The 3-3-1 model with A_4 flavor symmetry

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(Received 26 January 2010; revised manuscript received 9 February 2010; published 8 March 2010)

We argue that the A_4 symmetry as required by three flavors of fermions may well-embed in the $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ gauge model. The new neutral fermion singlets as introduced in a canonical seesaw mechanism can be combined with the standard model lepton doublets to perform $SU(3)_L$ triplets. Various leptoscalar multiplets such as singlets, doublets, and triplets as played in the models of A_4 are unified in single $SU(3)_L$ antisextets. As a result, naturally light neutrinos with various kinds of mass hierarchies are obtained as a combination of type I and type II seesaw contributions. The observed neutrino mixing pattern in terms of the Harrison-Perkins-Scott proposal is obtained by enforcing the A_4 group. The quark masses and Cabibbo-Kobayashi-Maskawa mixing matrix are also discussed. By virtue of very heavy antisextets, the nature of the vacuum alignments of scalar fields can be given.

DOI: 10.1103/PhysRevD.81.053004 PACS numbers: 14.60.Pq, 11.30.Hv, 12.60.−i, 14.60.St

I. INTRODUCTION

The explanation of the smallness of the neutrino masses and the profile of their mixing as required by experiment [1–3] have been a great puzzle in particle physics beyond the standard model (SM). The current experimental data are consistent with the tribimaximal form as proposed by Harrison-Perkins-Scott [4], which, apart from phase redefinitions, is given by

$$U^{\mathrm{HPS}} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}. \quad (1)$$

It is an interesting challenge to formulate dynamical principles that can lead to the tribimaximal mixing pattern given in a completely natural way as a first approximation. Along these lines the flavor symmetries have been extensively studied. For the first time, Ma and Rajasekaran [5] have advocated choosing A_4, the symmetry group of a tetrahedron, as a family symmetry group. An incomplete list of interesting works that came later include Refs. [6–10]. The key to its success is that the patterns of symmetry breaking with preserved subgroups are $A_4 \to Z_3$ and $A_4 \to Z_2$ in the two different sectors—the charged lepton sector and the neutrino sector, respectively. This misalignment can further be explained by auxiliary symmetries and particles or even in the context of extra dimensions (see, for example, [7,8]).

Here we would like to extend the above application to the $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ (3-3-1) gauge model [11–13], because it can give a partial explanation of the existence of just three fermion families in nature as a result of the gauge anomaly cancellations required by the A_4 symmetry. There are two typical variants of the 3-3-1 model as far as the lepton sectors are concerned. In the minimal version, three $SU(3)_L$ lepton triplets are of the form $(\nu_L, e_L, e^c_R)_{i=1,2,3}$, where e_{iR} are ordinary right-handed charged leptons [11]. In the second version, the third components of the lepton triplets include right-handed neutrinos, respectively, $(\nu_L, e_L, \nu^c_R)_{i=1,2,3}$ [12]. Note that Ref. [10] has considered the A_4 symmetry in the 3-3-1 model with heavy charged leptons, which is a modification of the minimal version.

In this work we will pay attention to the second version and try to recover the tribimaximal form. By analysis, a possibility close to the typical version is when we replace the right-handed neutrinos by those with vanishing lepton number [5,14,15]. The neutrinos thus gain masses only from contributions of $SU(3)_L$ scalar antisextets. After considering the quark sector, the scalar sector is completed. In this model, the antisextets contain tiny vacuum expectation values (VEVs) in the first components, as in the case of the standard model with scalar triplets. To avoid the decay of the Z boson into the Majorons associated with these components, the lepton-number violating scalar potential should be taken into account. Therefore, the lepton number is no longer of an exact symmetry; i.e. the Majorons can get large enough masses to escape from the decay of Z [14]. If the antisextets are supposed to be very heavy, the potential minimization conditions can naturally give an explanation of the expected vacuum alignments, and also the smallness of the lepton-number violating VEVs as well as the mentioned ones. Note that this dangerous decay

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channel of the Z boson has not been fully evaluated in the versions of the 3-3-1 model that include the antisextets [16].

The paper is organized as follows: In Sec. II, we introduce the A_4 family symmetry into the model and obtain the mass mechanisms and mixing matrix of leptons. Section III discusses the quark masses. The scalar sector is then completed. Section IV is devoted to the scalar potential, vacuum alignment problem for the scalar fields. In the last section, Sec. V, we summarize our results and make conclusions. Finally, the appendixes provide the basics of A_4 symmetry and the general scalar potential used in the text.

II. LEPTONS

The particle content of the 3-3-1 model under consideration is collected from Ref. [5]. We will show that this selection, with an appropriate A_4 flavor symmetry, provides, in the framework, a consistent mixing pattern and masses for the neutrinos. The leptons, under $(\mathrm{SU}(3)_L, \mathrm{U}(1)_X, A_4)$ symmetries, transform as

$$\psi_L = \begin{pmatrix} \nu_L \\ e_L \\ \nu_R^c \end{pmatrix} \sim (3, -1/3, \underline{3}), \tag{2}$$

$$e_{1R} \sim (1, -1, \underline{1}), \qquad e_{2R} \sim (1, -1, \underline{1}'), \qquad e_{3R} \sim (1, -1, \underline{1}''), \tag{3}$$

where ν_{iR} $(i = 1, 2, 3)$ are three right-handed fermions which are singlets under the standard model symmetry and have zero lepton number, $L(\nu_R) = 0$. The X charge of the $\mathrm{U}(1)_X$ group is related to the electric charge operator as $Q = T_3 - \frac{1}{\sqrt{3}} T_8 + X$, where T_a $(a = 1, 2, \ldots, 8)$ are $\mathrm{SU}(3)_L$ charges. Our model is therefore a type of the ones given in [11,12].

The lepton number in this model does not commute with the gauge symmetry. It is thus better to work with a new lepton charge $\mathcal{L}$ related to the lepton number L by diagonal matrices $L = xT_3 + yT_8 + \mathcal{L}$. Applying L to the lepton triplet, the coefficients are defined as $x = 0$, $y = 2/\sqrt{3}$, and thus $L = \frac{2}{\sqrt{3}} T_8 + \mathcal{L}$ [17]. The $\mathcal{L}$ charges for the multiplets are as follows:

Multiplet	ψ_L	e_{1R}	e_{2R}	e_{3R}
$\mathcal{L}$	2/3	1	1	1

To generate masses for the charged leptons, we introduce the following scalar fields:

$$\phi = \begin{pmatrix} \phi_1^+ \\ \phi_2^0 \\ \phi_3^+ \end{pmatrix} \sim (3, 2/3, \underline{3}, -1/3). \tag{4}$$

The first three quantum numbers are well defined as before. The last one is the $\mathcal{L}$ charge for ϕ such that the following Yukawa interaction is conserved:

$$\mathcal{L}_l = -h_{ijk} \bar{\psi}_{iL} \phi_j e_{kR} + \text{H.c.}, \tag{5}$$

where

$$h_{ij1} = h_1 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad h_{ij2} = h_2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix},$$
$$h_{ij3} = h_3 \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}, \tag{6}$$

with $\omega = e^{2\pi i/3}$. The lepton number for the components of ϕ, including the additional scalars as shown below, is explicitly given in Appendix B.

