Neutrino mass matrices with a texture zero and a vanishing minor

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We study the implications of the simultaneous existence of a texture zero and a vanishing minor in the neutrino mass matrix. There are 36 possible texture structures of this type, 21 of which reduce to two texture zero cases which have, already, been extensively studied. Of the remaining 15 textures only six are allowed by the current data. We examine the phenomenological implications of the allowed texture structures for Majorana type *CP*-violating phases, 1-3 mixing angle, and Dirac type *CP*-violating phase. All these possible textures can be generated through the seesaw mechanism and realized in the framework of discrete Abelian flavor symmetry. We present the symmetry realization of these texture structures.

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I. INTRODUCTION

During the last several years enormous progress has been made in the determination of the neutrino masses and mixings and in studies of the neutrino mass matrix. The main theoretical challenge is to understand the dynamics behind the observed pattern of neutrino masses and mixing. It is expected that detailed information of the neutrino mass spectrum and lepton mixing may eventually shed light on the origin of lepton masses, quark lepton symmetry, and the fermion mass problem. The main objectives of the neutrino physics include the determination of the absolute mass scale of neutrinos, their mass spectra/mass hierarchy and also the subdominant structure of mixing: namely, 1-3 mixing, deviation of 2-3 mixing from maximality, and the *CP*-violating phases. However, there exist a large number of the possible structures of neutrino mass matrix. Several proposals have been made in the literature to restrict the form of the neutrino mass matrix and to reduce the number of free parameters which include the presence of texture zeros [1–6], the requirement of zero determinant [7], and the zero trace condition [8]. In addition, the presence of vanishing minors [9] and the simultaneous existence of a texture zero and an equality [10] has been studied in the literature. However, the current neutrino oscillation data are consistent with only a limited number of texture schemes. A detailed phenomenological analysis of the two texture zeros [1–6] has been done in the past. The seesaw mechanism for understanding the scale of neutrino masses is regarded as the prime candidate not only due to its simplicity but also due to its theoretical appeal. In the framework of the type I seesaw mechanism [11], the effective Majorana mass matrix M_ν is given by

$$M_\nu = -M_D M_R^{-1} M_D^T, \tag{1}$$

where M_D is the Dirac neutrino mass matrix and M_R is the right-handed Majorana mass matrix. It has been noted by many authors [12,13] that the zeros of the Dirac neutrino mass matrix M_D and the right-handed Majorana mass matrix M_R are the progenitors of zeros in the effective Majorana mass matrix M_ν. Thus, the analysis of zeros in M_D and M_R is more basic than the study of zeros in M_ν. However, the zeros in M_D and M_R may not only show as zeros in the effective neutrino mass matrix. Another interesting possibility is that these zeros show as a vanishing minor in the effective mass matrix M_ν. Phenomenological analysis of the case where the zeros of M_R show as a vanishing minor in M_ν for diagonal M_D has been done recently [9,13]. This, however, is not the most general case. In the present work we explore the more general possibility of simultaneous existence of a texture zero and a vanishing minor in M_ν. Such texture structures are realized via the seesaw mechanism when there are texture zeros in M_D and M_R. Simultaneous existence of a texture zero and a vanishing minor restricts the form of the neutrino mass matrix and hence reduces the number of free parameters to five. These texture structures can be generated through the seesaw mechanism and realized by a discrete Abelian flavor symmetry [14]. We present the detailed analysis for such texture structures and examine their phenomenological implications. It is found that there are 36 such texture structures, 21 of which reduce to two texture zero forms which have been extensively studied in the literature. We analyze the remaining 15 texture structures and find that only six structures are consistent with the available data. We study the phenomenological implications for these texture structures and present their symmetry realization.

II. NEUTRINO MASS MATRIX

We reconstruct the neutrino mass matrix in the flavor basis (where the charged lepton mass matrix is diagonal) assuming also that neutrinos are Majorana particles. In this basis a complex symmetric neutrino mass matrix can be diagonalized by a unitary matrix V as

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$$M_\nu = V M_\nu^{\rm diag} V^T, \tag{2}$$

where

$$M_\nu^{\rm diag} = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}.$$

The matrix M_ν can be parametrized in terms of three neutrino mass eigenvalues (m_1, m_2, m_3), three neutrino mixing angles $(\theta_{12}, \theta_{23}, \theta_{13})$ (solar, atmospheric, and the reactor neutrino mixing angles, respectively), and the Dirac type CP-violating phase δ. The two additional phases α and β appear if neutrinos are Majorana particles. Here the matrix

$$V = UP, \tag{3}$$

where

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \tag{4}$$

with $s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \cos\theta_{ij}$ and

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i(\beta+\delta)} \end{pmatrix}$$

is the diagonal phase matrix with the two Majorana type CP-violating phases α, β and Dirac type CP-violating phase δ. The matrix V is called the neutrino mixing matrix or the Pontecorvo-Maki-Nakagawa-Sakata matrix [15]. Using Eqs. (3) and (4), the neutrino mass matrix can be written as

