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Calculable inverse-seesaw neutrino masses in supersymmetry

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We provide a scenario where naturally small and calculable neutrino masses arise from a supersymmetry-breaking renormalization-group-induced vacuum expectation value. The construction consists of an extended version of the next-to-minimal supersymmetric standard model and the mechanism is illustrated for a universal choice of the soft supersymmetry-breaking parameters. The lightest supersymmetric particle can be an isosinglet scalar neutrino state, potentially viable as WIMP dark matter through its Higgs new boson coupling. The scenario leads to a plethora of new phenomenological implications at accelerators including the Large Hadron Collider.

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Theory has no clue as to what causes the smallness of neutrino masses. It has become popular to ascribe it to the existence of a very high scale within the so-called minimal type-I seesaw [1]. Although this approach would fit naturally in unified schemes, no one to date has produced a convincing unified theory of flavor, where the observed pattern of quark and lepton masses and mixings is explained, especially the disparity between the small quark mixing angles and mixing angles [2] indicated by neutrino oscillation experiments. Moreover, if type-I seesaw is nature's way to understanding neutrinos one should give up hopes of ever obtaining its direct confirmation by accelerator experiments, such as the upcoming Large Hadron Collider (LHC).

Here we adopt as an alternative approach an $SU(3) \times SU(2) \times U(1)$ inverse-seesaw mechanism [3,4], which avoids introducing new states above the TeV scale. Neutrino masses arise well below the weak scale, thanks to a very small singlet mass term in whose presence lepton number is violated. Naturalness follows in t'Hooft's sense [5], namely, one is allowed to assume the smallness of parameters in whose absence the symmetry of the theory increases. Even though this is a perfectly valid and consistent procedure, it has not become as popular as the high-scale seesaw due to some discomfort in assuming by hand the smallness of an $SU(3) \times SU(2) \times U(1)$ invariant mass term. Rather than arguing that such theoretical prejudice is unjustified, here we provide a plausible mechanism where the origin of such small scale would, in addition, find a natural dynamical explanation.

Our basic assumption is supersymmetry, the leading framework to account for the stability of the weak interaction scale [6]. Here we show how the breaking of supersymmetry can spontaneously induce the radiative generation neutrino masses at very low scales. The mechanism requires the existence of a singlet sector, perhaps of stringy origin [7]. Such sector would be secluded from the standard model sector and hence hardly evolve under the renormalization group. "Calculable" neutrino masses then arise via the inverse-seesaw mechanism with dynamically generated mass parameters, in a scenario which can be considered an extended version of the next-to-minimal supersymmetric standard model (NMSSM).

In order to generate naturally small neutrino masses in our scheme we need to assume the vanishing of some of the new soft-trilinear parameters. For simplicity, we will illustrate this mechanism imposing universal conditions for the soft parameters, in a way analogous to the constrained next-to-minimal supersymmetric standard model (CNMSSM) [8]. However, as we will later clarify, our dynamical neutrino mass generation scenario need not rely on this universality. These initial conditions lead to a consistent phenomenological picture with an adequate electrically neutral dark matter candidate which is either a scalar neutrino or a spin $1/2$ neutralino, with suppressed couplings. Of these, here we focus on the first possibility. It has been shown that in this case the lightest superparticle (LSP) is likely to be a scalar neutrino whose relic abundance covers the range indicated by WMAP [9], and whose detection cross sections in nuclear recoil experiments can also be sizeable [10]. Moreover, the required magnitude of

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TABLE I. Multiplet content of the model.

	$\hat{Q}$	$\hat{U}^c$	$\hat{D}^c$	$\hat{L}$	$\hat{E}^c$	$\hat{\nu}^c$	$\hat{S}$	$\hat{H}^u$	$\hat{H}^d$	$\hat{\Phi}$	$\hat{\Delta}$	$\hat{\tilde{\Delta}}$
$SU(2)$	2	1	1	2	1	1	1	2	2	1	1	1
L	0	0	0	1	-1	-1	1	0	0	0	-2	1
R	1	1	1	1	1	1	1	0	0	0	0	0
Z_3	ω	ω	ω	ω	ω	ω	ω	ω	ω	ω	ω	ω

the supersymmetric Higgs mass parameter arises from the expectation value of the extra singlet field present in the NMSSM [11], avoiding the so-called μ problem [12,13], as recently advocated in Ref. [14].