The VEV of ϕ is (v_1, v_2, v_3) under A_4. The mass Lagrangian for the charged leptons reads $\mathcal{L}_l^{\text{mass}} = -(\bar{e}_{1L}, \bar{e}_{2L}, \bar{e}_{3L}) M_l (e_{1R}, e_{2R}, e_{3R})^T + \text{H.c.}$, where

$$M_l = \begin{pmatrix} h_1 v_1 & h_2 v_1 & h_3 v_1 \\ h_1 v_2 & h_2 \omega v_2 & h_3 \omega^2 v_2 \\ h_1 v_1 & h_2 \omega^2 v_2 & h_3 \omega v_3 \end{pmatrix}. \tag{7}$$

We put $v_1 = v_2 = v_3 = v$ so that A_4 is broken down to Z_3 (this is also a minimal condition for the Higgs potential as shown below). The mass matrix is then diagonalized,

$$U_L^\dagger M_l U_R = \begin{pmatrix} \sqrt{3} h_1 v & 0 & 0 \\ 0 & \sqrt{3} h_2 v & 0 \\ 0 & 0 & \sqrt{3} h_3 v \end{pmatrix} = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}, \tag{8}$$

where

$$U_L = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}, \qquad U_R = 1. \tag{9}$$

Notice that $\bar{\psi}_L^c \psi_L \phi$ is suppressed because of the $\mathcal{L}$-symmetry violation. Then $\bar{\psi}_L^c \psi_L$ can couple to $\mathrm{SU}(3)_L$ antisextets to generate masses for the neutrinos. The antisextets in this model transform as

$$\sigma = \begin{pmatrix} \sigma_{11}^0 & \sigma_{12}^+ & \sigma_{13}^0 \\ \sigma_{12}^+ & \sigma_{22}^{++} & \sigma_{23}^+ \\ \sigma_{13}^0 & \sigma_{23}^+ & \sigma_{33}^0 \end{pmatrix} \sim (6^*, 2/3, \underline{1}, -4/3), \tag{10}$$

$$s = \begin{pmatrix} s_{11}^0 & s_{12}^+ & s_{13}^0 \\ s_{12}^+ & s_{22}^{++} & s_{23}^+ \\ s_{13}^0 & s_{23}^+ & s_{33}^0 \end{pmatrix} \sim (6^*, 2/3, \underline{3}, -4/3). \tag{11}$$

The Yukawa interactions are

$$\mathcal{L}_\nu = -\tfrac{1}{2}x(\bar{\psi}^c_{1L}\psi_{1L} + \bar{\psi}^c_{2L}\psi_{2L} + \bar{\psi}^c_{3L}\psi_{3L})\sigma - y(\bar{\psi}^c_{2L}\psi_{3L}s_1 + \bar{\psi}^c_{3L}\psi_{1L}s_2 + \bar{\psi}^c_{1L}\psi_{2L}s_3) + \text{H.c.} \tag{12}$$

The VEV of s is set as $(\langle s_1\rangle, 0, 0)$ under A_4 (which is also a natural minimal condition for the Higgs potential). As such, the group is broken down to Z_2 in the neutrino sector, where

$$\langle s_1\rangle = \begin{pmatrix} u_1' & 0 & u_1 \\ 0 & 0 & 0 \\ u_1 & 0 & \Lambda_1 \end{pmatrix}. \tag{13}$$

The VEV of σ is

$$\langle \sigma\rangle = \begin{pmatrix} u' & 0 & u \\ 0 & 0 & 0 \\ u & 0 & \Lambda \end{pmatrix}. \tag{14}$$

The mass Lagrangian for the neutrinos is defined by

$$\mathcal{L}^{\text{mass}}_\nu = -\frac{1}{2}\bar{\chi}^c_L M_\nu \chi_L + \text{H.c.}, \qquad \chi_L \equiv \begin{pmatrix} \nu_L \\ \nu^c_R \end{pmatrix}, \qquad M_\nu \equiv \begin{pmatrix} M_L & M_D^T \\ M_D & M_R \end{pmatrix}, \tag{15}$$

where $\nu = (\nu_1, \nu_2, \nu_3)^T$. The mass matrices are then obtained by

$$M_{L,R,D} = \begin{pmatrix} a_{L,R,D} & 0 & 0 \\ 0 & a_{L,R,D} & b_{L,R,D} \\ 0 & b_{L,R,D} & a_{L,R,D} \end{pmatrix}, \tag{16}$$

with

$$a_L = xu', \quad a_D = xu, \quad a_R = x\Lambda, \quad b_L = yu_1', \quad b_D = yu_1, \quad b_R = y\Lambda_1. \tag{17}$$

Three active neutrinos gain masses via a combination of type I and type II seesaw mechanisms derived from (15) as

$$M^{\text{eff}} = M_L - M_D^T M_R^{-1} M_D = \begin{pmatrix} a' & 0 & 0 \\ 0 & a & b \\ 0 & b & a \end{pmatrix}, \tag{18}$$

where

$$\begin{aligned} a' &= a_L - \frac{a_D^2}{a_R}, \\ a &= a_L + 2a_D b_D \frac{b_R}{a_R^2 - b_R^2} - (a_D^2 + b_D^2)\frac{a_R}{a_R^2 - b_R^2}, \\ b &= b_L - 2a_D b_D \frac{a_R}{a_R^2 - b_R^2} + (a_D^2 + b_D^2)\frac{b_R}{a_R^2 - b_R^2}. \end{aligned} \tag{19}$$

We can diagonalize the mass matrix (18) as follows:

$$U_\nu^T M^{\text{eff}} U_\nu = \begin{pmatrix} a+b & 0 & 0 \\ 0 & a' & 0 \\ 0 & 0 & a-b \end{pmatrix} = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}, \tag{20}$$

where

$$U_\nu = \begin{pmatrix} 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{21}$$

Combined with (9), the lepton mixing matrix yields the tribimaximal mixing pattern as proposed by Harrison-Perkins-Scott (up to a phase):

$$U_L^\dagger U_\nu = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & i/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & -i/\sqrt{2} \end{pmatrix} = U^{\text{HPS}} P_\phi, \tag{22}$$

where the phase matrix $P_\phi = \text{diag}(1, 1, i)$ can be removed by absorbing it into the neutrino mass eigenstates. This is a main result of the paper.

