Tunneling cosmological state revisited: Origin of inflation with a nonminimally coupled standard model Higgs inflaton

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We suggest a path-integral formulation for the tunneling cosmological state, which admits a consistent renormalization and renormalization group improvement in particle physics applications of quantum cosmology. We apply this formulation to the inflationary cosmology driven by the standard model Higgs boson playing the role of an inflaton with a strong nonminimal coupling to gravity. In this way a complete cosmological scenario is obtained, which embraces the formation of initial conditions for the inflationary background in the form of a sharp probability peak in the distribution of the inflaton field and the ongoing generation of the cosmic microwave background (CMB) spectrum on this background. Formation of this probability peak is based on the same renormalization group mechanism which underlies the generation of the CMB spectrum which was recently shown to be compatible with the WMAP data in the Higgs mass range $135.6 \text{ GeV} \lesssim M_{\text{H}} \lesssim 184.5 \text{ GeV}$. This brings to life a convincing unification of quantum cosmology with the particle phenomenology of the standard model, inflation theory, and CMB observations.

I. INTRODUCTION

At the dawn of inflation theory two prescriptions for the quantum state of the Universe were seriously considered as a source of initial conditions for inflation. These are the so-called no-boundary [1] and tunneling [2,3] cosmological wave functions (see also [4] for a general review), whose semiclassical amplitudes are roughly inversely proportional to one another. In the model of chaotic inflation driven in the slow-roll approximation by the inflaton field φ with the potential $V(\varphi)$, these amplitudes read as $|\Psi_\pm(\varphi)| \simeq \exp(\mp S_E(\varphi)/2)$, where $\pm$ label, respectively, the no-boundary/tunneling wave functions. Here, $S_E(\varphi)$ is the Einstein action of the Euclidean de Sitter instanton S^4 with the effective cosmological constant given by the value of the inflaton field $\Lambda_{\text{eff}} = V(\varphi)/M_{\text{P}}^2$,

$$S_E(\varphi) \simeq -\frac{24\pi^2 M_{\text{P}}^4}{V(\varphi)}, \qquad (1)$$

in units of the reduced Planck mass $M_{\text{P}}^2 = 1/8\pi G$ ($\hbar = 1 = c$). The no-boundary state was originally formulated as a path integral over Euclidean four-geometries; the tunneling state in the form of a path integral over Lorentzian metrics was presented in [3,5], and both wave functions were also obtained as solutions of the minisuperspace Wheeler-DeWitt equation.

The no-boundary and tunneling states lead to opposite physical conclusions. In particular, in view of the negative value of the Euclidean de Sitter action the no-boundary state strongly enhances the contribution of empty universes with $V(\varphi) = 0$ in the full quantum state and, thus, leads to the very counterintuitive conclusion that infinitely large universes are infinitely more probable than those of a finite size—a property which underlies the once very popular but now nearly forgotten big-fix mechanism of Coleman [6]. On the other hand, the tunneling state favors big values of $V(\varphi)$ capable of generating inflationary scenarios. Thus, it would seem that the tunneling prescription is physically more preferable than the no-boundary one. However, the status of the tunneling prescription turns out to be not so simple and even controversial.

Naive attempts to go beyond the minisuperspace approximation lead to unnormalizable states in the sector of spatially inhomogeneous degrees of freedom for matter and metric and invalidate, in particular, the usual Wick rotation from the Lorentzian to the Euclidean spacetime. This problem was partly overcome by imposing the normalizability condition on the matter part of the solution of the Wheeler-DeWitt equation [7], but the situation remained controversial for the following reason.

Modulo the issue of quantum interference between the "contracting" and "expanding" branches of the cosmological wave function discussed, for example, in [4,7–9]; the amplitudes of the no-boundary and tunneling branches of such a semiclassical solution take the form

$$|\Psi_\pm(\varphi, \Phi(\mathbf{x}))| = \exp(\mp\tfrac{1}{2}S_E(\varphi))|\Psi_{\text{matter}}(\varphi, \Phi(\mathbf{x}))|, \quad (2)$$

where $\Phi(\mathbf{x})$ is a set of matter fields separate from the

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spatially homogeneous inflaton, and $\Psi_{\text{matter}}(\varphi, \Phi(\mathbf{x}))$ is their normalizable (quasi-Gaussian) part in the full wave function—in essence representing the Euclidean de Sitter invariant vacuum of linearized fields $\Phi(\mathbf{x})$ on the quasi–de Sitter background with $\Lambda_{\text{eff}} = V(\varphi)/M_{\text{P}}^2$. Quantum averaging over $\Phi(\mathbf{x})$ then leads to the following quantum distribution of the inflaton field:

$$\rho_{\pm}^{\text{1-loop}}(\varphi) = \int d[\Phi(\mathbf{x})] |\Psi_{\pm}(\varphi, \Phi(\mathbf{x}))|^2$$
$$= \exp(\mp S_E(\varphi) - S_E^{\text{1-loop}}(\varphi)), \qquad (3)$$

where $S_E^{\text{1-loop}}(\varphi) = (1/2)\,\text{Tr}\ln(\delta^2 S_E[\varphi, \Phi]/\delta\Phi(x)\delta\Phi(y))$ is the contribution of the UV divergent one-loop effective action [10–12]. With the aid of this algorithm a sharp probability peak was obtained in the *tunneling* distribution $\rho_{-}^{\text{1-loop}}(\varphi)$ for the model with a strong nonminimal coupling of the inflaton to gravity [10,13,14]. This peak was interpreted as generating the quantum scale of inflation—the initial condition for its inflationary scenario. Quite remarkably, for accidental reasons this result was free from the usual UV renormalization ambiguity. It did not require application of the renormalization scheme of absorbing the UV divergences into the redefinition of the coupling constants in the tree-level action $S_E(\varphi)$.