In order to illustrate the idea we consider the model defined by the supermultiplets given in Table I, where L denotes the global continuous lepton number and R denotes the R charge. We preserve the Z_3 symmetry of the NMSSM which forbids bilinear couplings, since we still want to generate an electroweak (EW) scale μ term in the same way as in the NMSSM. The superpotential is given as

$$\begin{aligned} W = {} & y^e_{ij}\hat{L}_i\hat{H}^d\hat{E}^c_j + y^u_{ij}\hat{Q}_i\hat{H}^u\hat{U}^c_j + y^d_{ij}\hat{Q}_i\hat{H}^d\hat{D}^c_j \\ & + \lambda_1\hat{\Phi}\hat{H}^u\hat{H}^d + \frac{1}{3!}\lambda_2\hat{\Phi}\,\hat{\Phi}\,\hat{\Phi} + y^\nu_{ij}\hat{L}i\hat{H}^u\hat{\nu}^c_j \\ & + \eta_{ij}\hat{\Phi}\hat{\nu}^c_i\hat{S}_j + \frac{1}{2}\xi_{ij}\hat{\Delta}\hat{S}_i\hat{S}_j + \frac{1}{2}\tilde{\xi}\,\hat{\Delta}\,\hat{\tilde{\Delta}}\,\hat{\tilde{\Delta}}, \end{aligned} \tag{1}$$

where the first three terms are standard, the next two account for the NMSSM extension, and the last four characterize our supersymmetric inverse-seesaw model. In contrast with the simplest versions employed in Ref. [15] we have only trilinear terms, thanks to the Z_3 symmetry. This is also natural in string constructions, where bilinear couplings are absent from the superpotential. The corresponding soft supersymmetry-breaking potential reads,

$$\begin{aligned} V_{\rm soft} = {} & a^u_{ij}\tilde{Q}_iH^u\tilde{u}^c{}_j + a^d_{ij}\tilde{Q}_iH^d\tilde{d}^c{}_j + a^e_{ij}\tilde{L}_iH^d\tilde{e}^c{}_j \\ & + a_{\Phi H}\Phi H^uH^d + \frac{1}{3!}a_\Phi\Phi^3 + a^\nu_{ij}\tilde{L}_iH^u\tilde{\nu}^c{}_j \\ & + a^\eta_{ij}\Phi\tilde{\nu}^c{}_iS_j + \frac{1}{2}a^S_{ij}\Delta\tilde{S}_i\tilde{S}_j + \frac{1}{2}a_\Delta\Delta\tilde{\Delta}^2 + \text{H.c.} \\ & + \Sigma_i m_i^2|\varphi_i|^2 + \frac{1}{2}\Sigma_iM_i\lambda_i\lambda_i, \end{aligned} \tag{2}$$

where the last two terms are the standard scalar and gaugino soft supersymmetry-breaking terms.

The minimization of the scalar potential leaves five equations which must be fulfilled for a successful radiative electroweak symmetry breaking. Of these, three are those obtained in the NMSSM and can be used to fix the values of the vacuum expectation value (vev) of the singlet field $\langle\Phi\rangle$, the coupling λ_2 and the mass-squared parameter m^2_Φ, a prescription which is usually adopted in the CNMSSM. The remaining two equations relate the vevs of Δ and $\tilde{\Delta}$ to the soft parameters in the secluded sector. These are given as

$$\begin{aligned} & \tilde{\xi}^2v^3_{\tilde{\Delta}}/4 + \tilde{\xi}^2v_{\tilde{\Delta}}v^2_\Delta/2 + a_\Delta v_{\tilde{\Delta}}v_\Delta/\sqrt{2} + m^2_{\tilde{\Delta}}v_{\tilde{\Delta}} \\ & \quad = 0\tilde{\xi}^2v^2_{\tilde{\Delta}}v_\Delta/2 + a_\Delta v^2_{\tilde{\Delta}}/(2\sqrt{2}) + m^2_\Delta v_\Delta = 0. \end{aligned} \tag{3}$$