With the aid of the results in (19), we identify u', u_1' as the VEVs of the type II seesaw mechanism. The mechanism works because, from Eq. (43) in Sec. IV, the spontaneous breaking of electroweak symmetry is already accomplished by v; hence u', u_1' may be small, as long as M is large. The parameter $\bar{\mu}_2$ (which has the dimension of mass) may also be naturally small, because its absence enhances the symmetry of $V^{s\sigma}$ [14]. On the other hand, u, u_1 are the VEVs of the type I seesaw mechanism. Similar to the case above, these VEVs are, however, much smaller than v. But they can be larger than u', u_1' because $v_\chi > v$ (notice that v_χ is the scale of the 3-3-1 symmetry breaking into the SM). The TeV scale type I seesaw mechanism can be achieved if we take $v_\chi = 10$ TeV, $\bar{\mu}_1 = 100\bar{\mu}_2$ [14].

It is noted that the lepton number L is really broken by the small VEVs of the antisextets s_1 and σ since their corresponding field components carry L; namely, the (11) has $L = -2$, the (13) has $L = -1$, but the (33) has $L = 0$. Now $u' \neq 0$ (or $u_1' \neq 0$) by itself means that L is broken by 2 units; hence $L \to (-)^L$, as lepton parity is still conserved. This is the case in most models of neutrino mass. The type I seesaw mechanism gives no contribution. However, if u (or u_1) is also nonzero, then L is broken completely. Both the seesaw mechanisms play this role.

III. QUARKS

It is well known that the 3-3-1 model is a good example of the fermion number problem: Why are there only three families of fermions in nature [11,12,17]? This perfectly meets the criteria of three-family symmetry theories such as A_4. The anomaly cancellation in the 3-3-1 models requires the number of $SU(3)_L$ triplets to be equal to the number of $SU(3)_L$ antitriplets; i.e., two families of quarks have to transform differently from the other one. Hence, the quark triplets and antitriplets of the three families cannot lie in a $\underline{3}$ representation of A_4. The right-handed exotic quarks are the same. Here, the following two situations exist.

The first situation is that the above scalar ϕ is *responsible* for generating quark masses. The quark content is obtained as follows:

$$Q_{3L} = \begin{pmatrix} u_{3L} \\ d_{3L} \\ T_L \end{pmatrix} \sim (3, 1/3, \underline{1}, -1/3), \tag{23}$$

$$Q_{1L} = \begin{pmatrix} d_{1L} \\ -u_{1L} \\ D_{1L} \end{pmatrix} \sim (3^*, 0, \underline{1}', 1/3), \quad Q_{2L} = \begin{pmatrix} d_{2L} \\ -u_{2L} \\ D_{2L} \end{pmatrix} \sim (3^*, 0, \underline{1}'', 1/3), \tag{24}$$

$$T_R \sim (1, 2/3, \underline{1}, -1), \qquad D_{1R} \sim (1, -1/3, \underline{1}'', 1), \qquad D_{2R} \sim (1, -1/3, \underline{1}', 1), \tag{25}$$

$$u_R \sim (1, 2/3, \underline{3}, 0), \qquad d_R \sim (1, -1/3, \underline{3}, 0). \tag{26}$$

From (23)–(25), it follows that the exotic quarks have a single lepton number, i.e. $L(T) = -1$ and $L(D) = +1$. Hence, in the considered model the exotic quarks are leptoquarks. With the above quark content, the scalar triplet ϕ is not enough to provide mass for all the quarks. Hence the following extra scalar fields are needed to provide masses for the remaining quarks [12]:

$$\eta = \begin{pmatrix} \eta_1^0 \\ \eta_2^- \\ \eta_3^0 \end{pmatrix} \sim (3, -1/3, \underline{3}, -1/3), \quad \chi = \begin{pmatrix} \chi_1^0 \\ \chi_2^- \\ \chi_3^0 \end{pmatrix} \sim (3, -1/3, \underline{1}, 2/3). \tag{27}$$

The Yukawa interactions are

$$\begin{aligned} -\mathcal{L}_q = {}& h_3^d \bar{Q}_{3L}(\phi d_R)_1 + h_1^u \bar{Q}_{1L}(\phi^* u_R)_{1''} + h_2^u \bar{Q}_{2L}(\phi^* u_R)_{1'} \\ & + h_3^u \bar{Q}_{3L}(\eta u_R)_1 + h_1^d \bar{Q}_{1L}(\eta^* d_R)_{1''} \\ & + h_2^d \bar{Q}_{2L}(\eta^* d_R)_{1'} + f_3 \bar{Q}_{3L}\chi T_R + f_1 \bar{Q}_{1L}\chi^* D_{1R} \\ & + f_2 \bar{Q}_{2L}\chi^* D_{2R} + \text{H.c.} \end{aligned} \tag{28}$$

Suppose that the VEVs of η and χ are (v', v', v') and v_χ, with $v' = \langle \eta_1^0 \rangle$, $v_\chi = \langle \chi_3^0 \rangle$, $\langle \eta_3^0 \rangle = 0$, and $\langle \chi_1^0 \rangle = 0$. The exotic quarks get masses directly from the VEV of χ: $m_T = f_3 v_\chi$, $m_{D_{1,2}} = f_{1,2} v_\chi$. In addition, v_χ has to be much larger than those of ϕ and η. The mass matrices for ordinary up quarks and down quarks are, respectively, obtained as follows:

$$M_u = \begin{pmatrix} -h_1^u v & -h_1^u \omega v & -h_1^u \omega^2 v \\ -h_2^u v & -h_2^u \omega^2 v & -h_2^u \omega v \\ h_3^u v' & h_3^u v' & h_3^u v' \end{pmatrix}, \quad M_d = \begin{pmatrix} h_1^d v' & h_1^d \omega v' & h_1^d \omega^2 v' \\ h_2^d v' & h_2^d \omega^2 v' & h_2^d \omega v' \\ h_3^d v & h_3^d v & h_3^d v \end{pmatrix}. \tag{29}$$

Let us put

$$A = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ \omega^2 & \omega & 1 \\ \omega & \omega^2 & 1 \end{pmatrix}. \tag{30}$$

We have then

$$\begin{aligned} M_u A &= \begin{pmatrix} -\sqrt{3} h_1^u v & 0 & 0 \\ 0 & -\sqrt{3} h_2^u v & 0 \\ 0 & 0 & \sqrt{3} h_3^u v' \end{pmatrix} = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}, \\ M_d A &= \begin{pmatrix} \sqrt{3} h_1^d v' & 0 & 0 \\ 0 & \sqrt{3} h_2^d v' & 0 \\ 0 & 0 & \sqrt{3} h_3^d v \end{pmatrix} = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}. \end{aligned} \tag{31}$$

The unitary matrices, which couple the left-handed up and down quarks to those in the mass bases, are $U_L^u = 1$ and $U_L^d = 1$, respectively. Therefore we get the Cabibbo-Kobayashi-Maskawa (CKM) matrix

$$U_{\text{CKM}} = U_L^{d\dagger} U_L^u = 1. \tag{32}$$

Note that the property in (32) is common for some models based on the A_4 group.