However, beyond the one-loop approximation and for other physical correlators the situation changes, and one has to implement a UV renormalization in full. This means the necessity to isolate the UV divergences as local counterterms and absorb them in the redefinition of the coupling constants in the bare action $S_E[\varphi, \Phi]$ whose on-shell value $S_E(\varphi)$ gives the dominant part of (3).[1] But with the $\mp S_E(\varphi)$ ambiguity in (3) this renormalization would be different for the tunneling and no-boundary states. For instance, an asymptotically free theory in the no-boundary case (associated with the usual Wick rotation to the Euclidean spacetime) will not be asymptotically free in the tunneling case. The tunneling versus no-boundary gravitational modification of the theory will contradict basic field-theoretical results in flat spacetime. This strongly invalidates a naive construction of the tunneling state of the above type. In particular, it does not allow one to go beyond the one-loop approximation in the model of nonminimally coupled in-

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[1]Einstein gravity theory is perturbatively nonrenormalizable by a finite set of counterterms, so under a consistent renormalization we understand the possibility to unambiguously absorb all UV divergences into an infinite set of quasilocal operators initially contained in $S_E[\varphi, \Phi]$, or the truncated set of those when the higher-dimensional operators are suppressed by the effective theory cutoff. The latter case holds for heavy massive fields admitting a local expansion of their quantum effective action. In particular, this includes the model considered in Sec. III for which the renormalization reduces, modulo certain modifications, to the RG analysis of the SM in the external gravitational field.

flaton and perform its renormalization group (RG) improvement.

Here we suggest a solution of this problem by formulating a new path-integral prescription for the tunneling state of the Universe. This formulation is based on a recently suggested construction of the cosmological density matrix [15] which describes a microcanonical ensemble of cosmological models [16]. The statistical sum of this ensemble was calculated in a spatially closed model with a generic set of scalar, spinor, and vector fields conformally coupled to gravity. It was obtained in the saddle-point approximation dominated by the contribution of the thermal cosmological instantons of topology $S^3 \times S^1$. These instantons also include the vacuum S^4 topology treated as a limiting case of the compactified time dimension S^1 in $S^3 \times S^1$ being ripped in the transition from $S^3 \times S^1$ to S^4. This limiting case exactly recovers the Hartle-Hawking state of [1], so that the whole construction of [15,16] can be considered as a generalization of the vacuum no-boundary state to the quasithermal no-boundary ensemble. The basic physical conclusion for this ensemble was that it exists in a bounded range of values of the effective cosmological constant, that it is capable of generating a big-boost scenario of the cosmological acceleration [17], and that its vacuum Hartle-Hawking member does not really contribute because it is suppressed by the infinite *positive* value of its action. This is a genuine effect of the conformal anomaly of quantum fields [18–20], which qualitatively changes the tree-level action (1).

Below we shall show that the above path integral actually has another saddle point corresponding to the negative value of the lapse function $N < 0$, which is gauge inequivalent to $N > 0$. In the main, this leads to the inversion of the sign of the action in the exponential of the statistical sum and, therefore, deserves the label "tunneling." In this tunneling state the thermal part vanishes and its instanton turns out to be a purely vacuum one. Finally, this construction no longer suffers from the above-mentioned controversy with the renormalization. A full quantum effective action is supposed to be calculated and renormalized by the usual set of counterterms on the background of a generic metric and then the result should be analytically continued to $N < 0$ and taken at the tunneling saddle point of the path integral over the lapse function N.

Below we shall apply this construction to a cosmological model for which the Lagrangian of the graviton-inflaton sector reads

$$\boldsymbol{L}\,(g_{\mu\nu}, \Phi) = \tfrac{1}{2}(M_{\text{P}}^2 + \xi|\Phi|^2)R - \tfrac{1}{2}|\nabla\Phi|^2 - V(|\Phi|), \quad (4)$$

$$V(|\Phi|) = \frac{\lambda}{4}(|\Phi|^2 - \nu^2)^2, \qquad |\Phi|^2 = \Phi^\dagger\Phi, \quad (5)$$

where Φ is the standard model (SM) Higgs boson, whose expectation value plays the role of an inflaton and which is

assumed here to possess a strong nonminimal curvature coupling with $\xi \gg 1$. Here, as above, M_P is a reduced Planck mass, λ is a quartic self-coupling of Φ, and v is an electroweak (EW) symmetry breaking scale.

The early motivation for this model with a grand unified theory (GUT) type boson Φ [21,22] was to avoid an exceedingly small quartic coupling λ by invoking a nonminimal coupling with a large ξ. This was later substantiated by the hope to generate the no-boundary/tunneling initial conditions for inflation [13,14]. This theory but with the SM Higgs boson Φ instead of the abstract GUT setup of [13,14] was suggested in [23], extended in [24] to the one-loop level, and considered regarding its reheating mechanism in [25]. The RG improvement in this model has predicted CMB parameters—the amplitude of the power spectrum and its spectral index—compatible with WMAP observations in a finite range of values of the Higgs mass, which is close to the widely accepted range dictated by the EW vacuum stability and perturbation theory bounds [26–31].

