frac{x_B}{x_1}-\frac{2x_B^2}{x_1 x}-\frac{x_B}{x}-1\Big) + \frac{C_A}{2}\frac{(x_B^2+xx_1)(2x_B-x-x_1)}{(x_B-x_1)(x-x_1)x_1}\Big\}. \quad (54)$$

In the above results, there are two terms with the color factor C_F. Clearly, one term will be combined with that of $\tilde{T}_\partial$ in Eq. (44) to form the contribution from the gauge invariant function $\tilde{g}(x)$, similar to the case for the above derivative contributions. The other term will be combined with that of $\tilde{T}_{\partial^+ A_\perp}$ from Eqs. (47) and (49) to form the contribution from the gauge invariant function $\tilde{T}_F(x, x_1)$.

Similarly, we can calculate the contributions from the matrix element $T_{\partial_\perp A^+}$,

$$g_{1T}(x_B, k_\perp)|_{T_{\partial_\perp A^+}} = \frac{\alpha_s}{\pi^2}\frac{1}{k_\perp^4}\int\frac{dxdx_1}{x}\frac{1}{x-x_1}T_{\partial_\perp A^+}(x,x_1) \times\left\{C_F\left(\frac{x_B^2}{x^2}+\frac{x_B}{x_1}-\frac{x_B}{x}-1\right) + \frac{C_A}{2}\frac{x_B^2-xx_1}{(x_1-x_B)x_1}\right\}. \quad (55)$$

Again, this result will combine with that from $T_{\partial^+ A_\perp}$ from Eqs. (48) and (49) to form the gauge invariant result in terms of $T_F(x, x_1)$.

Combining all these results together, we will obtain the final results for the TMD quark distribution $g_{1T}(x_B, k_\perp)$ at large transverse momentum,

$$g_{1T}(x_B, k_\perp) = \frac{\alpha_s}{\pi^2}\frac{M^2}{(k_\perp^2)^2}\int\frac{dx}{x}\left\{A_{g_{1T}} + C_F\tilde{g}(x)\delta(\xi-1)\times\left(\ln\frac{x_B^2\zeta^2}{k_\perp^2}-1\right)\right\}, \quad (56)$$

where $A_{g_{1T}}$ is given by

$$A_{g_{1T}} = \int dx_1\left\{\tilde{g}(x)C_F\left[\frac{\xi(1+\xi^2)}{(1-\xi)_+}\right]\delta(x-x_1) + P\frac{1}{x-x_1}\tilde{T}_F(x,x_1)\times\left[C_F\left(\frac{x_B^2}{x^2}+\frac{x_B}{x_1}-\frac{2x_B^2}{x_1 x}-\frac{x_B}{x}-1\right) + \frac{C_A}{2}\frac{(x_B^2+xx_1)(2x_B-x-x_1)}{(x_B-x_1)(x-x_1)x_1}\right] + P\frac{1}{x-x_1}T_F(x,x_1)\left[C_F\left(\frac{x_B^2}{x^2}+\frac{x_B}{x_1}-\frac{x_B}{x}-1\right) + \frac{C_A}{2}\frac{x_B^2-xx_1}{(x_1-x_B)x_1}\right]\right\}, \quad (57)$$

and we have partially integrated the derivative terms. Using the identities Eqs. (12) and (13), $A_{g_{1T}}$ can be transformed into the D-type correlation functions,

$$A_{g_{1T}} = \int dx_1\Big\{\tilde{g}(x)\left[C_F\frac{1+\xi^2}{(1-\xi)_+}-\frac{C_A}{2}\frac{1+\xi^2}{1-\xi}\right]\delta(x-x_1) + \tilde{G}_D(x,x_1)\left[C_F\left(\frac{x_B^2}{x^2}+\frac{x_B}{x_1}-\frac{2x_B^2}{x_1 x}-\frac{x_B}{x}-1\right)+\frac{C_A}{2}\times\frac{(x_B^2+xx_1)(2x_B-x-x_1)}{(x_B-x_1)(x-x_1)x_1}\right] + G_D(x,x_1)\times\left[C_F\left(\frac{x_B^2}{x^2}+\frac{x_B}{x_1}-\frac{x_B}{x}-1\right)+\frac{C_A}{2}\frac{x_B^2-xx_1}{(x_1-x_B)x_1}\right]\Big\}. \quad (58)$$

This result can also be obtained by directly combining the contributions from $\tilde{T}_{\partial_\perp}$ in Eq. (45), $\tilde{T}_{A_\perp}$ in Eq. (47), and $T_{A_\perp}$ in Eq. (48).

