# *CP*-phase effects on the effective neutrino mass $m_{ee}$ in the case of quasidegenerate neutrinos

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We study the possibility that the three mass states of the ordinary active neutrinos actually split into pairs of quasidegenerate states, with $\Delta m^2_{kk'} \sim 10^{-12}$ eV$^2$ or less, as a result of mixing of active neutrinos with sterile neutrinos. While in laboratory experiments these quasidegenerate pairs will look identical to single active states, the *CP* phase factors associated with active-sterile mixing might cause cancellations in the effective electron neutrino mass $m_{ee}$ measured in the neutrinoless double beta decay experiments thereby revealing the split nature of states.

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The possible existence of active-sterile neutrino mixing with a high mass degeneracy will not be revealed in oscillation experiments probing atmospheric, solar, or laboratory produced neutrinos if the degeneracy is of the order of $\Delta m^2 \sim 10^{-12}$ eV$^2$ or less [1]. However, as we have pointed out earlier, one might obtain hints of such degeneracy through the effects it may have on the measured fluxes of the standard active neutrinos originating in active galactic nuclei [2] or in supernovae [3,4] (for a recent study, see, e.g. [5]). For example, the ratio $\nu_e$:$\nu_\mu$:$\nu_\tau$ = 1:1:1 of neutrino fluxes from active galactic nuclei's one would expect in the standard three-neutrino-scheme [6] could be substantially distorted by the mixing of the active neutrinos with their quasidegenerate sterile counterparts.

In this paper we will examine another possible indication of the existence of quasidegenerate neutrino pairs, namely, the effects these pairs may have, through the extra *CP* phases associated with their mixing, on the so-called effective electron neutrino mass $m_{ee}$ probed in neutrinoless double beta decay ($(\beta\beta)_{0\nu}$) experiments. The neutrinoless double beta decay, which breaks the conservation of lepton number by two units, is a particularly interesting process as it is the only practical way to reveal the possible Majorana nature of neutrinos. Furthermore, the process is at the moment the only viable way to obtain information about the absolute mass scale of light neutrinos.

Generally, the effective mass $m_{ee}$ is given by

$$m_{ee} = \left|\sum_i U_{ei}^2 m_i\right|, \tag{1}$$

where the sum runs over all the mass states $\nu_i$ that has a $\nu_e$ component, $U$ is the neutrino mixing matrix ($\nu_\ell = \sum_i U_{\ell i}\nu_i$), and $m_i$ is the mass of the state $\nu_i$. The value of $m_{ee}$ depends on the phase factors of the matrix $U$, and the cancellations between different terms of the sum are possible. As a result of the cancellations, $m_{ee}$ could be smaller than any of the $m_i$. The effect of *CP* phases on the $m_{ee}$ has been previously studied in the standard three-neutrino case, e.g. in [7] and in the four-neutrino case in [8].

We will consider a scenario where there exist, in addition to the three active neutrino flavors $\nu_e$, $\nu_\mu$, $\nu_\tau$, three sterile neutrino flavors $\nu_{s1}$, $\nu_{s2}$, $\nu_{s3}$. In the standard three-neutrino case the three active neutrinos mix with each other and form three mass eigenstates. We will denote these states as $\hat{\nu}_k$ ($k = 1, 2, 3$). In our model these states are not mass eigenstates but we assume each of them to mix with a sterile state and to form with it two orthogonal superpositions that are quasidegenerate in mass. In the experiments not sensitive enough to recognize the mass difference of the states, such quasidegenerate states would look like a single state, the corresponding active state $\hat{\nu}_k$.

The mass eigenstates that result from the mixing of the active state $\hat{\nu}_k$ and the sterile state $\nu_{sk}$ are the following:

$$\nu_k = \cos\varphi_k \hat{\nu}_k - e^{i\delta_k}\sin\varphi_k \nu_{sk}, \tag{2}$$

$$\nu_{k'} = e^{i\delta_k}\sin\varphi_k \hat{\nu}_k + \cos\varphi_k \nu_{sk}. \tag{3}$$

The *CP* phases $\delta_k$ can have the values 0 or $\pi$ if *CP* is conserved; in the*CP* violating case the phase angle has some other value. The mass squared difference within each pair is assumed to be $\Delta m^2_{kk'} \sim 10^{-12}$ eV$^2$, and the mass differences between different pairs are the same as those of the mass states in the standard three-neutrino model.

The matrix elements $m_{\alpha\beta}$ between two neutrino flavors has the following general expression in terms of the neutrino masses [9]:

$$m_{\alpha\beta} = \left|\sum_{k=1}^{3} \tilde{U}^*_{\alpha k}\tilde{U}^*_{\beta k} m_k + \sum_{k'=1}^{3} \tilde{U}^*_{\alpha k'}\tilde{U}^*_{\beta k'} m_{k'}\right|, \tag{4}$$

where $m_k$, $m_{k'}$ are the masses of the neutrino mass eigenstates and $\tilde{U}_{\alpha k}$ the elements of the unitary $6 \times 6$ mixing matrix $\tilde{U}$, which diagonalizes the $6 \times 6$ neutrino mass matrix. When considering the quantity $m_{ee}$ only the matrix elements of first row of $\tilde{U}$ are relevant. They are given by

 

$$\tilde{U}_e = (\tilde{U}_{e1} \quad \tilde{U}_{e2} \quad \tilde{U}_{e3} \quad \tilde{U}_{e1'} \quad \tilde{U}_{e2'} \quad \tilde{U}_{e3'}) = (\cos\varphi_1 U_{e1}, \quad \cos\varphi_2 U_{e2}, \quad \cos\varphi_3 U_{e3}, \quad e^{-i\delta_1}\sin\varphi_1 U_{e1}, \quad e^{-i\delta_2}\sin\varphi_2 U_{e2}, \quad e^{-i\delta_3}\sin\varphi_3 U_{e3}),$$

where $U_{ek}$ are elements of the standard $3 \times 3$ neutrino mixing matrix as defined in [2].