The purpose of our paper is to extend the results of [29,30] by suggesting that this model does not only have WMAP-compatible CMB perturbations, but can also generate the initial conditions for the inflationary background upon which these perturbations propagate. These initial conditions are realized in the form of a *sharp probability peak* in the tunneling distribution function of the inflaton.

Our paper is organized as follows. In Sec. II we present the path-integral formulation for the tunneling state and derive the relevant distribution in the space of values of the cosmological constant. In Sec. III we apply this distribution to the gravitating SM model with the graviton-inflaton sector (4) and obtain the probability peak in the distribution of the initial value of the Higgs inflaton. Section IV contains a short discussion.

II. TUNNELING COSMOLOGICAL WAVE FUNCTION WITHIN THE PATH-INTEGRAL FORMULATION

The path integral for the microcanonical statistical sum in cosmology [16] can be cast into the form of an integral over a minisuperspace lapse function $N(\tau)$ and scale factor $a(\tau)$ of a spatially closed Euclidean Friedmann-Robertson-Walker (FRW) metric $ds^2 = N^2(\tau)d\tau^2 + a^2(\tau)d^2\Omega^{(3)}$,

$$e^{-\Gamma} = \int D[a, N]e^{-S_{\text{eff}}[a,N]}, \qquad (6)$$

$$e^{-S_{\text{eff}}[a,N]} = \int D\Phi(x)e^{-S[a,N;\Phi(x)]}. \qquad (7)$$

Here, $S_{\text{eff}}[a, N]$ is the Euclidean effective action of all inhomogeneous "matter" fields $\Phi(x) = (\phi(x), \psi(x), A_\mu(x), h_{\mu\nu}(x), \ldots)$ (which include also metric perturbations) on the minisuperspace background of the FRW metric, $S[a, N; \Phi(x)]$ is the classical Euclidean action,

and the integration runs over periodic fields on the Euclidean spacetime with a compactified time τ (of $S^1 \times S^3$ topology).

It is important that the integration over the lapse function N runs along the imaginary axis from $-i\infty$ to $+i\infty$ because this Euclidean path integral represents, in fact, the transformed version of the integral over metrics with Lorentzian signature. This transformation is the usual Wick rotation which can be incorporated by the transition from the Lorentzian lapse function N_L to the Euclidean one N by the relation $N_L = iN$ [16]. The Lorentzian path integral, in turn, fundamentally follows from the definition of the microcanonical ensemble in quantum cosmology which includes all *true* physical configurations satisfying the quantum first-class constraints—the Wheeler-DeWitt equations. The projector onto these configurations is realized in the integrand of the path integral by the delta functions of the Hamiltonian (and momentum) constraints. The Fourier representation of these delta functions in terms of the integral over the conjugated Lagrange multipliers—the lapse N_L (and shift) functions—implies an integration with limits at infinity, $-\infty < N_L < \infty$, which explains the range of integration over the Euclidean N.

It should be mentioned that a full nonperturbative evaluation of the path integral would require a careful inspection of the infinite contours in the complex N plane that render the integral convergent; see, for example, [32]. However, such an inspection is not needed here because we are dealing with a semiclassical approximation in which only the vicinity of the saddle point enters.

The convenience of writing the path integral (6) in the Euclidean form follows from the needs of the semiclassical approximation. In this approximation, it is dominated by the contribution of a saddle point, $\Gamma_0 = S_{\text{eff}}[a_0, N_0]$, where $a_0 = a_0(\tau)$ and $N_0 = N_0(\tau)$ solve the equation of motion for $S_{\text{eff}}[a, N]$ and satisfy periodicity conditions dictated by the definition of the statistical sum. Such periodic solutions exist in the Euclidean domain with real N rather than in the Lorentzian one with the imaginary lapse. This means that the contour of integration over N along the imaginary axis should be deformed into the complex plane to traverse the real axis at some $N_0 \neq 0$ corresponding to the Euclidean solution of the equations of motion for the minisuperspace action.

The residual one-dimensional diffeomorphism invariance of this action (which is gauged out by the gauge-fixing procedure implicit in the integration measure $D[a, N]$) allows one to fix the ambiguity in the choice of N_0. There remains only a double-fold freedom in this choice actually inherited from the sign indefiniteness of the integration range for N_L. This freedom is exhausted by either positive, $N_0 > 0$, or negative, $N_0 < 0$, values of the lapse, because, on the one hand, all values in each of these equivalence classes are gauge equivalent and, on the other hand, no continuous family of nondegenerate diffeomor-

phisms exists relating these classes to one another. Without loss of generality one can choose as representatives of these classes $N_\pm = \pm 1$ and label the relevant solutions and on-shell actions, respectively, as $a_\pm(\tau)$ and

$$\Gamma_\pm = S_{\text{eff}}[a_\pm(\tau), \pm 1]. \qquad (8)$$

Gauge inequivalence of these two cases, $\Gamma_- \neq \Gamma_+$, is obvious because, for example, all local contributions to the effective action are odd functionals of N, $S_{\text{local}}[a, N] = -S_{\text{local}}[a, -N]$.[2] Thus we can heuristically identify the statistical sums $\Gamma_\pm$ correspondingly with the "no-boundary" and tunneling prescriptions for the quantum state of the Universe,

$$\exp(-\Gamma_{\text{no boundary/tunnel}}) = e^{-\Gamma_\pm}. \qquad (9)$$

In other words, we use this equation to define no boundary and tunneling in the first place. This result shows that for both prescriptions a full quantum effective action as a whole sits in the exponential of the partition function without any splitting into the minisuperspace and matter contributions weighted by different sign factors as in (3). This means that the usual renormalization scheme is applicable to the calculation of (8)—generally covariant UV counterterms should be calculated on the background of a generic metric and afterward evaluated at the FRW metric with $N = \pm 1$, depending on the choice of either the no-boundary or tunneling prescription. Below we demonstrate how this procedure works for the system dominated by quantum fields with heavy masses, whose effective action admits a local expansion in powers of the spacetime curvature and matter field gradients.

