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# Aetherlike Lorentz-breaking actions

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We show that *CPT*-even aetherlike Lorentz-breaking actions, for the scalar and electromagnetic fields, are generated via their appropriate Lorentz-breaking coupling to spinor fields, in three, four, and five space-time dimensions. Besides, we also show that aetherlike terms for the spinor field can be generated as a consequence of the same couplings. We discuss the dispersion relations in the theories with aetherlike Lorentz-breaking terms and find the tree-level effective (Breit) potential for fermion scattering and the one-loop effective potential corresponding to the action of the scalar field.

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## I. INTRODUCTION

The possibility of Lorentz symmetry breaking [1] has attracted a great deal of attention during the recent years. One of the most important directions in this study is the investigation of possible Lorentz-violating extensions of field theory models. A general review of acceptable forms of such extensions was presented in [2]. The most known examples of Lorentz-breaking modifications of field theories are *CPT*-odd since the Lorentz symmetry breaking in these models is implemented through a constant vector field which is known to break the *CPT* symmetry [3]. The well-known examples of Lorentz and *CPT* breaking terms are the Carroll-Field-Jackiw term in electrodynamics [4], its non-Abelian generalization [5] and the gravitational Chern-Simons term [6]. Also, the Lorentz-*CPT* breaking terms which deserve to be mentioned are the Chern-Simons like term for the scalar fields (in two dimensions) [7] and the one-derivative Chern-Simons like term in the linearized gravity [8].

However, the *CPT*-odd terms do not exhaust the possibilities for Lorentz-breaking terms. Alternative types of Lorentz-breaking terms are *CPT*-even ones. Some of their forms were originally introduced in [9] within the context of the Lorentz-breaking extension of the standard model. Recently, the interest in such terms was increased due to the development of the aether concept [10] and to the study of extra dimensions [11]. Therefore, a natural problem is the study of the generation and physical impacts of the aetherlike terms, which can be treated as the simplest examples of *CPT*-even Lorentz-breaking terms [12]. Some aspects related to such terms, including the study of the dispersion relations in theories involving *CPT*-even Lorentz-breaking terms, the generation of such terms via the gauge embedding method, impacts of these terms at finite temperature and some experimental estimations for such terms, were considered in [13]. Also, in the papers [12], the modifications of dispersion relations of the scalar, spinor and electromagnetic fields due to *CPT*-even couplings of these fields with the aether field were studied.

In this paper, we show that the *CPT*-even couplings of the scalar, spinor and electromagnetic fields with the aether field proposed in [12] can arise in three, four and five-dimensional space-times as radiative corrections generated by appropriate *CPT*-odd Lorentz-breaking interactions of these fields between themselves and with a constant aether field. We also discuss their physical impacts, coming from the analysis of dispersion relations and the effective potential.

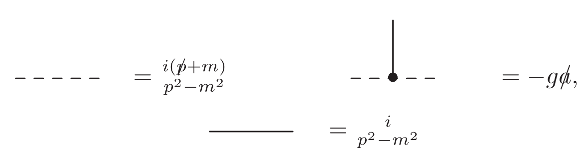
## II. AETHER TERMS IN THE SCALAR FIELD THEORY

We start with the model of a spinor field coupled to a scalar matter in a Lorentz-breaking manner:

$$S = \int d^D x [\bar{\psi}(i\not{\partial} - m)\psi + \tfrac{1}{2}(\partial^b \phi \partial_b \phi + m^2 \phi^2) - g\bar{\psi}\not{a}\psi\phi]. \tag{1}$$

Here $a^b$ is a constant vector implementing the Lorentz symmetry breaking. The coupling introduced in this way is a natural Lorentz-breaking generalization of the Yukawa coupling. We consider this model in three-, four-, and five-dimensional space-times. The Lorentz-breaking coupling in (1) is similar to the general structure of Eq. (6) of Ref. [9], with the identification $g\bar{\psi}a/\psi\phi = -[(G_L)_{AB}\bar{L}_A\phi R_B + (G_U)_{AB}\bar{Q}_A\phi^c U_B + (G_D)_{AB}\bar{Q}_A\phi D_B] +$ H.c.; notice however that in our case there is only one spinor (not chiral) field and no gauge interaction. As $\phi$ is real, we have $\phi^c = \phi$.

In this theory, the following Feynman rules take place:

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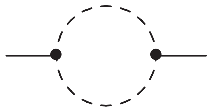
Thus, the lowest order contribution to the two-point vertex function of the scalar field is given by the following diagram:



Here, the simple line is for the external $\phi$ field, and the dashed line for the propagator of the spinor $\psi$ field. This diagram produces the following contribution to the effective action:

$$\begin{aligned} S_2(p) &= \frac{g^2}{2}\phi(p)\phi(-p)\int\frac{d^Dk}{(2\pi)^D}\,\mathrm{tr}[\not{a}S(k)\not{a}S(k+p)]. \\ &= -\frac{g^2}{2}\phi(p)\phi(-p)\int\frac{d^Dk}{(2\pi)^D}\,\mathrm{tr}[\not{a}(\not{k}+m) \\ &\quad\times\not{a}(\not{k}+\not{p}+m)]\frac{1}{[k^2-m^2][(k+p)^2-m^2]}. \end{aligned} \tag{2}$$

The key problem now consists in calculating the matrix traces. First of all, in five-dimensional space the gamma matrices are $4\times 4$ with $\gamma^0\ldots\gamma^3$ being the same as in four dimensions, and the five-dimensional $\gamma^4$ coinciding with the four-dimensional chirality matrix $\gamma_5$ (indeed, $\gamma_5$ anticommutes with each $\gamma^a$, $a=0\ldots 3$, and $(\gamma_5)^2=-\mathbf{1}$). The gamma matrices defined in this way satisfy the definition $\{\gamma^a,\gamma^b\}=2\eta^{ab}$, with $\eta^{ab}=diag(+-\ldots-)$. In three-dimensional space the gamma matrices are $(\gamma^0)^\alpha{}_\beta=\sigma^2$, $(\gamma^1)^\alpha{}_\beta=i\sigma^1$, $(\gamma^2)^\alpha{}_\beta=i\sigma^3$.