$$M_\nu = U P M_\nu^{\rm diag} P^T U^T. \tag{5}$$

The CP violation in neutrino oscillation experiments can be described through a rephasing invariant quantity, J_{CP} [16] with $J_{CP} = {\rm Im}(U_{e1}U_{\mu 2}U_{e2}^* U_{\mu 1}^*)$. In our parametrization, J_{CP} is given by

$$J_{CP} = s_{12}s_{23}s_{13}c_{12}c_{23}c_{13}^2 \sin\delta. \tag{6}$$

The observation of neutrinoless double beta decay would signal lepton number violation and imply a Majorana nature of neutrinos. The effective Majorana mass of the electron neutrino M_{ee} which determines the rate of neutrinoless double beta decay is given by

$$M_{ee} = |m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 e^{2i\alpha} + m_3 s_{13}^2 e^{2i\beta}|. \tag{7}$$

The experimental constraints on neutrino parameters at 1, 2, and 3σ [17] are given as

$$\Delta m_{12}^2 = 7.67^{(+0.16,+0.34,+0.52)}_{(-0.19,-0.36,-0.53)} \times 10^{-5}\ {\rm eV}^2,$$
$$\Delta m_{23}^2 = \pm 2.39^{(+0.11,+0.27,+0.47)}_{(-0.8,-0.20,-0.33)} \times 10^{-3}\ {\rm eV}^2,$$
$$\theta_{12} = 33.96^{\circ(+1.16,+2.43,+3.80)}_{(-1.12,-2.13,-3.10)},$$
$$\theta_{23} = 43.05^{\circ(+4.18,+7.83,+10.32)}_{(-3.35,-5.82,-7.93)}, \qquad \theta_{13} < 12.38^\circ (3\sigma). \tag{8}$$

The upper bound on θ_{13} is given by the CHOOZ experiment.

III. NEUTRINO MASS MATRICES WITH ONE TEXTURE ZERO AND ONE VANISHING MINOR

The simultaneous existence of a texture zero and a vanishing minor in the neutrino mass matrix gives

$$M_{\nu(xy)} = 0, \qquad M_{\nu(pq)}M_{\nu(rs)} - M_{\nu(tu)}M_{\nu(vw)} = 0. \tag{9}$$

These two conditions yield two complex equations viz.

$$m_1 X + m_2 Y e^{2i\alpha} + m_3 Z e^{2i(\beta+\delta)} = 0, \tag{10}$$

where $X = U_{x1}U_{y1}$, $Y = U_{x2}U_{y2}$, $Z = U_{x3}U_{y3}$, and

$$\sum_{l,k=1}^{3} (U_{pl}U_{ql}U_{rk}U_{sk} - U_{tl}U_{ul}U_{vk}U_{wk}) m_l m_k = 0. \tag{11}$$

We denote the minor corresponding to the (ij)th element by C_{ij}. The equation of vanishing minor becomes

$$m_1 m_2 A_3 e^{2i\alpha} + m_2 m_3 A_1 e^{2i(\alpha+\beta+\delta)} + m_3 m_1 A_2 e^{2i(\beta+\delta)} = 0, \tag{12}$$

where

$$A_h = (U_{pl}U_{ql}U_{rk}U_{sk} - U_{tl}U_{ul}U_{vk}U_{wk}) + (l \leftrightarrow k) \tag{13}$$

with (h, l, k) as the cyclic permutation of (1, 2, 3). These two complex equations (10) and (12) involve nine physical parameters m_1, m_2, m_3, θ_{12}, θ_{23}, θ_{13} and three CP-violating phases α, β, and δ. The masses m_2 and m_3 can be calculated from the mass-squared differences Δm_{12}^2 and Δm_{23}^2 using the relations

$$m_2 = \sqrt{m_1^2 + \Delta m_{12}^2}, \tag{14}$$

and

$$m_3 = \sqrt{m_2^2 + \Delta m_{23}^2}. \tag{15}$$

Using the experimental inputs of the two mass-squared differences and the two mixing angles we can constrain the other parameters. Thus, in the above two complex equations we are left with five unknown parameters m_1, α, β, δ, and θ_{13} which are, obviously, correlated.

Simultaneously solving Eqs. (10) and (12) for the two mass ratios, we obtain

$$\frac{m_1}{m_3}e^{-2i\beta} = -\frac{(XA_1 - YA_2 + ZA_3 \pm \sqrt{X^2A_1^2 + (YA_2 - ZA_3)^2 - 2XA_1(YA_2 + ZA_3)})}{2XA_3}e^{2i\delta}, \tag{16}$$

and

$$\frac{m_1}{m_2}e^{-2i\alpha} = \frac{(-XA_1 - YA_2 + ZA_3 \pm \sqrt{X^2A_1^2 + (YA_2 - ZA_3)^2 - 2XA_1(YA_2 + ZA_3)})}{2XA_2}. \tag{17}$$