For such a system the Euclidean effective action takes the form

$$S_{\text{eff}}[g_{\mu\nu}] = \int d^4 x\, g^{1/2}\Big(M_{\text{P}}^2\Lambda - \frac{M_{\text{P}}^2}{2}R(g_{\mu\nu}) + \cdots\Big), \qquad (10)$$

where we disregard the terms of higher orders in the curvature and derivatives of the mean values of matter fields. Here the cosmological term and the (reduced) Planck mass squared $M_{\text{P}}^2 = 1/8\pi G$ can be considered as functions of these mean values and treated as constants in the approximation of slowly varying fields. This effective action does not contain the thermal part characteristic of the statistical ensemble [15] because for heavy quanta the

radiation bath is not excited. This is justified by the fact that the effective temperature of this bath turns out to be vanishing.

In fact, the minisuperspace action functional for (10) reads in units of $m_{\text{P}}^2 = 3\pi/4G = 6\pi^2 M_{\text{P}}^2$ as

$$S_{\text{eff}}[a, N] = m_{\text{P}}^2 \int d\tau N(-aa'^2 - a + H^2 a^3), \qquad (11)$$

where $a' \equiv da/Nd\tau$, and we use the notation for the cosmological constant $\Lambda = 3H^2$ in terms of the effective Hubble factor H. Then the saddle point for the path integral (6)—the stationary configuration with respect to variations of the lapse function $\delta S_{\text{eff}}[a, N]/\delta N = 0$—satisfies the Euclidean Friedmann equation

$$a'^2 = 1 - H^2 a^2. \qquad (12)$$

It has one turning point at $a_+ = 1/H$ below which the real solution interpolates between $a_- \equiv a(0) = 0$ and a_+. In the gauge $N = \pm 1$ for both no-boundary/tunneling cases this solution describes the Euclidean de Sitter metric, that is, one hemisphere of S^4,

$$a_\pm(\tau) = \frac{1}{H}\sin(H\tau). \qquad (13)$$

After the bounce from the equatorial section of the maximal scale factor a_+, this solution spans at the contraction phase the rest of the full four-sphere.[3] Thus, this solution is not periodic and in the terminology of [15] describes a purely vacuum contribution to the statistical sum (6). As shown in [15], the effective temperature of this state is determined by the inverse of the full period of the instanton solution measured in units of the conformal time η. Therefore, for (13) it vanishes because this period between the poles of this spherical instanton is divergent,

$$\eta = 2\int_0^{\pi/2H} \frac{d\tau}{a(\tau)} \longrightarrow \infty. \qquad (14)$$

This justifies the absence of the thermal part in (10).

Thus, with $N = \pm 1$ the no-boundary and tunneling on-shell actions (8) read

$$\Gamma_\pm = \mp \frac{8\pi^2 M_{\text{P}}^2}{H^2}, \qquad (15)$$

and the object of major interest here—the tunneling partition function in the space of positive values of $H^2 = \Lambda/3$—is given by

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[2]This property can be manifestly seen in the Arnowitt-Deser-Misner (ADM) form of the Einstein action under the $(3 + 1)$ splitting of spacetime, cf. Eq. (11). It is important that it is exactly this form which follows from the microcanonical definition of the path integral [16] rather than the manifestly covariant metric expression with the square root $\sqrt{^4g}$ (which hides an implicit necessity of the sign choice under an analytic continuation). The microcanonical definition of the path integral is based on the Hamiltonian form of the gravitational action and yields after the Gaussian integration over momenta the path integral (6) and (7) with the action explicitly featuring an overall odd factor N in the ADM Lagrangian.

[3]The formal analytic extension from $N_0 = 1$ to $N_0 = -1$ should not, of course, be applied to $a(\tau) = \sin(N_0 H\tau)/H$ to give a negative $a(\tau)$ instead of (13), because in contrast to the sign-indefinite Lagrange multiplier N the path integration over $a(\tau)$ in (6) semiclassically always runs in the vicinity of its positive geometrically meaningful value. For this reason, $a(\tau)$ never brings sign factors into the on-shell action even though it enters the action with odd powers.

$$\rho_{\text{tunnel}}(\Lambda) = \exp\left(-\frac{24\pi^2 M_P^2}{\Lambda}\right), \qquad \Lambda > 0. \qquad (16)$$

It coincides with the semiclassical tunneling wave function of the Universe [2], $|\Psi_{\text{tunnel}}|^2 \simeq \exp(-8\pi^2 M_P^2/H^2)$, derived from the Wheeler-DeWitt equation in the tree-level approximation.

At the turning point a_+, the solution (13) can be analytically continued to the Lorentzian regime, $a_L(t) = a(\pi/2H + it)$. The scale factor then expands eternally as

$$a_L(t) = \frac{1}{H} \cosh(Ht), \qquad (17)$$

which can be interpreted as representing the distributions of scale factors in the quantum ensemble (after decoherence) of de Sitter models distributed according to (16). Note that the attempt to extend this ensemble to negative Λ fails, because Eq. (12) with $H^2 < 0$ does not have turning points with nucleating real Lorentzian geometries. Moreover, virtual cosmological models with Euclidean signature are also forbidden in the tunneling state because their positive Euclidean action diverges to infinity, so that $\rho_{\text{tunnel}}(\Lambda) = 0$ for $\Lambda < 0$.

