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Invisible Higgs boson decay with $B \to K\nu\bar{\nu}$ constraint

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If the Higgs boson were the only particle within the LHC accessible range, precision measurement of the Higgs's properties would play a unique role in studying electroweak symmetry breaking as well as possible new physics. We try to use low energy experiments such as rare B decay to constrain a challenging decay mode of Higgs, in which a Higgs decays to a pair of light ($\approx 1 \sim 2$ GeV) SM singlet S and becomes invisible. By using the current experimental bound of rare decay $B \to K\nu\bar{\nu}$ and computing the contribution of $B \to KSS$ to (the) $B \to K + \not{E}$, we obtain an upper bound on the Higgs coupling to such light singlet. It is interesting that the partial width of the invisible decay mode $h \to SS$ by taking the upper bound value of coupling is at a comparable level with $h \to WW/ZZ$ or $WW^{(*)}$ decay modes, making the Higgs identifiable but with a different predicted decay branching ratio from [the] standard model Higgs decay. It will then have an impact on precision measurement of the Higgs's properties. We also study the implication for cosmology from such a light singlet and propose a solution to the potential problem.

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I. INTRODUCTION

Searching the Higgs boson, the last missing piece in the standard model (SM) of particle physics, is one of the essential goals of the CERN Large Hadron Collider (LHC). The minimal Higgs boson model is the simplest solution to electroweak symmetry breaking and also the most economic one to be consistent with existing precision measurements. However, theoretical considerations suggest that the minimal Higgs boson model may not be complete. Being a fundamental scalar, the Higgs boson receives quantum corrections of quadratic divergence. To solve this, there have been many theoretical proposals which predict various new physics at $\mathcal{O}$(TeV). Direct evidences of new resonances at the LHC can determine what is the new physics model. However, if the Higgs boson were the only particle at the LHC accessible range, we will have to rely on precision measurements. The precision measurement of Higgs boson properties can play an important role to confirm electroweak symmetry breaking mechanism [1] and test new physics [2]. For instance, measurement on top quark Yukawa coupling y_t is crucial to probe the origin of fermion mass generation while $gg \to h$ production due to the top quark loop directly depends on the coupling y_t. On the other hand, the contribution from new physics may also change the $gg \to h$ production rate significantly. One interesting scenario will be that at the LHC one does discover the conventional Higgs search channels, confirm it is the Higgs and measure its mass but the observed event number is much smaller than what we expect for the SM Higgs of that mass.

However, a new decay mode of Higgs boson that cannot be easily identified will lead to the same consequence when the new decay width is comparable with the conventional SM Higgs width at the same mass [3]. For instance, if Higgs decay has an invisible mode, it is impossible to fully reconstruct such resonance and is very challenging to identify at the hadron colliders [4].

In this paper, we want to consider the invisible decay of Higgs to a pair of hidden sector scalar (S) particles in the minimal extension of the SM [5–8]. As the scalar particle is a singlet of the SM interactions it can only directly couple to the Higgs by the interaction Lagrangian

$$\frac{\lambda}{2v_0} H^\dagger H S^2 \equiv \frac{\hat{\lambda}}{2} H^\dagger H S^2, \tag{1}$$

where λ is a dimension one coupling constant and v_0 the vacuum expectation value (vev) of the Higgs boson. It is a challenge to identify such invisible Higgs at collider experiments and obtain any bound on invisible Higgs. The only controlled experiments at this moment that can put constraints on such decay mode are through low energy processes such as rare B or K decays. In these processes the Higgs is virtual, not interacting directly to B or K, but to top quark and S. Therefore, the only difference is CKM factor, for K it is about 10^{-5} smaller than B, so we would need more than 10^{10} K's. Therefore, we just focus on rare B decays in this work.

In Table I, we show the theoretical estimates of branching ratios (BRs) within the SM [9–12] and their current experimental bounds at B factories [13–15] for the decays $B \to M\nu\bar{\nu}$. The errors of the SM estimates in Table I are

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TABLE I. Expected BRs in the SM and experimental bounds (90% C.L.) in units of 10^{-6}. The SM values for K, π, K^* include the long distance contributions through intermediate on-shell τ, which can be dominant for π case [9].

