Baryon fields with $U_L(3) \times U_R(3)$ chiral symmetry: Axial currents of nucleons and hyperons

Hua-Xing Chen (陈 华星),[1,*] V. Dmitrašinović,[2,†] and Atsushi Hosaka (保坂 淳)[3,‡]

[1]*Department of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China*
[2]*Vinča Institute of Nuclear Sciences, lab 010, P.O. Box 522, 11001 Beograd, Serbia*
[3]*Research Center for Nuclear Physics, Osaka University, Ibaraki 567-0047, Japan*

(Received 21 December 2009; published 8 March 2010)

We use the conventional F and D octet and decimet generator matrices to reformulate chiral properties of local (nonderivative) and one-derivative nonlocal fields of baryons consisting of three quarks with flavor $SU(3)$ symmetry that were expressed in $SU(3)$ tensor form by Chen *et al.* [Phys. Rev. D **78**, 054021 (2008)]. We show explicitly the chiral transformations of the $[(6,3) \oplus (3,6)]$ chiral multiplet in the "$SU(3)$ particle basis," for the first time to our knowledge, as well as those of the $(3,\bar{3}) \oplus (\bar{3},3)$, $(8,1) \oplus (1,8)$ multiplets, which have been recorded before by Bardeen and Lee [Phys. Rev. **177**, 2389 (1969)] and Lee [Phys. Rev. **170**, 1359 (1968)]. We derive the vector and axial-vector Noether currents, and show explicitly that their zeroth (chargelike) components close the $SU_L(3) \times SU_R(3)$ chiral algebra. We use these results to study the effects of mixing of (three-quark) chiral multiplets on the axial current matrix elements of hyperons and nucleons. We show, in particular, that there is a strong correlation, indeed a definite relation between the flavor-singlet (i.e. the zeroth), the isovector (the third), and the eighth flavor component of the axial current, which is in decent agreement with the measured ones.

DOI: 10.1103/PhysRevD.81.054002 PACS numbers: 14.20.−c, 11.30.Rd, 11.40.Dw

I. INTRODUCTION

Axial current "coupling constants" of the baryon flavor octet [1] are well known by now; see Refs. [2,3]. The zeroth (timelike) components of these axial currents are generators of the $SU_L(3) \times SU_R(3)$ chiral symmetry that is one of the fundamental symmetries of QCD. The general flavor $SU_F(3)$ symmetric form of the nucleon axial current contains two free parameters, the so-called F and D couplings, which are empirically determined as $F = 0.459 \pm 0.008$ and $D = 0.798 \pm 0.008$; see Ref. [2]. The conventional models of (linearly realized) chiral $SU_L(3) \times SU_R(3)$ symmetry, Refs. [5,6], on the other hand appear to fix these parameters at either ($F = 0$, $D = 1$), which case goes by the name of $[(3,\bar{3}) \oplus (\bar{3},3)]$, or at ($F = 1$, $D = 0$), which case goes by the name of $[(8,1) \oplus (1,8)]$ representation. Both of these chiral representations suffer from the shortcoming that $F + D = 1 \neq g_A^{(3)} = 1.267$ without derivative couplings. But, even with derivative interactions, one cannot change the value of the vanishing coupling, e.g. of $F = 0$, in $[(3,\bar{3}) \oplus (\bar{3},3)]$, or of $D = 0$, in $[(8,1) \oplus (1,8)]$. Rather, one can only renormalize the nonvanishing coupling to 1.267.

Attempts at a reconciliation of the measured values of axial couplings with the (broken) $SU_L(3) \times SU_R(3)$ chiral symmetry go back at least 40 years [5–11], but, none have been successful to our knowledge thus far. As noted above, perhaps the most troublesome problem are the $SU(3)$ axial current's F, D values, which problem has repercussions for the meson-baryon interaction F, D values, with far-reaching consequences for hypernuclear physics and even astrophysics. Another, perhaps equally important and difficult problem is that of the flavor-singlet axial coupling of the nucleon [12]. This is widely thought of as being disconnected from the F, D problem, but we shall show that the three-quark interpolating fields cast some perhaps unexpected light on this problem. We shall attack both of these problems from Weinberg's [11] point of view, viz. chiral representation mixing, extended to the $SU_L(3) \times SU_R(3)$ and $U_L(1) \times U_R(1)$ chiral symmetries, with added input from three-quark baryon interpolating fields [13] that are ordinarily used in QCD calculations.

The basic idea is simple: a mixture of two baryon fields belonging to different chiral representations/multiplets has axial couplings that lie between the extreme values determined by the two chiral multiplets that are being mixed, and depend on the mixing angle, of course. Weinberg used this idea to fit the isovector axial coupling of the nucleon using the $[(1/2,0) \oplus (0,1/2)]$ and $[(1,1/2) \oplus (1/2,1)]$ multiplets of the $SU_L(2) \times SU_R(2)$ chiral symmetry, but the same idea may be used on any baryon belonging to the same octet, e.g. for the Λ, Σ, and Ξ hyperons. In other words, the F and D values of the mixture can be determined from the F and D values of the $SU_L(3) \times SU_R(3)$ representations corresponding to the $[(1/2,0) \oplus (0,1/2)]$ and $[(1,1/2) \oplus (1/2,1)]$ multiplets, viz. $[(3,\bar{3}) \oplus (\bar{3},3)]$ or $[(8,1) \oplus (1,8)]$, and $[(6,3) \oplus (3,6)]$, respectively. The same principle holds for the $U_L(1) \times U_R(1)$ symmetry "multiplets" and the value(s) of the flavor-singlet axial charge.

The $SU_L(3) \times SU_R(3)$ and $U_L(1) \times U_R(1)$ chiral transformation properties of three-quark baryon interpolating

*hxchen@rcnp.osaka-u.ac.jp
†dmitra@vinca.rs
‡hosaka@rcnp.osaka-u.ac.jp

fields, that are commonly used in various QCD (lattice, sum rules) calculations, and that have recently been determined in Ref. [13] will be used here as input into the chiral mixing formalism, so as to deduce as much phenomenological information about the axial currents of hyperons and nucleons as possible. As a result we find three "optimal" scenarios all with identical F, D values (see Sec. IV).

First we recast our previous results [13] into the language that is conventional for axial currents, i.e. in terms of octet F and D couplings. A large part of the present paper is devoted to this notational conversion (change of basis) and the subsequent check whether and how the resulting chiral charges actually satisfy the $SU(3) \times SU(3)$ chiral algebra. That is a nontrivial task for the $[(3, \bar{3}) \oplus (\bar{3}, 3)]$ and $(6, 3) \oplus (3, 6)$ representations, because they involve off-diagonal terms, and in the latter case one of the diagonal terms in the axial current is multiplied by a fractional coefficient, that appears to spoil the closure of the $SU(3) \times SU(3)$ chiral algebra; the off-diagonal terms in the axial current make crucial contributions that restore the closure. Thus, the aforementioned fractional coefficient is uniquely determined.

We use these results to study the effects of mixing of (three-quark) chiral multiplets on the axial current matrix elements of hyperons and nucleons. We show, in particular, that there is a strong correlation between the flavor-singlet (i.e. the zeroth), the isovector (the third), and the eighth flavor component of the axial current. There are, in principle, three independent observables here: the flavor-singlet (i.e. the zeroth), the isovector (the third), and the eighth flavor component of the axial current of the nucleon. By fitting just one mixing angle to one of these values, e.g. the (best known) isovector coupling, we predict the other two. These predictions may differ widely depending on the field that one assumes to be mixed with the $(6, 3) \oplus (3, 6)$ field (which must be present if the isovector axial coupling has any chance of being fit). If one assumes mixing of three fields (again, always keeping the $(6, 3) \oplus (3, 6)$ as one of the three) and fits the flavor-singlet and the isovector axial couplings, then one finds a *unique* prediction for the F, D values, which is in decent agreement with the measured ones, modulo $SU(3)$ symmetry breaking corrections, which may be important (for a recent fit, see Ref. [2]). The uniqueness of this result is a consequence of a remarkable relation, $g_A^{(0)} = 3F - D$, that holds for all three (five) chiral multiplets involved here, and which leads to the relation $g_A^{(0)} = \sqrt{3} g_A^{(8)}$; see Sec. IV.

Most of the ideas used in this paper, such as that of chiral multiplet mixing, have been presented in mid- to late 1960's, Refs. [8–11], with the (obvious) exception of the use of QCD interpolating fields, which arrived only a decade later, and the (perhaps less obvious) question of baryons' flavor-singlet axial current (also known as the $U_A(1)$), which was (seriously) raised yet another decade later.

The present paper consists of five parts: after the present Introduction, in Sec. II we define the $SU(3) \times SU(3)$ chiral transformations of three-quark baryon fields, with special emphasis on the $SU(3)$ phase conventions that ensure standard $SU(2)$ isospin conventions for the isospin submultiplets, and we define the [$SU(3)$ symmetric] vector and axial-vector Noether currents of three-quark baryon fields. In Sec. III we prove the closure of the chiral $SU_L(3) \times SU_R(3)$ algebra. In Sec. IV we apply chiral mixing formalism to the hyperons' axial currents and discuss the results. Finally, in Sec. V we offer a summary and an outlook on future developments.