We can verify that the well-known four-dimensional relation for the trace of the product $\gamma^a\gamma^b\gamma^c\gamma^d$, can be generalized also at least in three and five space-time dimensions as

$$\begin{aligned} \mathrm{tr}\,(\gamma^a\gamma^b\gamma^c\gamma^d) &= d(\eta^{ab}\eta^{cd}-\eta^{ac}\eta^{bd}+\eta^{ad}\eta^{bc}), \\ \mathrm{tr}(\gamma^a\gamma^b) &= d\eta^{ab}. \end{aligned} \tag{3}$$

where $d$ is the dimension of the gamma matrix in the corresponding space-time. The trace of the product of three gamma matrices cannot produce an aetherlike term giving either zero (in four or five dimensions) or the Levi-Civita symbol (in three dimensions), and $a^m a_m=a^2$ which does not generate a Lorentz-breaking term. Thus, omitting the $a^2$ terms, we find the only Lorentz-breaking contribution from (2):

$$\begin{aligned} S_2(p) \simeq & -\frac{d}{2}g^2\phi(p)\phi(-p)[\eta^{ab}\eta^{cd} \\ & +\eta^{ad}\eta^{bc}]a_a a_c\int\frac{d^Dk}{(2\pi)^D} \\ & \times\frac{k_b(k_d+p_d)}{[k^2-m^2][(k+p)^2-m^2]}. \end{aligned} \tag{4}$$

Then, one may use the Feynman representation with the parameter $x$ which yields

$$\begin{aligned} S_2(p) \simeq & -dg^2\phi(p)\phi(-p)a^b a^d\int\frac{d^Dk}{(2\pi)^D} \\ & \times\int_0^1 dx\frac{k_bk_d-p_bp_dx(1-x)}{[k^2-m^2+p^2x(1-x)]^2}. \end{aligned} \tag{5}$$

The term proportional to $k_bk_d$ after integration gives $\eta_{bd}$, thus, the corresponding term in the effective action will be proportional to $a^2$ which does not break the Lorentz symmetry. The desired Lorentz-breaking correction, after Wick rotation, takes the form

$$\begin{aligned} S_2(p) \simeq & -idg^2\phi(p)\phi(-p)(a\cdot p)^2\int\frac{d^Dk_E}{(2\pi)^D} \\ & \times\int_0^1 dxx(1-x)\frac{1}{[k^2+m^2+p^2x(1-x)]^2}. \end{aligned} \tag{6}$$

We find that in five dimensions this contribution is finite only within the framework of the dimensional regularization:

$$S_2^{D=5}(p)=\frac{ig^2|m|}{24\pi^2}\phi(p)\phi(-p)(a\cdot p)^2. \tag{7}$$

Thus, we conclude that in five dimensions the desired finite aetherlike term [10] of the form

$$S_{\mathrm{aether}}^{D=5}=\frac{1}{M}\int d^5x\phi(a\cdot\partial)^2\phi \tag{8}$$

is generated, with $\frac{1}{M}=\frac{|m|g^2}{24\pi^2}$. The parameters $a^b$ and $M$ are related with the parameter $(k_{\phi\phi})^{bc}$ from [9] as $\frac{1}{2}(k_{\phi\phi})^{bc}=-\frac{a^ba^c}{M}$.

In three space-time dimensions the aetherlike term is explicitly finite without any regularization being also of the form (8), but with $\frac{1}{M}=\frac{g^2}{16\pi|m|}$.

Finally, in four space-time dimensions we find

$$S_{\mathrm{aether}}^{D=4}=\frac{g^2}{12\pi^2\epsilon}\left(1-\frac{\epsilon}{2}\ln\frac{m^2}{\mu^2}\right)\int d^4x\phi(a\cdot\partial)^2\phi, \tag{9}$$

where the divergence can only be removed by adding a counterterm of the form $S_{\mathrm{aether}}^{\mathrm{ct}}=-\frac{g^2}{12\pi^2\epsilon}\int d^4x\phi(a\cdot\partial)^2\phi$. Thus, first, we find that the aether term should be present in the theory from the very beginning to provide a consistent one-loop renormalization and second, the renormalized one-loop aether contribution looks like

$$S_{\mathrm{aether}}^{D=4}=-\frac{g^2}{24\pi^2}\ln\frac{m^2}{\mu^2}\int d^4x\phi(a\cdot\partial)^2\phi, \tag{10}$$

which reproduces the desired aetherlike structure, with $\frac{1}{M}=-\frac{g^2}{24\pi^2}\ln\frac{m^2}{\mu^2}$. We note that in four dimensions the aetherlike contribution is renormalizable by dimensional reasons.

After including the additive aether term (8), the action of the scalar field takes the form

$$S = -\frac{1}{2}\int d^D x \phi \left[\partial_b \partial^b + m^2 + \frac{1}{M}(a\cdot\partial)^2\right]\phi, \quad (11)$$

which implies in the following dispersion relations:

(i) Timelike $a^b$: $E = \pm\sqrt{\frac{\vec{k}^2+m^2}{1+\frac{a_0^2}{M}}}$. In this case there is no restrictions on the dynamics.

(ii) Spacelike $a^b$: $E = \pm\sqrt{\vec{k}^2 + m^2 - \frac{(\vec{a}\cdot\vec{k})^2}{M}}$. In this case the dynamics is well defined only at small $|\vec{a}|$.

The propagator of this model has the form

$$G(k) = -\frac{1}{k^2 - m^2 + \frac{1}{M}(a\cdot k)^2}. \quad (12)$$

Following [14], we can find the tree-level Breit potential corresponding to fermion-fermion scattering in the theory where fermions are coupled via Yukawa interaction to a scalar fields with this propagator. Indeed, in the nonrelativistic limit the incoming (outcoming) fermions with mass $M$ have initial (final) momenta $p$ and $k$ (respectively $p'$ and $k'$) whose explicit form is $p = (M, \vec{p})$, $k = (M, \vec{k})$ etc. The momentum exchanged is $q = p' - p = (0, \vec{p}' - \vec{p})$.