mode	BRs in the SM [9–12]	Experimental bounds
$B \to K\nu\bar{\nu}$	5.1 ± 0.8	<14 [13]
$B \to \pi\nu\bar{\nu}$	9.7 ± 2.1	<100 [14]
$B \to K^*\nu\bar{\nu}$	8.4 ± 1.4	<80 [15]
$B \to \rho\nu\bar{\nu}$	$0.49^{+0.61}_{-0.38}$	<150 [13]

mainly due to the hadronic transition form factors and the CKM matrix elements, since those decay channels are among the cleanest SM processes due to only involving electroweak penguin diagrams [16], except for $B \to \pi\nu\bar{\nu}$ [9]. Please note that by taking the ratios such as $\mathrm{Br}(B \to \pi\nu\bar{\nu})/\mathrm{Br}(B \to \pi l\nu)$, $\mathrm{Br}(B \to K^*\nu\bar{\nu})/\mathrm{Br}(B \to \rho l\nu)$, we can reduce considerably the uncertainties related to the hadronic form factors [17]. For $B \to K\nu\bar{\nu}$, similarly one may consider the ratio $\mathrm{Br}(B \to K\nu\bar{\nu})/\mathrm{Br}(B \to K\ell^+\ell^-)$ where the uncertainties from the hadronic form factors are canceled to a large extent [12].

Here we will focus on $B^+ \to K^+\nu\bar{\nu}$ decay as its experimental upper bound is closest to the SM prediction as shown in Table I. Using the SM expectation value

$$\mathrm{Br}_{\mathrm{SM}}(B^+ \to K^+\nu\bar{\nu}) = 5.1 \pm 0.8 \times 10^{-6}, \tag{2}$$

and the current upper bound from BELLE [13] on this final state as

$$\mathrm{Br}\,(B \to K + \not{E}) < 14 \times 10^{-6}, \tag{3}$$

we can derive the corresponding constraint on Higgs invisible decay width.

To be kinematically allowed in $B \to KSS$, the singlet scalar cannot be heavier than 1–2 GeV. Therefore, the scalar can be easily thermalized through the Higgs interactions in the early universe. We first discuss its cosmological bound in the next section. The third section is the discussion on B decay. After taking into all the constraints, we discuss its implication in Higgs in the Sec. IV and finally present the conclusion in Sec. V.

II. COSMOLOGICAL BOUND AND DECAY OF A HIDDEN SECTOR SCALAR

If we assume the renormalizability of the theory and allow the mass term quartic self-interaction term and the quartic interaction term with the Higgs, the Lagrangian of the scalar sector is written as

$$\mathcal{L}_{\mathrm{scalar}} = \frac{1}{2}(\partial S)^2 - \frac{1}{2}m_S^2 S^2 - \frac{\lambda_S}{4!}S^4 - \frac{\hat{\lambda}}{2}S^2 H^\dagger H. \tag{4}$$

The Lagrangian respects the Z_2 symmetry ($S \to -S$) thus S is a stable particle. Indeed this scalar particle can be a good candidate of dark matter. The scalar particle could be in thermal equilibrium with the SM sector through interaction with Higgs boson in early universe and finally its relic still may survive in the current universe in the form of dark matter [5,18]. The relic density is determined by annihilation cross section of the scalar particle to the SM particles as [19]

$$\Omega_S h^2 \simeq \frac{0.1\ \mathrm{pb}}{\langle \sigma_S v_{\mathrm{rel}} \rangle}, \tag{5}$$

where σ_S is the annihilation cross section of S to the standard model particles through s-channel Higgs exchange diagrams and v is relative velocity between annihilating Ss. Since we are mainly interested in GeV scale particle, available channels are mainly to light leptons $(e, \mu, (\tau))$ and quarks $(u, d, s, c(, b))$ and the cross section is obtained as

$$\langle \sigma_S v_{\mathrm{rel}} \rangle = \frac{\hat{\lambda}^2 m_S^2}{\pi m_h^4}\Phi(m_S). \tag{6}$$

The precise value of $\Phi(m_S) \simeq \sum_f x_f^2(1 - x_f^2)^{3/2}$ where $x_f = m_f/m_S$ depends on the actual mass of scalar particle and the kinematically allowed channels. We found a stringent constraints on the annihilation cross section considering the WMAP data $\Omega_c h^2 = 0.1131 \pm 0.0034$ [20] as

$$\frac{\hat{\lambda}^2 m_S^2}{\pi m_h^4} \gtrsim \frac{0.1\ \mathrm{pb}}{\Omega h^2|_{\mathrm{WMAP\,5\,yr}}} \Rightarrow \hat{\lambda} \gtrsim 3.5 \times \left(\frac{1\ \mathrm{GeV}}{m_S}\right) \times \left(\frac{m_h}{150\ \mathrm{GeV}}\right)^2. \tag{7}$$

If $m_h \simeq 150(115)$ GeV and $m_S \simeq 1$ GeV we get $\hat{\lambda} \gtrsim 3.5(1.2)$, respectively, which is within the strong coupling regime where the perturbative description of the model is not available.