The magnitude of the two mass ratios is given as

$$\rho = \left|\frac{m_1}{m_3}e^{-2i\beta}\right|, \tag{18}$$

$$\sigma = \left|\frac{m_1}{m_2}e^{-2i\alpha}\right|, \tag{19}$$

while the CP-violating Majorana phases α and β are given by

$$\alpha = -\frac{1}{2}\arg\left(\frac{(-XA_1 - YA_2 + ZA_3 \pm \sqrt{X^2A_1^2 + (YA_2 - ZA_3)^2 - 2XA_1(YA_2 + ZA_3)})}{2XA_2}\right), \tag{20}$$

$$\beta = -\frac{1}{2}\arg\left(-\frac{(XA_1 - YA_2 + ZA_3 \pm \sqrt{X^2A_1^2 + (YA_2 - ZA_3)^2 - 2XA_1(YA_2 + ZA_3)})}{2XA_3}e^{2i\delta}\right). \tag{21}$$

Since Δm_{12}^2 and Δm_{23}^2 are known experimentally, the values of mass ratios (ρ, σ) from Eqs. (18) and (19) can be used to calculate m_1. This can be done by inverting Eqs. (14) and (15) to obtain the two values of m_1, viz.

$$m_1 = \sigma\sqrt{\frac{\Delta m_{12}^2}{1 - \sigma^2}}, \tag{22}$$

and

$$m_1 = \rho\sqrt{\frac{\Delta m_{12}^2 + \Delta m_{23}^2}{1 - \rho^2}}. \tag{23}$$

We vary the oscillation parameters within their known experimental ranges. However, the Dirac type CP-violating phase δ is varied within its full range and θ_{13} is varied in its 3σ range given by the CHOOZ bound. The two values of m_1 obtained from the mass ratios ρ and

TABLE I. Thirty-six allowed texture structures of M_ν with a texture zero and one vanishing minor.

		A	B	C	D	E	F
1	$\begin{pmatrix} 0 & b & c \\ b & d & e \\ c & e & f \end{pmatrix}$	$df - e^2 = 0$	$bf - ec = 0$	$be - cd = 0$	Two zero	Two zero	Two zero
2	$\begin{pmatrix} a & 0 & c \\ 0 & d & e \\ c & e & f \end{pmatrix}$	$df - e^2 = 0$	Two zero	Two zero	$af - c^2 = 0$	Two zero	Two zero
3	$\begin{pmatrix} a & b & 0 \\ b & d & e \\ 0 & e & f \end{pmatrix}$	$df - e^2 = 0$	Two zero	Two zero	Two zero	Two zero	$ad - b^2 = 0$
4	$\begin{pmatrix} a & b & c \\ b & 0 & e \\ c & e & f \end{pmatrix}$	Two zero	$bf - ec = 0$	Two zero	$af - c^2 = 0$	$ae - bc = 0$	Two zero
5	$\begin{pmatrix} a & b & c \\ b & d & 0 \\ c & 0 & f \end{pmatrix}$	Two zero	Two zero	Two zero	$af - c^2 = 0$	Two zero	$ad - b^2 = 0$
6	$\begin{pmatrix} a & b & c \\ b & d & e \\ c & e & 0 \end{pmatrix}$	Two zero	Two zero	$be - dc = 0$	Two zero	$ae - bc = 0$	$ad - b^2 = 0$

σ, respectively, must be equal to within the errors of the oscillation parameters for the simultaneous existence of a texture zero and a vanishing minor.

There are in total 36 possible structures of neutrino mass matrix (Table I) with a single texture zero and a vanishing minor. As can be seen from Table I, 21 structures correspond to two texture zero cases which have, already, been studied extensively. We examine the phenomenological viability of all the remaining texture structures and also present detailed phenomenological implications for the viable structures.

IV. RESULTS AND DISCUSSION

As pointed out earlier, the two values of m_1 obtained from Eqs. (22) and (23) must be equal, of course, to within the errors of the oscillation parameters, for the simultaneous existence of a texture zero and a vanishing minor. Correspondingly, we obtain two regions of solutions. The viability of the simultaneous existence of a texture zero and a vanishing minor in the neutrino mass matrix is studied for both these regions. The known oscillation parameters are varied within their experimental ranges while the Dirac type CP-violating phase δ is varied within its full possible range. The 1-3 mixing angle is varied over the range limited by the 3σ CHOOZ bound. It is found that out of the 15 possible structures of the neutrino mass matrix with a texture zero and a vanishing minor, only six are allowed at 99% C.L., while the remaining nine are phenomenologically disallowed. We examine the phenomenological implications of all six viable structures. We present the lower bound on the effective Majorana mass M_{ee} for the viable cases. This important parameter determines the rate of neutrinoless double beta decay and will help decide the nature of neutrinos. The analysis of M_{ee} will be significant as many neutrinoless double beta decay experiments will constrain this parameter. A most stringent constraint on the value of M_{ee} was obtained in the ^{76}Ge Heidelberg-Moscow experiment [18] $|M_{ee}| < 0.35$ eV. There are a large number of projects such as SuperNEMO [19], CUORE [20], CUORICINO [20], and GERDA [21] which aim to achieve a sensitivity below 0.01 eV to M_{ee}. A forthcoming experiment SuperNEMO, in particular, will explore $M_{ee} <$ 0.05 eV [22]. In addition we also predict the bounds on the 1-3 mixing angle, and deviation of the 2-3 mixing angle from maximality for some viable cases. The precise measurements of the mixing angles and, in particular, searches for the deviations of the 1-3 mixing from zero and 2-3 mixing from maximality are crucial for understanding the underlying physics. The proposed experiments such as Double CHOOZ plan to explore $\sin^2 2\theta_{13}$ down to 0.06 in phase I (0.03 in phase II) [23]. Daya Bay has a higher sensitivity and plans to observe $\sin^2 2\theta_{13}$ down to 0.01 [24]. Next, we present the detailed numerical analysis for the six viable texture structures of neutrino mass matrix with a texture zero and a vanishing minor. In addition we study classes 2A and 3A analytically as these two classes give strongly hierarchical mass spectrum (IH). We also present the correlation plots between different parameters at 3σ C.L., and obtain interesting constraints on some of the parameters which are testable in the near future. We generate about 10^6 random points in our numerical analysis, and make direct use of the two mass-squared differences [Eqs. (22) and (23)], thus, making our analysis more reliable.