Thus, the matrix element for the scattering process is (cf. [14])

$$i\mathcal{M}(q) = \frac{-ig^2}{q^2 + (u\cdot q)^2 - m^2}(2M\delta^{rr'})(2M\delta^{ss'}), \quad (13)$$

where the delta symbols correspond to the spins, and $u^b = \frac{a^b}{\sqrt{M}}$. The Breit effective potential is found via Fourier transform of the spatial part:

$$U(\vec{r}) = -g^2 \int \frac{d^d\vec{q}}{(2\pi)^d}\frac{e^{i\vec{q}\cdot\vec{r}}}{\vec{q}^2 - (\vec{u}\cdot\vec{q})^2 + m^2}. \quad (14)$$

Here $d = D - 1$. To perform the integration, we diagonalize the matrix in the denominator via an appropriate rotation of the coordinates. Afterwards, this expression takes the form (cf. [14])

$$U(\vec{r}) = -g^2 \int \frac{d^d\vec{q}}{(2\pi)^d}\frac{e^{i\vec{q}\cdot\vec{r}}}{\sum_i S_{ii}q_i^2 + m^2}, \quad (15)$$

where $S_{ij} = \delta_{ij} - u_i u_j$. Let us carry out changes of variables similar to [14], that is, $\tilde{q}_i = \sqrt{S_{ii}}q_i$, $\tilde{r}_i = \frac{r_i}{\sqrt{S_{ii}}}$. We find

$$U(\vec{r}) = -\frac{g^2}{\det S}\int \frac{d^d\tilde{q}}{(2\pi)^d}\frac{e^{i\tilde{q}\cdot\tilde{r}}}{\tilde{q}^2 + m^2}, \quad (16)$$

where $\tilde{q}^2 = \sum_i \tilde{q}_i^2$. This integral yields

$$U(\vec{r}) = \frac{|m|^{d/2-1}g^2}{(2\pi)^{d/2}\sqrt{\det S}}\frac{K_{d/2-1}(m\tilde{r})}{|\tilde{r}|^{d/2-1}}, \quad (17)$$

where $K_n(x)$ is the modified Bessel function. Suggesting the vectors $u_i$ to be small we find $\det S = 1 - \vec{u}^2$, $(S^{-1})_{ij} = \delta_{ij} + u_i u_j$, so, we get $\bar{r} = \sqrt{r^2 + (u\cdot r)^2}$, and

$$U(\vec{r}) = \frac{|m|^{d/2-1}g^2}{(2\pi)^{d/2}\sqrt{1-u^2}}\frac{K_{d/2-1}(m\sqrt{r^2+(u\cdot r)^2})}{(\sqrt{r^2+(u\cdot r)^2})^{d/2-1}}. \quad (18)$$

The asymptotics of the function $K_n(x)$ is

$$K_n(x)|_{x\to\infty} \simeq \sqrt{\frac{\pi}{2x}}e^{-x} \quad (19)$$

Thus, we find that at large distances, the effective potential displays exponential decay with the distance in any space-time dimension:

$$U(\vec{r})|_{r\to\infty} = \frac{\sqrt{m}g^2}{4\pi^2\sqrt{1-u^2}}\sqrt{\frac{\pi}{2}}\frac{e^{-m\sqrt{r^2+(u\cdot r)^2}}}{[r^2+(u\cdot r)^2]^{3/4}}. \quad (20)$$

At small distances, the leading term in the Breit potential, for example, in the case $d = 4$ (that is, $D = 5$) is

$$U(\vec{r})|_{r\to 0} = \frac{mg^2}{4\pi^2\sqrt{1-u^2}}\frac{1}{r^2+(u\cdot r)^2}. \quad (21)$$

In the case $d = 3$ ($D = 4$), $d/2 - 1 = 1/2$, and $K_{1/2}(x) = \sqrt{\frac{\pi}{2x}}e^{-x}$ exactly. Hence, the expression (20) in this space-time dimension is valid both at small and large distances. Finally, in the case $d = 2$ ($D = 3$), the expression (20) is valid at large distances whereas at small ones, the effective potential grows logarithmically with the distance. In all cases, the Breit potential is anisotropic. However, in the limit $\vec{u}\to 0$, this potential reproduces Yukawa's (notice that for large enough $\vec{u}$, the Lorentz-breaking term cannot be treated as a small perturbation).

Next, we study the one-loop effective potential for the aether model (11), to which an arbitrary scalar potential of the scalar field $V(\phi)$ is added. Following a common procedure, we can split the field $\phi$ into the sum of the background, $\Phi$, and the quantum, $\chi$, fields. The quadratic action for $\chi$ looks like

$$S_2[\chi] = -\frac{1}{2}\int d^D x\chi\left[\partial_b\partial^b + m^2 + \frac{1}{M}(a\cdot\partial)^2 + V''(\Phi)\right]\chi. \quad (22)$$

The corresponding one-loop effective action of the $\Phi$ background field is

$$\Gamma^{(1)} = \frac{i}{2}\operatorname{tr}\ln\left[\partial_b\partial^b + m^2 + \frac{1}{M}(a\cdot\partial)^2 + V''(\Phi)\right] = -\int d^D x V_{\rm eff}^{(1)}(\Phi). \quad (23)$$

To find the effective potential we carry out the Fourier transform and the Wick rotation and treate the background field as a constant. As a result, we get

$$V^{(1)}_{\rm eff}(\Phi) = -\frac{1}{2}\int\frac{d^Dk_E}{(2\pi)^D}\ln\left[k^2+m^2-\frac{1}{M}(a\cdot k)^2 + V''(\Phi)\right] \tag{24}$$

Because of symmetry properties, we can replace $k_ak_b \to \frac{k^2}{D}\delta_{ab}$. Thus, we get

$$V^{(1)}_{\rm eff}(\Phi) = -\frac{1}{2}\int\frac{d^Dk_E}{(2\pi)^D}\ln\left[k^2\left(1-\frac{a^2}{DM}\right)+m^2 + V''(\Phi)\right], \tag{25}$$

which is equal to

$$V^{(1)}_{\rm eff}(\Phi) = -\Gamma\left(-\frac{D}{2}\right)\left(\frac{m^2+V''(\Phi)}{4\pi(1-u^2/D)}\right)^{D/2}. \tag{26}$$

Thus, we have shown that the one-loop effective potential is finite for $D=5$ and $D=3$ within the dimensional regularization framework and it is free of Lorentz symmetry breaking. In four dimensions, however, it is divergent being equal to

$$V^{(1)}_{\rm eff}(\Phi) = -\frac{1}{16\pi^2\epsilon}(m^2+V''(\Phi))^2+\frac{1}{16\pi^2}\times\frac{(m^2+V''(\Phi))^2}{(1-u^2/D)^2}\ln\left(\frac{m^2+V''(\Phi)}{\mu^2}\right), \tag{27}$$

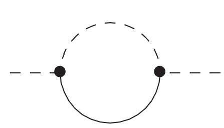
where the constants, including the explicit $u^a$ dependence, are absorbed into a redefinition of $\mu$, hence the one-loop effective potential is Lorentz symmetric.