In Fig. 1, we presented the allowed parameter space in $(\hat{\lambda} = \lambda/v_0, m_S)$ plane by the 5 yr WMAP data on the CDM component with various values of Higgs mass (115, 150, 185) GeV taking threshold effects into account. Basically a GeV scale mass range, only in which range $B \to KSS$ is allowed, is not compatible with the cosmological observations.[1] On the other hand, if the scalar is heavier ($m_S >$ 2 GeV) even though the scalar cannot contribute to the B-decays but can be a successful dark matter candidate, if the λ coupling is properly chosen.

However, we can easily avoid this cosmological constraint provided that the singlet actually decays into light

[1] In Ref. [21], a scalar field in the mass range of 1 GeV has been considered and the authors reached the same conclusion with ours: a large coupling constant is required in order to avoid overabundance. However, this large coupling constant is ruled out by the $B \to K\nu\bar{\nu}$ data, as we consider in Sec. III.

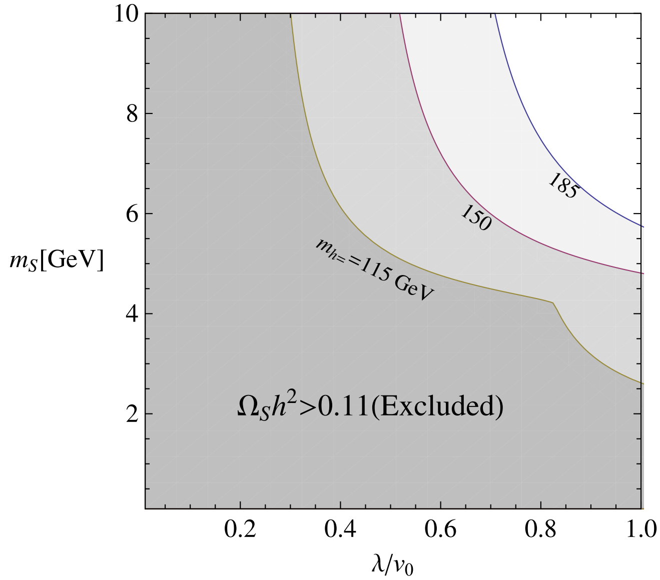

FIG. 1 (color online). Cosmological constraints for a stable S from the relic abundance. Allowed parameter space in ($\hat{\lambda} = \lambda/v_0$, m_S) plane with $m_h = 115$, 150 and 185 GeV, respectively.

particles since only (absolutely) stable particles can significantly contribute to the dark matter density of the current universe. As the longevity of the scalar particle is inherited by the Z_2 symmetry, a mechanism of breaking Z_2 symmetry leads to a natural way out. Indeed there is a very promising source of the symmetry breaking. Quantum gravity effect actually allows higher order operators and some of them might break global symmetries such as Z_2. For instance, the scalar particle may decay to a pair of photons or gluons through dimension five operators:

$$C_1 \frac{S F_{\mu\nu} F^{\mu\nu}}{\Lambda} + C_2 \frac{S G^a_{\mu\nu} G_a^{\mu\nu}}{\Lambda}, \tag{8}$$

where $C_1 \sim C_2 \sim O(1)$ are (unknown) parameters. One should notice that both operators respect gauge symmetry but break Z_2 symmetry. The life time of the scalar is suppressed by a large cutoff scale ($\Lambda \sim M_{\rm Planck}$) but certainly much shorter than the age of universe so that we can avoid the strong constraint from the relic density measurements.

III. $B \to KSS$ AND INVISIBLE HIGGS

In this section we study the constraint on the interaction term between the Higgs boson and the SM singlet from B decays. Specifically we will look at $B \to KSS$ decay which currently has the most stringent experimental upper bound 14×10^{-6} [13].