Because of their different charges with respect to the global lepton number symmetry, Δ and $\tilde{\Delta}$ behave differently. On the one hand, from the second equation v_Δ we have

$$v_\Delta = -\frac{1}{\sqrt{2}}a_\Delta\frac{v^2_{\tilde{\Delta}}}{\tilde{\xi}^2v^2_{\tilde{\Delta}} + 2m^2_\Delta}. \tag{4}$$

The light neutrino mass is controlled by v_Δ through the inverse-seesaw mechanism [3,4] which applied to the Lagrangian obtained by the superpotential of Eq. (1) gives

$$m^\nu = -y^\nu v^u(\eta v_\Phi)^{-1}(\xi v_\Delta)((\eta v_\Phi)^T)^{-1}(y^\nu v^u)^T. \tag{5}$$

From Eq. (4) we see that $v_\Delta \propto a_\Delta$, so it depends exclusively on the trilinear soft parameter of the singlet sector. On the other hand, the equation involving $v_{\tilde{\Delta}}$ gives in first approximation

$$v^2_{\tilde{\Delta}} = -4\frac{m^2_{\tilde{\Delta}}}{\tilde{\xi}^2} + O(a_\Delta). \tag{6}$$

This is controlled by the soft parameter $m^2_{\tilde{\Delta}}$ and does not vanish in the limit in which the trilinear soft term a_Δ goes to zero. Thus under the assumption that the trilinear soft terms of the singlet sector vanish at the unification scale, minimizing the tree-level Higgs potential one finds that $\tilde{\Delta}$ develops a vacuum expectation value $v_{\tilde{\Delta}}$ while Δ does not. Therefore, in this tree-level limit neutrinos are still massless.

One can derive the corresponding logarithm renormalization group evolution equations (RGEs) for all masses and couplings. These contain the RGEs of the NMSSM, supplemented with the evolution of the new parameters of the secluded sector. Imposing vanishing trilinear soft terms at the unification scale, the evolution of the corresponding trilinear soft parameters can be approximated by the following set of equations:

$$\begin{aligned} & \beta_{a_{\Phi H}} = 6\lambda_1(g_2^2M_2 + g_1^2M_1/5), \qquad \beta_{a_\Phi} = 12a_{\Phi H}\lambda_1^*\lambda_2, \\ & \beta_{a_\eta} = 4a_{\Phi H}\lambda_1^*\eta, \qquad \beta_{a_S} = 2a_\eta\eta^*\xi, \\ & \beta_{a_\Delta} = a_S\xi^*\tilde{\xi}, \end{aligned} \tag{7}$$

where we have defined $16\pi^2da_i/dt = \beta_{a_i}$. The first line above encodes the fact that gaugino soft supersymmetry-breaking terms are the seed for the low-scale dynamical neutrino mass generation mechanism proposed here, as illustrated in Fig. 1.

The breaking of supersymmetry due to the nonvanishing gaugino mass is sequentially transmitted to all the soft trilinear terms of the singlet sector, the smallest being a_Δ. Their approximated order of magnitude can be easily read from the RGEs given in Eq. (7) and are given by

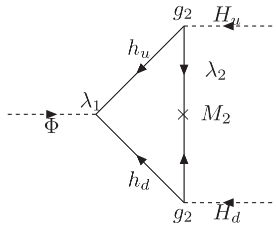

FIG. 1. Supersymmetry breaking as seed for low-scale dynamical neutrino mass generation.

$$
\begin{aligned}
a_{\Phi H} &\sim M_{1/2} O(g^2)\left(\frac{\log \mu / M_0}{16\pi^2}\right),\\
a_{\Phi}, a_{\eta} &\sim M_{1/2} O(g^2)\left(\frac{\log \mu / M_0}{16\pi^2}\right)^2,\\
a_S &\sim M_{1/2} O(g^2)\left(\frac{\log \mu / M_0}{16\pi^2}\right)^3,\\
a_{\Delta} &\sim M_{1/2} O(g^2)\left(\frac{\log \mu / M_0}{16\pi^2}\right)^4.
\end{aligned}
\tag{8}
$$