The complete one-loop corrected effective potential is

$$V_{\rm eff}(\Phi) = V(\Phi) + V^{(1)}_{\rm eff}(\Phi). \tag{28}$$

Now let us obtain the aether term for the spinor field. To do it, we start with the action (1) and obtain the two-point function of the spinor field.

The corresponding Feynman diagram looks like



Its contribution has the form

$$S_2^{sp} = g^2\bar{\psi}(-p)\int\frac{d^Dk}{(2\pi)^D}\times\frac{\slashed{a}(\slashed{k}+m)\slashed{a}}{(k^2-m^2)[(k+p)^2-m^2]}\psi(p). \tag{29}$$

Simplifying the product of matrices, we find

$$S_2^{sp} = g^2\bar{\psi}(-p)\int\frac{d^Dk}{(2\pi)^D}\times\frac{2(a\cdot k)\slashed{a}-a^2\slashed{k}+ma^2}{(k^2-m^2)[(k+p)^2-m^2]}\psi(p). \tag{30}$$

Here, the mass of scalar and spinor fields are chosen to be equal for simplicity, however, the case of different masses is treated in the same way.

Then we employ Feynman representation, carry out change of variables and Wick rotation and disregard $p^2$ in the denominator. As a result, the leading term (which will produce the contribution with no more than one derivative) looks like

$$S_2^{sp} = ig^2\bar{\psi}(-p)\int\frac{d^Dk_E}{(2\pi)^D}\times\int_0^1 dx\frac{-2x(a\cdot p)\slashed{a}+xa^2\slashed{p}+ma^2}{(k_E^2+m^2)^2}\psi(p). \tag{31}$$

Integrating over $d^4k_E$ and $dx$, one finds

$$S_2^{sp} = ig^2\Gamma(2-D/2)\bar{\psi}(-p)\times\frac{-(a\cdot p)\slashed{a}+\frac{1}{2}a^2\slashed{p}+ma^2}{(4\pi)^{D/2}(m^2)^{2-D/2}}\psi(p). \tag{32}$$

One finds that the term proportional to $(a\cdot p)$ exactly reproduces the aether term introduced in [12]. Returning to the Minkowski space and to the coordinate representation, one finds

$$S_2^{sp} = g^2\frac{\Gamma(2-D/2)}{(4\pi)^{D/2}(m^2)^{2-D/2}}\bar{\psi}\left\{-i\slashed{a}(a\cdot\partial)+\frac{i}{2}a^2\slashed{\partial}+ma^2\right\}\psi. \tag{33}$$

Thus, we succeeded to generate the aetherlike term for the spinor field, that is, the first term in the expression above. It is clear that this term is finite in an odd-dimensional space-time. The relevant, Lorentz-breaking part of this term is

$$S_2^{sp} = -i\alpha\bar{\psi}\slashed{a}(a\cdot\partial)\psi, \tag{34}$$

where $\alpha = g^2\frac{\Gamma(2-D/2)}{(4\pi)^{D/2}(m^2)^{2-D/2}}$. Following [9], one can write down the following generic form of the aetherlike term

$$S_2^{sp} = \frac{i}{2}\bar{\psi}_A\{(c_Q)_{\mu\nu AB}+(c_U)_{\mu\nu AB}+(c_D)_{\mu\nu AB}\}\gamma^\mu\overleftrightarrow{\partial}^\nu\psi_B. \tag{35}$$

To compare with our model, we should restrict ourselves to the case of only one spinor field,that is, $A=B=1$. In this situation, disregarding the Lorentz-invariant terms proportional to $a^2$, we should identify $((c_Q)_{\mu\nu AB}+(c_U)_{\mu\nu AB}+(c_D)_{\mu\nu AB}) = -\alpha a_\mu a_\nu$.

If we consider the spinor action with the additive aether term, that is,

$$S^{sp} = \int d^D x \bar{\psi}(i\not{\partial} - m - i\not{a}(a\cdot\partial))\psi, \qquad (36)$$

one can find that the dispersion relations corresponding to this action look like

$$p^2 + (a\cdot p)^2 a^2 + 2(a\cdot p)^2 = m^2. \qquad (37)$$

One can see that for the timelike $a^b = (a_0, \vec{0})$, one finds $E^2(1+a_0^2)^2 = \vec{p}^2 + m^2$, which is evidently consistent at any $a_0$. At the spacelike $a^b = (0, \vec{a})$, one finds $E^2 = \vec{p}^2 + m^2 + (a\vec{p})^2\vec{a}^2 - 2(\vec{a}\cdot\vec{p})^2$ whose behavior can be nonphysical in a certain interval of $|\vec{a}|$.

One should note that an attempt to generate the aether term via the alternative Lorentz-breaking coupling of scalar field to the spinor one, that is, via the action

$$S' = \int d^D x \bar{\psi}(i\not{\partial} - m - \not{b} - g\phi)\psi, \qquad (38)$$

in four dimensions can be shown to give zero result in the one-loop approximation.

