PPN parameter γ and solar system constraints of massive Brans-Dicke theories

L. Perivolaropoulos

Department of Physics, University of Ioannina, Greece

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Previous solar system constraints of the Brans-Dicke (BD) parameter ω have either ignored the effects of the scalar field potential (mass terms) or assumed a highly massive scalar field. Here, we interpolate between the above two assumptions and derive the solar system constraints on the BD parameter ω for *any* field mass. We show that for $\omega = O(1)$ the solar system constraints relax for a field mass $m \gtrsim 20 \times m_{\rm AU} = 20 \times 10^{-27}$ GeV.

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Scalar-tensor (ST) theories [1] constitute a fairly generic extension of general relativity (GR) where the gravitational constant is promoted to a field whose dynamics is determined by the following action [1,2]

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}(F(\Phi)R - Z(\Phi)g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - 2U(\Phi)) + S_m[\psi_m; g_{\mu\nu}]. \tag{1}$$

where G is the bare gravitational constant, R is the scalar curvature of the metric $g_{\mu\nu}$ and S_m is the action of matter fields. The variation of the dimensionless function $F(\Phi)$ describes the variation of the effective gravitational constant. This variation (spatial or temporal) is severely constrained by solar system experiments [3–5]. The GR limit of ST theories is obtained either by fixing $F(\Phi) = \Phi_0 \simeq 1$ (Φ_0 is a constant) or by freezing the dynamics of Φ using the function $Z(\Phi)$ or the potential $U(\Phi)$. For example a large and steep $Z(\Phi)$ makes it very costly energetically for Φ to develop a kinetic term while a steep $U(\Phi)$ (massive Φ) can make it very costly energetically for Φ to develop potential energy. In both cases we have an effective *freezing* of the dynamics which reduces the ST theory to GR.

ST theories have attracted significant attention recently as a potentially physical mechanism [2,6,7] for generating the observed accelerating expansion of the universe (see Ref. [8,9] and references therein). A significant advantage of this mechanism is that it can naturally generate an accelerating expansion rate corresponding to an effective equation of state parameter $w_{\rm eff}$ that crosses the phantom divide line $w = -1$ [6,10,11]. Such a crossing is consistent with cosmological observations and is difficult to obtain in the context of GR [12]. In addition ST theories naturally emerge in the context of string theories [13] and in Kaluza-Klein [14] theories with compact extra dimensions [15].

A special case of ST theories is the Brans-Dicke (BD) theory [16] where

$$F(\Phi) = \Phi, \tag{2}$$

$$Z(\Phi) = \frac{\omega}{\Phi}. \tag{3}$$

For a massive BD theory we also assume a potential of the form

$$U(\Phi) = \frac{1}{2}m^2(\Phi - \Phi_0)^2. \tag{4}$$

Clearly, the spatial dynamics of Φ can freeze for $\omega \gg 1$ or for $m \gg r^{-1}$ where r is the scale of the experiment or observation testing the dynamics of Φ. For solar system scale observations, the relevant scale is the Astronomical Unit (AU $\simeq 10^8$ km) corresponding to a mass scale $m_{\rm AU} \simeq 10^{-27}$ GeV. Even though this scale is small for particle physics considerations, it is still much larger than the Hubble mass scale $m_{H_0} \simeq 10^{-42}$ GeV required for nontrivial cosmological evolution of Φ [7,17].

Current solar system constraints [4,18] of the BD parameter ω have been obtained under one of the following assumptions:

(i) *Negligible mass of the field* Φ ($m \ll m_{\rm AU}$): In this case the relation between the observable Post-Newtonian parameter γ (measuring how much space curvature is produced by a unit rest mass) [18] and ω is of the form [4,19,20]

$$\gamma(\omega) = \frac{1+\omega}{2+\omega}. \tag{5}$$

This relation combined with the solar system constraints of the Cassini mission [5]

$$\gamma_{\rm obs} - 1 = (2.1 \pm 2.3) \times 10^{-5} \tag{6}$$

which constraint γ close to its GR value $\gamma = 1$, leads to the constraint on ω

$$\omega > 4 \times 10^4 \tag{7}$$

at the 2σ confidence level. Equation (5) however should not be used in the case of massive BD theories as was attempted recently in version 2 (v.2) of Ref. [21] (this error has been corrected in v.3 of Ref. [21]).