A. Class 2A

This texture structure has zero (1, 2) element and zero minor corresponding to (1, 1) entry ($C_{11} = 0$). This class has a clear inverted hierarchy. Here, $A_1 = c_{12}^2 c_{13}^2$, $A_2 = c_{13}^2 s_{12}^2$, and $A_3 = s_{13}^2 e^{2i\delta}$. We obtain the following analytical approximations for the mass ratios in the leading order of s_{13} as

$$\rho = \left|\frac{m_1}{m_3}\right| \approx \frac{1}{s_{13}^2} + O\left(\frac{1}{s_{13}}\right), \tag{24}$$

$$\sigma = \left|\frac{m_1}{m_2}\right| \approx 1 - \frac{\cos\delta s_{13} s_{23}}{c_{12} c_{23} s_{12}} + O(s_{13}^2). \tag{25}$$

It can be seen from Eq. (24) that ρ is always greater than 1, which gives inverted mass hierarchy of neutrino masses. Since the mass ratio σ is always smaller than 1 we find from Eq. (25) that $\cos\delta$ is always positive i.e. δ lies in the first and fourth quadrants. Figure 1 gives the correlation plots for this texture. There exists a lower bound on effective Majorana neutrino mass $M_{ee} > 0.042$ eV and $\theta_{13} > 0.35°$. We get highly constrained parameter space for the two Majorana type CP-violating phases α and β [Fig. 1(b)]. Larger values of θ_{13} are allowed for δ near 90°, 270° as seen from Fig. 1(d). The Jarlskog rephasing invariant J_{CP} is within the range (−0.45)–(0.45) [Fig. 1(c)].

B. Class 3A

This texture structure has zero 13 element and zero minor corresponding to 11 entry ($C_{11} = 0$). Class 3A also gives clear inverted mass hierarchy of neutrino masses. This texture has same values of A_1, A_2, and A_3 as for class 2A since both have the same vanishing minor. The analytical approximations for the mass ratios in the leading order of s_{13} are given as

$$\rho = \left|\frac{m_1}{m_3}\right| \approx \frac{1}{s_{13}^2} + O\left(\frac{1}{s_{13}}\right), \tag{26}$$

$$\sigma = \left|\frac{m_1}{m_2}\right| \approx 1 + \frac{\cos\delta s_{13} c_{23}}{c_{12} s_{23} s_{12}} + O(s_{13}^2). \tag{27}$$

It can be seen from Eq. (26) that ρ is always greater than 1, which gives inverted hierarchy of neutrino masses. Since the mass ratio σ is always smaller than 1 we find from Eq. (27) that $\cos\delta$ is always negative i.e. δ lies in the second and third quadrants. Figure 2 gives some interesting