II. $SU(3) \times SU(3)$ CHIRAL TRANSFORMATIONS OF THREE-QUARK BARYON FIELDS AND THEIR NOETHER CURRENTS

We must make sure that our conventions ensure that identical isospin multiplets in different $SU(3)$ multiplets, such as the octet and the decuplet, have identical isospin algebras/generators. That is a relatively simple matter of definition, but was not the case with the octet conventions used in Ref. [13]. Our new definitions of the octet and decuplet fields avoid these problems.

A. Octet and decuplet state definition

The new Ξ^- wave function comes with a minus sign: that is precisely the convention used in Eqs. (18) and (19) in Sect. 18 of Gasiorowicz's textbook [14]. But then we must also adjust the 8×10 $SU(3)$-spurion matrices for this modification.

$$\Sigma^{\mp} \sim \frac{\pm 1}{\sqrt{2}}(N^1 \pm iN^2), \qquad N^3 \sim \Sigma^0, \qquad N^8 \sim \Lambda_8,$$
$$\begin{pmatrix} \Xi^- \\ p \end{pmatrix} \sim \frac{\mp 1}{\sqrt{2}}(N^4 \pm iN^5), \qquad \begin{pmatrix} \Xi^0 \\ n \end{pmatrix} \sim \frac{1}{\sqrt{2}}(N^6 \pm iN^7). \tag{1}$$

$$\begin{pmatrix} p \\ n \\ \Sigma^+ \\ \Sigma^0 \\ \Sigma^- \\ \Xi^0 \\ \Xi^- \\ \Lambda_8 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{\sqrt{2}} & \frac{-i}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & \frac{-i}{\sqrt{2}} & 0 \\ \frac{-1}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & \frac{-1}{\sqrt{2}} & \frac{-i}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} N^1 \\ N^2 \\ N^3 \\ N^4 \\ N^5 \\ N^6 \\ N^7 \\ N^8 \end{pmatrix}, \tag{2}$$

or put them into the 3×3 baryon matrix as follows:

$$\mathfrak{N} = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^8}{\sqrt{6}} & -\Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^8}{\sqrt{6}} & n \\ -\Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda^8 \end{pmatrix}. \tag{3}$$

Note the minus signs in front of Ξ^- and Σ^+. We also use a new normalization of the decuplet fields:

$$\begin{aligned} &\Delta^1 \sim -\frac{1}{\sqrt{3}}\Delta^{++}, \qquad \Delta^7 \sim -\frac{1}{\sqrt{3}}\Delta^{-},\\ &\Delta^{10} \sim -\frac{1}{\sqrt{3}}\Omega^-, \qquad \Delta^2 \sim -\Delta^+, \qquad \Delta^4 \sim -\Delta^0,\\ &\Delta^3 \sim -\Sigma^{*+}, \qquad \Delta^8 \sim -\Sigma^{*-}, \qquad \Delta^6 \sim -\Xi^{*0},\\ &\Delta^9 \sim -\Xi^{*-}, \qquad \Delta^5 \sim -\sqrt{2}\Sigma^{*0}. \end{aligned} \tag{4}$$

For the singlet Λ, we use the normalization

$$\Lambda_1 = \Lambda_{\rm phy} = \frac{2\sqrt{2}}{\sqrt{3}}\Lambda. \tag{5}$$

For simplicity, we will just use Λ_1 instead of $\Lambda_{\rm phy}$ in the following sections.

We define the flavor-octet and -decuplet matrices/column vectors as

$$N = (p, n, \Sigma^+, \Sigma^0, \Sigma^-, \Xi^0, \Xi^-, \Lambda_8)^T, \tag{6}$$

$$\Delta = (\Delta^{++}, \Delta^+, \Delta^0, \Delta^-, \Sigma^{*+}, \Sigma^{*0}, \Sigma^{*-}, \Xi^{*0}, \Xi^{*-}, \Omega)^T. \tag{7}$$

In our previous paper, Ref. [13], we found that the baryon interpolating fields $N_+^a = N_1^a + N_2^a$ belong to the chiral representation $(\mathbf{8},\mathbf{1})\oplus(\mathbf{1},\mathbf{8})$; Λ and $N_-^a = N_1^a - N_2^a$ belong to the chiral representation $(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})$; N_μ^a and Δ_μ^P belong to the chiral representation $(\mathbf{6},\mathbf{3})\oplus(\mathbf{3},\mathbf{6})$; and $\Delta_{\mu\nu}^P$ belong to the chiral representation $(\mathbf{10},\mathbf{1})\oplus(\mathbf{1},\mathbf{10})$. Here N_1^a and N_2^a are the two independent kinds of nucleon fields. N_1^a contains the "scalar diquark" and N_2^a contains the "pseudoscalar diquark." Moreover, we calculated their chiral transformations in Ref. [13]. That form, however, is not conventionally used for the axial currents. So in the following subsections, we use different conventions, listed above, and display the chiral transformations in these bases.

B. Chiral transformations of three-quark interpolating fields

1. (8, 1) ⊕ (1, 8) *Chiral transformations*

This chiral representation contains the flavor-octet representation **8**. For the octet baryon field N^a ($a = 1, \cdots, 8$), chiral transformations are given by

$$\delta_5^{\vec b} N_+ = i\gamma_5 b^a \mathbf{F}^a_{(8)} N_+. \tag{8}$$

The $SU(3)$-spurion matrices $\mathbf{F}^a_{(8)}$ are listed in Appendix A 2. This corresponds to the chiral transformations of Ref. [13]:

$$\delta_5^{\vec b}(N_1^a + N_2^a) = \gamma_5 b^b f^{bac}(N_1^c + N_2^c).$$

The coefficients f^{abc} are the standard antisymmetric "structure constants" of $SU(3)$. For completeness' sake, we show the following equation which defines the f and d coefficients:

$$\begin{aligned} \lambda^a_{AB}\lambda^b_{BC} &= (\lambda^a\lambda^b)_{AC} = \frac{1}{2}\{\lambda^a, \lambda^b\}_{AC} + \frac{1}{2}[\lambda^a, \lambda^b]_{AC}\\ &= \frac{2}{3}\delta^{ab}\delta_{AC} + (d^{abc} + if^{abc})\lambda^c_{AC}. \end{aligned} \tag{9}$$

2. $(3, \bar{3}) \oplus (\bar{3}, 3)$ *Chiral transformations*

This chiral representation contains the flavor-octet and -singlet representations $\bar{\mathbf{3}}\otimes\mathbf{3} = \mathbf{8}\oplus\mathbf{1} \sim (N^a, \Lambda)$. These two flavor representations are mixed under chiral transformations as

$$\begin{aligned} \delta_5^{\vec b}\Lambda_1 &= i\gamma_5 b^a \sqrt{\frac{2}{3}}\mathbf{T}^a_{1/8} N_-,\\ \delta_5^{\vec b} N_- &= i\gamma_5 b^a\Big(\mathbf{D}^a N_- + \sqrt{\frac{2}{3}}\mathbf{T}^{a\dagger}_{1/8}\Lambda_1\Big), \end{aligned} \tag{10}$$

where $\mathbf{D}^a$ are defined in Appendix A 1. The $SU(3)$-spurion matrices $\mathbf{T}^a_{1/8}$ have the following properties:

$$\mathbf{T}^a_{1/8}\mathbf{T}^{a\dagger}_{1/8} = 8, \qquad \mathbf{T}^{a\dagger}_{1/8}\mathbf{T}^a_{1/8} = \mathbf{1}_{8\times 8}, \tag{11}$$

and are listed in Appendix A 4. Here $\mathbf{1}_{8\times 8}$ is a unit matrix of 8×8 dimensions.

3. (6, 3) ⊕ (3, 6) *Chiral transformations*

This chiral representation contains flavor-octet and -decuplet representations $\mathbf{6}\otimes\mathbf{3} = \mathbf{8}\oplus\mathbf{10} \sim (N^a, \Delta^b)$. For their chiral transformations we use the results from Ref. [13], where they were expressed in terms of coefficients g, g', g'', and g''' that were tabulated in Table II. For off-diagonal terms (between octet and decuplet), there is a (new) factor $\frac{1}{6}$, which comes from the different normalization of octet and decuplet. Here we show the final result:

$$\begin{aligned} \delta_5^{\vec b} N &= i\gamma_5 b^a\Big(\Big(\mathbf{D}^a + \frac{2}{3}\mathbf{F}^a_{(8)}\Big)N + \frac{2}{\sqrt{3}}\mathbf{T}^a\Delta\Big),\\ \delta_5^{\vec b}\Delta &= i\gamma_5 b^a\Big(\frac{2}{\sqrt{3}}\mathbf{T}^{a\dagger}N + \frac{1}{3}\mathbf{F}^a_{(10)}\Delta\Big). \end{aligned} \tag{12}$$

These $SU(3)$-spurion matrices $\mathbf{T}^a$ (sometimes we use $\mathbf{T}^a_{10/8}$) and $\mathbf{F}^a_{(10)}$ have the following properties:

$$\begin{aligned} \mathbf{F}^a_{(10)} = -if^{abc}\mathbf{T}^{b\dagger}\mathbf{T}^c, \qquad \mathbf{T}^a\mathbf{T}^{a\dagger} = \frac{5}{2}\mathbf{1}_{8\times 8},\\ \mathbf{T}^{a\dagger}\mathbf{T}^a = 2\,\mathbf{1}_{10\times 10}. \end{aligned} \tag{13}$$

These transition matrices $\mathbf{T}^c$ and the decuplet generators $\mathbf{F}^a_{(10)}$ are listed in Appendixes A 2 and A 3, respectively.