The presence of a small but nonvanishing a_Δ, which arises effectively at four-loops,[1] induces in turn a naturally small vev v_Δ according to Eq. (4). As a result v_Δ is naturally expected to be very small, of order MeV or even smaller. This shows how the small calculable parameter v_Δ acts as the seed of the light neutrino mass according to Eq. (5). The existence of the secluded sector therefore plays an important role in our proposal of a novel mechanism in terms of which to understand the origin of neutrino mass. As we have already explained, the main assumption is that the trilinear parameters associated to the singlet sector, $a_{\Phi H}$, a_Φ, a_η, and a_S vanish at the unification scale M_0, while the rest of the soft parameters are unconstrained. Vanishing trilinear parameters are possible in the supergravity scenarios obtained as the low-energy limit of string theory. More specifically in some phenomenologically appealing constructions, such as *D*-brane compactifications of the type-I string or orbifold scenarios in the Heterotic string, some trilinear couplings vanish for specific choices of the parameters which define supergravity breaking (see e.g. some of the examples in Refs. [16]). Notice that the fields in the new singlet sector in our model have different quantum numbers than the MSSM fields and must therefore correspond to different string modes. Thus the vanishing of the trilinear terms associated to these singlets does not necessarily imply that all the trilinears are null.

Let us now illustrate this mechanism with a specific example. Since the role of the MSSM trilinear parameters is not relevant for our discussion, for simplicity we will

[1] Because of the singlet nature of Δ and $\tilde{\Delta}$ and the specific couplings in Eq. (1) the RGE two-loop order contributions to a_Δ are absent. We did not explicitly check that all the three-loop order ones vanish, even if we expect so. However, even if the latter were induced, one could arrange the arbitrary tree-level parameters to obtain v_Δ of the correct order of magnitude.

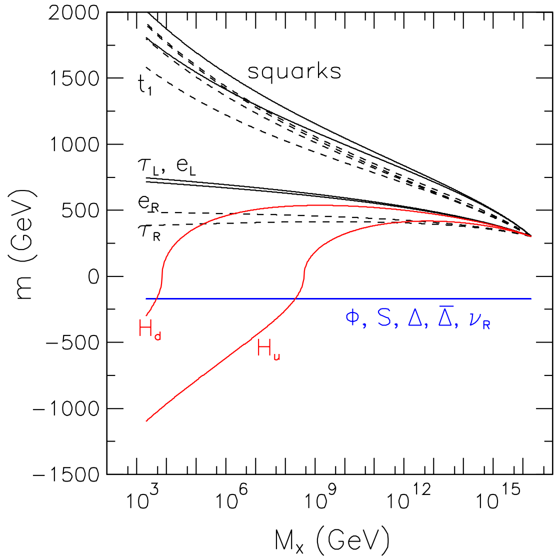

FIG. 2 (color online). Renormalization group evolution of the scalar masses (with $m \equiv \text{sign}(m_o)\sqrt{|m_0^2|}$) for a representative choice of parameters (see text).

impose universal conditions just as in the CNMSSM and assume that all trilinear soft breaking terms vanish at M_0. We also consider a universal gaugino mass parameter, $m_{1/2}$, and a positive scalar squared-mass parameter, m_0^2, for all gauge nonsinglet scalars. For the singlet scalars we assume them to be all equal to m_Φ^2 and use the minimization conditions to fix its value, which typically results in $m_\Phi^2 < 0$. This choice could be understood, e.g., if the singlet fields had a different origin from the NMSSM fields in a more fundamental theory. Negative squared-mass parameters for scalars were also considered in the MSSM [17].

In Fig. 2 we display the evolution of the masses of the scalar fields of the model for a concrete example in which we choose $m_{1/2} = 1$ TeV, $m_o = 300$ GeV, $\tan\beta = 40$, $\lambda_1 = 0.01$, $\xi = 0.1$ and $\eta = \tilde{\xi} = 0.0053$. The minimization conditions impose $\mu = 1140$ GeV, $m_\Phi^2 = -2.94 \times 10^4$ GeV^2 and $\lambda_2 = -0.0011$ at the EW scale.[2] We note that the evolution of the parameters concerning the states in the NMSSM are not substantially different from what is expected and that, due to the smallness of λ_2, the soft masses of the singlets practically do not deviate from their value at M_0.