(ii) *Very massive scalar field* Φ ($m \gg m_{\rm AU}$): In this case the spatial dynamics of Φ is frozen on solar system scales by the potential term and all values of ω are observationally acceptable even though rapid oscillations of the field can lead to interesting non-trivial effects [22–24].

In this study we fill the gap between the above two assumptions and derive the form of the predicted *effective* parameter γ for all values of the field mass m. In particular, we derive the form of $\gamma(\omega, m, r)$ where r is the scale of the experiment-observation constraining γ. We then use the current solar system constraints (6) to obtain the (ω, m) parameter regions allowed by observations at the 1σ and 2σ confidence level.

The dynamical equations obtained for the field Φ and the metric $g_{\mu\nu}$ by variation of the action (1) in the massive BD case defined by Eqs. (3) and (4) are of the form [2]

$$\Phi\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right) = 8\pi G T_{\mu\nu} + \frac{\omega}{\Phi}\left(\partial_\mu\Phi\partial_\nu\Phi - \frac{1}{2}g_{\mu\nu}(\partial_\alpha\Phi)^2\right) + \nabla_\mu\partial_\nu F(\Phi) - g_{\mu\nu}\Box\Phi - g_{\mu\nu}\frac{1}{2}m^2(\Phi - \Phi_0)^2, \tag{8}$$

$$(2\omega + 3)\Box\Phi = 8\pi G T + 2m^2((\Phi - \Phi_0)^2 + (\Phi - \Phi_0)\Phi). \tag{9}$$

Considering the physical setup of the solar system involving a weak gravitational field we expand around a constant-uniform background field Φ_0 and a Minkowski metric $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$[1]

$$\Phi = \Phi_0 + \varphi \tag{10}$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{11}$$

The resulting equations for φ and $h_{\mu\nu}$ obtained from (8), (9), (11), and (10) in the gauge $h^\mu_{\nu},_\mu - \frac{1}{2}h^\mu_{\mu},_\nu = \frac{1}{\Phi_0}\varphi,_\nu$ are

$$\left(\Box - \frac{2m^2\Phi_0}{2\omega + 3}\right)\varphi = -8\pi G\frac{\rho - 3p}{2\omega + 3}, \tag{12}$$

$$-\frac{\Phi_0}{2}\left[\Box\left(h_{\mu\nu} - \eta_{\mu\nu}\frac{h}{2}\right)\right] = 8\pi G T_{\mu\nu} + \partial_\mu\partial_\nu\varphi - \eta_{\mu\nu}\Box\varphi, \tag{13}$$

where $T_{\mu\nu} = \mathrm{diag}(\rho, p, p, p)$ and $h = h^\mu_\mu$. Since we are interested in approximately static solutions corresponding to a gravitating mass such as the Sun or the Earth we ignore time derivatives and set $p \simeq 0$. Thus Eqs. (12) and (13) become

[1]Cosmological considerations would allow a slow evolution of $\Phi_0 = \Phi_0(t)$ on cosmological time scales but since these time scales are much larger than the solar system time scales we may ignore that evolution for our physical setup.

$$\nabla^2\varphi - \frac{2m^2\Phi_0}{2\omega + 3}\varphi = -8\pi G\frac{\rho}{2\omega + 3}, \tag{14}$$

$$\Phi_0\nabla^2 h_{00} - \nabla^2\varphi = -8\pi G\rho, \tag{15}$$

$$\Phi_0\nabla^2 h_{ij} - \delta_{ij}\nabla^2\varphi = -8\pi G\rho\delta_{ij}. \tag{16}$$

These equations are consistent with corresponding results of Ref. [22,23,25] even though our notation and assumptions are somewhat different. Setting $\rho = M_s\delta(r)$ we obtain the following solution

$$\varphi = \frac{2GM_s}{(2\omega + 3)r}e^{-\bar{m}(\omega)r}, \tag{17}$$

$$h_{00} = \frac{2GM_s}{\Phi_0 r}\left(1 + \frac{1}{2\omega + 3}e^{-\bar{m}(\omega)r}\right), \tag{18}$$

$$h_{ij} = \frac{2GM_s}{\Phi_0 r}\delta_{ij}\left(1 - \frac{1}{2\omega + 3}e^{-\bar{m}(\omega)r}\right), \tag{19}$$

where $\bar{m}(\omega) \equiv \sqrt{\frac{2\Phi_0}{2\omega+3}}m$ (Φ_0 is dimensionless). Using now the standard expansion of the metric in terms of the γ Post-Newtonian parameter

$$g_{00} = -1 + 2u, \tag{20}$$

$$g_{ij} = (1 + 2\gamma u)\delta_{ij}, \tag{21}$$

where u is the Newtonian potential we find (see also [25])

$$\gamma(\omega, m, r) = \frac{h_{ij}|_{i=j}}{h_{00}} = \frac{1 - \frac{e^{-\bar{m}(\omega)r}}{2\omega+3}}{1 + \frac{e^{-\bar{m}(\omega)r}}{2\omega+3}}. \tag{22}$$

In the special case of $m = 0$ we obtain the familiar result of Eq. (5).