C. Noether currents of the chiral $SU_L(3) \times SU_R(3)$ symmetry

The chiral $SU_L(3) \times SU_R(3)$ transformations of the baryon fields B_i define the eight components of the baryon isovector axial current $\mathbf{J}^a_{\mu 5}$, by way of Noether's theorem:

$$- \boldsymbol{b} \cdot \boldsymbol{J}_{\mu 5} = \sum_i \frac{\partial \mathcal{L}}{\partial \partial^\mu B_i} \delta_5^{\vec{b}} B_i. \tag{14}$$

Similarly, the flavor $SU(3)$ transformations $\delta^{\vec{a}} B_i$ define the Lorentz-vector baryon Noether (flavor-octet) current

$$- \boldsymbol{a} \cdot \boldsymbol{J}_{\mu} = \sum_i \frac{\partial \mathcal{L}}{\partial \partial^\mu B_i} \delta^{\vec{a}} B_i. \tag{15}$$

1. The axial current in the $(8, 1) \oplus (1, 8)$ *multiplet*

Equation (8), the chiral $SU_L(3) \times SU_R(3)$ transformation rules of the $B_i = N^i_+$ baryons in the $(8, 1) \oplus (1, 8)$ chiral multiplet, define the eight components of the (hyperon) flavor-octet axial current $\mathbf{J}^a_{\mu 5}$, by way of Noether's theorem, Eq. (14), where B_i are the octet N^i baryon fields. The axial Noether current $\mathbf{J}_{\mu 5}$ is

$$\mathbf{J}^{\,a}_{\mu 5} = \bar{N} \gamma_\mu \gamma_5 \mathbf{F}^a_{(8)} N, \tag{16}$$

where $\mathbf{F}^i_{(8)}$ are the $SU(3)$ octet matrices/generators, and the Lorentz-vector Noether (flavor-octet) current in this multiplet reads

$$\mathbf{J}^{\,a}_{\mu} = \bar{N} \gamma_\mu \mathbf{F}^a_{(8)} N. \tag{17}$$

Equations (16) and (17) for the two Noether currents hold when the interactions do not contain derivatives.

2. The axial current in the $(\bar{3}, 3) \oplus (3, \bar{3})$ *multiplet*

Equations (10), the chiral $SU_L(3) \times SU_R(3)$ transformation rules of the $B_i = (N^i_-, \Lambda)$ baryons in the $(\bar{3}, 3) \oplus (3, \bar{3})$ chiral multiplet, define the eight components of the (hyperon) flavor-octet axial current $\mathbf{J}^a_{\mu 5}$, by way of Noether's theorem, Eq. (14), where B_i are the flavor-octet N^i_- and the flavor-singlet Λ_1 baryon fields. The axial current $\mathbf{J}_{\mu 5}$ is

$$\mathbf{J}^{\,a}_{\mu 5} = \bar{N} \gamma_\mu \gamma_5 \left(\mathbf{D}^a N + \sqrt{\frac{2}{3}} \mathbf{T}^{a\dagger}_{1/8} \Lambda_1 \right) + \bar{\Lambda}_1 \gamma_\mu \gamma_5 \sqrt{\frac{2}{3}} \mathbf{T}^a_{1/8} N. \tag{18}$$

Here $\mathbf{D}^i$ are the $SU(3)$ octet matrices/generators. The Lorentz-vector Noether (flavor-octet) current in this multiplet reads

$$\mathbf{J}^{\,a}_{\mu} = \bar{N} \gamma_\mu \mathbf{F}^a_{(8)} N. \tag{19}$$

3. Axial current in the $(3, 6) \oplus (6, 3)$ *multiplet*

The chiral $SU_L(3) \times SU_R(3)$ transformation rules of the $B_i = (N^i, \Delta^j)$ baryons, Eqs. (12), in the $(3, 6) \oplus (6, 3)$ chiral multiplet, define the eight components of the (hyperon) flavor-octet axial current $\mathbf{J}^a_{\mu 5}$, by way of Noether's theorem [Eq. (14)], where B_i are the octet N^i and the decuplet Δ^j baryon fields. The axial current $\mathbf{J}_{\mu 5}$ is

$$\mathbf{J}^{\,a}_{\mu 5} = \bar{N} \gamma_\mu \gamma_5 \left(\left(\mathbf{D}^a + \frac{2}{3} \mathbf{F}^a_{(8)} \right) N + \frac{2}{\sqrt{3}} \mathbf{T}^a \Delta \right) + \bar{\Delta} \gamma_\mu \gamma_5 \left(\frac{2}{\sqrt{3}} \mathbf{T}^{a\dagger} N + \frac{1}{3} \mathbf{F}^a_{(10)} \Delta \right). \tag{20}$$

Here $\mathbf{D}^i$ and $\mathbf{F}^i_{(8)}$ are the $SU(3)$ octet matrices/generators $\mathbf{D}^a$ and $\mathbf{F}^a_{(8)}$, respectively; $\mathbf{F}^i_{(10)}$ are the $SU(3)$ decuplet generators; and $\mathbf{T}^i$ are the so-called $SU(3)$-spurion matrices. The Lorentz-vector Noether (flavor-octet) current in this multiplet reads

$$\mathbf{J}^{\,a}_{\mu} = (\bar{N} \gamma_\mu \mathbf{F}^a_{(8)} N) + (\bar{\Delta} \gamma_\mu \mathbf{F}^a_{(10)} \Delta). \tag{21}$$

III. CLOSURE OF THE CHIRAL $SU_L(3) \times SU_R(3)$ ALGEBRA

The $SU(3)$ vector charges $Q^a = \int d\mathbf{x} J^a_0(t, \mathbf{x})$ defined by Eq. (15), together with the axial charges $Q^a_5 = \int d\mathbf{x} J^a_{05}(t, \mathbf{x})$ defined by Eq. (14) ought to close the chiral algebra

$$[Q^a, Q^b] = i f^{abc} Q^c, \tag{22}$$

$$[Q^a_5, Q^b] = i f^{abc} Q^c_5, \tag{23}$$

$$[Q^a_5, Q^b_5] = i f^{abc} Q^c, \tag{24}$$

where f^{abc} are the $SU(3)$ structure constants. Equations (22) and (23) usually hold automatically, as a consequence of the canonical (anti)commutation relations between Dirac baryon fields B_i, whereas Eq. (24) is not trivial for the chiral multiplets that are different from the $[(8, 1) \oplus (1, 8)]$, because of the (nominally) fractional axial charges and the presence of the off-diagonal components. When taking a matrix element of Eq. (24) by baryon states in a certain chiral representation, the axial charge mixes different flavor states within the same chiral representation. This is an algebraic version of the Adler-Weisburger sum rule [11]. In the following we shall check and confirm the validity of Eq. (24) in the three multiplets of $SU_L(3) \times SU_R(3)$.

A. Closure of the chiral $SU_L(3) \times SU_R(3)$ algebra in the $(8, 1) \oplus (1, 8)$ multiplet

Because of the absence of fractional coefficients in the $(8, 1) \oplus (1, 8)$ multiplet's axial charge $Q^a_5 = \int d\mathbf{x} J^a_{05}(t, \mathbf{x})$ defined by the current given in Eq. (16), the vector charge $Q^a = \int d\mathbf{x} J^a_0(t, \mathbf{x})$ defined by the current given in Eq. (17) and the axial charge close the chiral algebra defined by Eqs. (22), (23) and (24). The same comments holds for the

(10, 1) ⊕ (1, 10) chiral multiplet for the same reasons as in the example shown above.