After numerically solving the RGEs, the particle spectrum can be calculated at the EW scale. The inclusion of the secluded sector has no effect on the masses of most of the NMSSM particles. However, the new singlet S mixes with the right- and left-handed sneutrino states, giving rise to three sneutrino states.

[2] As noted in [18], the resulting value of λ_2 after solving the minimization conditions in the CNMSSM is typically 1 order of magnitude smaller than λ_1.

One can see that in most cases the lightest sneutrino is a combination of the two singlet states $\tilde{\nu}^c$ and $\tilde{s}$. Since all trilinear couplings vanish at the unification scale, the trilinears involving only gauge singlet fields run very slowly so v_Δ is very small compared to the other vevs. Thus for the above choice the sneutrino mass matrix can be approximated as

$$M^2_{\tilde{\nu}_{ri}} \sim \begin{pmatrix} m_L^2 & 0 & 0 \\ 0 & m_0^2 + \alpha_{ri} v^2 & \pm\delta v^2 \\ 0 & \pm\delta v^2 & m_0^2 + \beta_{ri} v^2 \end{pmatrix}.$$

with $0 < \alpha_{ri}, \beta_{ri} \sim \mathcal{O}(1)$ while $\delta \sim \mathcal{O}(0.1)$ and where we have used $m^2_{\nu^c}, m^2_S \tilde{m}^2_\Phi < 0$. Given that $m_L^2 > 0$, the lightest sneutrino is typically the CP-even or CP-odd combination of the singlet states.

As illustrated in Fig. 3, due to these mixing effects it is likely that the lightest supersymmetric particle is a mainly a mixture of the singlet scalars in ν^c and S, instead of the neutralino or stau. This is analogous to the construction of Ref. [10], although now the sneutrino has virtually no left-handed component. In this sense, this model is similar to the scenario of Refs. [14,19], since in both the right-handed sneutrino component couples directly to the NMSSM Higgs sector through the singlet Φ. As shown there, this makes it possible to fulfil the WMAP result, thereby making the sneutrino a viable WIMP. A similar effect is expected in the present model, this time through the η coupling in Eq. (1). Thus, the scheme proposed here opens yet new alternative ways to understand supersymmetric dark matter.

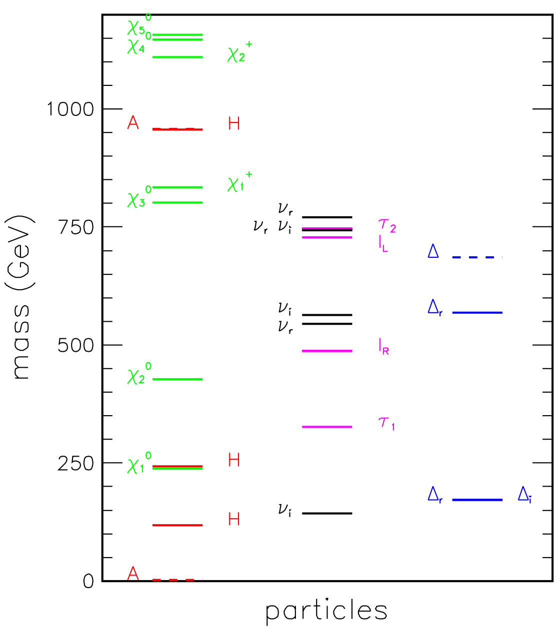

FIG. 3 (color online). Supersymmetric spectrum for the same choice of parameters as in Fig. 2. The real(imaginary) sneutrino state is labeled as $\nu_r(\nu_i)$, the scalar(pseudoscalar) of the secluded sector is indicated as $\Delta_r(\Delta_i)$ and the singlino as Δ. Gluino and squark masses are larger than 1.5 TeV and not shown.

In this example, the smallness of the λ_2 parameter implies the quasirestoration of a U(1) Peccei-Quinn symmetry in the superpotential. This entails the occurrence a very light CP-odd Higgs (in our example $m_A = 3$ GeV) as the pseudo-Goldstone boson of this broken symmetry [20]. Since the latter is almost a pure singlet it is consistent with existing phenomenological bounds.