For TMD distribution $h_{1L}(x, k_\perp)$, the perturbative calculation follows a similar procedure. It receives contributions from $\tilde{T}_F^{(\sigma)}(x, x_1)$, $\tilde{h}(x)$. We skip the detailed derivation, and list the final result,

$$h_{1L}(x_B, k_\perp) = \frac{\alpha_s}{\pi^2}\frac{M^2}{(k_\perp^2)^2}\int\frac{dx}{x}\left\{A_{h_{1L}} + C_F\tilde{h}(x)\delta(\xi-1)\times\left(\ln\frac{x_B^2\zeta^2}{k_\perp^2}-1\right)\right\}, \quad (59)$$

where $A_{h_{1L}}$ is defined by

$$A_{h_{1L}} = \int dx_1\left\{C_F\left[\tilde{h}(x)\frac{2\xi^2}{(1-\xi)_+}\right]\delta(x_1-x) + P\frac{1}{x-x_1}\tilde{T}_F^{(\sigma)}(x,x_1)\left[C_F\frac{2(x-x_1-x_B)}{x_1} + \frac{C_A}{2}\frac{2x_B(x_Bx+x_Bx_1-x^2-x_1^2)}{(x_B-x_1)(x-x_1)x_1}\right]\right\}. \quad (60)$$

Similarly, it can be expressed in terms of the D-type function as

$$A_{h_{1L}} = \int dx_1\left\{\tilde{h}(x)\left[C_F\frac{2\xi}{(1-\xi)_+}-\frac{C_A}{2}\frac{2\xi}{1-\xi}\right]\delta(x_1-x) + H_D(x,x_1)\left[C_F\frac{2(x-x_1-x_B)}{x_1} + \frac{C_A}{2}\frac{2x_B(x_Bx+x_Bx_1-x^2-x_1^2)}{(x_B-x_1)(x-x_1)x_1}\right]\right\}. \quad (61)$$

Indeed, the above results for the naive-time-reversal-even but $k_\perp$-odd distributions also have the same structure as those discussed in Sec. III.

V. SINGLE SPIN A_{UT} AND DOUBLE SPIN A_{LT} ASYMMETRIES IN THE DRELL-YAN LEPTON PAIR PRODUCTION

In this section, we will calculate the angular distribution of the lepton pair in the polarized Drell-Yan process, especially the A_{UT} and A_{LT} asymmetries: One of the incident hadrons is transversely polarized and another is unpolarized or longitudinally polarized. We focus on the lepton pair production in hadronic scattering,

$$H_a + H_b \to \gamma^* + X \to \ell^+\ell^- + X, \tag{62}$$

where the lepton pair comes from the virtual photon decays. In the leading order, the virtual photon is produced through quark-antiquark annihilation, $q\bar{q} \to \gamma^*$ in the parton picture [34]. In the rest frame of the lepton pair, we can define two angles [35]: one is the polar angle θ between one lepton momentum and the incident hadron's momentum, and the azimuthal angle ϕ is defined as the angle between the hadronic plane and the lepton plane. The general formalism has been worked out for the angular distributions in the polarized Drell-Yan process [22]. For our calculations of A_{UT} and A_{LT}, one can write down the following structure in the CS frame [22]:

$$\begin{aligned}\frac{d\sigma_{(UT,LT)}}{d^4q d\Omega} = \frac{\alpha_{em}^2}{2(2\pi)^4 S^2 Q^2}\{|S_{aT}|[\sin\phi_a((1+\cos^2\theta)W_T^{UT} + (1-\cos^2\theta)W_L^{UT} + \sin 2\theta\cos\phi W_\Delta^{UT} + \sin^2\theta\cos 2\phi W_{\Delta\Delta}^{UT})\\ + \cos\phi_a(\sin 2\theta \sin\phi W_\Delta^{\prime UT} + \sin^2\theta \sin 2\phi W_{\Delta\Delta}^{\prime UT})] + |S_{aT}|S_{bL}[\cos\phi_a((1+\cos^2\theta)W_T^{LT} + (1-\cos^2\theta)W_L^{LT}\\ + \sin 2\theta\cos\phi W_\Delta^{LT} + \sin^2\theta\cos 2\phi W_{\Delta\Delta}^{LT}) + \sin\phi_a(\sin 2\theta\sin\phi W_\Delta^{\prime LT} + \sin^2\theta \sin 2\phi W_{\Delta\Delta}^{\prime LT})]\},\end{aligned} \tag{63}$$

where the orientation of the transverse polarization of the hadron a is expressed through the CS angle ϕ_a, and ϕ and θ have been introduced above. In the above expressions, S_{aT} and S_{bL} are the hadron a transverse polarization vector and hadron b longitudinal polarization vector, respectively. The angular-integrated cross section is expressed in terms of the W_T^{UT}, W_L^{UT}, W_T^{LT}, and W_L^{LT} as [22]

$$\begin{aligned}\frac{d\sigma_{(UT,LT)}}{d^4q} = \frac{\alpha_{em}^2}{12\pi^3 S^2 Q^2}\{|S_{aT}|\sin\phi_a(2W_T^{UT} + W_L^{UT})\\ + |S_{aT}|S_{bL}\cos\phi_a(2W_T^{LT} + W_L^{LT})\}.\end{aligned} \tag{64}$$

In particular, the structure function $2W_T^{UT} + W_L^{UT}$ has been calculated [31], which represents the Sivers contribution to the single transverse spin asymmetry. From the above expression, we also see that the Sivers contribution is the only contribution to the single spin asymmetry when we integrate out the lepton angles.

In the following, we will calculate the structure functions W^{UT} and W^{LT} in Eq. (63) in the intermediate transverse momentum region $\Lambda_{\rm QCD} \ll q_\perp \ll Q$, and will compare the predictions from the collinear factorization and the transverse momentum dependent approaches. For the single spin asymmetry W^{UT}, we follow the previous calculations [31]. The only difference is that the calculation in [31] is equivalent to the virtual photon production in single transversely polarized nucleon-nucleon scattering, whereas we will contract the hadronic tensor to the lepton tensor to obtain the angular distributions of the lepton pair in this process. However, the technique is the same. Again, as discussed in [31], we will have soft pole and hard pole contributions,[6] and they will have cancellation in the intermediated transverse momentum region. For the double spin asymmetry part W^{LT}, we will follow the procedure in Sec. IV to calculate the twist-three contributions. The corresponding correlation functions will be G_D (T_F), $\tilde{G}_D$ ($\tilde{T}_F$), $\tilde{g}$, and so on. The similar Feynman diagrams can be drawn accordingly, which can be organized into the contributions from the twist-three matrix elements $\langle\psi\partial_\perp\psi\rangle$, $\langle\psi A_\perp\psi\rangle$, and $\langle\psi\partial_\perp A^+\psi\rangle$. These contributions are then combined into the contributions from the gauge invariant correlation functions G_D, $\tilde{G}_D$, $\tilde{g}$, and so on. In order to calculate the different terms in the angular distribution Eq. (62), we choose the Collins-Soper frame [35], where four orthogonal unit vectors are defined as [52,53]

$$\begin{aligned}&T^\mu = \frac{q^\mu}{\sqrt{q^2}}, \qquad Z^\mu = \frac{2}{\sqrt{Q^2+Q_\perp^2}}[q_{\bar p}\tilde{P}^\mu - q_p\tilde{\bar{P}}^\mu],\\ &X^\mu = -\frac{Q}{Q_\perp}\frac{2}{\sqrt{Q^2+Q_\perp^2}}[q_{\bar p}\tilde{P}^\mu + q_p\tilde{\bar{P}}^\mu],\\ &Y_\mu = \epsilon^{\mu\nu\alpha\beta}T_\nu Z_\alpha X_\beta,\end{aligned} \tag{65}$$