III. QUANTUM ORIGIN OF THE UNIVERSE WITH THE SM HIGGS INFLATON NONMINIMALLY COUPLED TO CURVATURE

The partition function of the above type can serve as a source of initial conditions for inflation only when the cosmological constant $\Lambda = 3H^2$ becomes a composite field capable of a decay at the exit from inflation. Usually this is a scalar inflaton field whose quantum mean value φ is nearly constant in the slow-roll regime, and its effective potential $V(\varphi)$ plays the role of the cosmological constant driving the inflation. When the contribution of the inflaton gradients is small, the above formalism remains applicable also with the inclusion of this field whose ultimate effect reduces to the generation of the effective cosmological constant $\Lambda = V(\varphi)/M_P^2$ and the effective Planck mass.

These constants are the coefficients of the zeroth and first order terms of the effective action expanded in powers of the curvature, and they incorporate radiative corrections due to all quantum fields in the path integral (7). Now there is no mismatch between the signs of the tree-level and loop parts of the partition function. Therefore, one can apply the usual renormalization and, if necessary, the RG improvement to obtain the full effective action $S_{\text{eff}}[g_{\mu\nu}, \varphi]$ and then repeat the procedure of the previous section. In the slow-roll approximation the effective action has the general form

$$S_{\text{eff}}[g_{\mu\nu}, \varphi] = \int d^4x g^{1/2}\left(V(\varphi) - U(\varphi)R(g_{\mu\nu})\right.$$
$$\left. + \frac{1}{2} G(\varphi)(\nabla\varphi)^2 + \cdots\right), \qquad (18)$$

where $V(\varphi)$, $U(\varphi)$, and $G(\varphi)$ are the coefficients of the derivative expansion, and we disregard the contribution of higher-derivative operators. With the slowly varying inflaton the coefficients $V(\varphi)$ and $U(\varphi)$ play the role of the effective cosmological and Planck mass constants, so that one can identify in (10) and (11) the effective $M_P^2 = m_P^2/6\pi^2$ and H^2, respectively, with $2U(\varphi)$ and $V(\varphi)/6U(\varphi)$. Therefore, the tunneling partition function (16) becomes the following distribution of the field φ:

$$\rho_{\text{tunnel}}(\varphi) = \exp\left(-\frac{24\pi^2 M_P^4}{\hat{V}(\varphi)}\right), \qquad (19)$$

$$\hat{V}(\varphi) = \left(\frac{M_P^2}{2}\right)^2 \frac{V(\varphi)}{U^2(\varphi)}, \qquad (20)$$

where $\hat{V}(\varphi)$ in fact coincides with the potential in the Einstein frame of the action (18) [29,30].

Now we apply this formalism to the model (4) of inflation driven by the SM Higgs inflaton $\varphi = (\Phi^\dagger\Phi)^{1/2}$. As shown in [29,30], the one-loop RG improved action in this model has for large φ the form (18) with the coefficient functions

$$V(\varphi) = \frac{\lambda(t)}{4} Z^4(t)\varphi^4, \qquad (21)$$

$$U(\varphi) = \tfrac{1}{2}(M_P^2 + \xi(t)Z^2(t)\varphi^2), \qquad (22)$$

$$G(\varphi) = Z^2(t), \qquad (23)$$

determined in terms of the running couplings $\lambda(t)$ and $\xi(t)$, and the field renormalization $Z(t)$. They incorporate a summation of powers of logarithms and belong to the solution of the RG equations which at the inflationary stage with a large $\varphi \sim M_P/\sqrt{\xi}$ and large $\xi \gg 1$ read as (see [29,30] for details)

$$\frac{d\lambda}{dt} = \frac{A}{16\pi^2}\lambda - 4\gamma\lambda, \qquad (24)$$

$$\frac{d\xi}{dt} = \frac{6\lambda}{16\pi^2}\xi - 2\gamma\xi, \qquad (25)$$

and $dZ/dt = \gamma Z$. Here, γ is the anomalous dimension of the Higgs field, the running scale $t = \ln(\varphi/M_t)$ is normalized at the top quark mass $\mu = M_t$, and $A = A(t)$ is the running parameter of the *anomalous scaling*. This quantity was introduced in [10] as the prelogarithm coefficient of the overall effective potential of all SM physical particles and Goldstone modes. Because of their quartic, gauge, and Yukawa couplings with φ, they acquire masses $m(\varphi) \sim \varphi$

and for large φ give rise to the asymptotic behavior of the Coleman-Weinberg potential,

$$V^{\text{1-loop}}(\varphi) = \sum_{\text{particles}} (\pm 1) \frac{m^4(\varphi)}{64\pi^2} \ln \frac{m^2(\varphi)}{\mu^2}$$

$$\simeq \frac{\lambda A}{128\pi^2} \varphi^4 \ln \frac{\varphi^2}{\mu^2}, \tag{26}$$

which can serve as a definition of A.