## III. AETHER TERM IN THE ELECTRODYNAMICS

We start with the model of the spinor field coupled to the electromagnetic field in a Lorentz-breaking manner in three, four and five space-time dimensions. We start with the introduction of a generic form of the magnetic coupling of the electromagnetic field to the spinor one, characterized by the action

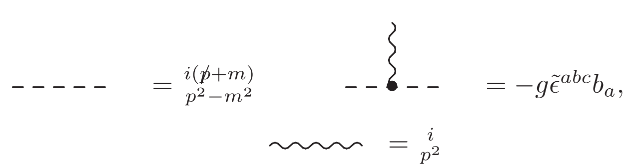
$$S = \int d^D x\left[\bar{\psi}(i\not{\partial} - m - \tilde{\epsilon}^{abc} b_a F_{bc})\psi - \frac{1}{4}F_{ab}F^{ab}\right], \qquad (39)$$

where $\tilde{\epsilon}^{abc}$ is a matrix-valued object antisymmetric with respect to its Lorentz indices and defined as $\tilde{\epsilon}^{abc} = \epsilon^{abc}$, in $D=3$; $\tilde{\epsilon}^{abc} \equiv \epsilon^{abcd}\gamma_d$, in $D=4$, and $\tilde{\epsilon}^{abc} = \epsilon^{abcde}\sigma_{de}$, in $D=5$. Here $b_a$ is a vector implementing the Lorentz symmetry breaking. We note that in the three- and five-dimensional space-times the one-loop contribution will be finite within the framework of the dimensional regularization.

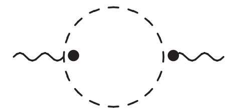
In this theory, after imposing the Feynman gauge via adding the usual gauge fixing term

$$S_{gf} = \tfrac{1}{2}A_a\partial^a\partial^b A_b. \qquad (40)$$

the following Feynman rules take place:




Thus, we find the lowest order contribution to the two-point vertex function of the electromagnetic field is given by the following diagram:



Here the wavy line is for the external $F_{mn}$ field, and the dashed line is for the propagator of the spinor $\psi$ field. In five dimensions, this diagram produces the following contribution to the effective action:

$$S_2(p) = \frac{g^2}{2}\epsilon^{abcde}\epsilon^{a'b'c'd'e'} b_a F_{bc}(p) b_{a'} F_{b'c'}(-p) \times \int \frac{d^5k}{(2\pi)^5}\,\mathrm{tr}[\sigma_{de}S(k)\sigma_{d'e'}S(k+p)] \qquad (41)$$

Thus, we can write down the following explicit expression for the new, Lorentz-breaking contribution to the quadratic effective action of the theory:

$$S_2(p) = -\frac{g^2}{2}\epsilon^{abcde}\epsilon^{a'b'c'd'e'} b_a F_{bc}(p) b_{a'} F_{b'c'}(-p) \times \int \frac{d^5k}{(2\pi)^5}\,\mathrm{tr}[\sigma_{de}(\not{k}+m)\sigma_{d'e'}(\not{k}+\not{p}+m)] \times \frac{1}{[k^2-m^2][(k+p)^2-m^2]}. \qquad (42)$$

Here the matrices are defined just as in the previous section.

The aetherlike terms do not involve derivatives of $F_{ab}$, hence we can impose condition $p_a = 0$ in the internal lines from the very beginning. As a result, we have

$$S_2(p) = -\frac{g^2}{2}\epsilon^{abcde}\epsilon^{a'b'c'd'e'} b_a F_{bc}(p) b_{a'} F_{b'c'}(-p) \times \int \frac{d^5k}{(2\pi)^5}\frac{1}{[k^2-m^2]^2}\,\mathrm{tr}[m^2\sigma_{de}\sigma_{d'e'} + k^m k^n \gamma_m\sigma_{de}\gamma_n\sigma_{d'e'}]. \qquad (43)$$

Here, we disregarded the terms proportional to a trace of an odd number of gamma matrices, (which is either zero or produces irrelevant terms). This expression can be rewritten as

$$S_2(p) = S_2^{(1)}(p) + S_2^{(2)}(p), \qquad (44)$$

where

$$S_2^{(1)}(p) = -\frac{g^2m^2}{2}\epsilon^{abcde}\epsilon^{a'b'c'd'e'} b_a F_{bc}(p) b_{a'} F_{b'c'}(-p) \times \mathrm{tr}[\sigma_{de}\sigma_{d'e'}]\int \frac{d^5k}{(2\pi)^5}\frac{1}{[k^2-m^2]^2}, \qquad (45)$$

$$S_2^{(2)}(p) = -\frac{g^2}{2}\epsilon^{abcde}\epsilon^{a'b'c'd'e'} b_a F_{bc}(p) b_{a'} F_{b'c'}(-p) \times \mathrm{tr}[\gamma_m\sigma_{de}\gamma_n\sigma_{d'e'}]\int \frac{d^5k}{(2\pi)^5}\frac{k^m k^n}{[k^2-m^2]^2}. \qquad (46)$$

By applying the relation (3) and using the definition $\sigma_{ab} = \frac{i}{2}[\gamma_a, \gamma_b]$, we get

$$\mathrm{tr}\,(\sigma_{de}\sigma_{d'e'}) = 4(\eta_{dd'}\eta_{ee'} - \eta_{de'}\eta_{d'e}). \tag{47}$$

As a result, taking into account the relation

$$\epsilon^{abcde}\epsilon^{a'b'c'd'e'}\eta_{dd'}\eta_{ee'} = 2(\eta^{aa'}\eta^{bb'}\eta^{cc'} - \eta^{aa'}\eta^{bc'}\eta^{cb'} + \eta^{ab'}\eta^{bc'}\eta^{ca'} - \eta^{ac'}\eta^{bb'}\eta^{ca'} + \eta^{ac'}\eta^{ba'}\eta^{cb'} - \eta^{ab'}\eta^{ba'}\eta^{cc'}), \tag{48}$$

and, consequently,

$$\epsilon^{abcde}\epsilon^{a'b'c'd'e'}\eta_{dd'}\eta_{ee'}b_aF_{bc}b_{a'}F_{b'c'} = -8b^aF_{ac}b_bF^{bc} \equiv -8(b^aF_{ab})^2, \tag{49}$$

we can calculate the trace in the first term, so

$$S_2^{(1)}(p)|_{p=0} = -2\frac{g^2 i}{\pi^2}|m|^3(b^aF_{ab})^2. \tag{50}$$

Also, we can find the integral in $S_2^{(2)}$:

$$\int \frac{d^5k}{(2\pi)^5}\frac{k^ak^b}{[k^2-m^2]^2} = -i\eta^{ab}\frac{|m|^3}{48\pi^2}, \tag{51}$$

thus,

$$S_2^{(2)}(p)|_{p=0} = -\frac{g^2 i|m|^3}{96\pi^2}\epsilon^{abcde}\epsilon^{a'b'c'd'e'}b_aF_{bc}b_{a'}F_{b'c'} \times \mathrm{tr}[\gamma_m\sigma_{de}\gamma^m\sigma_{d'e'}]. \tag{52}$$