where q^μ is the virtual photon momentum, P and $\bar{P}$ are momenta for the incoming hadrons, and we further define $\tilde{P}^\mu = [P^\mu - (P\cdot q)/q^2 q^\mu]/\sqrt{S}$, $\tilde{\bar{P}}^\mu = [\bar{P}^\mu - (\bar{P}\cdot q)/q^2 q^\mu]/\sqrt{S}$ with $q_p \equiv P\cdot q/\sqrt{S}$, $q_{\bar p} = \bar{P}\cdot/\sqrt{S}$ and S is the total hadronic center of mass energy square. The structure function can be obtained by contracting the hadronic tensor $W^{\mu\nu}$ with six symmetric tensors constructed by the four orthogonal vectors [52,53],

$$\begin{aligned}W_T &= \tfrac{1}{2}(X^\mu X^\nu + Y^\mu Y^\nu)W_{\mu\nu},\\ W_L &= Z^\mu Z^\nu W_{\mu\nu},\\ W_\Delta &= -\tfrac{1}{2}(Z^\mu X^\nu + Z^\nu X^\mu)W_{\mu\nu},\\ W_{\Delta\Delta} &= \tfrac{1}{2}(-X^\mu X^\nu + Y^\mu Y^\nu)W_{\mu\nu},\\ W_\Delta^{\prime UT} &= -\tfrac{1}{2}(Y^\mu Z^\nu + Y^\nu Z^\mu)W_{\mu\nu},\\ W_{\Delta\Delta}^{\prime UT} &= \tfrac{1}{2}(Y^\mu X^\nu + Y^\nu X^\mu)W_{\mu\nu},\end{aligned} \tag{66}$$

and similar expressions hold for the UT and LT structure functions.

Furthermore, we are interested in the cross section contributions in the intermediate transverse momentum region, $\Lambda_{\rm QCD} \ll Q_\perp \ll Q$, where we only keep the

[6]The so-called soft-fermion pole will also contribute to the cross sections [31,51] which have been neglected in our calculations. However, these contributions do not change the conclusions of the consistency between the two approaches [31].

leading terms in $Q_\perp/Q$, and neglect all higher order terms. With this power counting expansion, six leading order structure functions survive and can be simplified as

$$W_T^{UT} = \frac{\alpha_s M}{\pi Q_\perp^3}\sum_q \frac{e_q^2}{N_c}\int \frac{dx}{x}\frac{dz}{z}\{A_{f_{1T}^\perp}(x)\delta(1-\hat{\xi}) + B_{f_{1T}^\perp}(x)\delta(1-\xi)\}\bar{f}(z), \tag{67}$$

$$W_{\Delta\Delta}^{UT} = \frac{\alpha_s M}{\pi Q_\perp^3}\sum_q \frac{e_q^2}{N_c}\int \frac{dx}{x}\frac{dz}{z}\{A_{h_1^\perp}(z)\delta(1-\xi) + B_{h_1^\perp}(z)\delta(1-\hat{\xi})\}h_1(x), \tag{68}$$

$$W_{\Delta\Delta}^{\prime UT} = -\frac{\alpha_s M}{\pi Q_\perp^3}\sum_q \frac{e_q^2}{N_c}\int \frac{dx}{x}\frac{dz}{z}\{A_{h_1^\perp}(z)\delta(1-\xi) + B_{h_1^\perp}(z)\delta(1-\hat{\xi})\}h_1(x), \tag{69}$$

$$W_T^{LT} = -\frac{\alpha_s M}{\pi^2 Q_\perp^3}\sum_q \frac{e_q^2}{N_c}\int \frac{dx}{x}\frac{dz}{z}\{A_{g_{1T}^\perp}(x)\delta(1-\hat{\xi}) + B_{g_{1T}^\perp}(x)\delta(1-\xi)\}\bar{g}_1(z), \tag{70}$$

$$W_{\Delta\Delta} = \frac{\alpha_s M}{\pi^2 Q_\perp^3}\sum_q \frac{e_q^2}{N_c}\int \frac{dx}{x}\frac{dz}{z}\{A_{h_{1L}}(z)\delta(1-\xi) + B_{h_{1L}}(z)\delta(1-\hat{\xi})\}h_1(x), \tag{71}$$