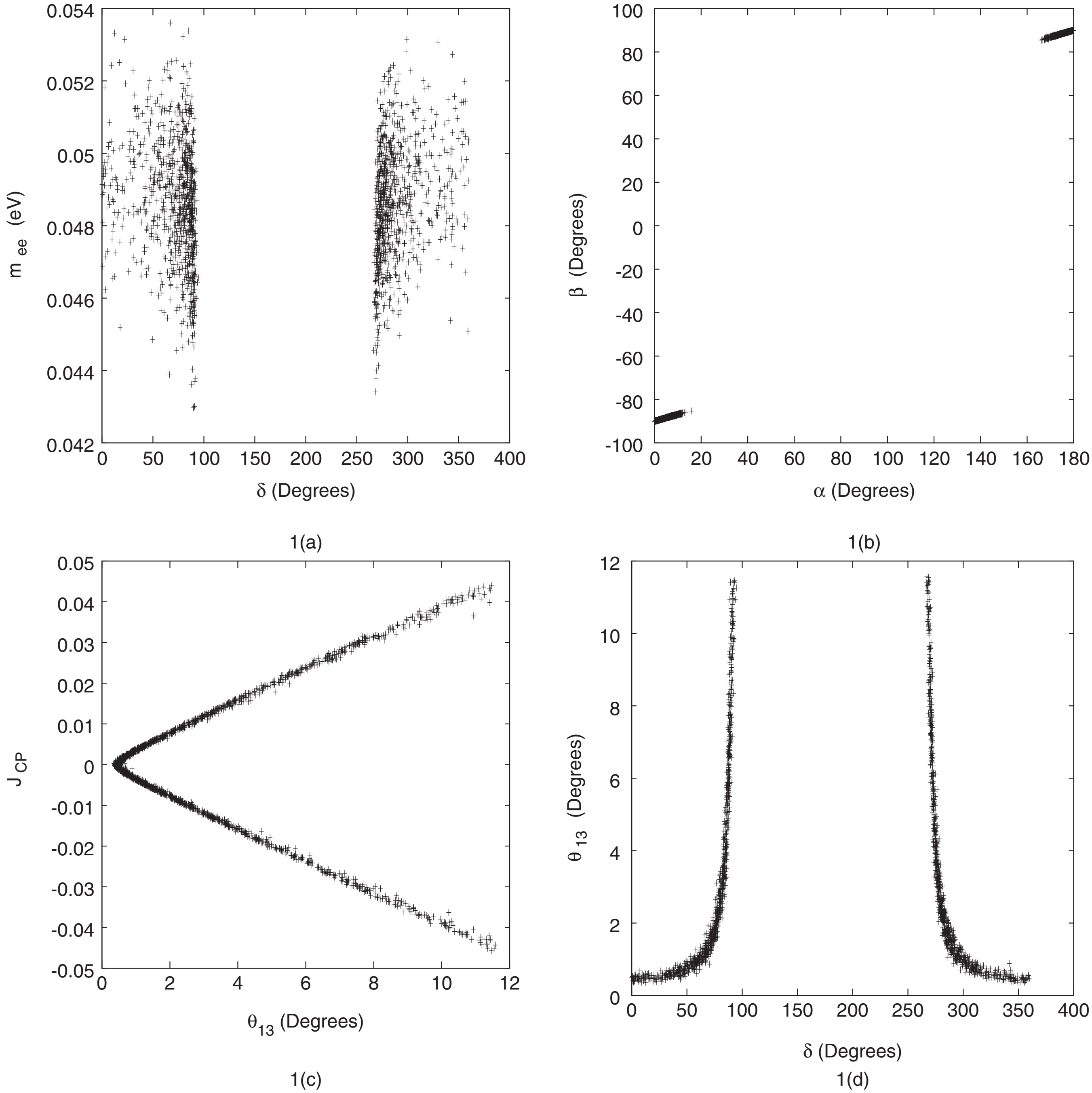

FIG. 1. Correlation plots for class 2A.

correlation plots for this texture structure. A lower bound on $M_{ee} > 0.044$ eV and 1-3 mixing angle ($\theta_{13} > 0.3°$) is obtained for this class. Larger values of θ_{13} are allowed near 90°, 270°. The rephasing invariant, J_{CP} lies in the range (−0.045)–(0.045).

It is found that the limit of a vanishing mass eigenvalue i.e. $m_3 = 0$ can be reached for both classes 2A and 3A. As can be seen from Fig. 2(b) that very small range ($0.3° \leq \theta_{13} \leq 1.1°$) is allowed for the limit of vanishing mass eigenvalue $m_3 = 0$.

It is interesting to note that class 3A is transformed to class 2A and vice versa by the transformation $\delta \longrightarrow \delta + \pi$, $\theta_{23} \longrightarrow (\frac{\pi}{2} - \theta_{23})$. Therefore, the predictions for neutrino mass matrices for these two classes will be identical for all neutrino parameters except θ_{23} and/or δ.

C. Remaining viable classes

The remaining viable texture structures which have phenomenological implications are 2D, 3F, 4B, and 6C.