B. Closure of the chiral $SU_L(3) \times SU_R(3)$ algebra in the $(3, \bar{3}) \oplus (\bar{3}, 3)$ multiplet

The vector charge $Q^a = \int d\mathbf{x} J_0^a(t, \mathbf{x})$ defined by the current given in Eq. (19), together with the axial charge $Q_5^a = \int d\mathbf{x} J_{05}^a(t, \mathbf{x})$ defined by the current given in Eq. (18) ought to close the chiral algebra defined by Eqs. (22) and (23). Equations (22) and (23) hold here, whereas Eq. (24) is the nontrivial one: the diagonal D charge of N ($Q_{5D}^a(N)$) axial charge,

$$Q_{5D}^a(N) = \int d\mathbf{x}(\bar{N}\gamma_0\gamma_5\mathbf{D}^a N), \tag{25}$$

$$Q_D^a(N) = \int d\mathbf{x}(\bar{N}\gamma_0\mathbf{D}^a N), \tag{26}$$

lead to

$$[Q_{5D}^a(N), Q_{5D}^b(N)] = \int d\mathbf{x}(\bar{N}\gamma_0(\mathbf{D}^a\mathbf{D}^b - \mathbf{D}^b\mathbf{D}^a)N). \tag{27}$$

It turns out that the off-diagonal terms in the axial charge,

$$Q_5^a(N, \Lambda) = \int d\mathbf{x}\left(\sqrt{\frac{2}{3}}(\bar{N}\gamma_0\gamma_5\mathbf{T}_{1/8}^{a\dagger}\Lambda + \bar{\Lambda}\gamma_0\gamma_5\mathbf{T}_{1/8}^a N)\right), \tag{28}$$

play a crucial role in the closure of the chiral commutator Eq. (24). The additional terms in the commutator add up to

$$[Q_5^a(N, \Delta), Q_5^b(N, \Delta)] = \frac{2}{3}\int d\mathbf{x}\bar{N}\gamma_0(\mathbf{T}_{1/8}^{a\dagger}\mathbf{T}_{1/8}^b - \mathbf{T}_{1/8}^{b\dagger}\mathbf{T}_{1/8}^a)N, \tag{29}$$

which provide the "missing" factors due to the following properties of the off-diagonal isospin operators $\mathbf{T}_{1/8}^i$ and $\mathbf{D}^i$ matrices

$$if^{ijk}(\mathbf{F}_{(8)}^k) = (\mathbf{D}^i\mathbf{D}^j - \mathbf{D}^j\mathbf{D}^i) + \frac{2}{3}(\mathbf{T}_{1/8}^{i\dagger}\mathbf{T}_{1/8}^j - \mathbf{T}_{1/8}^{j\dagger}\mathbf{T}_{1/8}^i). \tag{30}$$

Therefore, the chiral algebra Eqs. (22)–(24) close.

C. Closure of the chiral $SU_L(3) \times SU_R(3)$ algebra in the $(3, 6) \oplus (6, 3)$ multiplet

The vector charge $Q^a = \int d\mathbf{x} J_0^a(t, \mathbf{x})$ defined by the current in Eq. (21), together with the axial charge $Q_5^a = \int d\mathbf{x} J_{05}^a(t, \mathbf{x})$ defined by the current in Eq. (20) ought to close the chiral algebra defined by Eqs. (22)–(24). Equations (22) and (23) hold here, whereas Eq. (24) is once again the nontrivial one: the fractions $\frac{2}{3}$ and $\frac{1}{3}$ in the diagonal F charge of N ($Q_5^a(N)$) and Δ axial charges, respectively, and the diagonal D charge of N ($Q_5^a(N)$):

$$Q_{5F}^a(N) = \frac{2}{3}\int d\mathbf{x}(\bar{N}\gamma_0\gamma_5\mathbf{F}_{(8)}^a N), \tag{31}$$

$$Q_{5F}^a(\Delta) = \frac{1}{3}\int d\mathbf{x}(\bar{\Delta}\gamma_0\gamma_5\mathbf{F}_{(10)}^a\Delta), \tag{32}$$

$$Q_{5D}^a(N) = \int d\mathbf{x}(\bar{N}\gamma_0\gamma_5\mathbf{D}^a N), \tag{33}$$

lead to

$$\begin{aligned}[Q_{5D+F}^a(N), Q_{5D+F}^b(N)] &= \int d\mathbf{x}\Big(\bar{N}\gamma_0\Big(\Big(\mathbf{D}^a + \frac{2}{3}\mathbf{F}_{(8)}^a\Big)\Big(\mathbf{D}^b + \frac{2}{3}\mathbf{F}_{(8)}^b\Big) \\ &\quad - \Big(\mathbf{D}^b + \frac{2}{3}\mathbf{F}_{(8)}^b\Big)\Big(\mathbf{D}^a + \frac{2}{3}\mathbf{F}_{(8)}^a\Big)\Big)N\Big),\end{aligned} \tag{34}$$

$$[Q_{5F}^a(\Delta), Q_{5F}^b(\Delta)] = if^{abc}\frac{1}{9}Q^c(\Delta), \tag{35}$$

which lead to "only" one part of the N and Δ vector charges, respectively, on the right-hand side of Eqs. (34) and (35).

Once again, it turns out that the off-diagonal terms in the axial charge

$$Q_5^a(N, \Delta) = \int d\mathbf{x}\left(\frac{2}{\sqrt{3}}(\bar{N}\gamma_0\gamma_5\mathbf{T}^a\Delta + \bar{\Delta}\gamma_0\gamma_5\mathbf{T}^{a\dagger}N)\right), \tag{36}$$

play a crucial role in the closure of the chiral algebra Eq. (24). The additional terms in the commutator add up to

$$\begin{aligned}[Q_5^a(N, \Delta), Q_5^b(N, \Delta)] = \frac{4}{3}\int d\mathbf{x}(\bar{N}\gamma_0(\mathbf{T}^a\mathbf{T}^{b\dagger} - \mathbf{T}^b\mathbf{T}^{a\dagger})N \\ + \bar{\Delta}\gamma_0(\mathbf{T}^{a\dagger}\mathbf{T}^b - \mathbf{T}^{b\dagger}\mathbf{T}^a)\Delta),\end{aligned} \tag{37}$$

which provide the missing factors due to the following properties of the off-diagonal flavor operators $\mathbf{T}^i$ and $\mathbf{D}^i$ matrices

$$\begin{aligned}if^{ijk}(\mathbf{F}_{(8)}^k) &= \Big(\Big(\mathbf{D}^i + \frac{2}{3}\mathbf{F}_{(8)}^i\Big)\Big(\mathbf{D}^j + \frac{2}{3}\mathbf{F}_{(8)}^j\Big) \\ &\quad - \Big(\mathbf{D}^j + \frac{2}{3}\mathbf{F}_{(8)}^j\Big)\Big(\mathbf{D}^i + \frac{2}{3}\mathbf{F}_{(8)}^i\Big)\Big) \\ &\quad + \frac{4}{3}(\mathbf{T}_{10/8}^i\mathbf{T}_{10/8}^{j\dagger} - \mathbf{T}_{10/8}^j\mathbf{T}_{10/8}^{i\dagger}), \\ i\frac{2}{3}f^{ijk}\mathbf{F}_{(10)}^k &= \mathbf{T}_{10/8}^{i\dagger}\mathbf{T}_{10/8}^j - \mathbf{T}_{10/8}^{j\dagger}\mathbf{T}_{10/8}^i.\end{aligned} \tag{38}$$

Therefore, the chiral algebra Eqs. (22)–(24) closes in spite, or perhaps because of the apparent fractional axial charges ($\frac{2}{3}$ and $\frac{1}{3}$).

IV. CHIRAL MIXING AND THE AXIAL CURRENT

A unique feature of the use of the linear chiral representation is that the axial coupling is determined by the chiral representations, as given by the coefficients of the axial transformations. For the nucleon (proton and neutron), chiral representations of $SU_L(2) \times SU_R(2)$, $(\frac{1}{2}, 0) \times (\sim(8,1), (3,\bar{3}))$ and $(1, \frac{1}{2})(\sim(6,3))$ provide the nucleon isovector axial coupling $g_A^{(3)} = 1$ and $5/3$ respectively. Therefore, the mixing of chiral $(\frac{1}{2}, 0)$ and $(1, \frac{1}{2})$ nucleons leads to the axial coupling

$$\begin{aligned} 1.267 &= g^{(1)}_{A(\frac{1}{2},0)} \cos^2\theta + g^{(1)}_{A(1,\frac{1}{2})} \sin^2\theta \\ &= g^{(1)}_{A(\frac{1}{2},0)} \cos^2\theta + \frac{5}{3} \sin^2\theta. \end{aligned} \tag{39}$$

Three-quark nucleon interpolating fields in QCD have also well-defined, if perhaps unexpected $U_A(1)$ chiral transformation properties (see Table I) that can be used to predict the isoscalar axial coupling $g^{(0)}_{A\text{mix.}}$,

$$\begin{aligned} g^{(0)}_{A\text{mix.}} &= g^{(0)}_{A(\frac{1}{2},0)} \cos^2\theta + g^{(0)}_{A(1,\frac{1}{2})} \sin^2\theta \\ &= g^{(0)}_{A(\frac{1}{2},0)} \cos^2\theta + \sin^2\theta, \end{aligned} \tag{40}$$

together with the mixing angle θ extracted from Eq. (39). Note, however, that due to the different (bare) non-Abelian $g_A^{(1)}$ and Abelian $g_A^{(0)}$ axial couplings, (see Table I) the mixing formulas Eq. (40) give substantially different predictions from one case to another (see Table II). We can see in Table II that the two best candidates are cases I and IV, with $g_A^{(0)} = -0.2$ and $g_A^{(0)} = 0.4$, respectively, the latter being within the error bars of the measured value $g^{(0)}_{A\text{expt.}} = 0.33 \pm 0.08$ [12,19]. Moreover, this scheme predicts the F and D values, as well:

$$F = F_{(\frac{1}{2},0)} \cos^2\theta + F^{(1)}_{(1,\frac{1}{2})} \sin^2\theta = F_{(\frac{1}{2},0)} \cos^2\theta + \frac{2}{3} \sin^2\theta, \tag{41}$$

$$D = D_{(\frac{1}{2},0)} \cos^2\theta + D_{(1,\frac{1}{2})} \sin^2\theta = D_{(\frac{1}{2},0)} \cos^2\theta + \sin^2\theta, \tag{42}$$

where we have used the F and D values for different chiral multiplets as listed in Table I.