It remains to find $\mathrm{tr}[\gamma_m\sigma_{de}\gamma^m\sigma_{d'e'}]$. One can find that $\mathrm{tr}[\gamma_m\sigma_{de}\gamma^m\sigma_{d'e'}] = \mathrm{tr}[\sigma_{de}\sigma_{d'e'}]$. Substituting this result to the above expression, we find

$$S_2^{(2)}(p)|_{p=0} = \frac{2g^2 i}{3\pi^2}|m|^3(b^aF_{ab})^2. \tag{53}$$

The final five-dimensional result being the sum of (50) and (53), after returning to the Minkowski space, is

$$S_2(p)|_{p=0} = -\frac{4g^2}{3\pi^2}|m|^3(b^aF_{ab})^2. \tag{54}$$

Let us consider other dimensions. In three space-time dimensions the only way to apply this approach (without use of the derivative expansion) is based on the action

$$S = \int d^3x\bar{\psi}(i\partial\!\!\!/ - m - g\epsilon^{abc}b_aF_{bc})\psi. \tag{55}$$

The corresponding correction is

$$S_2(p) = -\frac{g^2}{2}\epsilon^{abc}\epsilon^{a'b'c'}b_aF_{bc}(p)b_{a'}F_{b'c'}(-p) \times \int\frac{d^3k}{(2\pi)^3}\frac{1}{[k^2-m^2]^2}\mathrm{tr}[m^2 + k^mk^n\gamma_m\gamma_n]. \tag{56}$$

Calculating the trace and carrying out the Wick rotation, one finds

$$S_2(p) = -ig^2\epsilon^{abc}\epsilon^{a'b'c'}b_aF_{bc}(p)b_{a'}F_{b'c'}(-p) \times \int\frac{d^3k}{(2\pi)^3}\frac{-k^2+m^2}{[k^2+m^2]^2}. \tag{57}$$

After a straightforward calculation of the integral we find

$$S_2(p) = -ig^2\frac{|m|}{2\pi}\epsilon^{abc}\epsilon^{a'b'c'}b_aF_{bc}(p)b_{a'}F_{b'c'}(-p). \tag{58}$$

We can multiply the Levi-Civita symbols which, after returning to the Euclidean space, yields

$$S_2(p) = 4g^2\frac{|m|}{2\pi}(b^aF_{ab})^2. \tag{59}$$

Finally, let us consider the four-dimensional space. In this case, the spinor matter is coupled to the electromagnetic field via the action

$$S = \int d^4x\bar{\psi}(i\partial\!\!\!/ - m - g\epsilon^{abcd}b_aF_{bc}\gamma_d)\psi, \tag{60}$$

and the Lorentz-breaking contribution to the quadratic effective action of the electromagnetic field looks like

$$S_2(p) = \frac{g^2}{2}\epsilon^{abcd}\epsilon^{a'b'c'd'}b_aF_{bc}(p)b_{a'}F_{b'c'}(-p) \times \int\frac{d^4k}{(2\pi)^4}\mathrm{tr}[\gamma_dS(k)\gamma_{d'}S(k+p)] \tag{61}$$

The explicit form of this expression, at $p = 0$, is

$$S_2(p) = -\frac{g^2}{2}\epsilon^{abcd}\epsilon^{a'b'c'd'}b_aF_{bc}(p)b_{a'}F_{b'c'}(-p)\int\frac{d^4k}{(2\pi)^4} \times \frac{1}{[k^2-m^2]^2}\mathrm{tr}[m^2\gamma_d\gamma_{d'} + k^mk^n\gamma_m\gamma_d\gamma_n\gamma_{d'}]. \tag{62}$$

Proceeding with calculation of the trace, we arrive at

$$S_2(p) = -2g^2\epsilon^{abcd}\epsilon^{a'b'c'd'}b_aF_{bc}(p)b_{a'}F_{b'c'}(-p) \times \int\frac{d^4k}{(2\pi)^4}\frac{1}{[k^2-m^2]^2}[m^2\eta_{dd'} + k^mk^n \times (\eta_{md}\eta_{nd'} - \eta_{mn}\eta_{dd'} + \eta_{md'}\eta_{nd})]. \tag{63}$$

We employ the relation $k^mk^n \to \frac{1}{4}\eta^{mn}k^2$. After Wick rotation we find

$$S_2(p) = -ig^2\epsilon^{abcd}\epsilon^{a'b'c'd'}b_aF_{bc}(p)b_{a'}F_{b'c'}(-p)\eta_{dd'} \times \int\frac{d^4k}{(2\pi)^4}\frac{k^2+2m^2}{[k^2+m^2]^2}. \tag{64}$$

Despite the integral over $k$ is quadratically divergent, within the dimensional regularization it yields finite result:

$$\int \frac{d^{4-\epsilon}k}{(2\pi)^{4-\epsilon}} \frac{k^2+2m^2}{[k^2+m^2]^2} = \frac{m^{2(1-\epsilon/2)}}{(4\pi)^{2-\epsilon/2}} \times [\Gamma(\epsilon/2)+\Gamma(-1+\epsilon/2)] = -\frac{m^2}{16\pi^2} + O(\epsilon). \quad (65)$$

Collecting all together, using the properties of product of the Levi-Civita symbols and returning to the Minkowski space, we arrive at

$$S_2(p) = g^2 \frac{m^2}{4\pi^2}(b^a F_{ab})^2. \quad (66)$$

We note that, had we done choice $k^m k^n \to \frac{1}{d}\eta^{mn}k^2$ with the subsequent choice $d = 4-\epsilon$ as it has been done in [5], we would obtain the same result (66). Therefore, differently from the Lorentz-breaking Chern-Simons term [4,5], this result appears to be unambiguously determined.