Let us denote the mass difference between eigenstates $\nu_k$ and $\nu_{k'}$ as the $\epsilon_k$, that is, $m_{k'} = m_k + \epsilon_k$. We will then have

$$m_{ee} = \left| \sum_{k=1}^{3} (\cos^2\varphi_k + e^{2i\delta_k}\sin^2\varphi_k) U_{ek}^2 m_k + \sum_{k=1}^{3} e^{2i\delta_k}\sin^2\varphi_k U_{ek}^2 \epsilon_k \right|. \quad (5)$$

As the mass difference $\epsilon_k$ is according to our assumption very small, we can approximate it as

$$\epsilon_k \approx \frac{\Delta m_{kk'}^2}{2m_k}. \quad (6)$$

Generally, there may be $CP$ phases in the standard $3 \times 3$ active neutrino mixing matrix $U$ also. Let us assume that this is not the case here but that all $CP$ phases arise from the mixings of the active and sterile states. The values of the mixing angles and the squared mass differences are most stringently constrained by cosmology. If they are too large, neutrino oscillations would bring sterile neutrinos into thermal equilibrium with other particles leading to a conflict between theoretical and observational results concerning the primordial nucleosynthesis [10]. Because the mass differences between degenerate states are in our case very small, this constraint will not cause any problems, and the active-sterile mixing angles $\varphi_k$ can have any value between 0 and $\pi/4$.

The $CP$ phases associated with the active-sterile mixing may cause cancellations in $m_{ee}$. As one can see, a value of $\delta_k$ different from 0 causes a cancellation in the first terms of Eq. (5), and with $\delta_k = \pi/2$ the cancellation is complete if the active-sterile mixing is maximal, i.e. $\varphi_k = \pi/4$. In this situation the states $\nu_k$ and $\nu_k'$ would combine into a Dirac particle, providing their mass difference $\epsilon_k$ vanishes, and the neutrinoless double beta decay would be prohibited. If the mass difference does not vanish, the decay is still possible but the decay width would be tiny, dictated by the mass difference. The effect of the mixing of the states 3 and 3′ is very small due to fact that these states contain only a tiny $\nu_e$ component. In the case of the lighter states, on the other hand, the effect is noticeable, and can decrease the value of $m_{ee}$ by a factor of about 2/3 and 1/3 for the 1-1′ and 2-2′—mixings, respectively.

Considering the absolute mass scale of neutrinos, the most stringent limit comes from cosmology. According to the result of the WMAP Collaboration, the sum of masses of the light active neutrinos is constrained as [11]

$$\sum_k m_k < 0.67 \text{ eV}. \quad (7)$$

This also gives the absolute upper limit for the effective mass $m_{ee}$ in the limit of exact degeneracy, as one can infer from Eq. (1). Let us assume that this limit is saturated and that the values of the squared mass differences are small in comparison with the mass limit (7). We make an approximation that all the mass states are, more or less, degenerate. Then

$$m_k \approx 0.2 \text{ eV}, \quad (8)$$

where $k = 1, \ldots, 6$.

Using these numbers, we plot in Fig. 1 the value of $m_{ee}$ as a function of the active-sterile mixing angle $\varphi$. As can be seen from the plot, the combined effect of the all three mixings can cause a strong cancellation, decreasing the value of $m_{ee}$ almost to zero at large mixing angles. We remind that here we have ignored the possible cancellations arising from the phases appearing in the standard $3 \times 3$ mixing matrix. They could make the cancellation even stronger.

In conclusion, possible extra $CP$ phases, which come from the existence of extra neutrino states, could greatly affect the effective neutrino mass $m_{ee}$ via cancellations. With suitable $CP$ phases even quite small mixings between active and sterile neutrino states can decrease the value of $m_{ee}$ notably. This would affect the analyses of and conclusions made from the results of neutrinoless double beta decay experiment, which probe the value of $m_{ee}$. The measurement of $m_{ee}$ is currently the only way to get information about the absolute scale of neutrino masses. It is important to keep in mind this new possible source of cancellation when drawing conclusions from the data.

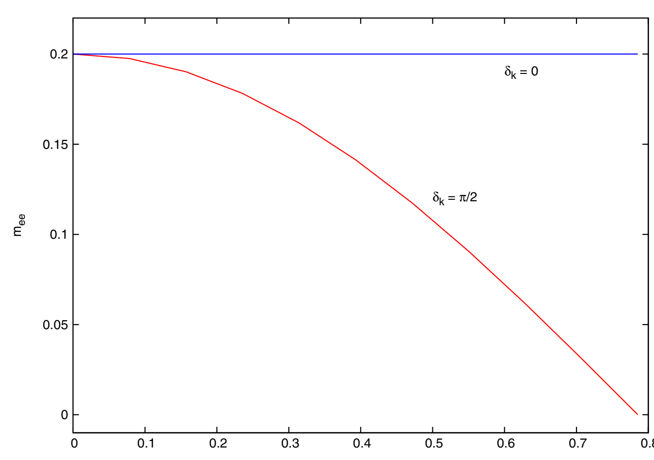


FIG. 1 (color online). The effect of all three active-sterile mixings on the value of $m_{ee}$. All the active-sterile mixing angles have the same value $\varphi_k = \varphi$, $k = 1, 2, 3$.

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