Cases I and IV, with $F/D = 0.267$ and 0.491, respectively, ought to be compared with $F/D = 0.571 \pm 0.005$ [20]. Case I is, of course, the well-known "Ioffe current," which reproduces the nucleon's properties in QCD lattice and sum rules calculations. The latter is a "mirror" opposite of the orthogonal complement to the Ioffe current, an interpolating field that, to our knowledge, has not been used in QCD thus far.

Manifestly, a linear superposition of any three fields (except for the mixtures of cases II and III, IV above, which yield complex mixing angles) should give a perfect fit to the central values of the experimental axial couplings and predict the F and D values. Such a three-field admixture introduces new free parameters (besides the two already introduced mixing angles, e.g. θ_1 and θ_4, we have the relative/mutual mixing angle θ_{14}, as the two nucleon fields I and IV may also mix). One may subsume the sum and the difference of the two angles θ_1 and θ_4 into the new angle θ, and define $\varphi \doteq \theta_{14}$ (this relationship depends on the pre-

TABLE I. The Abelian and the non-Abelian axial charges (+ sign indicates naive, − sign mirror transformation properties) and the non-Abelian chiral multiplets of $J^P = \frac{1}{2}$, Lorentz representation $(\frac{1}{2}, 0)$ nucleon and Δ fields; see Refs. [15–18].

Case	Field	$g_A^{(0)}$	$g_A^{(1)}$	F	D	$SU_L(3) \times SU_R(3)$
I	$N_1 - N_2$	-1	$+1$	0	$+1$	$(3,\bar{3}) \oplus (\bar{3},3)$
II	$N_1 + N_2$	$+3$	$+1$	$+1$	0	$(8,1) \oplus (1,8)$
III	$N_1' - N_2'$	$+1$	-1	0	-1	$(\bar{3},3) \oplus (3,\bar{3})$
IV	$N_1' + N_2'$	-3	-1	-1	0	$(1,8) \oplus (8,1)$
0	$\partial_\mu(N_3^\mu + \frac{1}{3}N_4^\mu)$	$+1$	$+\frac{5}{3}$	$+\frac{2}{3}$	$+1$	$(6,3) \oplus (3,6)$

TABLE II. The values of the baryon isoscalar axial coupling constant predicted from the naive mixing and $g^{(1)}_{A\text{expt.}} = 1.267$; compare with $g^{(0)}_{A\text{expt.}} = 0.33 \pm 0.03 \pm 0.05$, $F = 0.459 \pm 0.008$, and $D = 0.798 \pm 0.008$, leading to $F/D = 0.571 \pm 0.005$; see Ref. [2].

Case	$(g_A^{(1)}, g_A^{(0)})$	$g^{(1)}_{A\text{mix.}}$	θ_i	$g^{(0)}_{A\text{mix.}}$	$g^{(0)}_{A\text{mix.}}$	F	F/D
I	$(+1, -1)$	$\frac{1}{3}(4 - \cos 2\theta)$	39.3^o	$-\cos 2\theta$	-0.20	0.267	0.267
II	$(+1, +3)$	$\frac{1}{3}(4 - \cos 2\theta)$	39.3^o	$(2\cos 2\theta + 1)$	2.20	0.866	2.16
III	$(-1, +1)$	$\frac{1}{3}(1 - 4\cos 2\theta)$	67.2^o	1	1.00	0.567	0.81
IV	$(-1, -3)$	$\frac{1}{3}(1 - 4\cos 2\theta)$	67.2^o	$-(2\cos 2\theta + 1)$	0.40	0.417	0.491

TABLE III. The values of the mixing angles obtained from the simple fit to the baryon axial coupling constants and the predicted values of axial F and D couplings. The experimental values are $F = 0.459 \pm 0.008$ and $D = 0.798 \pm 0.008$, leading to $F/D = 0.571 \pm 0.005$; see Ref. [2].

Case	$g^{(3)}_{A\rm expt.}$	$g^{(0)}_{A\rm expt.}$	θ	φ	F	D	F/D
I-II	1.267	0.33	39.3^o	$28.0^o \pm 2.3^o$	0.399 ± 0.02	0.868 ∓ 0.02	0.460 ± 0.04
I-III	1.267	0.33	$50.7^o \pm 1.8^o$	$23.9^o \pm 2.9^o$	0.399 ± 0.02	0.868 ∓ 0.02	0.460 ± 0.04
I-IV	1.267	0.33	$63.2^o \pm 4.0^o$	$54^o \pm 23^o$	0.399 ± 0.02	0.868 ∓ 0.02	0.460 ± 0.04

cise definition of the mixing angles θ_1, θ_4 and θ_{14}); thus we find two equations with two unknowns of the general form:

$$\frac{5}{3}\sin^2\theta + \cos^2\theta(g_A^{(1)}\cos^2\varphi + g_A^{(1)\prime}\sin^2\varphi) = 1.267, \quad (43)$$

$$\sin^2\theta + \cos^2\theta(g_A^{(0)}\cos^2\varphi + g_A^{(0)\prime}\sin^2\varphi) = 0.33 \pm 0.08. \quad (44)$$

The solutions to these equations (the values of the mixing angles θ, φ) provide, at the same time, input for the prediction of F and D:

$$\cos^2\theta(F\cos^2\varphi + F'\sin^2\varphi) + \frac{2}{3}\sin^2\theta = F, \quad (45)$$

$$\cos^2\theta(D\cos^2\varphi + D'\sin^2\varphi) + \sin^2\theta = D. \quad (46)$$

The values of the mixing angles (θ, φ) obtained from this straightforward fit to the baryon axial coupling constants are shown in Table III. Note that all three admissible scenarios [i.e. choices of pairs of fields admixed to the (6, 3) one that lead to real mixing angles] yield the same values of F and D. This is due to the fact that all three-quark baryon fields satisfy the following relation $g_A^{(0)} = 3F - D = \sqrt{3}g_A^{(8)}$ [21]. The first relation $g_A^{(0)} = 3F - D$ was not expected, as the flavor-singlet properties, such as $g_A^{(0)}$ are generally expected to be independent of the flavor-octet ones, such as F, D. Yet, it is not unnatural, either, as it indicates the absence of polarized $s\bar{s}$ pairs in these $SU(3)$ symmetric, three-quark nucleon interpolators. In order to show that, we define $g_A^{(0)} = \Delta u + \Delta d + \Delta s$ and $g_A^{(8)} = \frac{1}{\sqrt{3}}(\Delta u + \Delta d - 2\Delta s)$, where Δq are the (corresponding flavor) quark contributions to the matrix element of the nucleon's axial-vector current $\Delta q = \langle N|\bar{q}\gamma_\mu\gamma_5 q|N\rangle$. We see that $g_A^{(0)} \sim g_A^{(8)}$ only if $\Delta s = 0$.

Thus, the relation $g_A^{(0)} = 3F - D$ appears to depend on the choice of three-quark interpolating fields as a source of admixed mirror fields and may well change when one considers other interpolating fields, such as the five-quark (pentaquark) ones, for example [22]. In that sense a deviation of the measured values of $g_A^{(0)}$ and $g_A^{(8)} = \frac{1}{\sqrt{3}}(3F - D)$ from this relation may well be seen as a measure of the contribution of higher-order configurations to the baryon ground state. It seems very difficult, however, to evaluate F and D for specific higher-order configurations without going through the procedure outlined in Ref. [13] for the pentaquark interpolator chiral multiplets [23].