The importance of this quantity and its modification due to the RG running of the nonminimal coupling $\xi(t)$,

$$A_I = A - 12\lambda \tag{27}$$

[A_I gives the running of the ratio λ/ξ^2, $16\pi^2(d/dt) \times (\lambda/\xi^2) = A_I(\lambda/\xi^2)$] is that for $\xi \gg 1$ mainly these parameters determine the quantum inflationary dynamics [14,33] and yield the parameters of the CMB generated during inflation [24]. In particular, the value of φ at the beginning of the inflationary stage of duration N in units of the e-folding number turns out to be [24]

$$\varphi^2 = -\frac{64\pi^2 M_P^2}{\xi A_I(t_{\text{end}})} (1 - e^x), \tag{28}$$

$$x \equiv \frac{N A_I(t_{\text{end}})}{48\pi^2}, \tag{29}$$

where a parameter x has been introduced which directly involves $A_I(t_{\text{end}})$ taken at the end of inflation, $t_{\text{end}} = \ln(\varphi_{\text{end}}/M_t)$, $\varphi_{\text{end}} \simeq 2M_P/\sqrt{3\xi}$. This parameter also enters simple algorithms for the CMB power spectrum $\Delta_\zeta^2(k)$ and its spectral index $n_s(k)$. As shown in [29,30], the application of these algorithms under the observational constraints $\Delta_\zeta^2(k_0) \simeq 2.5 \times 10^{-9}$ and $0.94 < n_s(k_0) < 0.99$ (the combined WMAP+BAO+SN data at the pivot point $k_0 = 0.002$ Mpc^{-1} corresponding to $N \simeq 60$ [34]) gives the CMB-compatible range of the Higgs mass 135.6 GeV $\lesssim M_H \lesssim 184.5$ GeV, both bounds being determined by the lower bound on the CMB spectral index.

Now we want to show that, in addition to the good agreement of the spectrum of cosmological perturbations with the CMB data, this model can also describe the mechanism of generating the cosmological *background* itself upon which these perturbations exist. This mechanism consists in the formation of the initial conditions for inflation in the form of a sharp probability peak in the distribution function (19) at some appropriate value of the inflaton field φ_0 with which the Universe as a whole starts its evolution. The shape and the magnitude of the potential (20) depicted in Fig. 1 for several values of the Higgs mass clearly indicates the existence of such a peak.

Indeed, the negative of the inverse potential damps to zero after exponentiation of the probability of those values of φ at which $\hat{V}(\varphi) = 0$ and, vice versa, enhances the probability at the positive maxima of the potential. The

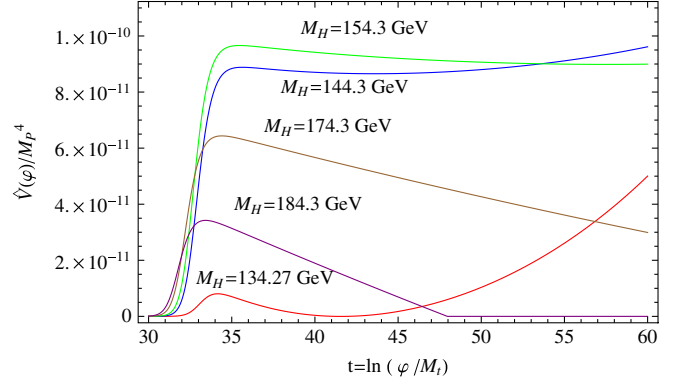
FIG. 1 (color online). The succession of effective potential graphs above the EW vacuum instability threshold $M_H^{\text{inst}} = 134.27$ GeV up to $M_H = 184.3$ GeV showing the occurrence of a metastable vacuum followed for high M_H by the formation of a negative slope branch. Local peaks of $\hat{V}$ situated at $t = 34$–35 grow with M_H for $M_H \lesssim 160$ GeV and start decreasing for larger M_H [29].

pattern of this behavior with growing Higgs mass M_H is as follows.

As is known, for low M_H the SM has a domain of unstable EW vacuum, characterized by negative values of running $\lambda(t)$ at certain energy scales. Thus we begin with the EW vacuum instability threshold [35,36] which exists in this gravitating SM at $M_H^{\text{inst}} \approx 134.27$ GeV [29,30] and which is slightly lower than the CMB-compatible range of the Higgs mass (M_H^{inst} is chosen in Fig. 2 and for the lowest curve in Fig. 1). The potential $\hat{V}(\varphi)$ drops to zero at $t_{\text{inst}} \simeq 41.6$, $\varphi_{\text{inst}} \sim 80 M_P$, and forms a false vacuum [29,30] separated from the EW vacuum by a large peak at $t \simeq 34$. Correspondingly, the probability of creation of the Universe with the initial value of the inflaton field at the EW scale $\varphi = v$ and at the instability scale φ_{inst} is damped to zero, while the most probable value belongs to this peak.
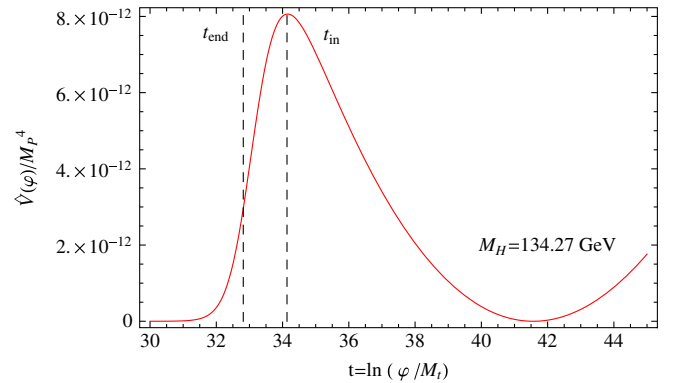