The effective Hamiltonian for this decay can be expressed as

$$H_{\rm eff} = \frac{\lambda V_{tb}^* V_{ts}}{2m_h^2} C_s \bar{s}(1 + \gamma_5) b S S. \tag{9}$$

Intuitively, $b \to sSS$ decay can be divided into two processes: first b quark decays to s quark plus a off-shell Higgs boson h, and subsequently h decays to two light singlets. From the interaction Lagrangian term $\lambda H^+ H S^2/2v_0$, with $H^+ = (\phi^-, (v_0 + h - i\phi^0)/\sqrt{2})$, it is easy to show that the Higgs boson decay $h \to SS$ can proceed through a trilinear term $\lambda h S S/2$. But as we will see later, another term $\lambda \phi^+ \phi^- S^2/2v_0$ is also crucial to guarantee the gauge independence of the decay amplitude.

To evaluate the decay amplitude, the Wilson coefficient C_s at scale $\mu_b = \mathcal{O}(m_b)$ should be known, which can be obtained by matching the full theory to the effective theory at scale around m_W to obtain $C_s(m_W)$ and then evolving down to μ_b. As the above operator does not mix with other effective operators, the QCD running effects can be obtained straightforwardly with the calculation of the anomalous dimension of $\bar{s}(1 + \gamma_5) b$ [22]:

$$C_s(\mu_b) = \left(\frac{\alpha_s(\mu_b)}{\alpha_s(m_W)}\right)^{12/23} C_s(m_W). \tag{10}$$

$C_s(m_W)$ can be obtained by calculating the diagrams in Fig. 2. Notice that the Higgs boson does not couple to s-quark by taking $m_s = 0$.

In Fig. 2, the first eight diagrams represent exactly the intuitive picture that first $b \to sh$, and then $h \to SS$. Since the later one is a trivial tree level process, one may first focus on the construction of an one-loop effective bsh vertex

$$\mathcal{L}_{bsh} = C_{bsh} V_{tb}^* V_{ts} \bar{s}(1 + \gamma_5) b h \tag{11}$$

with the coefficient in t'Hooft-Feynman gauge as [23,24]

$$C_{bsh}(m_W) = \frac{g^2}{(4\pi)^2} \frac{m_b x_t}{8 v_0} \left(3 + x_h \times \frac{(3 - x_t)(1 - x_t) + 2x_t(2 - x_t)\ln x_t}{(1 - x_t)^3}\right), \tag{12}$$

where $x_t \equiv m_t^2/m_W^2$, $x_h \equiv m_h^2/m_W^2$ with the approximation $m_b^2/(m_W^2, m_t^2, m_h^2) \simeq 0$. Notice that this expression is gauge-dependent as the Higgs boson is off shell. Although the calculation itself is straightforward, the issues about gauge dependence and renormalization scheme ambiguities are a bit subtle which were finally settled down by several groups a few years later [25].

But for the decay amplitude $b \to sSS$ to be gauge invariant, the last diagram in Fig. 2, i.e. Fig. 2(i), has to be included which (surprisingly at first look) does not contain virtual Higgs boson exchange at all. Actually Fig. 2(i) arises from the interaction term $\lambda \phi^+ \phi^- S^2/2v_0$. Therefore strictly speaking, $b \to sSS$ cannot be factorized into $b \to sh$ and $h \to SS$.

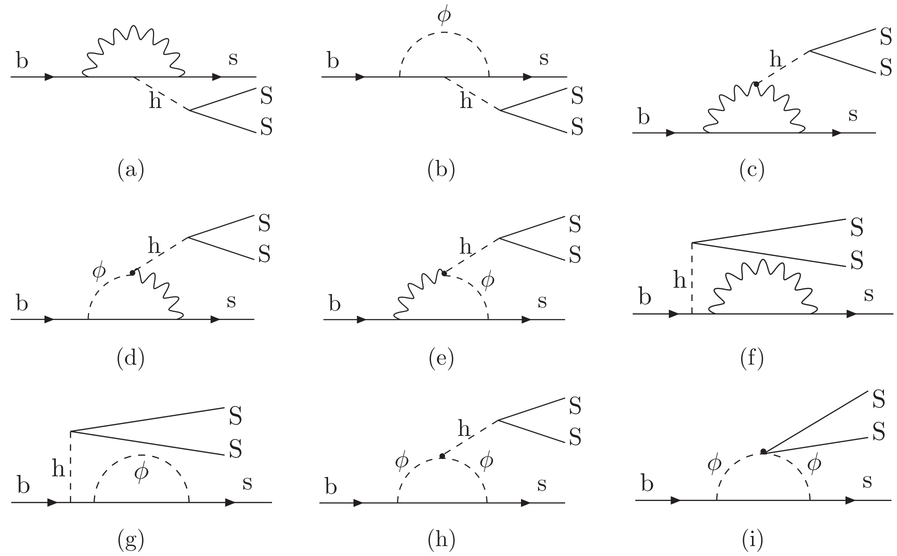

FIG. 2. b-quark decays to s-quark plus two light singlets. The internal quark lines represent up, charm or top quarks, while the internal dashed lines denote Higgs boson (h) or unphysical charged Goldstone bosons (ϕ).