In the last situation the mentioned scalar field ϕ *is not responsible* for the quark masses. The ordinary right-handed quarks are therefore in singlets under A_4. In this case, we might introduce three extra $SU(3)_L$ Higgs triplets such as

$$\eta = \begin{pmatrix} \eta_1^0 \\ \eta_2^- \\ \eta_3^0 \end{pmatrix} \sim (3, -1/3, \underline{1}, -1/3), \quad \rho = \begin{pmatrix} \rho_1^+ \\ \rho_2^0 \\ \rho_3^+ \end{pmatrix} \sim (3, 2/3, \underline{1}, -1/3), \tag{33}$$

and χ, as in the first situation. A combination of such Higgs scalar fields will give mass for all the quarks [12].

However, all these scalar triplets as well as the quarks lie in $\underline{1}$ representations of A_4. It is easy to check that all quarks get masses in the same ordinary 3-3-1 model; namely, $v_\eta = \langle \eta_1^0 \rangle$ provides the mass for u_3, d_1, and d_2 quarks, $v_\rho = \langle \rho_2^0 \rangle$ for d_3, u_1, and u_2 quarks, and $v_\chi = \langle \chi_3^0 \rangle$ for exotic quarks T, D_1, and D_2.

Notice that, for both situations, if the lepton parity $(-)^L$ is broken, i.e. the lepton number L is broken completely, then there is no longer a symmetry which protects η_3^0 ($L = -1$) and χ_1^0 ($L = 1$) from acquiring VEVs. This will induce mixing between the leptoquarks and the usual quarks, which may lead to the effects of flavor changing neutral currents. This kind of mixing in the 3-3-1 model has been studied in a number of papers [18], so we will not discuss it further. Anyway, the solution corresponding to the residual symmetry $(-)^L$ should be more natural.

In this model the first situation is quite natural because the A_4 triplet η, which may strongly couple to ϕ via some potential, will be aligned in the (1, 1, 1) VEV direction of ϕ, as assumed. Namely, we can check that those VEV structures for ϕ and η are an automatic solution from the potential minimization conditions; no misalignment solution appears. But, in the following we will consider the scalar and quark content of the second situation. The results obtained can be similarly derived for the first situation. The scalar content and general scalar potential in the case of interest are summarized in Appendix B. Note that, in Ref. [10], only the lepton sector has been considered, and the quark sector has not been mentioned.

IV. VACUUM ALIGNMENT

There are several scalar sectors where ϕ is responsible for charged lepton masses, σ and s are responsible for neutrino masses, and η, ρ, χ for quark masses, with the vacuum structures shown above. If the first two sectors such as ϕ and s are strongly coupled, i.e. the couplings of $V(s, \phi)$ in (B22) are turned on with enough strength, such vacuum alignments for ϕ and s would be broken. To resolve this problem, we might include extra dimensions as in [7] or supersymmetry as in [8]. However, in this work we will provide an alternative explanation, following [5,6].

At the low-energy limit, the antisextets σ and s are decomposed into the ones of standard model symmetry. Noting that $6^* = 3^* \oplus 2^* \oplus 1$ under $SU(2)_L$ we get

$$\sigma = \begin{pmatrix} \sigma_{11}^0 & \sigma_{12}^+ \\ \sigma_{12}^+ & \sigma_{22}^{++} \end{pmatrix} \oplus \begin{pmatrix} \sigma_{13}^0 \\ \sigma_{23}^+ \end{pmatrix} \oplus \sigma_{33}^0, \qquad s = \begin{pmatrix} s_{11}^0 & s_{12}^+ \\ s_{12}^+ & s_{22}^{++} \end{pmatrix} \oplus \begin{pmatrix} s_{13}^0 \\ s_{23}^+ \end{pmatrix} \oplus s_{33}^0, \tag{34}$$

where the antitriplets have the lepton number $L = -2$, antidoublets $L = -1$, and singlets $L = 0$. Our effective theory thus plays the same role as the previous well-known proposals of A_4, such as in Refs. [5,6]. The dynamics of the antitriplets and antidoublets can further be found in [14]. Similar to those cases, σ and s in the model may be very heavy and are all integrated away, so they do not appear as physical particles at or below the TeV scale. They have interactions among themselves similar to those of the potentials for ϕ as shown below. Only their imprint at low energy shows the VEV structures as given.

To see this, let us suppose that the antisextets σ and s are heavy, with masses μ_σ and μ_s, respectively, and consider the minimization conditions of a potential $V^{s\sigma}$ concerning these antisextets. To obtain the desirable solution $\langle \sigma \rangle \neq 0$, $\langle s_1 \rangle \neq 0$, and $\langle s_2 \rangle = \langle s_3 \rangle = 0$, the lepton number $\mathcal{L}$, as well as A_4, must be broken as given in (B31). The new observation is that the following choice of soft scalar terms of (B31) works in the $V^{s\sigma}$ potential:

$$V^{s\sigma} = V(s) + V(\sigma) + V(s, \sigma) + (\bar{\mu}_1 \eta^T \sigma \chi + \bar{\mu}_2 \eta^T \sigma \eta + \bar{\lambda}_1 \eta^\dagger s_1^\dagger \chi \rho + \bar{\lambda}_2 \eta^\dagger s_1^\dagger \eta \rho + \bar{\lambda}_3 \chi^\dagger s_1^\dagger \chi \rho + \text{H.c.}). \tag{35}$$

From $V^{s\sigma}$, one solution to the minimization conditions is $\langle s_2 \rangle = \langle s_3 \rangle = 0$, and

$$\langle s_1 \rangle = \begin{pmatrix} u_1' & 0 & u_1 \\ 0 & 0 & 0 \\ u_1 & 0 & \Lambda_1 \end{pmatrix}, \qquad \langle \sigma \rangle = \begin{pmatrix} u' & 0 & u \\ 0 & 0 & 0 \\ u & 0 & \Lambda \end{pmatrix}. \tag{36}$$

Here $\langle s_1 \rangle$ and $\langle \sigma \rangle$ are the root of the $\partial V_{\min}^{s\sigma} / \partial \langle s_1 \rangle^* = 0$ and $\partial V_{\min}^{s\sigma} / \partial \langle \sigma \rangle^* = 0$ (with $V_{\min}^{s\sigma}$ the minimum of $V^{s\sigma}$), whereas other similar conditions vanish due to $\langle s_2 \rangle = \langle s_3 \rangle = 0$. This is also an important result of our paper.