Some of the ideas used above have also been used in some of the following early papers: two-chiral-multiplet mixing was considered long ago by Harari [8] and by Weinberg [11], for example. Moreover, special cases of three-field/configuration chiral mixing have been considered by Harari [9], and by Gerstein and Lee [10] in the context of the (''collinear'') $U(3) \times U(3)$ current algebra at infinite momentum. One (obvious) distinction from these early precedents is our use of QCD interpolating fields, which appeared only in the early 1980's, and the (perhaps less obvious) issue of baryons' flavor-singlet axial current ([also known as the $U_A(1)$], that was (seriously) raised yet another decade later. We emphasize here that our results are based on the $U_L(3) \times U_R(3)$ chiral algebra of space-integrated time components of currents, without any assumptions about saturation of this algebra by one-particle states, or its dependence on any one particular frame of reference. Indeed, our nucleon interpolating fields transform as the $(\frac{1}{2}, 0) + (0, \frac{1}{2})$ representation of the Lorentz group, thus making the Noether currents (fully) Lorentz covariant, so that our results hold in any frame.

V. SUMMARY AND OUTLOOK

We have reorganized the results of our previous paper [13] into the (perhaps more) conventional form for the baryon octet using F and D coupling [$SU(3)$ structure constants]. This means that, *inter alia*, we have explicitly written down (perhaps for the first time) the chiral transformations of the $(6, 3) \oplus (3, 6)$ octet and decimet fields in the $SU(3)$ particle (octet and decimet) basis.

In the process we have independently constructed $SU(3)$ generators of the decimet and derived a set of $SU(3)$ Clebsch-Gordan coefficients in the ''natural'' convention, which means that all isospin $SU(2)$ submultiplets of the octet and the decimet have standard isospin $SU(2)$ generators.

Then we used the above mentioned $SU(3)$ Clebsch-Gordan coefficients to explicitly check the closure of the $SU_L(3) \times SU_R(3)$ chiral algebra in the $SU(3)$ particle basis, which forms an independent check/confirmation of the calculation.

Next, we investigated the phenomenological consequence for the baryon axial currents, of the chiral $[(6,3)\oplus(3,6)]$ multiplet mixing with other three-quark baryon field multiplets, such as the $[(3,\bar{3})\oplus(\bar{3},3)]$ and $[(8,1)\oplus(1,8)]$. The results of the three-field ("two-angle") mixing are interesting: all permissible combinations fields lead to the same F/D prediction, that is in reasonable agreement with experiment. This identity of results is a consequence of the relation $g_A^{(0)} = 3F - D$ between the flavor-singlet axial coupling $g_A^{(0)}$ and the (previously unrelated) flavor-octet F and D values. That relation is a unique feature of the three-quark interpolating fields and any potential departures from it may be attributed to fields with a number of quarks higher than three.

The next step, left for the future, is to investigate $SU_L(3)\times SU_R(3)$ chiral invariant interactions and the $SU(3)\times SU(3)\to SU(2)\times SU(2)$ symmetry breaking/reduction and to the study of the chiral $SU(2)\times SU(2)$ properties of hyperons. Then one may consider explicit chiral symmetry breaking corrections to the axial and the vector currents, which are related to the $SU(3)\times SU(3)$ symmetry breaking meson-nucleon derivative interactions, not just the explicit $SU(3)$ symmetry breaking ones that have been considered thus far (see Ref. [2] and the previous subsection, above).

ACKNOWLEDGMENTS

We wish to thank Professors Daisuke Jido, Akira Ohnishi, and Makoto Oka for valuable conversations regarding the present work. One of us (V. D.) wishes to thank the RCNP, Osaka University, under whose auspices this work was begun, and the Yukawa Institute for Theoretical Physics, Kyoto (molecular workshop "Algebraic aspects of chiral symmetry for the study of excited baryons"), where it was finished, for kind hospitality and financial support.

APPENDIX A: $SU(3)$ OCTET, DECIMET GENERATORS AND 8×10 TRANSITION MATRICES

1. Octet "generator" 8×8 matrices $\mathbf{D}^a$, $\mathbf{F}^a_{(8)}$ in the particle basis

$$\left(\mathbf{D}^1 + \frac{2}{3}\mathbf{F}^1_{(8)}\right) = \begin{pmatrix} 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & 0 & 0 & p \\ \frac{5}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & n \\ 0 & 0 & 0 & \frac{\sqrt{2}}{3} & 0 & 0 & 0 & -\frac{1}{\sqrt{6}} & \Sigma^+ \\ 0 & 0 & \frac{\sqrt{2}}{3} & 0 & \frac{\sqrt{2}}{3} & 0 & 0 & 0 & \Sigma^0 \\ 0 & 0 & 0 & \frac{\sqrt{2}}{3} & 0 & 0 & 0 & \frac{1}{\sqrt{6}} & \Sigma^- \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{6} & 0 & \Xi^0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{6} & 0 & 0 & \Xi^- \\ 0 & 0 & -\frac{1}{\sqrt{6}} & 0 & \frac{1}{\sqrt{6}} & 0 & 0 & 0 & \Lambda_8 \\ p & n & \Sigma^+ & \Sigma^0 & \Sigma^- & \Xi^0 & \Xi^- & \Lambda_8 & \end{pmatrix} \tag{A1}$$

$$\mathbf{D}^1 = \begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{\sqrt{6}} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{6}} \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & -\frac{1}{\sqrt{6}} & 0 & \frac{1}{\sqrt{6}} & 0 & 0 & 0 \end{pmatrix} \tag{A2}$$

$$\mathbf{F}^1_{(8)} = \begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{A3}$$

$$
\left(\mathbf{D}^2 + \frac{2}{3}\mathbf{F}^2_{(8)}\right) = \begin{pmatrix}
0 & -\frac{5i}{6} & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{5i}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{i\sqrt{2}}{3} & 0 & 0 & 0 & \frac{i}{\sqrt{6}} \\
0 & 0 & \frac{i\sqrt{2}}{3} & 0 & -\frac{i\sqrt{2}}{3} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{i\sqrt{2}}{3} & 0 & 0 & 0 & \frac{i}{\sqrt{6}} \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{6} & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{i}{6} & 0 & 0 \\
0 & 0 & -\frac{i}{\sqrt{6}} & 0 & -\frac{i}{\sqrt{6}} & 0 & 0 & 0
\end{pmatrix} \tag{A4}
$$

$$
\mathbf{D}^2 = \begin{pmatrix}
0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{\sqrt{6}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{\sqrt{6}} \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 \\
0 & 0 & -\frac{i}{\sqrt{6}} & 0 & -\frac{i}{\sqrt{6}} & 0 & 0 & 0
\end{pmatrix} \tag{A5}
$$

$$
\mathbf{F}^2_{(8)} = \begin{pmatrix}
0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{i}{\sqrt{2}} & 0 & -\frac{i}{\sqrt{2}} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix} \tag{A6}
$$

$$
\left(\mathbf{D}^3 + \frac{2}{3}\mathbf{F}^3_{(8)}\right) = \begin{pmatrix}
\frac{5}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{5}{6} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{3}} \\
0 & 0 & 0 & 0 & -\frac{2}{3} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{1}{6} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 \\
0 & 0 & 0 & \frac{1}{\sqrt{3}} & 0 & 0 & 0 & 0
\end{pmatrix} \tag{A7}
$$

$$
\mathbf{D}^3 = \begin{pmatrix}
\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{3}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\
0 & 0 & 0 & \frac{1}{\sqrt{3}} & 0 & 0 & 0 & 0
\end{pmatrix} \tag{A8}
$$

$$\mathbf{F}^3_{(8)} = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{A9}$$

$$\left(\mathbf{D}^4 + \frac{2}{3}\mathbf{F}^4_{(8)}\right) = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6\sqrt{2}} & 0 & 0 & 0 & -\frac{\sqrt{\frac{3}{2}}}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{5}{6} & 0 & 0 \\ \frac{1}{6\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & -\frac{5}{6\sqrt{2}} & 0 \\ 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{5}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{5}{6\sqrt{2}} & 0 & 0 & 0 & -\frac{1}{2\sqrt{6}} \\ -\frac{\sqrt{\frac{3}{2}}}{2} & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2\sqrt{6}} & 0 \end{pmatrix} \tag{A10}$$

$$\mathbf{D}^4 = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2\sqrt{2}} & 0 & 0 & 0 & -\frac{1}{2\sqrt{6}} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 \\ \frac{1}{2\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2\sqrt{2}} & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2\sqrt{2}} & 0 & 0 & 0 & \frac{1}{2\sqrt{6}} \\ -\frac{1}{2\sqrt{6}} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2\sqrt{6}} & 0 \end{pmatrix} \tag{A11}$$

$$\mathbf{F}^4_{(8)} = \begin{pmatrix} 0 & 0 & 0 & -\frac{1}{2\sqrt{2}} & 0 & 0 & 0 & -\frac{\sqrt{\frac{3}{2}}}{2} \\ 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 \\ -\frac{1}{2\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2\sqrt{2}} & 0 \\ 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2\sqrt{2}} & 0 & 0 & 0 & -\frac{\sqrt{\frac{3}{2}}}{2} \\ -\frac{\sqrt{\frac{3}{2}}}{2} & 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{\frac{3}{2}}}{2} & 0 \end{pmatrix} \tag{A12}$$