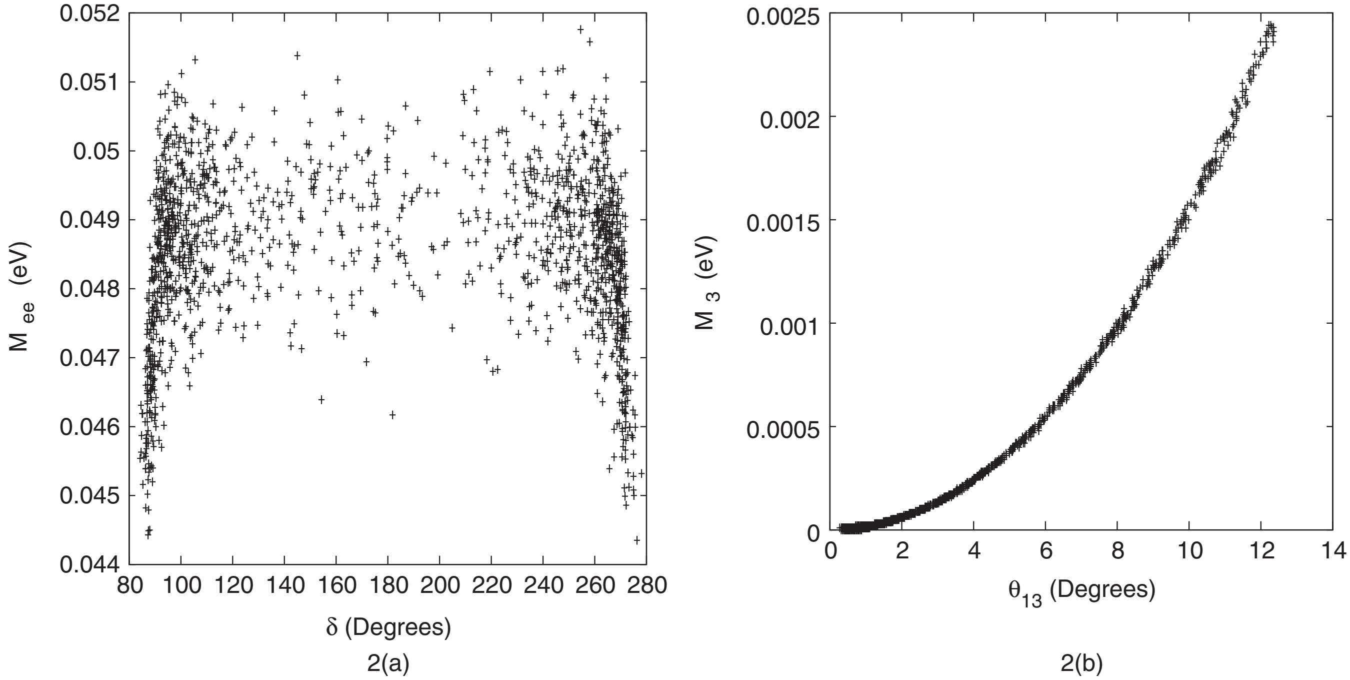

FIG. 2. Correlation plots for class 3A.

The corresponding values of A_1, A_2, and A_3 for these texture structures are given in Table II. All these textures give normal, inverted, and quasidegenerate mass spectra. For class 2D(NH, QD), we get unconstrained parameter space, i.e., there are no strong predictions. However, for 2D (IH), the Dirac type *CP*-violating phase δ is constrained to the range 90°–270°, while the Majorana type *CP*-violating phase α takes the value 0°, 180°. A lower bound on the effective Majorana mass $M_{ee} > 0.05$ eV is obtained. The atmospheric mixing angle θ_{23} lies below maximality i.e. $\theta_{23} < 45°$ [Fig. 3(a)]. Class 3F(IH) has similar predictions and bounds for all parameters as in class 2D(IH) except θ_{23} which is above maximal. For classes 4B and 6C, unconstrained parameter space is obtained for inverted and quasidegenerate mass hierarchy. However, 4B(NH) and 6C (NH) give some strong predictions for parameters under investigation. A lower bound $M_{ee} > 0.01$ eV is obtained for both these cases. The atmospheric mixing angle θ_{23} is above maximal for 6C(NH) and below maximal for 4B (NH). There are some projects like Tokai-to-Kamioka-Korea which plans to resolve the octant degeneracy of θ_{23} (i.e. $\theta_{23} < 45°$ or $\theta_{23} > 45°$) [25].

V. SYMMETRY REALIZATION

All the phenomenologically viable textures with a texture zero and a zero minor in M_ν discussed in this work can be realized in a simple way in models based on the seesaw mechanism with an Abelian flavor symmetry [14]. For constructing the required leptonic mass matrices, we consider three left-handed standard model lepton doublets D_{La} ($a = 1, 2, 3$) and three right-handed charged lepton singlets l_{Ra}. To this we add three right-handed neutrino singlets ν_{Ra} in order to enable the seesaw mechanism for suppressing the neutrino masses. For each nonzero entry in M_l or M_D we need one Higgs doublet. Similarly, for each nonzero matrix element of M_R we need a scalar singlet χ_{ab}. Here, we present the symmetry realization for the texture structure 2D which can be generated through the seesaw mechanism. One of the possible texture structures of M_D and M_R which reproduce class 2D is

$$M_D = \begin{pmatrix} 0 & 0 & a \\ b & c & 0 \\ 0 & d & e \end{pmatrix}, \qquad M_R = \begin{pmatrix} 0 & A & 0 \\ A & B & 0 \\ 0 & 0 & C \end{pmatrix}. \quad (28)$$

The resulting M_ν generated through the seesaw mechanism

TABLE II. A_1, A_2, and A_3 for the remaining viable textures.