Finally, summing all the diagrams, we obtain[2]

$$C_s(m_W) = \frac{g^2}{(4\pi)^2}\frac{3m_b x_t}{8v_0} \tag{13}$$

The calculation details can be found in the appendix. Here m_b should be evaluated at the scale m_W, but interestingly when combined with the QCD evolution effect of Eq. (10), one has[3]

$$m_b(m_W)\left(\frac{\alpha_s(\mu_b)}{\alpha_s(m_W)}\right)^{12/23} = m_b(m_b). \tag{14}$$

Please also note that in [11] the authors considered $b \to sSS$ in an effective theory approach, however, with C_s as model independent free parameters.

To get the decay amplitude, the hadronic matrix element $\langle K^-|\bar{s}(1+\gamma_5)b|B^-\rangle$ is needed as input, which can be related to the known form factors through equation of motion,

$$\begin{aligned}\langle K^-|\bar{s}(1+\gamma_5)b|B^-\rangle &= \frac{q^\mu}{m_b}\langle K^-|\bar{s}\gamma_\mu b|B^-\rangle \\ &= \frac{q^\mu}{m_b}\bigg(f_+(q^2)(p+l)_\mu + (f_0(q^2) \\ &\quad - f_+(q^2))\frac{m_B^2 - m_K^2}{q^2}q_\mu\bigg),\end{aligned} \tag{15}$$

with the light-cone sum rules (LCSR) estimation [26]

$$\begin{aligned} f_+(q^2) &= \frac{0.162}{1 - q^2/5.41^2} + \frac{0.173}{(1 - q^2/5.41^2)^2} \\ f_0(q^2) &= \frac{0.33}{1 - q^2/37.46}\end{aligned} \tag{16}$$

As discussed in [26], the uncertainty of the q^2 dependence of the form factors have not been fully analyzed in LCSR but likely to be smaller than that at $q^2 = 0$ which is about 12%. Thus as an rough error estimation we assign a universal 12% uncertainty to the above form factors.

Then, the branching ratio can then be obtained

$$\begin{aligned}\mathrm{Br}(B \to KSS) &= \frac{\lambda^2|V_{tb}^* V_{ts}|^2}{512\pi^3 m_B^3 \Gamma_B m_h^4} C_s^2(m_b) \int_{4m_S^2}^{(m_B - m_K)^2} dq^2 \\ &\quad \times \langle K^-|\bar{s}(1+\gamma_5)b|B^-\rangle^2 \sqrt{q^2 - 4m_S^2} \\ &\quad \times \sqrt{\frac{(m_B^2 - q^2 - m_K^2)^2}{q^2} - 4m_K^2}.\end{aligned} \tag{17}$$

Taking as illustration

$$m_h = 130\ \mathrm{GeV}, \qquad m_S = 1\ \mathrm{GeV}, \tag{18}$$

and with the values [27]

[2]This expression has been obtained in [7]. However, in the derivation, they divided the process $b \to sSS$ into $b \to sh$ and $h \to SS$. They then evaluate the bsh vertex with the approximation of vanishing Higgs boson mass. But even with these unrigorousness or approximations, they do obtain finally the correct expression which is due to the almost completely cancellation between Fig. 2(h) and 2(i) up to $O(m_b^2/m_h^2)$. However, generally this kind of cancellation does not happen and the summation should be of order m_h^2/m_W^2, as pointed out by Botella and Lim in [25].

[3]We thank the referee for pointing this out to us.

$$m_b(m_b) = 4.2\ \text{GeV}, \qquad m_t = 171.3\ \text{GeV}, \qquad A = 0.814, \qquad \lambda_{\text{CKM}} = 0.2257 \tag{19}$$

and $V_{ts} = -A\lambda_{\text{CKM}}^2$, we can obtain the branching ratio

$$\text{Br}(B \to KSS) = (0.82 \pm 0.20) \times \left(\frac{\lambda}{1\ \text{GeV}}\right)^2 \left(\frac{130\ \text{GeV}}{m_h}\right)^4 \times 10^{-10}, \tag{20}$$

where only the form factor uncertainty has been included in the error estimation.