Examples with a larger value of λ_1 generally lead to a larger λ_2 when the radiative electroweak symmetry-breaking (REWSB) conditions are solved. In these cases the lightest CP-odd Higgs is not as light as in the example above and, apart from of some new heavy states, the spectrum resembles that of the MSSM. However, the increase in λ_1 also entails a more negative value of m^2_Φ as a solution to the REWSB equations, and this in turn makes it more complicated to obtain nontachyonic sneutrinos. Only through an increase in η can this be achieved but this has as a consequence a significant increase in the masses of the particles in the secluded sector (Δ and $\Delta_{r,i}$). For completeness, in Fig. 4 we display the resulting spectrum for $m_{1/2} = 300$ GeV, $m_o = 1$ TeV, $\tan\beta = 2$, $\lambda_1 = 1$, $\xi = \tilde{\xi} = 0.07$ and $\eta = 2.8$. The minimization conditions now lead to $\mu = 996$ GeV, $m^2_\Phi = -1.17 \times 10^6$ GeV^2 and $\lambda_2 = -0.4$ at the EW scale. As in the previous example, the imaginary sneutrino is now the LSP and this has a significant mixture of the scalars in ν^c and S. Notice that the particles in the secluded sector are heavier than 1.7 TeV.

Last, but not least, in addition to a new supersymmetric dark matter scenario, our model leads to different phenomenological implications for the LHC and other accelerator experiments. The most interesting of these follow directly or indirectly from the new gauge singlet fermions

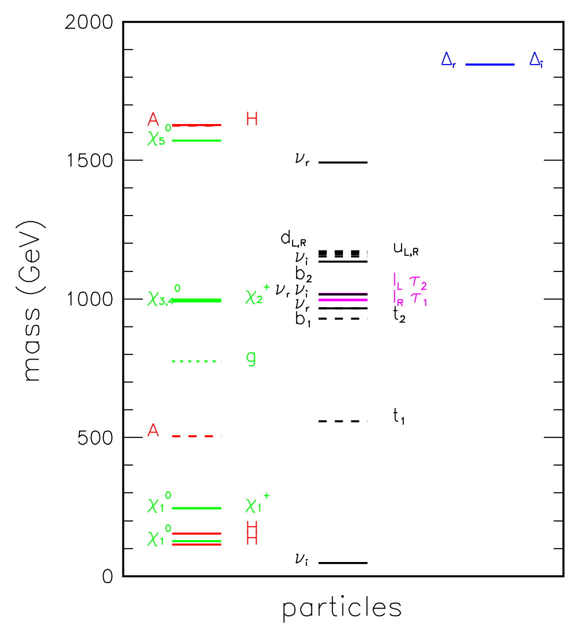

FIG. 4 (color online). Same as in Fig. 3, but for a different choice of input parameters, with larger λ_1 and λ_2, and a small $\tan\beta$, as described in the text.

at the TeV-scale. Apart from the possibility of direct production through mixing in the standard model weak currents [21], their exchange can induce lepton-flavor violating (LFV) [22] as well as leptonic CP violating effects [23], leading to processes such as $\mu^- \to e^- \gamma$, nuclear $\mu^- - e^-$ conversion [24] and LFV tau decays [25]. These processes can proceed even in the limit of decoupled supersymmetry, and even in the absence of neutrino masses. As a result their expected rates can be sizeable [15]. In addition supersymmetry brings in the possibility of observing lepton-flavor violation at high energies, in the decays of supersymmetric states, opening the possibility that LHC can directly probe the underlying physics [26].

Finally $v_{\hat{\Delta}}$ spontaneously breaks lepton number at the TeV scale, generating a pseudoscalar Goldstone boson, called Majoron [27]. Its couplings with ordinary matter are tiny, evading stellar energy loss constraints.

In summary, we have described a framework in which supersymmetry breaking can provide the dynamical origin for small neutrino masses through the inverse-seesaw mechanism. The seed for neutrino masses is a small renormalization-group-induced $SU(3) \times SU(2) \times U(1)$ singlet vacuum expectation value, while the μ problem is also dynamically solved as in the NMSSM. A mixed singlet sneutrino arises as a natural candidate for WIMP dark matter, in addition to a plethora of new phenomenological implications.

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