Texture	A_1	A_2	A_3
2D	$e^{-2i\delta}(e^{i\delta}s_{12}c_{23} + s_{13}s_{23}c_{12})^2$	$e^{-2i\delta}(e^{i\delta}c_{12}c_{23} - s_{13}s_{23}s_{12})^2$	$c_{13}^2 s_{23}^2$
3F	$e^{-2i\delta}(-e^{i\delta}s_{12}s_{23} + s_{13}c_{23}c_{12})^2$	$e^{-2i\delta}(e^{i\delta}c_{12}s_{23} + s_{13}c_{23}s_{12})^2$	$c_{13}^2 c_{23}^2$
4B	$c_{12}c_{13}(e^{-i\delta}c_{12}s_{23}s_{13} + c_{23}s_{12})$	$s_{12}c_{13}(e^{-i\delta}s_{12}s_{23}s_{13} - c_{23}c_{12})$	$-e^{i\delta}(c_{13}s_{23}s_{13})$
6C	$c_{12}c_{13}(-e^{-i\delta}c_{12}c_{23}s_{13} + s_{23}s_{12})$	$s_{12}c_{13}(-e^{-i\delta}s_{12}c_{23}s_{13} - s_{23}c_{12})$	$e^{i\delta}s_{13}c_{23}c_{13}$

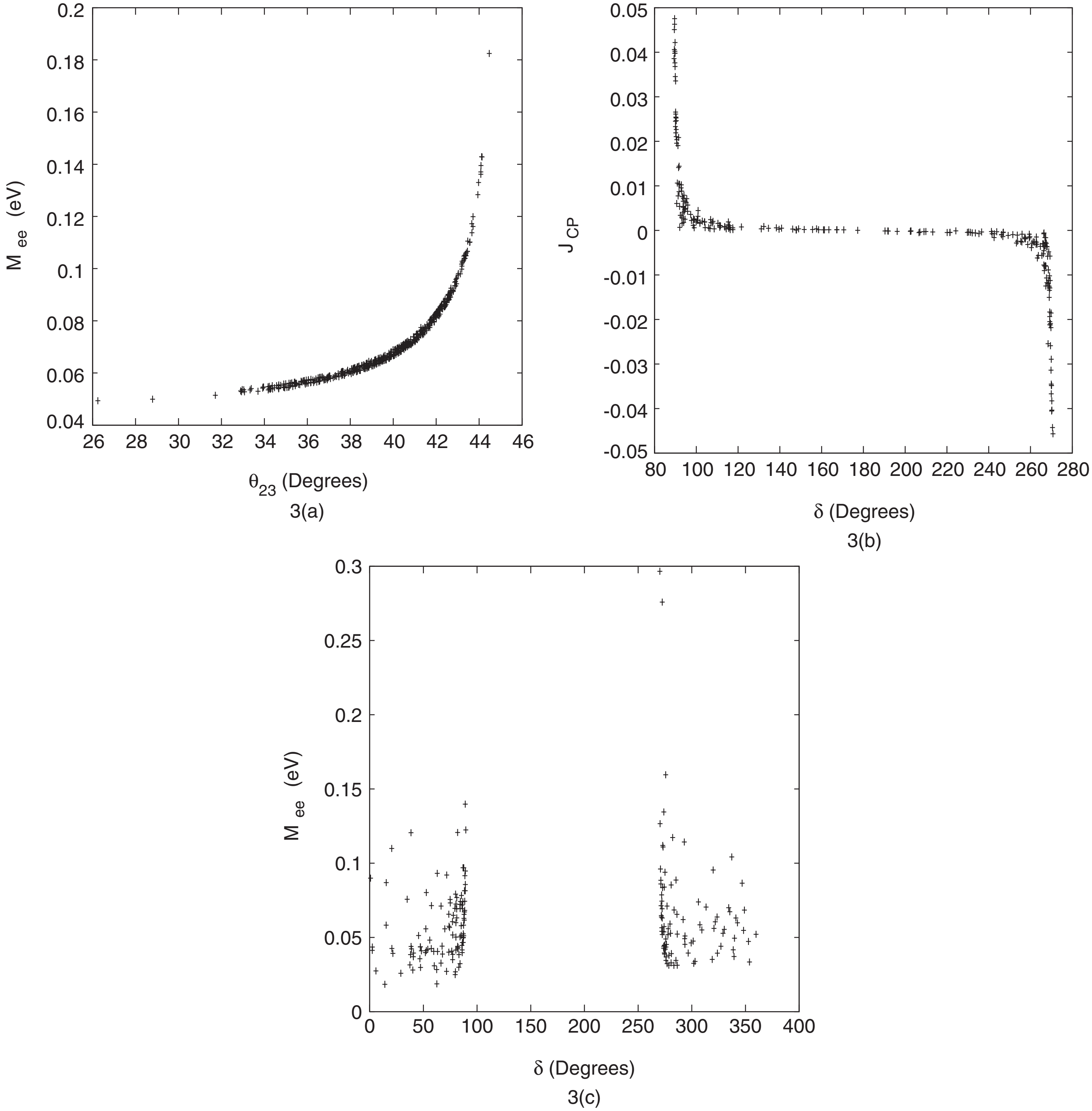

FIG. 3. Correlation plots for classes 2D (IH), 3F (IH), and 6C (NH), respectively.