Since Λ, Λ_1 are much larger than u, u', u_1, u_1', from the minimization conditions $\partial V_{\min}^{s\sigma} / \partial \Lambda_1^* = 0$ and $\partial V_{\min}^{s\sigma} / \partial \Lambda^* = 0$ we derive

$$\begin{aligned} \Lambda_1^2 \simeq {} & [2(\lambda^\sigma + \lambda'^s)\mu_s^2 - (2\lambda_3'^{s\sigma} + 2\lambda_3^{s\sigma} + \lambda_1'^{s\sigma} + \lambda_1^{s\sigma} \\ & + \lambda_2^{s\sigma} + \lambda_2'^{s\sigma})\mu_\sigma^2]/[(2\lambda_3'^{s\sigma} + 2\lambda_3^{s\sigma} + \lambda_1'^{s\sigma} + \lambda_1^{s\sigma} \\ & + \lambda_2^{s\sigma} + \lambda_2'^{s\sigma})(\lambda_2'^{s\sigma} + \lambda_1'^{s\sigma} + \lambda_2^{s\sigma} + \lambda_1^{s\sigma} + 2\lambda_3'^{s\sigma} \\ & + 2\lambda_3^{s\sigma}) - 4(\lambda^\sigma + \lambda'^s) \times (\lambda_1'^s + \lambda_2'^s + \lambda_1^s + \lambda_2^s)], \end{aligned} \tag{37}$$

$$\begin{aligned} \Lambda^2 \simeq {} & [2(\lambda_1'^s + \lambda_2'^s + \lambda_1^s + \lambda_2^s)\mu_\sigma^2 - (\lambda_2'^{s\sigma} + \lambda_1'^{s\sigma} + \lambda_2^{s\sigma} \\ & + \lambda_1^{s\sigma} + 2\lambda_3'^{s\sigma} + 2\lambda_3^{s\sigma})\mu_s^2]/[(2\lambda_3'^{s\sigma} + 2\lambda_3^{s\sigma} + \lambda_1'^{s\sigma} \\ & + \lambda_1^{s\sigma} + \lambda_2^{s\sigma} + \lambda_2'^{s\sigma})(\lambda_2'^{s\sigma} + \lambda_1'^{s\sigma} + \lambda_2^{s\sigma} + \lambda_1^{s\sigma} \\ & + 2\lambda_3'^{s\sigma} + 2\lambda_3^{s\sigma}) - 4(\lambda^\sigma + \lambda'^s)(\lambda_1'^s + \lambda_2'^s \\ & + \lambda_1^s + \lambda_2^s)], \end{aligned} \tag{38}$$

i.e. Λ and Λ_1 are on the scale of the antisextet masses μ_σ, μ_s. However, u, u_1, u', and u_1' get very small values [6,14] derived from the remaining minimization conditions as given by

$$u' \simeq \frac{[\mu_s^2 + 2(\lambda_1^{\prime s} + \lambda_2^{\prime s})\Lambda_1^2 + \lambda_1^{\prime s\sigma}\Lambda^2]v_\eta^2\bar{\mu}_2 + (\lambda_2^{\prime s\sigma} + 2\lambda_3^{\prime s\sigma})\Lambda\Lambda_1 v_\eta v_\rho v_\chi \bar{\lambda}_1}{(\mu_\sigma^2 + 2\lambda^{\prime\sigma}\Lambda^2 + \lambda_1^{\prime s\sigma}\Lambda_1^2)[\mu_s^2 + 2(\lambda_1^{\prime s} + \lambda_2^{\prime s})\Lambda_1^2 + \lambda_1^{\prime s\sigma}\Lambda^2] - (\lambda_2^{\prime s\sigma} + 2\lambda_3^{\prime s\sigma})^2\Lambda^2\Lambda_1^2}, \tag{39}$$

$$u_1' \simeq \frac{-(\lambda_2^{\prime s\sigma} + 2\lambda_3^{\prime s\sigma})\Lambda\Lambda_1 v_\eta^2\bar{\mu}_2 - (\mu_\sigma^2 + 2\lambda^{\prime\sigma}\Lambda^2 + \lambda_1^{\prime s\sigma}\Lambda_1^2)v_\eta v_\rho v_\chi \bar{\lambda}_1}{(\mu_\sigma^2 + 2\lambda^{\prime\sigma}\Lambda^2 + \lambda_1^{\prime s\sigma}\Lambda_1^2)[\mu_s^2 + 2(\lambda_1^{\prime s} + \lambda_2^{\prime s})\Lambda_1^2 + \lambda_1^{\prime s\sigma}\Lambda^2] - (\lambda_2^{\prime s\sigma} + 2\lambda_3^{\prime s\sigma})^2\Lambda^2\Lambda_1^2}, \tag{40}$$

$$\begin{aligned} u \simeq & \{[2\mu_s^2 + 4(\lambda_1^s + \lambda_1^{\prime s} + \lambda_2^s + \lambda_2^{\prime s})\Lambda_1^2 + (\lambda_1^{s\sigma} + 2\lambda_1^{\prime s\sigma} + \lambda_2^{s\sigma})\Lambda^2]v_\eta v_\chi \bar{\mu}_1 - (\lambda_1^{s\sigma} + \lambda_2^{s\sigma} + 2\lambda_2^{\prime s\sigma} + 4\lambda_3^{s\sigma} + 4\lambda_3^{\prime s\sigma}) \\ & \times \Lambda\Lambda_1 v_\rho(v_\eta^2\bar{\lambda}_2 - v_\chi^2\bar{\lambda}_3)\}/\{[2\mu_\sigma^2 + 4(\lambda^\sigma + \lambda^{\prime\sigma})\Lambda^2 + (\lambda_1^{s\sigma} + 2\lambda_1^{\prime s\sigma} + \lambda_2^{s\sigma})\Lambda_1^2][2\mu_s^2 + 4(\lambda_1^s + \lambda_1^{\prime s} + \lambda_2^s + \lambda_2^{\prime s})\Lambda_1^2 \\ & + (\lambda_1^{s\sigma} + 2\lambda_1^{\prime s\sigma} + \lambda_2^{s\sigma})\Lambda^2] - (\lambda_1^{s\sigma} + \lambda_2^{s\sigma} + 2\lambda_2^{\prime s\sigma} + 4\lambda_3^{s\sigma} + 4\lambda_3^{\prime s\sigma})^2\Lambda^2\Lambda_1^2\}, \end{aligned} \tag{41}$$

$$\begin{aligned} u_1 \simeq & \{[2\mu_\sigma^2 + 4(\lambda^\sigma + \lambda^{\prime\sigma})\Lambda^2 + (\lambda_1^{s\sigma} + 2\lambda_1^{\prime s\sigma} + \lambda_2^{s\sigma})\Lambda_1^2]v_\rho(v_\eta^2\bar{\lambda}_2 - v_\chi^2\bar{\lambda}_3) - (\lambda_1^{s\sigma} + \lambda_2^{s\sigma} + 2\lambda_2^{\prime s\sigma} + 4\lambda_3^{s\sigma} + 4\lambda_3^{\prime s\sigma}) \\ & \times \Lambda\Lambda_1 v_\eta v_\chi \bar{\mu}_1\}/\{[2\mu_\sigma^2 + 4(\lambda^\sigma + \lambda^{\prime\sigma})\Lambda^2 + (\lambda_1^{s\sigma} + 2\lambda_1^{\prime s\sigma} + \lambda_2^{s\sigma})\Lambda_1^2][2\mu_s^2 + 4(\lambda_1^s + \lambda_1^{\prime s} + \lambda_2^s + \lambda_2^{\prime s})\Lambda_1^2 \\ & + (\lambda_1^{s\sigma} + 2\lambda_1^{\prime s\sigma} + \lambda_2^{s\sigma})\Lambda^2] - (\lambda_1^{s\sigma} + \lambda_2^{s\sigma} + 2\lambda_2^{\prime s\sigma} + 4\lambda_3^{s\sigma} + 4\lambda_3^{\prime s\sigma})^2\Lambda^2\Lambda_1^2\}. \end{aligned} \tag{42}$$