IV. INVISIBLE HIGGS

If there exists such light SM singlet scalar, the Higgs decay can be significantly modified. For $m_S = 1$ GeV, we take the upper bound on λ derived from the $B \to K\not{E}$ as

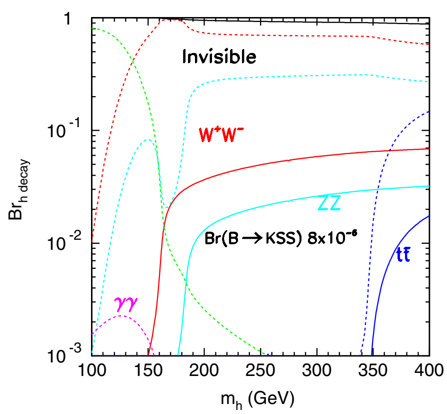

FIG. 3 (color online). Higgs boson decay BR with Invisible decay mode predicted from current upper bound of $B \to K\nu\bar{\nu}$ in solid lines. *(For comparison, dashed lines are for SM Higgs decay BR.)*

$$\begin{aligned}\text{Br}(B \to KSS) &= 0.82 \times 10^{-10}\left(\frac{\lambda}{1\ \text{GeV}}\right)^2\left(\frac{130\ \text{GeV}}{m_h}\right)^4 \\ &\leq \text{Br}_{\text{exp}}(B \to K\not{E}) - \text{Br}_{\text{SM}}(B \to K\nu\bar{\nu}) \\ &\simeq 8 \times 10^{-6}\end{aligned} \tag{21}$$

and compute the upper bound of partial width for $h \to SS$. The partial width of Higgs decaying into two scalar is

$$\Gamma(h \to SS) = \frac{\lambda^2}{32\pi m_h}\left(1 - \frac{4m_S^2}{m_h^2}\right)^{1/2}, \tag{22}$$

where λ is the dimension one coupling and m_h, m_S are the Higgs boson mass and hidden sector scalar mass, respectively. To illustrate the feature, we scan m_h and plot in Fig. 3 how the Higgs decay BR will be changed due to the $h \to SS$ decay. The partial width of $h \to SS$ is obtained by taking $m_S = 1$ GeV and the λ upper bound value computed for that m_h point.

If $m_h < 150$ GeV, $h \to SS$ completely dominates the Higgs decay and Higgs is only invisible. Even though the traditional invisible Higgs search can be applied to search for such modes, it is impossible to identify the resonance through invisible modes at the LHC.

When $m_h > 150$ GeV, the partial width of $h \to SS$ is comparable to the partial widths of conventional channels, such as $h \to W^+W^-$ or $h \to ZZ$. The multilepton searches for Higgs resonance are still valid but the decay BRs significantly decrease. If the measured event numbers of $h \to W^+W^-$ or $h \to ZZ$ are below the expected numbers. There are several possibilities:

(i) There are more than one Higgs boson responsible for the W gauge boson mass M_W. The vacuum expectation value for the lightest Higgs boson is much smaller than v_0 so that the coupling W^+W^-H is gv'.

(ii) The production of Higgs boson is suppressed due to new physics. For instance, $gg \to H$ production is less due to the top quark partner in the triangle loop and significantly cancels the top quark loop.

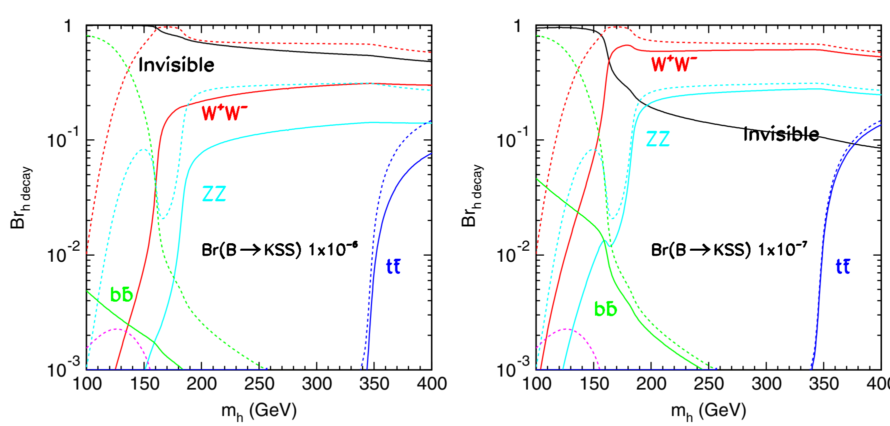

FIG. 4 (color online). Solid lines correspond to the Higgs BR with invisible decay mode predicted from the upper bound value for $m_S = 1$ GeV and $\text{Br}(B \to KSS) = 1 \times 10^{-6}$ or 1×10^{-7} respectively. Dashed lines are the standard SM Higgs decay BR.