The effective mass $\bar{m}(\omega)$ imposes a range $\bar{m}(\omega)^{-1}$ to the gravitational interaction in BD theories. In these theories, the Newtonian potential is

$$h_{00} = 2u = \frac{2G_{\mathrm{eff}}M_s}{r} \tag{23}$$

with

$$G_{\mathrm{eff}} = \frac{G}{\Phi_0}\left(1 + \frac{1}{2\omega + 3}e^{-\bar{m}(\omega)r}\right). \tag{24}$$

The dependence of the effective parameter γ on the scale should be interpreted as a dependence on the scale of the experiment-observation imposing a bound on γ. For example, for solar system constraints we have $r \simeq 1\mathrm{AU} \simeq 10^8$ km which corresponds to the mass scale $m_{\mathrm{AU}} \simeq 10^{-27}$ GeV. For $\bar{m}(\omega) \gtrsim m_{\mathrm{AU}}$, the value of γ predicted by BD theories for solar system scale observations (Eq. (22)) is significantly different from the standard expression (5). The other major Post-Newtonian parameter β (measuring how much 'nonlinearity' there is in the superposition law of gravity) is not discussed in this study but it

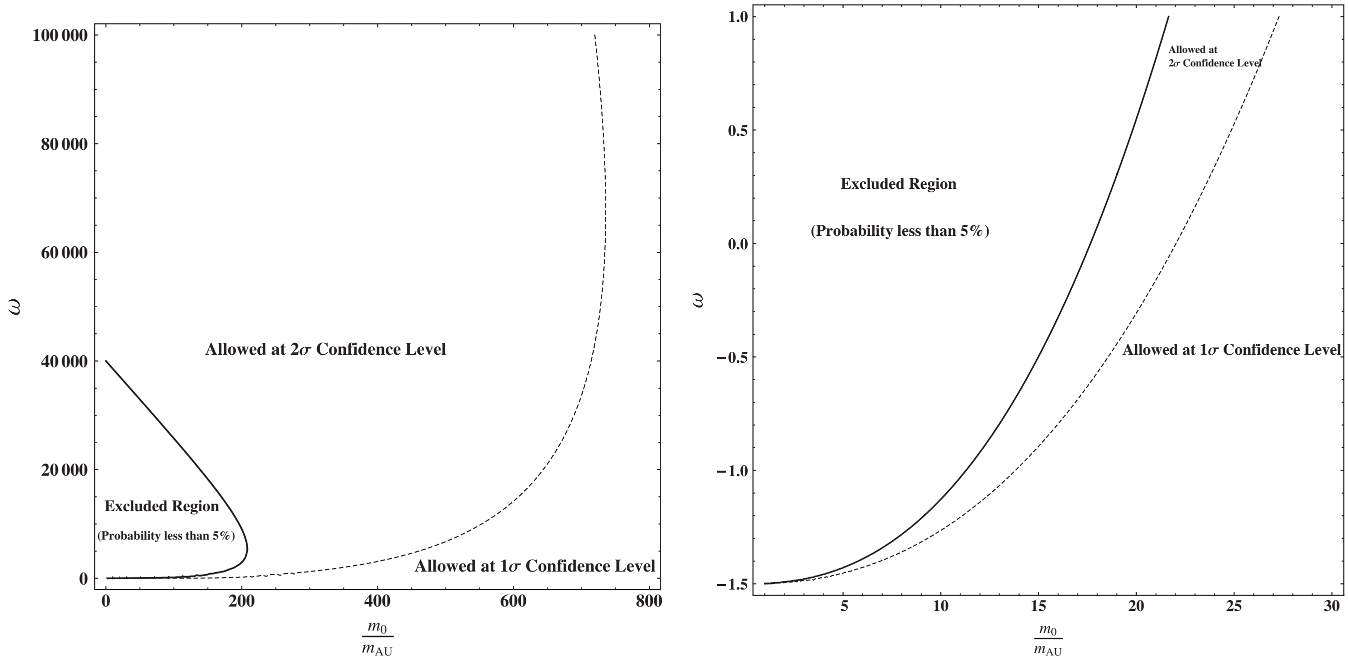

FIG. 1. (a) The observationally allowed regions for the parameters ω and $m_0 \equiv \sqrt{2\Phi_0}m \simeq m$ at 1σ 68% confidence level (above and right of dashed line) and 2σ 95% confidence level (above and right of thick line). Notice that for $\frac{m_0}{m_{AU}} \gtrsim 200$ all values of ω are observationally allowed at the 2σ level. (b) Same a Fig. 1(a) focused on a region close to the origin. Notice that for $\omega = O(1)$ solar system constraints relax for $\frac{m_0}{m_{AU}} \gtrsim 20$ at the 2σ level (thick line).

is anticipated to remain at its GR value $\beta = 1$ as in the case of massless BD theories [4] since the mass term can only improve the consistency with GR.

In order to constrain the allowed $\omega - m$ parameter region we use the recent observational estimates of Eq. (6) obtained by the Cassini spacecraft delay into the radio waves transmission near the solar conjuction [5]. Equation (6) implies lower bound constraints on the parameter γ i.e.

$$\gamma(\omega, m, m_{AU}^{-1}) > 1 - 0.2 \times 10^{-5} \tag{25}$$

$$\gamma(\omega, m, m_{AU}^{-1}) > 1 - 2.5 \times 10^{-5} \tag{26}$$