Let us put Λ, Λ_1, μ_σ, $\mu_s \sim M$. Suppose that all the terms existing in the same numerator are the same order, i.e. $v_\eta \sim v_\rho(\sim v)$, $v_\chi\bar{\lambda}_1 \sim \bar{\mu}_2$, and $v_\chi\bar{\lambda}_3 \sim \bar{\mu}_1$. We derive

$$u' \sim u_1' \sim \frac{\bar{\mu}_2 v^2}{M^2}, \qquad u \sim u_1 \sim \frac{\bar{\mu}_1 v v_\chi}{M^2}. \tag{43}$$

(See also the remarks in Sec. II for completion.)

The potential concerning ϕ, after integrating out over the heavy fields as mentioned, can be identified as $V^\phi = V(\phi) + V(\phi, \rho) + V(\phi, \eta) + V(\phi, \chi)$. The minimum of the potential is given by

$$\begin{aligned} V_{\min}^\phi = & (m^2 - 2\lambda_3^{\phi\rho}v_\rho^2)(|v_1|^2 + |v_2|^2 + |v_3|^2) + \lambda_1^\phi(|v_1|^2 + |v_2|^2 + |v_3|^2)^2 + \lambda_2^\phi(|v_1|^2 + \omega^2|v_2|^2 + \omega|v_3|^2) \\ & \times (|v_1|^2 + \omega|v_2|^2 + \omega^2|v_3|^2) + \lambda_3^\phi(|v_2|^2|v_3|^2 + |v_3|^2|v_1|^2 + |v_1|^2|v_2|^2) + \{\lambda_4^\phi(v_2^*v_3)^2 + (v_3^*v_1)^2 \\ & + (v_1^*v_2)^2 + \lambda_3^{\phi\rho}v_\rho^2[(v_1^*)^2 + (v_2^*)^2 + (v_3^*)^2] + (\lambda_4^{\phi\rho} + \lambda_5^{\phi\rho})v_\rho^*[v_1^*v_2v_3 + v_1v_2^*v_3 + v_1v_2v_3^*] + \text{c.c.}\}. \end{aligned} \tag{44}$$

Here we have defined $m^2 = \mu_\phi^2 + \lambda_1^{\phi\eta}|v_\eta|^2 + \lambda_1^{\phi\chi}|v_\chi|^2 + (\lambda_1^{\phi\rho} + \lambda_2^{\phi\rho})|v_\rho|^2 + 2\lambda_3^{\phi\rho}v_\rho^2$, with $v_\eta = \langle\eta\rangle$, $v_\rho = \langle\rho\rangle$, and $v_\chi = \langle\chi\rangle$. The minimization conditions on v_i are given by

$$\begin{aligned} \frac{\partial V_{\min}^\phi}{\partial v_1^*} = & (m^2 - 2\lambda_3^{\phi\rho}v_\rho^2)v_1 + 2\lambda_1^\phi v_1(|v_1|^2 + |v_2|^2 + |v_3|^2) + \lambda_2^\phi v_1(2|v_1|^2 - |v_2|^2 - |v_3|^2) + \lambda_3^\phi v_1(|v_2|^2 + |v_3|^2) \\ & + 2\lambda_4^\phi v_1^*(v_2^2 + v_3^2) + 2\lambda_3^{\phi\rho}v_\rho^2 v_1^* + (\lambda_4^{\phi\rho} + \lambda_5^{\phi\rho})[v_\rho^*v_2v_3 + v_\rho(v_2^*v_3 + v_2v_3^*)], \end{aligned} \tag{45}$$

and other similar equations. One solution to these equations is

$$v_1 = v_2 = v_3 = \frac{-3v_\rho(\lambda_4^{\phi\rho} + \lambda_5^{\phi\rho}) + \sqrt{9|v_\rho|^2(\lambda_4^{\phi\rho} + \lambda_5^{\phi\rho})^2 - 8m^2(3\lambda_1^\phi + \lambda_3^\phi + 2\lambda_4^\phi)}}{4(3\lambda_1^\phi + \lambda_3^\phi + 2\lambda_4^\phi)}. \tag{46}$$

Let us note that such vacuum alignment does not change when the terms ϕ in (B31), except for those coupled to s, are included.

V. CONCLUSIONS

We have constructed the $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ gauge model based on A_4 flavor symmetry. This 3-3-1 model is different from previous proposals [10–12] because it includes the new neutral fermion singlets with zero lepton number following [5] into the third components of the $SU(3)_L$ lepton triplets, as well as the scalar antisextets as required to generate the masses for the neutrinos.

The charged leptons gain masses from the Yukawa interactions of the $SU(3)_L$ triplet ϕ. The neutrinos and neutral fermion singlets gain masses from contributions of the antisextets σ and s. The three active neutrinos have naturally small masses as a result of the interplay of type I and II seesaw mechanisms. The quark masses exist in one of the two cases. The first case is induced by contributions from ϕ, where the CKM matrix may be unity at the first approximation. In contrast, the second case is due to a discriminative scalar sector of the η, ρ, χ triplets. The resulting masses and mixing matrix of quarks are the same as in the ordinary 3-3-1 model.

The separation of the two A_4 triplets ϕ and s, which generate masses for charged leptons and neutrinos, respectively, is evaluated. We have shown that if the antitriplets σ and s are heavy, small lepton-number violating vacuum expectation values may be induced via the lepton-number violating scalar potentials as well as the scalar soft terms of A_4. The vacuum alignment for these antisextets exists as a result. The scalar potential concerning ϕ at or below the TeV scale is obtained by integrating out from the very heavy antisextets, which naturally yields the vacuum structures as expected. Remember that in this case the type I seesaw scale is very large, corresponding to those of the antisextets. To achieve a TeV seesaw scale, other mechanisms, such as ones [7,8] for separating ϕ and s, should be used.

Finally, since in our model one family of quarks is different from the other two, other flavor symmetry groups which contain $\underline{2}$ representations such as S_4 may be preferred. This subject is dedicated to future studies.

ACKNOWLEDGMENTS

This work was supported in part by the National Foundation for Science and Technology Development (NAFOSTED) of Vietnam under Grant No. 103.01.15.09.

APPENDIX A: A_4 SYMMETRY

For three families of fermions, we should look for a group with an irreducible $\underline{3}$ representation which acts on the family indices, the simplest of which is A_4, the group of even permutation of four objects. It is also the symmetry group of a regular tetrahedron.