$$\left(\mathbf{D}^5 + \frac{2}{3}\mathbf{F}^5_{(8)}\right) = \begin{pmatrix} 0 & 0 & 0 & -\frac{i}{6\sqrt{2}} & 0 & 0 & 0 & \frac{1}{2}i\sqrt{\frac{3}{2}} \\ 0 & 0 & 0 & 0 & -\frac{i}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{5i}{6} & 0 & 0 \\ \frac{i}{6\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & \frac{5i}{6\sqrt{2}} & 0 \\ 0 & \frac{i}{6} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{5i}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{5i}{6\sqrt{2}} & 0 & 0 & 0 & -\frac{i}{2\sqrt{6}} \\ -\frac{1}{2}i\sqrt{\frac{3}{2}} & 0 & 0 & 0 & 0 & 0 & \frac{i}{2\sqrt{6}} & 0 \end{pmatrix} \tag{A13}$$

$$\mathbf{D}^5 = \begin{pmatrix} 0 & 0 & 0 & -\frac{i}{2\sqrt{2}} & 0 & 0 & 0 & \frac{i}{2\sqrt{6}} \\ 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 \\ \frac{i}{2\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & \frac{i}{2\sqrt{2}} & 0 \\ 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{i}{2\sqrt{2}} & 0 & 0 & 0 & \frac{i}{2\sqrt{6}} \\ -\frac{i}{2\sqrt{6}} & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2\sqrt{6}} & 0 \end{pmatrix} \tag{A14}$$

$$\mathbf{F}^5_{(8)} = \begin{pmatrix} 0 & 0 & 0 & \frac{i}{2\sqrt{2}} & 0 & 0 & 0 & \frac{1}{2} i\sqrt{\frac{3}{2}} \\ 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 \\ -\frac{i}{2\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & \frac{i}{2\sqrt{2}} & 0 \\ 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{i}{2\sqrt{2}} & 0 & 0 & 0 & -\frac{1}{2} i\sqrt{\frac{3}{2}} \\ -\frac{1}{2} i\sqrt{\frac{3}{2}} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} i\sqrt{\frac{3}{2}} & 0 \end{pmatrix} \tag{A15}$$

$$\left(\mathbf{D}^6 + \frac{2}{3}\mathbf{F}^6_{(8)}\right) = \begin{pmatrix} 0 & 0 & -\frac{1}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{6\sqrt{2}} & 0 & 0 & 0 & -\frac{\sqrt{\frac{3}{2}}}{2} \\ -\frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{6\sqrt{2}} & 0 & 0 & 0 & -\frac{5}{6\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{5}{6} & 0 \\ 0 & 0 & 0 & -\frac{5}{6\sqrt{2}} & 0 & 0 & 0 & \frac{1}{2\sqrt{6}} \\ 0 & 0 & 0 & 0 & -\frac{5}{6} & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{\frac{3}{2}}}{2} & 0 & 0 & 0 & \frac{1}{2\sqrt{6}} & 0 & 0 \end{pmatrix} \tag{A16}$$

$$\mathbf{D}^6 = \begin{pmatrix} 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2\sqrt{2}} & 0 & 0 & 0 & -\frac{1}{2\sqrt{6}} \\ -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{2\sqrt{2}} & 0 & 0 & 0 & -\frac{1}{2\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & -\frac{1}{2\sqrt{2}} & 0 & 0 & 0 & -\frac{1}{2\sqrt{6}} \\ 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & -\frac{1}{2\sqrt{6}} & 0 & 0 & 0 & -\frac{1}{2\sqrt{6}} & 0 & 0 \end{pmatrix} \tag{A17}$$

$$\mathbf{F}^6_{(8)} = \begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2\sqrt{2}} & 0 & 0 & 0 & -\frac{\sqrt{\frac{3}{2}}}{2} \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2\sqrt{2}} & 0 & 0 & 0 & -\frac{1}{2\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & -\frac{1}{2\sqrt{2}} & 0 & 0 & 0 & \frac{\sqrt{\frac{3}{2}}}{2} \\ 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{\frac{3}{2}}}{2} & 0 & 0 & 0 & \frac{\sqrt{\frac{3}{2}}}{2} & 0 & 0 \end{pmatrix} \tag{A18}$$

$$(\mathbf{D}^7 + \frac{2}{3}\mathbf{F}^7_{(8)}) = \begin{pmatrix} 0 & 0 & \frac{i}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{i}{6\sqrt{2}} & 0 & 0 & 0 & \frac{1}{2}i\sqrt{\frac{3}{2}} \\ -\frac{i}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{i}{6\sqrt{2}} & 0 & 0 & 0 & \frac{5i}{6\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{5i}{6} & 0 \\ 0 & 0 & 0 & -\frac{5i}{6\sqrt{2}} & 0 & 0 & 0 & \frac{i}{2\sqrt{6}} \\ 0 & 0 & 0 & 0 & -\frac{5i}{6} & 0 & 0 & 0 \\ 0 & -\frac{1}{2}i\sqrt{\frac{3}{2}} & 0 & 0 & 0 & -\frac{i}{2\sqrt{6}} & 0 & 0 \end{pmatrix} \tag{A19}$$

$$\mathbf{D}^7 = \begin{pmatrix} 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{i}{2\sqrt{2}} & 0 & 0 & 0 & \frac{i}{2\sqrt{6}} \\ -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{i}{2\sqrt{2}} & 0 & 0 & 0 & \frac{i}{2\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 \\ 0 & 0 & 0 & -\frac{i}{2\sqrt{2}} & 0 & 0 & 0 & -\frac{i}{2\sqrt{6}} \\ 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 \\ 0 & -\frac{i}{2\sqrt{6}} & 0 & 0 & 0 & \frac{i}{2\sqrt{6}} & 0 & 0 \end{pmatrix} \tag{A20}$$

$$\mathbf{F}^7_{(8)} = \begin{pmatrix} 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{i}{2\sqrt{2}} & 0 & 0 & 0 & \frac{1}{2}i\sqrt{\frac{3}{2}} \\ \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{i}{2\sqrt{2}} & 0 & 0 & 0 & \frac{i}{2\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 \\ 0 & 0 & 0 & -\frac{i}{2\sqrt{2}} & 0 & 0 & 0 & \frac{1}{2}i\sqrt{\frac{3}{2}} \\ 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 \\ 0 & -\frac{1}{2}i\sqrt{\frac{3}{2}} & 0 & 0 & 0 & -\frac{1}{2}i\sqrt{\frac{3}{2}} & 0 & 0 \end{pmatrix} \tag{A21}$$

$$(\mathbf{D}^8 + \frac{2}{3}\mathbf{F}^2_{(8)}) = \begin{pmatrix} \frac{1}{2\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{3}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\sqrt{3}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{3}}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{\sqrt{3}} \end{pmatrix} \tag{A22}$$

$$\mathbf{D}^8 = \begin{pmatrix} -\frac{1}{2\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{2\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{3}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\sqrt{3}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{2\sqrt{3}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2\sqrt{3}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{\sqrt{3}} \end{pmatrix} \tag{A23}$$

$$\mathbf{F}^{8}_{(8)} = \begin{pmatrix} \frac{\sqrt{3}}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{3}}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{A24}$$

2. Octet-decimet 8×10 transition matrices T^a

$$T_1 = \begin{pmatrix} -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{6}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p \\ 0 & -\frac{1}{\sqrt{6}} & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & n \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2\sqrt{3}} & 0 & 0 & 0 & 0 & \Sigma^+ \\ 0 & 0 & 0 & 0 & \frac{1}{2\sqrt{3}} & 0 & \frac{1}{2\sqrt{3}} & 0 & 0 & 0 & \Sigma^0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2\sqrt{3}} & 0 & 0 & 0 & 0 & \Sigma^- \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{\sqrt{6}} & 0 & \Xi^0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{\sqrt{6}} & 0 & 0 & \Xi^- \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & \Lambda_8 \\ \Delta^{++} & \Delta^+ & \Delta^0 & \Delta^- & \Sigma^{*+} & \Sigma^{*0} & \Sigma^{*-} & \Xi^{*0} & \Xi^{*-} & \Omega & \end{pmatrix} \tag{A25}$$

$$T_2 = \begin{pmatrix} -\frac{i}{\sqrt{2}} & 0 & -\frac{i}{\sqrt{6}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{i}{\sqrt{6}} & 0 & -\frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{i}{2\sqrt{3}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{i}{2\sqrt{3}} & 0 & -\frac{i}{2\sqrt{3}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{i}{2\sqrt{3}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{\sqrt{6}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{6}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & \frac{i}{2} & 0 & 0 & 0 \end{pmatrix} \tag{A26}$$

$$T_3 = \begin{pmatrix} 0 & \sqrt{\frac{2}{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{\frac{2}{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\sqrt{6}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{\sqrt{6}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{\sqrt{6}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{6}} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 \end{pmatrix} \tag{A27}$$