FIG. 2 (color online). The effective potential for the instability threshold $M_H^{\text{inst}} = 134.27$ GeV. A false vacuum occurs at the instability scale $t_{\text{inst}} \simeq 41.6$, $\varphi \sim 80 M_P$. An inflationary domain for a $N = 60$ CMB perturbation (ruled out by the WMAP bounds) is marked by dashed lines [29].
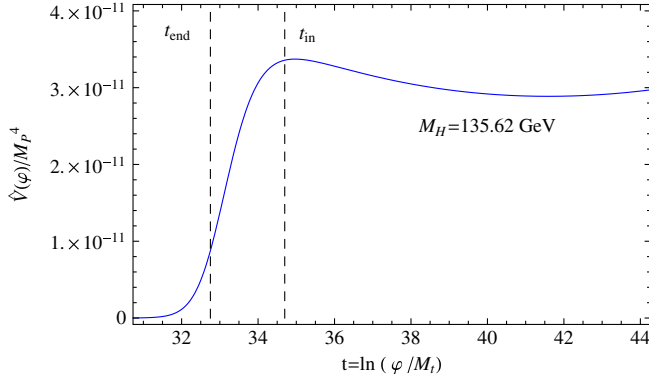
FIG. 3 (color online). Inflaton potential at the lowest CMB-compatible value of M_H with a metastable vacuum at $t \simeq 42$ [29].

The inflationary stage of the formation of the pivotal $N = 60$ CMB perturbation (from the moment t_{in} of the first horizon crossing until the end of inflation t_{end}), which is marked by dashed lines in Fig. 2, lies to the left of this peak. This conforms to the requirement of the chronological succession of the initial conditions for inflation and the formation of the CMB spectra.

The above case is, however, below the CMB-compatible range of M_H and was presented here only for illustrative purposes.[4] An important situation occurs at higher Higgs masses from the lower CMB bound on $M_H \simeq 135.6$ GeV until about 160 GeV. Here we get a family of a metastable vacua with $\hat{V} > 0$. An example is the plot for the lower CMB bound $M_H = 135.62$ GeV depicted in Fig. 3. Despite the shallowness of this vacuum its small maximum generates via (19) a sharp probability peak for the initial inflaton field. This follows from an extremely small value of $\hat{V}/M_P^4 \sim 10^{-11}$, the reciprocal of which generates a rapidly changing exponential of (19). The location of the peak again precedes the inflationary stage for a pivotal $N = 60$ CMB perturbation (also marked by dashed lines in Fig. 3).

For even larger M_H these metastable vacua get replaced by a negative slope of the potential which interminably decreases to zero at large t (at least within the perturbation theory range of the model); see Fig. 1. Therefore, for large M_H close to the upper CMB bound 185 GeV, the probability peak of (19) gets separated from the nonperturbative domain of large over-Planckian scales due to a fast drop of $\hat{V} \sim \lambda/\xi^2$ to zero. This, in turn, follows from the fact that

$\xi(t)$ grows much faster than $\lambda(t)$ when they both start approaching their Landau pole [29].

The location φ_0 of the probability peak and its quantum width can be found in analytical form, and their derivation shows the crucial role of the running $A_I(t)$ for the formation of initial conditions for inflation. Indeed, the exponential of the tunneling distribution (19) for $M_P^2/\xi\varphi^2 \ll 1$ reads as

$$\Gamma_-(\varphi) = 24\pi^2 \frac{M_P^4}{\hat{V}(\varphi)} \simeq 96\pi^2 \frac{\xi^2}{\lambda}\left(1 + \frac{2M_P^2}{\xi Z^2 \varphi^2}\right), \quad (30)$$

and in view of the RG equations (24) and (25) has an extremum satisfying the equation

$$\varphi \frac{d\Gamma}{d\varphi} = \frac{d\Gamma}{dt} = -\frac{6\xi^2}{\lambda}\left(A_I + \frac{64\pi^2 M_P^2}{\xi Z^2 \varphi^2}\right) = 0, \quad (31)$$

where we again neglect higher order terms in $M_P^2/\xi Z^2 \varphi^2$ and $A_I/64\pi^2$ (extending beyond the one-loop order). Here, A_I is the anomalous scaling introduced in (26) and (27)—the quantity that should be negative for the existence of the solution for the probability peak,

$$\varphi_0^2 = -\frac{64\pi^2 M_P^2}{\xi A_I Z^2}\bigg|_{t=t_0}. \quad (32)$$

As shown in [29,30], this quantity is indeed negative. In the CMB-compatible range of M_H its running starts from the range $-36 \lesssim A_I(0) \lesssim -23$ at the EW scale and reaches small but still negative values in the range $-11 \lesssim A_I(t_{end}) \lesssim -2$ at the inflation scale. Also, the running of $A_I(t)$ and $Z(t)$ is very slow—the quantities belonging to the two-loop order—and the duration of inflation is very short $t_0 \sim t_{in} \simeq t_{end} + 2$ [29,30]. Therefore, $A_I(t_0) \simeq A_I(t_{end})$, and these estimates apply also to $A_I(t_0)$. As a result, the second derivative of the tunneling on-shell action is positive and very large,

$$\frac{d^2\Gamma_-}{dt^2} \simeq -\frac{12\xi^2}{\lambda}A_I \gg 1, \quad (33)$$

which gives an extremely small value of the quantum width of the probability peak,

$$\frac{\Delta\varphi^2}{\varphi_0^2} = -\frac{\lambda}{12\xi^2}\frac{1}{A_I}\bigg|_{t=t_0} \sim 10^{-10}. \quad (34)$$