The group has 12 elements and four equivalence classes with three inequivalent one-dimensional representations and one three-dimensional one. Its character table is given in Table I. The multiplication rule for $\underline{3}$ representations is

$$\begin{aligned}\underline{3}\otimes\underline{3} &= \underline{1}(11+22+33)\oplus\underline{1}'(11+\omega^2 22+\omega 33)\\ &\oplus\underline{1}''(11+\omega 22+\omega^2 33)\oplus\underline{3}(23,31,12)\\ &\oplus\underline{3}(32,13,21).\end{aligned}\tag{A1}$$

TABLE I. Character table of A_4, where $\omega = e^{2\pi i/3}$ is the cube root of unity.

Class	n	χ_1	$\chi_{1'}$	$\chi_{1''}$	χ_3
C_1	1	1	1	1	3
C_2	4	1	ω	ω^2	0
C_3	4	1	ω^2	ω	0
C_4	3	1	1	1	-1

Further, we can denote, on the right-hand side, the first $\underline{3}$ as $\underline{3}_s$ and the second $\underline{3}$ as $\underline{3}_a$.

APPENDIX B: SCALAR SECTOR

1. Scalar content

Let us summarize the Higgs content of the model:

$$\phi = \begin{pmatrix}\phi_1^+\\ \phi_2^0\\ \phi_3^+\end{pmatrix} \sim (3, 2/3, \underline{3}, -1/3),\tag{B1}$$

$$\eta = \begin{pmatrix}\eta_1^0\\ \eta_2^-\\ \eta_3^0\end{pmatrix} \sim (3, -1/3, \underline{1}, -1/3),\tag{B2}$$

$$\rho = \begin{pmatrix}\rho_1^+\\ \rho_2^0\\ \rho_3^+\end{pmatrix} \sim (3, 2/3, \underline{1}, -1/3),\tag{B3}$$

$$\chi = \begin{pmatrix}\chi_1^0\\ \chi_2^-\\ \chi_3^0\end{pmatrix} \sim (3, -1/3, \underline{1}, 2/3),\tag{B4}$$

$$\sigma = \begin{pmatrix}\sigma_{11}^0 & \sigma_{12}^+ & \sigma_{13}^0\\ \sigma_{12}^+ & \sigma_{22}^{++} & \sigma_{23}^+\\ \sigma_{13}^0 & \sigma_{23}^+ & \sigma_{33}^0\end{pmatrix} \sim (6^*, 2/3, \underline{1}, -4/3),\tag{B5}$$

$$s = \begin{pmatrix}s_{11}^0 & s_{12}^+ & s_{13}^0\\ s_{12}^+ & s_{22}^{++} & s_{23}^+\\ s_{13}^0 & s_{23}^+ & s_{33}^0\end{pmatrix} \sim (6^*, 2/3, \underline{3}, -4/3),\tag{B6}$$

where the parentheses denote the quantum numbers based on $(SU(3)_L, U(1)_X, A_4, U(1)_{\mathcal{L}})$ symmetries, respectively. The subscripts to the component fields are indices of $SU(3)_L$. The $\underline{3}$ indices of A_4 for ϕ and s are discarded and understood. For convenience, we also list the lepton number (L) for the component particles:

Scalars	L
ϕ_1^+, ϕ_2^0, η_1^0, η_2^-, ρ_1^+, ρ_2^0, χ_3^0, σ_{33}^0, s_{33}^0	0
ϕ_3^+, η_3^0, ρ_3^+, χ_1^{0*}, χ_2^+, σ_{13}^0, σ_{23}^+, s_{13}^0, s_{23}^+	-1
σ_{11}^0, σ_{12}^+, σ_{22}^{++}, s_{11}^0, s_{12}^+, s_{22}^{++}	-2

2. Scalar potential

We can separate the general scalar potential into

$$V_{\text{scalar}} = V_1 + V_2 + \bar{V}_3,\tag{B7}$$

in which the first and second terms conserve the $\mathcal{L}$ charge whereas the third term violates this charge. Moreover, V_1 consists of all terms of ϕ, η, ρ, χ, without σ and s; V_2 is all the terms having at least a σ or s. V_1 is a sum of

$$\begin{aligned} V(\phi) = \mu_\phi^2(\phi^\dagger\phi)_{\underline{1}} + \lambda_1^\phi(\phi^\dagger\phi)_{\underline{1}}(\phi^\dagger\phi)_{\underline{1}} \\ + \lambda_2^\phi(\phi^\dagger\phi)_{\underline{1}'}(\phi^\dagger\phi)_{\underline{1}''} \\ + \lambda_3^\phi(\phi^\dagger\phi)_{\underline{3}_s}(\phi^\dagger\phi)_{\underline{3}_a} \\ + [\lambda_4^\phi(\phi^\dagger\phi)_{\underline{3}_s}(\phi^\dagger\phi)_{\underline{3}_s} + \text{H.c.}], \end{aligned} \tag{B8}$$

$$V(\eta) = \mu_\eta^2\eta^\dagger\eta + \lambda^\eta(\eta^\dagger\eta)^2, \tag{B9}$$

$$V(\rho) = \mu_\rho^2\rho^\dagger\rho + \lambda^\rho(\rho^\dagger\rho)^2, \tag{B10}$$

$$V(\chi) = \mu_\chi^2\chi^\dagger\chi + \lambda^\chi(\chi^\dagger\chi)^2, \tag{B11}$$

$$V(\phi,\eta) = \lambda_1^{\phi\eta}(\phi^\dagger\phi)_{\underline{1}}(\eta^\dagger\eta) + \lambda_2^{\phi\eta}(\phi^\dagger\eta)(\eta^\dagger\phi), \tag{B12}$$

$$\begin{aligned} V(\phi,\rho) = \lambda_1^{\phi\rho}(\phi^\dagger\phi)_{\underline{1}}(\rho^\dagger\rho) + \lambda_2^{\phi\rho}(\phi^\dagger\rho)(\rho^\dagger\phi) \\ + [\lambda_3^{\phi\rho}(\phi^\dagger\rho)(\phi^\dagger\rho) + \lambda_4^{\phi\rho}(\rho^\dagger\phi)(\phi^\dagger\phi)_{\underline{3}_s} \\ + \lambda_5^{\phi\rho}(\rho^\dagger\phi)(\phi^\dagger\phi)_{\underline{3}_a} + \text{H.c.}], \end{aligned} \tag{B13}$$

$$V(\phi,\chi) = \lambda_1^{\phi\chi}(\phi^\dagger\phi)_{\underline{1}}(\chi^\dagger\chi) + \lambda_2^{\phi\chi}(\phi^\dagger\chi)(\chi^\dagger\phi), \tag{B14}$$

$$V(\eta,\rho) = \lambda_1^{\eta\rho}(\eta^\dagger\eta)(\rho^\dagger\rho) + \lambda_2^{\eta\rho}(\eta^\dagger\rho)(\rho^\dagger\eta), \tag{B15}$$