$$W_{\Delta\Delta}^{\prime LT} = \frac{\alpha_s M}{\pi^2 Q_\perp^3}\sum_q \frac{e_q^2}{N_c}\int \frac{dx}{x}\frac{dz}{z}\{A_{h_{1L}}(z)\delta(1-\xi) + B_{h_{1L}}(z)\delta(1-\hat{\xi})\}h_1(x), \tag{72}$$

where $\xi = x_B/x$, $\hat{\xi} = z_B/z$, x_B, and z_B are defined as $x_B = Q/\sqrt{S}e^y$ and $z_B = Q/\sqrt{S}e^{-y}$ with y the rapidity of the lepton pair in the center of mass frame, respectively. The functions $A_{f_{1T}}$, $A_{h_1^\perp}$, $A_{g_{1T}}$, and $A_{h_{1L}}$ have been defined in Sec. III with appropriate variable replacements. The functions $B_{f_{1T}^\perp}$, $B_{h_1^\perp}$, $B_{g_{1T}}$, and $B_{h_{1L}}$ are given by

$$B_{f_{1T}^\perp}(x) = C_F T_F(x,x)\left[\frac{1+\hat{\xi}^2}{(1-\hat{\xi})_+} + 2\delta(\hat{\xi}-1)\ln\frac{Q^2}{Q_\perp^2}\right], \tag{73}$$

$$B_{h_1^\perp}(z) = C_F T_F^{(\sigma)}(z,z)\left[\frac{2\xi}{(1-\xi)_+} + 2\delta(\xi-1)\ln\frac{Q^2}{Q_\perp^2}\right], \tag{74}$$

$$B_{g_{1T}}(x) = C_F \tilde{g}(x)\left[\frac{1+\hat{\xi}^2}{(1-\hat{\xi})_+} + 2\delta(\hat{\xi}-1)\ln\frac{Q^2}{Q_\perp^2}\right], \tag{75}$$

$$B_{h_{1L}}(z) = C_F \tilde{h}(z)\left[\frac{2\xi}{(1-\xi)_+} + 2\delta(\xi-1)\ln\frac{Q^2}{Q_\perp^2}\right], \tag{76}$$

respectively.

Meanwhile, the transverse momentum dependent factorization can be applied at low transverse momentum $Q_\perp \ll Q$. The relevant structure functions can be written as [22]

$$W_T^{UT} = \int \frac{\hat{q}_\perp \cdot k_{a\perp}}{M_a} f_{1T}^\perp(x_B, k_{a\perp})\bar{f}_1(z_B, k_{b\perp}), \tag{77}$$

$$\frac{W_{\Delta\Delta}^{\prime UT} - W_{\Delta\Delta}^{UT}}{2} = -\int \frac{\hat{q}_\perp \cdot k_{b\perp}}{M_a} h_1(x_B, k_{a\perp})\bar{h}_1^\perp(z_B, k_{b\perp}), \tag{78}$$

$$W_T^{LT} = -\int \frac{\hat{q}_\perp \cdot k_{a\perp}}{M_a} g_{1T}(x_B, k_{a\perp})\bar{g}_{1L}(z_B, k_{b\perp}), \tag{79}$$

$$\frac{W_{\Delta\Delta}^{\prime LT} + W_{\Delta\Delta}^{LT}}{2} = \int \frac{\hat{q}_\perp \cdot k_{b\perp}}{M_a} h_1(x_B, k_{a\perp})\bar{h}_{1L}(z_B, k_{b\perp}), \tag{80}$$

where the integral symbol represents a convolution in transverse momentum

$$\int = \frac{1}{N_c}\sum_q e_q^2 \int d^2k_{a\perp} d^2k_{b\perp} d^2\lambda_\perp (S(\lambda_\perp))^{-1} H(Q^2) \times \delta^2(k_{a\perp} + k_{b\perp} + \lambda_\perp - q_\perp),$$

and $S(\lambda_\perp)$ and $H(Q^2)$ are the soft and hard factors, respectively. In addition, the combinations of structure functions

$$\frac{W_{\Delta\Delta}^{UT} + W_{\Delta\Delta}^{\prime UT}}{2}, \qquad \frac{W_{\Delta\Delta}^{LT} - W_{\Delta\Delta}^{\prime LT}}{2}$$

also receive the leading power contribution from the product of TMD distributions $h_{1T}^\perp \times \bar{h}_1^\perp$ and $h_{1T}^\perp \times \bar{h}_{1L}^\perp$, which is however beyond the scope of the present paper.