at the 1σ and 2σ levels, respectively.[2] Using Eqs. (22), (25), and (26) we may find the observationally allowed range of ω for each value of m (measured in units of $m_{AU} \simeq 10^{-27}$ GeV) at the 1σ and 2σ confidence levels. This allowed range at 2σ confidence level is shown in Fig. 1 (regions above and on the right of the thick line). The thick line of Fig. 1 is obtained by equating the expression of $\gamma(\omega, m, r_{AU}) = \gamma(\omega, m, m_{AU}^{-1})$ [Eq. (22)] with the 2σ limit of Eq. (26) and plotting the corresponding contour in the $(\omega, m/m_{AU})$ parameter space. The dashed line of Fig. 1 is obtained in a similar way using the 1σ limit of Eq. (25). Clearly, for $\omega = O(1)$ and $\frac{m}{m_{AU}} \gtrsim 20$ the solar system constraints relax and values of $\omega = O(1)$ are allowed by solar system observations at the 2σ level. For $m = 0$ we reobtain the familiar bound $\omega \gtrsim 40000$ at 2σ level while for $m \gtrsim 200 m_{AU}$ all values of ω are allowed. The plot of Fig. 1 can be used for any experiment-observation constraining the parameter γ on a scale r by proper reinterpretation of the units of the m axis.

In conclusion, we have used solar system constraints of the Post-Newtonian parameter γ to find the allowed (ω, m) parameter region of massive BD theories for all values of the scalar field mass m including the mass scale m_{AU} corresponding to the solar system distance scale. This result, fills a gap in the literature where only the cases $m \ll m_{AU}$ and $m \gg m_{AU}$ had been considered. We have found that for $m \simeq m_{AU} \simeq 10^{27}$ GeV, the observationally allowed range of ω at the 2σ level is practically identical to the corresponding range corresponding to the $m = 0$ range of Eq. (7). However, for $m \gtrsim 200 m_{AU}$ all values of $\omega > -3/2$ are observationally allowed.

An interesting extension of the present study would be the generalization of the well known expression of γ in scalar-tensor theories

$$\gamma(F, Z) - 1 = -\frac{(dF(\Phi)/d\Phi)^2}{Z(\Phi)F(\Phi) + 2(dF(\Phi)/d\Phi)^2} \tag{27}$$

which like Eq. (5) ignores the possible stabilizing effects of

[2] Eqs. (25) and (26) are obtained by subtracting the 1σ error ($\delta\gamma = 2.3 \times 10^{-5}$) and the 2σ error ($2\delta\gamma = 4.6 \times 10^{-5}$), respectively, from the mean value of $\bar{\gamma}_{obs} = 1 + 2.1 \times 10^{-5}$ of Eq. (6).

the scalar field potential. Such a generalization would lead to an expression of the effective scale dependent parameter γ in terms of $F(\Phi)$, $Z(\Phi)$ and $U(\Phi)$.

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