$$V(\eta,\chi) = \lambda_1^{\eta\chi}(\eta^\dagger\eta)(\chi^\dagger\chi) + \lambda_2^{\eta\chi}(\eta^\dagger\chi)(\chi^\dagger\eta), \tag{B16}$$

$$V(\rho,\chi) = \lambda_1^{\rho\chi}(\rho^\dagger\rho)(\chi^\dagger\chi) + \lambda_2^{\rho\chi}(\rho^\dagger\chi)(\chi^\dagger\rho), \tag{B17}$$

$$V(\eta,\rho,\chi) = \mu_1\eta\rho\chi + \text{H.c.} \tag{B18}$$

V_2 is a sum of

$$\begin{aligned} V(s) = \text{Tr}\{V(\phi\to s) + \lambda_1^{\prime s}(s^\dagger s)_{\underline{1}}\,\text{Tr}(s^\dagger s)_{\underline{1}} \\ + \lambda_2^{\prime s}(s^\dagger s)_{\underline{1}'}\,\text{Tr}(s^\dagger s)_{\underline{1}''} + \lambda_3^{\prime s}(s^\dagger s)_{\underline{3}_s}\,\text{Tr}(s^\dagger s)_{\underline{3}_a} \\ + [\lambda_4^{\prime s}(s^\dagger s)_{\underline{3}_s}\,\text{Tr}(s^\dagger s)_{\underline{3}_s} + \text{H.c.}]\}, \end{aligned} \tag{B19}$$

$$V(\sigma) = \text{Tr}[V(\eta\to\sigma) + \lambda'^\sigma(\sigma^\dagger\sigma)\,\text{Tr}(\sigma^\dagger\sigma)], \tag{B20}$$

$$\begin{aligned} V(s,\sigma) = \text{Tr}\{V(\phi\to s,\rho\to\sigma) + \lambda_1^{\prime s\sigma}(s^\dagger s)_{\underline{1}}\,\text{Tr}(\sigma^\dagger\sigma) \\ + \lambda_2^{\prime s\sigma}(s^\dagger\sigma)\,\text{Tr}(\sigma^\dagger s) + [\lambda_3^{\prime s\sigma}(s^\dagger\sigma) \\ \times\text{Tr}(s^\dagger\sigma) + \lambda_4^{\prime s\sigma}(\sigma^\dagger s)\,\text{Tr}(s^\dagger s)_{\underline{3}_s} \\ + \lambda_5^{\prime s\sigma}(\sigma^\dagger s)(s^\dagger s)_{\underline{3}_a} + \text{H.c.}]\}, \end{aligned} \tag{B21}$$

$$\begin{aligned} V(s,\phi) = \text{Tr}\{\lambda_1^{\phi s}(\phi^\dagger\phi)_{\underline{1}}(s^\dagger s)_{\underline{1}} \\ + [\lambda_2^{\phi s}(\phi^\dagger\phi)_{\underline{1}'}(s^\dagger s)_{\underline{1}''} + \lambda_3^{\phi s}(\phi^\dagger\phi)_{\underline{3}_s}(s^\dagger s)_{\underline{3}_a} \\ + \lambda_4^{\phi s}(\phi^\dagger\phi)_{\underline{3}_s}(s^\dagger s)_{\underline{3}_s} + \text{H.c.}] \\ + \lambda_5^{\phi s}(\phi^\dagger s^\dagger)_{\underline{1}}(s\phi)_{\underline{1}} + \lambda_6^{\phi s}(\phi^\dagger s^\dagger)_{\underline{1}'}(s\phi)_{\underline{1}''} \\ + \lambda_7^{\phi s}(\phi^\dagger s^\dagger)_{\underline{3}_s}(s\phi)_{\underline{3}_a} + [\lambda_8^{\phi s}(\phi^\dagger s^\dagger)_{\underline{3}_s}(s\phi)_{\underline{3}_s} \\ + \text{H.c.}]\}, \end{aligned} \tag{B22}$$

$$V(s,\eta) = \text{Tr}[V(\phi\to s^\dagger,\eta\to\eta)], \tag{B23}$$

$$V(s,\rho) = \text{Tr}[V(\phi\to s^\dagger,\eta\to\rho)], \tag{B24}$$

$$V(s,\chi) = \text{Tr}[V(\phi\to s^\dagger,\eta\to\chi)], \tag{B25}$$

$$V(\sigma,\phi) = \text{Tr}[V(\phi\to\phi,\eta\to\sigma^\dagger)], \tag{B26}$$

$$V(\sigma,\eta) = \text{Tr}[V(\eta\to\eta,\rho\to\sigma^\dagger)], \tag{B27}$$

$$V(\sigma,\rho) = \text{Tr}[V(\eta\to\rho,\rho\to\sigma^\dagger)], \tag{B28}$$

$$V(\sigma,\chi) = \text{Tr}[V(\eta\to\chi,\rho\to\sigma^\dagger)] + [\mu_2\chi^T\sigma\chi + \text{H.c.}], \tag{B29}$$

$$V(s,\phi,\eta,\chi) = \lambda_1\chi^\dagger s^\dagger\eta\phi + \text{H.c.} \tag{B30}$$

Notice that $(\text{Tr}A)(\text{Tr}B) = \text{Tr}(A\,\text{Tr}B)$ and $V(X\to X_1, Y\to Y_1) \equiv V(X,Y)|_{X=X_1,Y=Y_1}$.

The third term $\bar{V}_3$ is given by

$$\begin{aligned} \bar{V}_3 = \bar{\mu}_1\eta^T\sigma\chi + \bar{\mu}_2\eta^T\sigma\eta + \bar{\lambda}_1\eta^\dagger s^\dagger\chi\phi + \bar{\lambda}_2\eta^\dagger s^\dagger\eta\phi \\ + \bar{\lambda}_3\chi^\dagger s^\dagger\chi\phi + \bar{\lambda}_4\eta^\dagger s^\dagger s\chi + \bar{\lambda}_5\eta^\dagger\sigma^\dagger\sigma\chi \\ + [\bar{\lambda}_6\,\text{Tr}(\sigma^\dagger\sigma) + \bar{\lambda}_7\,\text{Tr}(s^\dagger s) + \bar{\lambda}_8\eta^\dagger\chi + \bar{\lambda}_9\eta^\dagger\eta \\ + \bar{\lambda}_{10}\rho^\dagger\rho + \bar{\lambda}_{11}\chi^\dagger\chi + \bar{\lambda}_{12}\phi^\dagger\phi + \bar{\mu}_3^2]\eta^\dagger\chi \\ + \bar{\lambda}_{13}(\eta^\dagger\rho)(\rho^\dagger\chi) + \bar{\lambda}_{14}(\eta^\dagger\phi)(\phi^\dagger\chi) + \text{H.c.} \end{aligned} \tag{B31}$$

There may exist soft terms in $\bar{V}$ explicitly violating the A_4 symmetry. But, only some of them are mentioned in the text.

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