As a result, within the dimensional regularization scheme the aetherlike term is finite in all dimensions from three to five. Its generic form is

$$L_{\rm aether} = c(b^a F_{ab})^2, \quad (67)$$

where $c$ is a some constant depending on the space-time dimension. This term is equivalent to the term introduced in [9]:

$$L_{\rm even} = -\tfrac{1}{4}(k_F)_{\kappa\lambda\mu\rho}F^{\kappa\lambda}F^{\mu\rho}, \quad (68)$$

with the constant parameters $(k_F)_{\kappa\lambda\mu\rho}$, $c$, $b_a$ related in the following way: $(k_F)_{\kappa\lambda\mu\rho} = -c(\eta_{\kappa\mu}b_\lambda b_\rho + \eta_{\lambda\rho}b_\kappa b_\mu - \eta_{\kappa\rho}b_\mu b_\lambda + \eta_{\lambda\mu}b_\kappa b_\rho)$.

Thus, we have succeeded to generate an effective Lagrangian with the aetherlike term, which has the form

$$L = -\tfrac{1}{4}F_{ab}F^{ab} + \tfrac{1}{2}u^a u^b \eta^{cd}F_{ac}F_{bd}, \quad (69)$$

where $u^a = \sqrt{|c|}b^a$. The equations of motion for this model are

$$\partial_a F^{ab} + 2u^a u^c \partial_a F^b{}_c = 0. \quad (70)$$

The energy-momentum tensor for this theory is evidently conserved due to the homogeneity of the space-time, being equal to

$$T^a_b = \frac{\partial L}{\partial(\partial_a A_c)}\partial_b A_c - L\delta^a_b, \quad (71)$$

which in this case is

$$T^a_b = (F^{ac} - 2u^a u^d F^c_d)\partial_b A_c - \delta^a_b(-\tfrac{1}{4}F_{cd}F^{cd} + \tfrac{1}{2}u^m u^n \eta^{cd}F_{mc}F_{nd}). \quad (72)$$

However, due to the preferential direction in the space-time introduced by the $u^a$ vector, the angular momentum is not conserved.

Let us obtain the propagator in the model (69). In the Feynman gauge, the action takes the form

$$S = \int d^4x \frac{1}{2}A_a(\eta^{ab}\partial^2 - u^a u^b\partial^2 - \eta^{ab}(u\cdot\partial)^2 + (u^a\partial^b + u^b\partial^a)(u\cdot\partial))A_b. \quad (73)$$

We can obtain the inverse operator for

$$\Delta^{ab} = \eta^{ab}\partial^2 - u^a u^b \partial^2 - \eta^{ab}(u\cdot\partial)^2 + (u^a\partial^b + u^b\partial^a)(u\cdot\partial). \quad (74)$$

The most convenient way to do that is the undetermined coefficients method. We suggest

$$\Delta^{-1}_{bc} = k_1\eta_{bc} + k_2\partial_b\partial_c + k_3 u_b u_c + k_4 u_b\partial_c + k_5 u_c\partial_b, \quad (75)$$

with $\Delta^{ab}\Delta^{-1}_{bc} = \delta^b_c$. It is clear that $k_1 = [\partial^2 - (u\cdot\partial)^2]^{-1}$. Then,

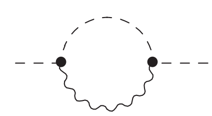
$$\begin{aligned} k_2 &= \frac{k_1(u\cdot\partial)^2}{\partial^4(1-u)^2} = [\partial^2 - (u\cdot\partial)^2]^{-1}\frac{(u\cdot\partial)^2}{\partial^4(1-u)^2}, \\ k_3 &= \frac{k_1}{1-u^2} = [\partial^2 - (u\cdot\partial)^2]^{-1}\frac{1}{1-u^2}, \\ k_4 &= -\frac{k_1(u\cdot\partial)}{\partial^2(1-u^2)} = [\partial^2 - (u\cdot\partial)^2]^{-1}\frac{(u\cdot\partial)}{\partial^2(1-u^2)}, \\ k_5 &= -\frac{k_1(u\cdot\partial)}{\partial^2(1-u^2)} = -[\partial^2 - (u\cdot\partial)^2]^{-1}\frac{(u\cdot\partial)}{\partial^2(1-u^2)}. \end{aligned} \quad (76)$$

The dispersion relations can be found from the poles of the denominators $[\partial^2 - (u\cdot\partial)^2]^{-1}$ and $(\partial^2)^{-1}$. The second one is the usual Lorentz-invariant one, whereas the first one yields the following cases for $(+-\ldots-)$ signature:

(i) The $u^a$ is spacelike. We get $E^2 = \vec{p}^2 - (\vec{u}\cdot\vec{p})^2$. This possibility is consistent only for small $|\vec{u}|$.

(ii) The $u^a$ is timelike. We get $E^2(1+u_0^2) = \vec{p}^2$. This case is consistent for any $u_0$.

It is interesting also to generate the aetherlike term for the spinor field from its coupling with the electromagnetic field (39). The lowest-order Feynman diagram looks like



Its contribution is

$$I = 4g^2 \int \frac{d^D k}{(2\pi)^D}\bar\psi(-p)\tilde\epsilon^{abc}[\gamma^d(k_d+p_d) - m] \times \tilde\epsilon^{a'b'c'}\eta_{cc'}b_a b_{a'}\frac{k_b k_{b'}}{k^2[(k+p)^2 - m^2]}\psi(p). \quad (77)$$

Then we use the Feynman representation which allows us to write

$$I = 4g^2 \int \frac{d^D k}{(2\pi)^D} \int_0^1 dx \bar{\psi}(-p)\tilde{\epsilon}^{abc}\eta_{cc'}b_a b_{a'} \times [\gamma^d(k_d + p_d(1-x)) - m]\tilde{\epsilon}^{a'b'c'} \times \frac{(k_b - p_b x)(k_{b'} - p_{b'}x)}{[k^2 + p^2 x(1-x) - m^2 x]^2}\psi(p). \tag{78}$$