$$T_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{1}{2\sqrt{3}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{6}} & 0 & 0 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{6}} & 0 & 0 \\ 0 & -\frac{1}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2\sqrt{3}} & 0 \\ 0 & 0 & -\frac{1}{\sqrt{6}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\sqrt{6}} & 0 & 0 & 0 & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2\sqrt{3}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 \end{pmatrix} \tag{A28}$$

$$T_5 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -\frac{i}{2\sqrt{3}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{6}} & 0 & 0 & 0 \\ -\frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{6}} & 0 & 0 \\ 0 & -\frac{i}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2\sqrt{3}} & 0 \\ 0 & 0 & -\frac{i}{\sqrt{6}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{i}{\sqrt{6}} & 0 & 0 & 0 & 0 & \frac{i}{\sqrt{2}} \\ 0 & 0 & 0 & 0 & 0 & \frac{i}{2\sqrt{3}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 \end{pmatrix} \tag{A29}$$

$$T_6 = \begin{pmatrix} 0 & 0 & 0 & 0 & -\frac{1}{\sqrt{6}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{2\sqrt{3}} & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{\sqrt{6}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{\sqrt{3}} & 0 & 0 & 0 & 0 & \frac{1}{2\sqrt{3}} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{6}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2\sqrt{3}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{6}} & 0 & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix} \tag{A30}$$

$$T_7 = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{i}{\sqrt{6}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{i}{2\sqrt{3}} & 0 & 0 & 0 & 0 \\ 0 & -\frac{i}{\sqrt{6}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{i}{\sqrt{3}} & 0 & 0 & 0 & 0 & -\frac{i}{2\sqrt{3}} & 0 & 0 \\ 0 & 0 & 0 & -\frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{6}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{i}{2\sqrt{3}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{\sqrt{6}} & 0 & 0 & \frac{i}{\sqrt{2}} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 \end{pmatrix} \tag{A31}$$

$$T_8 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{A32}$$

3. Decimet generator 10×10 matrices $\mathbf{F}^a_{(10)}$

$$\mathbf{F}^1_{(10)} = \begin{pmatrix} 0 & \frac{\sqrt{3}}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Delta^{++} \\ \frac{\sqrt{3}}{2} & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Delta^{+} \\ 0 & 1 & 0 & \frac{\sqrt{3}}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \Delta^{0} \\ 0 & 0 & \frac{\sqrt{3}}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Delta^{-} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & \Sigma^{*+} \\ 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \Sigma^{*0} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & \Sigma^{*-} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \Xi^{*0} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \Xi^{*-} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Omega \\ \Delta^{++} & \Delta^{+} & \Delta^{0} & \Delta^{-} & \Sigma^{*+} & \Sigma^{*0} & \Sigma^{*-} & \Xi^{*0} & \Xi^{*-} & \Omega & \end{pmatrix} \tag{A33}$$

$$\mathbf{F}^2_{(10)} = \begin{pmatrix} 0 & -\frac{i\sqrt{3}}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{i\sqrt{3}}{2} & 0 & -i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & i & 0 & -\frac{i\sqrt{3}}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{i\sqrt{3}}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{i}{\sqrt{2}} & 0 & -\frac{i}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{A34}$$

$$\mathbf{F}^3_{(10)} = \begin{pmatrix} \frac{3}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{3}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{A35}$$

$$\mathbf{F}^4_{(10)} = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{\sqrt{3}}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{2} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{2} & 0 & 0 \end{pmatrix} \tag{A36}$$

$$\mathbf{F}^{5}_{(10)} = \begin{pmatrix} 0 & 0 & 0 & 0 & -\frac{i\sqrt{3}}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{i\sqrt{3}}{2} & 0 & 0 & 0 & 0 & 0 & 0 & -i & 0 & 0 \\ 0 & \frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{2}} & 0 \\ 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & i & 0 & 0 & 0 & 0 & -\frac{i\sqrt{3}}{2} \\ 0 & 0 & 0 & 0 & 0 & \frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i\sqrt{3}}{2} & 0 & 0 \end{pmatrix} \tag{A37}$$

$$\mathbf{F}^{6}_{(10)} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{3}}{2} & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \frac{\sqrt{3}}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{2} & 0 \end{pmatrix} \tag{A38}$$

$$\mathbf{F}^{7}_{(10)} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i\sqrt{3}}{2} & 0 & 0 & 0 \\ 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & \frac{i\sqrt{3}}{2} & 0 & 0 & 0 & 0 & -i & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & i & 0 & 0 & -\frac{i\sqrt{3}}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i\sqrt{3}}{2} & 0 \end{pmatrix} \tag{A39}$$

$$\mathbf{F}^{8}_{(10)} = \begin{pmatrix} \frac{\sqrt{3}}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\sqrt{3}}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{3}}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{3}}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\sqrt{3} \end{pmatrix} \tag{A40}$$

4. Singlet-octet 1×8 transition matrices $T^a_{1/8}$

$$\mathbf{T}^1_{1/8} = \begin{pmatrix} 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \Lambda_1 \\ p & n & \Sigma^+ & \Sigma^0 & \Sigma^- & \Xi^0 & \Xi^- & \Lambda_8 & \end{pmatrix} \tag{A41}$$

$$\mathbf{T}^2_{1/8} = \begin{pmatrix} 0 & 0 & -\frac{i}{\sqrt{2}} & 0 & -\frac{i}{\sqrt{2}} & 0 & 0 & 0 \end{pmatrix} \tag{A42}$$

$$\mathbf{T}^3_{1/8} = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{A43}$$

$$\mathbf{T}^4_{1/8} = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 \end{pmatrix} \tag{A44}$$

$$\mathbf{T}^5_{1/8} = \begin{pmatrix} \frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & \frac{i}{\sqrt{2}} & 0 \end{pmatrix} \tag{A45}$$

$$\mathbf{T}^6_{1/8} = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \end{pmatrix} \tag{A46}$$

$$\mathbf{T}^7_{1/8} = \begin{pmatrix} 0 & \frac{i}{\sqrt{2}} & 0 & 0 & 0 & -\frac{i}{\sqrt{2}} & 0 & 0 \end{pmatrix} \tag{A47}$$

$$\mathbf{T}^8_{1/8} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{A48}$$

[1] L. B. Okun, *Leptons And Quarks* (North-Holland, Amsterdam, Netherlands, 1982), p. 361.
[2] T. Yamanishi, Phys. Rev. D **76**, 014006 (2007).
[3] For history and other references, see Chap. 6.7 of Okun's book [1] and PDG tables [4].
[4] C. Amsler *et al.* (Particle Data Group), Phys. Lett. B **667**, 1 (2008).
[5] W. A. Bardeen and B. W. Lee, Phys. Rev. **177**, 2389 (1969).
[6] B. W. Lee, Phys. Rev. **170**, 1359 (1968).
[7] Y. Hara, Phys. Rev. **139**, B134 (1965).
[8] H. Harari, Phys. Rev. Lett. **16**, 964 (1966).
[9] H. Harari, Phys. Rev. Lett. **17**, 56 (1966).
[10] I. S. Gerstein and B. W. Lee, Phys. Rev. Lett. **16**, 1060 (1966).
[11] S. Weinberg, Phys. Rev. **177**, 2604 (1969).
[12] S. D. Bass, *The Spin structure of the proton* (World Scientific, Singapore, 2007), p. 212 (ISBN 978-981-270-946-2 and ISBN 978-981-270-947-9).
[13] H. X. Chen, V. Dmitrašinović, A. Hosaka, K. Nagata, and S. L. Zhu, Phys. Rev. D **78**, 054021 (2008).
[14] S. Gasiorowicz, *Elementary Particle Physics* (J. Wiley, New York, 1966).
[15] V. Dmitrašinović, K. Nagata, and A. Hosaka, Mod. Phys. Lett. A **23**, 2381 (2008).
[16] K. Nagata, A. Hosaka, and V. Dmitrašinović, Eur. Phys. J. C **57**, 557 (2008).
[17] V. Dmitrašinović, A. Hosaka, and K. Nagata, Mod. Phys. Lett. A **25**, 233 (2010).
[18] V. Dmitrašinović, A. Hosaka, and K. Nagata, Int. J. Mod. Phys. E **19**, 91 (2010).
[19] E. S. Ageev *et al.* (Compass Collaboration), Phys. Lett. B **647**, 330 (2007).
[20] Note that the Ref. [2] values add up to $F + D = 1.312 \pm 0.002$, more than 2-σ away from the experimental constraint $\neq 1.269 \pm 0.002$.
[21] We thank D. Jido for pointing out the relation $g_A^{(0)} = 3F - D$.
[22] Note, however, that five- and more quark, and derivative interpolating fields are not the only ones that can produce mirror fields; so can the one-gluon–three-quark "hybrid baryon" interpolators, which necessarily have the same chiral properties as the corresponding three-quark fields.
[23] If one were to assign one particular source of mirror fields, for example, some pentaquark interpolators, then one could try to determine the contribution of $s\bar{s}$ pairs to the flavor-singlet axial coupling.