takes the form

$$M_\nu = \begin{pmatrix} \frac{a^2}{C} & 0 & \frac{ae}{C} \\ 0 & \frac{b(-(bB)+2Ac)}{A^2} & \frac{bd}{A} \\ \frac{ae}{C} & \frac{bd}{A} & \frac{e^2}{C} \end{pmatrix}. \tag{29}$$

This M_ν has a zero (1, 2) element and a zero minor corresponding to the (2, 2) element ($C_{22} = 0$) as required. The symmetry realization for this texture structure could be done through a generic choice of the Abelian symmetry group $Z_{12} \times Z_2$ discussed in [14] which however may not be the most economic way. Under Z_{12} the leptonic fields transform as

$$\begin{aligned} &\bar{l}_{R1} \to \omega \bar{l}_{R1}, \quad \bar{\nu}_{R1} \to \omega \bar{\nu}_{R1}, \quad D_{L1} \to \omega D_{L1}, \\ &\bar{l}_{R2} \to \omega^2 \bar{l}_{R2}, \quad \bar{\nu}_{R2} \to \omega^2 \bar{\nu}_{R2}, \quad D_{L2} \to \omega^3 D_{L2}, \\ &\bar{l}_{R3} \to \omega^5 \bar{l}_{R3}, \quad \bar{\nu}_{R3} \to \omega^5 \bar{\nu}_{R3}, \quad D_{L3} \to \omega^8 D_{L3}, \end{aligned} \tag{30}$$

where $\omega = \exp(i\pi/6)$. The bilinears $\bar{l}_{Ra}D_{Lb}$ and $\bar{\nu}_{Ra}D_{Lb}$, relevant for $(M_l)_{ab}$ and $(M_D)_{ab}$ transform as

$$\begin{pmatrix} \omega^2 & \omega^4 & \omega^9 \\ \omega^3 & \omega^5 & \omega^{10} \\ \omega^6 & \omega^8 & \omega \end{pmatrix},$$

while the bilinears $\bar{\nu}_{Ra}C\bar{\nu}_{Rb}^T$, relevant for $(M_R)_{ab}$, transform as

$$\begin{pmatrix} \omega^2 & \omega^3 & \omega^6 \\ \omega^3 & \omega^4 & \omega^7 \\ \omega^6 & \omega^7 & \omega^{10} \end{pmatrix}.$$

To obtain a diagonal charged lepton mass matrix, only three Higgs doublets are needed, viz. ϕ_{11}, ϕ_{22}, and ϕ_{33}. Under Z_{12} these scalar doublets get, respectively, multiplied by ω^{10}, ω^7, and ω^{11} so that the charged lepton mass term remains invariant. The nondiagonal entries of M_l remain zero in the absence of any further Higgs doublets. Similarly nonzero entries of M_D and M_R in Eq. (28) can be obtained by introducing scalar Higgs doublets $\tilde{\phi}_{13}$, $\tilde{\phi}_{21}$, $\tilde{\phi}_{22}$, $\tilde{\phi}_{32}$, and $\tilde{\phi}_{33}$ being multiplied by ω^3, ω^9, ω^7, ω^4, and ω^{11}, respectively, under Z_{12} for M_D and by introducing scalar singlet fields, namely, χ_{12}, χ_{22}, and χ_{33} which get multiplied by ω^9, ω^8, and ω^2 under Z_{12} for M_R. It is important to note that the scalar Higgs doublets acquire vacuum expectation values (vevs) at the electroweak scale, while scalar singlets acquire vevs at the seesaw scale. Under Z_2 the $\tilde{\phi}_{ab}$ and the neutrino singlets ν_{Ra} change sign, while all other multiplets remain invariant. The symmetry realization for different M_D and M_R giving M_ν corresponding to our viable textures can be similarly performed.

VI. CONCLUSIONS

We presented a comprehensive phenomenological analysis for the Majorana neutrino mass matrices with a texture zero and a vanishing minor. All these texture structures can be generated through the seesaw mechanism when there are texture zeros in M_D and M_R and realized in the framework of discrete Abelian flavor symmetry. It is found that out of a total of 36 texture structures, 21 reduce to two zero texture structures which have been extensively studied in the past. The viability of the simultaneous existence of a texture zero and a vanishing minor in the neutrino mass matrix is studied for the two regions of solutions. Nine out of the remaining textures are disallowed by the current data and we presented the numerical analysis for the remaining six texture structures. Analytical framework for the two classes with strongly hierarchical mass spectrum is, also, given. Predictions for the 1-3 mixing angle and the Dirac type *CP*-violating phase are given for the allowed texture structures. These parameters are expected to be measured in the forthcoming neutrino oscillation experiments. We, also, obtained the lower bound on the effective Majorana mass for different classes. In the end, we presented the symmetry realization of these texture structures which are generated via the seesaw mechanism. However, the evolution of the Yukawa coupling matrices of the mass operators in models with multi-Higgs doublets is an important issue since the evolution of the coupling matrices from the seesaw scale down to the electroweak scale may alter the predictions of the models under consideration. However, these corrections, in general, are expected to be negligibly small except perhaps for the case of a degenerate neutrino mass spectrum which needs to be investigated carefully.

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