Afterwards, let us again restrict ourselves by the terms involving at most first order in external momenta $p_a$ (that is, the terms whose form is similar to those ones from the Dirac Lagrangian). Moreover, the term proportional to $m$ can be easily shown to generate only Lorentz-invariant contribution proportional to $b^a b_a$ (indeed, this term does not contain explicit momenta $p^a$, hence the $b^a$ can be contracted only among themselves), hence we disregard this term and also neglect $p^2$ in the denominator and rest with

$$I = -4g^2 \int \frac{d^D k}{(2\pi)^D} \int_0^1 dx \bar{\psi}(-p)\tilde{\epsilon}^{abc}\eta_{cc'}b_a b_{a'} \times \frac{(\gamma^d k_d k_b p_{b'}x + \gamma^d k_d k_{b'} p_b x - \gamma^d p_d(1-x)k_b k_{b'})\tilde{\epsilon}^{a'b'c'}}{[k^2 - m^2 x]^2} \times \psi(p). \tag{79}$$

Then, we replace $k_a k_b \to \eta_{ab}\frac{k^2}{D}$ and carry out the Wick rotation $k_0 \to ik_{0E}$ (hence, $k^2 \to k_E^2$), as a result we arrive at

$$I = \frac{4i}{D}g^2 \int \frac{d^D k_E}{(2\pi)^D}\frac{k_E^2}{[k_E^2 + m^2 x]^2}\int_0^1 dx \bar{\psi}(-p)\tilde{\epsilon}^{abc}\eta_{cc'}b_a b_{a'} \times (\gamma_b p_{b'}x + \gamma_{b'}p_b x - \gamma^d p_d(1-x)\eta_{bb'})\tilde{\epsilon}^{a'b'c'}\psi(p). \tag{80}$$

Integrating over $k$ together with the subsequent returning to the Minkowski space, we find

$$I = \frac{2g^2}{(4\pi)^{D/2}}|m|^{D-2}\Gamma\left(1 - \frac{D}{2}\right)\bar{\psi}(-p)\tilde{\epsilon}^{abc}\eta_{cc'}b_a b_{a'} \times \int_0^1 dx x^{D/2-1}(\gamma_b p_{b'}x + \gamma_{b'}p_b x - \gamma^d p_d \eta_{bb'}(1-x))\tilde{\epsilon}^{a'b'c'}\psi(p). \tag{81}$$

Afterwards, we integrate over $x$ and arrive at

$$I = \frac{2g^2}{(4\pi)^{D/2}}|m|^{D-2}\Gamma\left(1 - \frac{D}{2}\right)\bar{\psi}(-p)\tilde{\epsilon}^{abc}\eta_{cc'}b_a b_{a'} \times \left[\frac{2}{D}(\gamma_b p_{b'} + \gamma_{b'}p_b) - \left(\frac{2}{D} - \frac{2}{2+D}\right)\gamma^d p_d \eta_{bb'}\right] \times \tilde{\epsilon}^{a'b'c'}\psi(p). \tag{82}$$

It remains to simplify the matrix products. For example, at $D = 3$ we have

$$I = \frac{2g^2}{3\pi}|m|\bar{\psi}(-p)\gamma^a p^b b_a b_b \psi(p), \tag{83}$$

which in the coordinate representation yields

$$I = \frac{2i|m|g^2}{3\pi}\bar{\psi}\not{b}(b\cdot\partial)\psi, \tag{84}$$

that is, the aetherlike term of the same form as in (34). The calculations at $D = 4$, 5 do not principally differ. So, in $D = 4$ the contribution (82) diverges being, at $D = 4 + \epsilon$, of the form

$$I = \frac{7im^2 g^2}{24\pi^2\epsilon}\bar{\psi}\not{b}(b\cdot\partial)\not{b}\psi + \text{fin}, \tag{85}$$

which again reproduces the form (34), however, for the renormalizability such a term should present in the Lagrangian from the very beginning.

Finally, one can show that at $D = 5$ the term (33) also arises, and it is finite in the dimensional regularization framework. Calculating the products of matrices in (82) in this case, we find

$$I = -\frac{484g^2}{105\pi^2}i|m|^3\bar{\psi}\not{b}(b\cdot\partial)\psi. \tag{86}$$

Therefore, we succeeded in generating the aetherlike term also in this case, and it is finite within the dimensional regularization prescription. By analogy with the case of the coupling with the scalar field, one can write down the one-loop quantum contribution to the aether term for the spinor field in all space-time dimensions as

$$I = i\tilde{\alpha}\,\bar{\psi}\,\not{b}(b\cdot\partial)\psi, \tag{87}$$

where $\tilde{\alpha}$ is a some constant depending on the dimension of the space-time. Comparing this term with the standard form of the aetherlike action for the spinors (35) originally introduced in [9], we conclude that the constant parameters in [9] and those used in this paper are related as $((c_Q)_{\mu\nu AB} + (c_U)_{\mu\nu AB} + (c_D)_{\mu\nu AB}) = -\tilde{\alpha}b_\mu b_\nu$.

## IV. SUMMARY

We succeeded in generating $CPT$-even Lorentz-breaking models with aetherlike terms for the scalar and vector fields through appropriate couplings to the spinor matter and studied their different physical aspects. We also generated aetherlike terms for the spinor field from the same interactions. We have shown that the aetherlike terms, arising as perturbative corrections, are finite within the dimensional regularization framework in three and five dimensions, and in the case of the electrodynamics—also in four dimensions, which is highly nontrivial since this contribution is superficially quadratically divergent. We note that the last result is unambiguously determined unlike the four-dimensional Chern-Simons term [1,4,5] which reflects naturally the fact that the four-dimensional Chern-Simons term is related with the ABJ anomaly [5] whereas there is no known analog of this anomaly for the aether term.

We have calculated the tree-level Breit effective potential of scalar-spinor coupling which was shown to be anisotropic, as well as the one-loop effective potential for the scalar field which is shown to be free of Lorentz symmetry breaking. We also studied the dispersion relations for the modified scalar, spinor and electromagnetic field models.

The natural continuation of this study would consist in the implementation of the condition of compactness for the extra dimension and the study of the physical impacts of the extra compact dimensions for these terms (some studies for the phenomenological aspects of the compact extra dimensions can be found in [10]). We are planning to carry out this study in a forthcoming paper.

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