When the transverse momenta $k_{a\perp}$ and $k_{b\perp}$ are of order $\Lambda_{\rm QCD}$, the TMD quark distribution functions are entirely nonperturbative objects. But in the large transverse momentum region $k_{a,b\perp} \gg \Lambda_{\rm QCD}$, we can calculate the transverse momentum dependence, as shown in Sec. III. To compare with the results from the collinear factorization calculation, we let one of the transverse momenta $k_{a\perp}$, $k_{b\perp}$, $\lambda_\perp$ be of order $q_\perp$, and the others are much smaller. After integrating the delta function, we will obtain

$$W_T^{UT} = \frac{\alpha_s M}{\pi Q_\perp^3}\sum_q \frac{e_q^2}{N_c}\int \frac{dx}{x}\frac{dz}{z}\{A_{f_{1T}^\perp}(x)\delta(1-\hat{\xi}) + B_{f_{1T}^\perp}(x)\delta(1-\xi)\}\bar{f}(z), \tag{81}$$

$$\frac{W_{\Delta\Delta}^{\prime UT} - W_{\Delta\Delta}^{UT}}{2} = -\frac{\alpha_s M}{\pi Q_\perp^3} \sum_q \frac{e_q^2}{N_c} \int \frac{dx}{x} \frac{dz}{z} \{A_{h_1^\perp}(z) \delta(1-\xi) + B_{h_1^\perp}(z) \delta(1-\hat{\xi})\} h_1(x), \tag{82}$$

$$W_T^{LT} = -\frac{\alpha_s M}{\pi^2 Q_\perp^3} \sum_q \frac{e_q^2}{N_c} \int \frac{dx}{x} \frac{dz}{z} \{A_{g_{1T}^\perp}(x) \delta(1-\hat{\xi}) + B_{g_{1T}^\perp}(x) \delta(1-\xi)\} \bar{g}_1(z), \tag{83}$$

$$\frac{W_{\Delta\Delta}^{\prime LT} + W_{\Delta\Delta}^{LT}}{2} = \frac{\alpha_s M}{\pi^2 Q_\perp^3} \sum_q \frac{e_q^2}{N_c} \int \frac{dx}{x} \frac{dz}{z} \{A_{h_{1L}}(z) \delta(1-\xi) + B_{h_{1L}}(z) \delta(1-\hat{\xi})\} h_1(x). \tag{84}$$

It is evident that the above results reproduce the differential cross sections we derived in the collinear factorization framework.

VI. SUMMARY

In this paper, we have calculated the naive-time-reversal-even but $k_\perp$-odd TMD distributions $g_{1T}(x, k_\perp)$, $h_{1L}(x, k_\perp)$ at large transverse momentum, and they are related to a class of collinear twist-three matrix elements. We further studied the angular distribution of the lepton pair produced in the polarized Drell-Yan process for the single spin asymmetry A_{UT} and double spin asymmetry A_{LT}. We found that the transverse momentum dependent and twist-three collinear approaches are consistent in the intermediated transverse momentum region.

These calculations are not straightforward extensions of the previous studies for the naive-time-reversal-odd TMD quark distributions [31]. In the previous case, we take the pole contribution from the initial/final state interactions, which simplifies the calculations. In this paper, we have to deal with more complicated kinematics, similar to next-to-leading order perturbative calculations for the g_T structure function [41]. To carry out the calculations, we have set up the twist expansion framework, and,, in particular, we have shown that the contributions from twist-three matrix elements will combine into the gauge invariant form. This shall encourage further developments in the higher-twist calculations. For example, an extension to calculate the remaining TMD distributions $h_{1T}^\perp(x, k_\perp)$ would be possible. The formalism we developed in this paper can also be extended to other semi-inclusive processes, such as the semi-inclusive deep inelastic scattering and back-to-back two hadron production in e^+e^- annihilation processes. We will address these issues in future publications.

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