(iii) There exists unknown Higgs decay mode which cannot be easily identified. The invisible Higgs mode that we discuss here falls into this category. Another example is the $h \to \nu N$ decay in some TeV neutrino models [3].

We expect the SuperB or SuperBelle will improve the measurement significantly and reduce the allowed region of $\mathrm{Br}(B \to KSS) = \mathrm{Br}_{\mathrm{exp}}(B \to K\not{E}) - \mathrm{Br}_{\mathrm{SM}}(B \to K\nu\bar{\nu})$. In Fig. 4, we plot how the Higgs decay BR will change accordingly for $m_S = 1$ GeV and improved bound on $\mathrm{Br}(B \to KSS)$. As can be seen, if the value of $\mathrm{Br}(B \to KSS)$ becomes smaller than 2×10^{-6}, it will only change the Higgs decay before on-shell WW threshold and will not significantly change the heavy Higgs decay.

V. CONCLUSION

We have studied the contribution of virtual Higgs in $B \to K\not{E}$ by assuming Higgs coupling to a light SM singlet scalar S, $B \to KSS$. For $M_S = 1$ GeV,

$$\mathrm{Br}(B \to KSS) = (0.82 \pm 0.20) \times \left(\frac{\lambda}{1\ \mathrm{GeV}}\right)^2 \times \left(\frac{130\ \mathrm{GeV}}{m_h}\right)^4 \times 10^{-10}.$$

Given the current experimental bound and subtracting the known SM contribution,

$$\mathrm{Br}_{\mathrm{exp}}(B \to K\not{E}) - \mathrm{Br}_{\mathrm{SM}}(B \to K\nu\bar{\nu}) \simeq 8 \times 10^{-6},$$

we obtain an upper bound on the coupling between the Higgs and singlet scalar S. We take the upper bound value of this coupling and compute the $h \to SS$ decay partial width. It is interesting that the partial width of $h \to SS$ decay is at comparable level when the Higgs mass is close to the WW threshold. Consequently, Higgs may still be discovered via the conventional Higgs search channels but with a smaller event number. This will have some impact on precision measurement of Higgs property. We expect that the SuperB or SuperBelle experiments can improve the $B \to K\not{E}$ measurement and put a stringent bound on possible invisible Higgs decay.

We have also studied the possible implication in cosmology from this scalar. It turns out that for the interesting region of couplings between h and S, such light scalar may not have enough annihilation cross section and will then over close the universe if it is a stable particle. We propose a scenario where S is not stable in the cosmological scale but only a stable particle in B decay or collider environments.

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APPENDIX: CALCULATION DETAILS ON $b \to sSS$

In the calculations, we use the t'Hooft-Feynman gauge $\xi = 1$. p, l and q denote the momentum of b-quark, s-quark and virtual Higgs boson, respectively. We have taken the approximation $q^2 - m_h^2 \simeq -m_h^2$ and dropped a common factor λ/m_h^2 in the following expressions. We get

$$\mathrm{Fig.\,2(a)} = \frac{-ig^2 V_{tb}^* V_{ts}}{(4\pi)^2} \bar{s}(l)(1+\gamma_5)b(p)\frac{m_b}{4v_0} \times \frac{x_t(x_t^2 - 1 - 2x_t \ln x_t)}{(x_t - 1)^3} \tag{A1}$$

with $x_t \equiv m_t^2/m_W^2$.

$$\mathrm{Fig.\,2(b)} = \frac{-ig^2 V_{tb}^* V_{ts}}{(4\pi)^2} \bar{s}(l)(1+\gamma_5)b(p)\frac{x_t m_b}{4v_0}\left(\frac{1}{\epsilon} - \gamma + \ln 4\pi - \frac{1}{2} - 2\int_{x+y\le 1} dxdy\left(\ln\frac{\Delta_1(x,y)}{\mu^2} + \frac{m_t^2 y}{\Delta_1(x,y)}\right)\right) \tag{A2}$$

with

$$\Delta_1(x,y) \simeq (1-x-y)m_W^2 + (x+y)m_t^2.$$

The divergence of Fig. 2(b) can be canceled by that of Fig. 2(g):

$$\mathrm{Fig.\,2(g)} = \frac{ig^2 V_{tb}^* V_{ts}}{(4\pi)^2} \bar{s}(l)(1+\gamma_5)b(p)\frac{x_t m_b}{4v_0}\left(\frac{1}{\epsilon} - \gamma + \ln 4\pi - \int_0^1 dx \ln\frac{x m_t^2 + (1-x)m_W^2}{\mu^2}\right). \tag{A3}$$