This width is about $(24\pi^2/|A_I|)^{1/2}$ times—1 order of magnitude—higher than the CMB perturbation at the pivotal wavelength $k^{-1} = 500$ Mpc (which we choose to correspond to $N = 60$). The point φ_{in} of the horizon crossing of this perturbation (and other CMB waves with different N's) easily follows from Eq. (28) which in view of $A_I(t_0) \simeq A_I(t_{end})$ takes the form

$$\frac{\varphi_{in}^2}{\varphi_0^2} = 1 - \exp\left(-N\frac{|A_I(t_{end})|}{48\pi^2}\right). \quad (35)$$

It indicates that for wavelengths longer than the pivotal one

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[4]Another interesting range of M_H is below the instability threshold M_H^{inst} where $\hat{V}$ becomes negative in the "true" high energy vacuum. As mentioned in the previous section, the tunneling state rules out aperiodic solutions of effective equations with $H^2 < 0$, which cannot contribute to the quantum ensemble of expanding Lorentzian signature models. Therefore, this range is semiclassically ruled out not only by the instability arguments, but also contradicts the tunneling prescription.

the instant of horizon crossing approaches the moment of "creation" of the Universe, but always stays chronologically succeeding it, as it must.

IV. CONCLUSIONS AND DISCUSSION

In this paper we have constructed the tunneling quantum state of the Universe based on the path integral for the microcanonical ensemble in cosmology. The corresponding apparent ensemble from the quantum state exists in the unbounded positive range of the effective cosmological constant, unlike the no-boundary state discussed in [15,16] whose apparent ensemble is bounded by the reciprocated coefficient of the topological term in the overall conformal anomaly. Also, in contrast to the no-boundary case, the tunneling state turns out to be a radiation-free vacuum one.

The status of the tunneling versus no-boundary states is rather involved. In fact, the formal Euclidean path integral (6) is a transformed version of the microcanonical path integral over Lorentzian metrics, so that its lapse function integration runs along the imaginary axis from $-i\infty$ to $+i\infty$ [16].[5] The absence of periodic solutions for stationary points of (6) with the Lorentzian signature makes one to distort the contour of integration over N into a complex plane, so that it traverses the real axis at the points $N = +1$ or $N = -1$ which give rise to no-boundary or tunneling states. One can show that the no-boundary thermal part of the statistical sum of [15] is not analytic in the full complex plane of N. The $N \gtrless 0$ domains are separated by the infinite sequence of its poles densely filling the imaginary axes of N. Therefore, the contour of integration passing through both points $N = \pm 1$ is impossible, and the no-boundary and tunneling states cannot be obtained by analytic continuation from one another.[6] They represent alternative solutions (quantum states) of the Wheeler-DeWitt equation.

The path-integral formulation of the tunneling state admits a consistent renormalization scheme and a RG resummation which is very efficient in cosmology according to a series of recent papers [26–31]. For this reason we have applied the obtained tunneling distribution to a recently considered model of inflation driven by the SM Higgs boson nonminimally coupled to curvature. In this way a complete cosmological scenario was obtained, embracing the formation of initial conditions for the inflationary background (in the form of a sharp probability peak in the inflaton field distribution) and the ongoing generation of the CMB perturbations on this background. As was shown in [29,30], the comparison of the CMB amplitude and the spectral index with the WMAP observations impose bounds on the allowed range of the Higgs mass. These bounds turn out to be remarkably consistent with the widely accepted EW vacuum stability and perturbation theory restrictions. Interestingly, the behavior of the running anomalous scaling $A_I(t) < 0$, being crucially important for these bounds, also guarantees the existence of the obtained probability peak. The quantum width of this peak is 1 order of magnitude higher than the amplitude of the CMB spectrum at the pivotal wavelength, which could entail interesting observational consequences. Unfortunately, this quantum width is hardly measurable directly because it corresponds to an infinite wavelength perturbation [a formal limit of $N \to \infty$ in (35)], but indirect effects of this quantum trembling of the cosmological background deserve further study.

We have entertained here the idea that we can obtain sensible predictions from peaks in the cosmological wave function. This is, of course, different from approaches based on the anthropic principle. We find it intriguing, however, that a consistent scenario based on our more traditional approach may be possible and even falsifiable.

To summarize, the obtained results bring to life a convincing unification of quantum cosmology with the particle phenomenology of the SM, inflation theory, and CMB observations. They support the hypothesis that an appropriately extended standard model [37,38] can be a consistent quantum field theory all the way up to quantum gravity and perhaps explain the fundamentals of all major phenomena in early and late cosmology.

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[5]This might seem to be equivalent to the tunneling path integral of [3,5], but the class of metrics integrated over is very different. We do not impose by hands $a_- = 0$ as the boundary condition, but derive it from the saddle-point approximation for the integral over formally periodic configurations. The fact that periodicity gets violated by the boundary condition $a_- = 0$ implies that the *a priori* postulated tunneling statistical ensemble is exhausted at the dynamical level by the contribution of a pure vacuum state [15,16].

[6]In the case of the vacuum no-boundary state when the vanishing thermal part of the effective action cannot present an obstacle to analytic continuation in the complex plane of N the situation stays the same. Indeed, any integration contour from $-i\infty$ to $+i\infty$ crosses the real axes an odd number of times, so that the contribution of only one such crossing survives, because any two (gauge-equivalent) saddle points traversed in opposite directions give contributions canceling one another.

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