The sum of Figs. 2(b) and 2(g) then gives (taking the scale $\mu = m_W$)

$$\mathrm{Fig.\,2(b+g)} = \frac{ig^2 V_{tb}^* V_{ts}}{(4\pi)^2} \bar{s}(l)(1+\gamma_5)b(p)\frac{x_t^2 m_b}{4v_0} \times \frac{(3x_t - 5)(x_t - 1) - 2(x_t - 2)\ln x_t}{(x_t - 1)^3}. \tag{A4}$$

It is clear that for Figs. 2(a), 2(b), and 2(g), the internal up and charm quarks contributions are suppressed at least by

$m_{u,c}^2/m_t^2$ compared to the virtual top quark contribution and can be safely neglected.

For Fig. 2(c), the internal top quark contribution is

$$\mathrm{Fig.\,2(c)}_t = \frac{-ig^2 V_{tb}^* V_{ts}}{(4\pi)^2} \bar{s}(l)(1+\gamma_5)b(p)\frac{m_b}{4v_0} \times \frac{2x_t^2 \ln x_t - (3x_t - 1)(x_t - 1)}{(x_t - 1)^3}. \quad \text{(A5)}$$

But here the internal up and charm quarks contributions are not suppressed, which can be obtained from the above expression by taking the limit $x_t \to 0$ and changing the corresponding CKM factors. We then obtain using the CKM unitarity condition,

$$\mathrm{Fig.\,2(c)} = \frac{-ig^2 V_{tb}^* V_{ts}}{(4\pi)^2} \bar{s}(l)(1+\gamma_5)b(p)\frac{m_b}{4v_0} \times \left(\frac{2x_t^2 \ln x_t - (3x_t - 1)(x_t - 1)}{(x_t - 1)^3} - 1\right). \quad \text{(A6)}$$

The virtual top quark contribution to Fig. 2(d) is

$$\mathrm{Fig.\,2(d)}_t = \frac{-ig^3 V_{tb}^* V_{ts}}{(4\pi)^2} \bar{s}(l)(1+\gamma_5)b(p)\frac{m_b}{8m_W} \times \left(\frac{1}{\epsilon} - \gamma + \ln 4\pi - \frac{1}{2} - \int_{x+y\le 1} dxdy \times \left(2\ln\frac{\Delta_2(x,y)}{\mu^2} + \frac{(1+x+y)m_t^2}{\Delta_2(x,y)}\right)\right) \quad \text{(A7)}$$

with

$$\Delta_2(x,y) \simeq x m_t^2 + (1-x)m_W^2. \quad \text{(A8)}$$

The divergence here can be canceled when the contributions from the internal up and charm quarks are included, then we get

$$\mathrm{Fig.\,2(d)} = \frac{ig^3 V_{tb}^* V_{ts}}{(4\pi)^2} \bar{s}(l)(1+\gamma_5)b(p)\frac{x_t m_b}{32 m_W} \times \frac{2x_t(5x_t - 6)\ln x_t - (9x_t - 11)(x_t - 1)}{(x_t - 1)^3}. \quad \text{(A9)}$$

For Fig. 2(e), we have

$$\mathrm{Fig.\,2(e)} = \frac{-ig^3 V_{tb}^* V_{ts}}{(4\pi)^2} \bar{s}(l)(1+\gamma_5)b(p)\frac{x_t m_b}{32 m_W} \times \frac{2x_t(3x_t - 2)\ln x_t - (7x_t - 5)(x_t - 1)}{(x_t - 1)^3}. \quad \text{(A10)}$$

Here the internal up and charm quarks contributions are again negligibly small due to the $\mathcal{O}(m_{u,c}^2/m_t^2)$ suppression. It is easy to show that the contribution of Fig. 2(f) vanishes by using the equation of motion $\bar{s}(l)/\!\!\!l = 0$. The cancellation between Fig. 2(h) and 2(i) is obvious by approximating the Higgs boson propagator $i/(q^2 - m_h^2) \simeq -i/m_h^2$.

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