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Viable Randall-Sundrum model for quarks and leptons with T' family symmetry

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We propose a Randall-Sundrum model with a bulk family symmetry based on the double tetrahedral group, T', which generates the tribimaximal neutrino mixing pattern and a realistic CKM matrix, including CP violation. Unlike 4D models where the generation of mass hierarchy requires additional symmetry, the warped geometry naturally gives rise to the fermion mass hierarchy through wave function localization. The T' symmetry forbids tree-level flavor-changing-neutral-currents in both the quark and lepton sectors, as different generations of fermions are unified into multiplets of T'. This results in a low first KK mass scale and thus the model can be tested at collider experiments.

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I. INTRODUCTION

The Randall-Sundrum (RS) Model [1], based on a nonfactorizable geometry in a slice of anti-de Sitter (AdS_5) space with a warped background metric, has been proposed as a nonsupersymmetry alternative solution to the gauge hierarchy problem. In addition to solving the gauge hierarchy problem, the model can accommodate the fermion mass hierarchy, when the standard model (SM) fermions and gauge bosons are allowed to propagate in the bulk [2]. By localizing different fermions at different points in the fifth dimension, the widely dispersed masses of the SM fermions can be accommodated with all 5D Yukawa coupling constants being order unity [3,4]. This in turn also leads to new ways to generate neutrino masses [3,5]. Having the SM particles in the bulk generally causes large contributions to the electroweak observables, unless the Kaluza-Klein (KK) mass scale is much higher than a TeV. To suppress these contributions, realistic models based on bulk custodial symmetry [6] or large brane kinetic terms [7] have also been built, in which the first KK mass scale $\sim$3 TeV is allowed by the electroweak precision data.

The presence of the 5D bulk mass parameters, which govern the localizations of the bulk fields, leads to flavor violations in addition to the contributions caused by the 5D Yukawa interactions. These two generically independent flavor violation sources can generate dangerously large flavor-changing-neutral-currents (FCNCs) already at the tree level through the exchange of the KK gauge bosons. Although these processes are suppressed by the built-in RS Glashow-Iliopoulos-Maiani mechanism [4,8,9], constraints from the CP-violating parameter ϵ_K for $K^0 - \bar{K}^0$ mixing in the quark sector still give a stringent bound on the first KK mass scale of $\mathcal{O}(10\ \text{TeV})$ [10], when a generic flavor structure is assumed. Lepton flavor violation (LFV) in various rare leptonic processes mediated by neutral KK gauge bosons also gives stringent constraints on the KK mass scale [11–13]. Even in the absence of neutrino masses, severe bounds on the first KK mass scale already arise from processes mediated by tree-level FCNCs, with generic anarchical 5D Yukawa couplings [13].

One way to avoid the tree-level FCNCs is by imposing minimal flavor violation (MFV) [14] which assumes that all flavor violation comes from the Yukawa sector. Implementation of MFV in the quark sector has been proposed [15]. Realizations in the lepton sector with [16] and without [17] a bulk lepton symmetry have also been suggested. In these implementations, the bulk mass matrices are properly aligned with the 5D Yukawa matrices as dictated by the $[U(3)]^6$ flavor symmetry. With such alignment, which can arise from a shining mechanism [18], tree-level FCNCs can be suppressed and a first KK mass scale of 2–3 TeV can be allowed, rendering the model testable at collider experiments [19]. It has also been shown that a low first KK mass scale can also be obtained with the so-called minimal flavor protection mechanism [20] which utilized an $U(3)$ flavor symmetry, with nonminimal representations for leptons under $SU(2)_R$ [21], or alternatively, by considering a bulk Higgs and modified value of the 5D strong coupling constant [22].

In this paper, we propose an alternative by imposing a bulk family symmetry. In [23], a bulk family symmetry based on A_4 has been utilized in the lepton sector. Because of the common bulk mass term for the three lepton doublets, which is required to generate tribimaximal (TBM) neutrino mixing [24] as suggested by the recent global fit [25], tree-level leptonic FCNCs are absent. While A_4 well describes the lepton sector, it does not gives rise to a realistic quark sector. Here we consider the double tetrahedral group [26–29], T', as the bulk family symmetry. In addition to simultaneously giving rise to TBM neutrino mixing and a realistic Cabibbo-Kobayashi-Maskawa (CKM) matrix, the complex Clebsch-Gordan (CG) coefficients of T' also give the possibility that CP violation is entirely geometrical in origin [28,29]. While the three lepton doublets form a T' triplet, as in the case of A_4, the three generations of quarks transform as $2 \oplus 1$, leading to realistic masses and mixing angles in the quark sector. This assignment also forbids tree-level FCNCs involving the first and second generations of quarks, which are the most severely constrained.

The paper is organized as follows. In Sec. II we review various sources of flavor violation in generic RS models. We then present in Sec. III an RS model with a bulk T' family symmetry, in which the tree-level FCNCs are avoided. This is followed by Sec. IV where our numerical results are summarized. Section V concludes the paper.

II. FLAVOR VIOLATION IN RS

In this section, we provide a brief review of flavor violation in generic Randall-Sundrum models. We adopt the RS1 framework, where the fifth dimension y is compactified on a $S^1/\mathbb{Z}_2$ orbifold. The resulting bulk geometry between the two orbifold fixed points corresponds to a slice of AdS_5 space of length πR. A 3-brane is located at each orbifold fixed point; the geometric warp factor separating the two branes affects two distinct scales, M_{Pl} and $M_{\mathrm{Pl}} e^{-\pi k R}$, where $k \sim \mathcal{O}(M_{\mathrm{Pl}})$ is the AdS_5 curvature scale. The electroweak scale naturally arises through the warp factor for $kR \sim 11$. Hence, the fundamental scale of the 3-brane located at $y = 0$ is on the order of M_{Pl}, while the fundamental scale of the 3-brane located at $y = \pi R$ is $\sim M_{\mathrm{Pl}} e^{-\pi k R}$, which is $\sim \mathcal{O}(1\ \mathrm{TeV})$. We confine the Higgs to the TeV brane and allow the SM fermions and gauge fields to propagate in the bulk. In this way, the observable fermion masses and mixings are determined by the respective wave function overlaps between the SM Higgs and other SM fields on the TeV brane. Here we implicitly assume the Goldberger-Wise mechanism [30] for stabilizing the extra dimension.

With SM fermions and gauge fields propagating in the bulk, the electroweak precision measurements place stringent constraints on the bulk masses of the SM fermions. To satisfy these constraints, among which the most stringent are the ρ parameter and the Z couplings of the fermions, the bulk mass parameters of the SM fermion are generally required to be greater than 0.5. To preserve the bulk custodial symmetry, we assume that the bulk obeys $SU(2)_L \times SU(2)_R \times U(1)_X$ symmetry [6]. In addition, to avoid large corrections to the Z couplings of the fermions, an additional $L \leftrightarrow R$ parity is required and the fermions must transform in the nonminimal representations under $SU(2)_L \times SU(2)_R$ [31]. With this assignment, the $Zb_L\bar{b}_L$ coupling is protected by the left-right parity, and consequently the associated bulk parameter can be allowed to be less than 0.5. While this leads to a shift in the Z coupling of t_L, such a deviation is allowed since experimentally the $Zt_L\bar{t}_L$ is not very constrained. For the lighter generations, we take all bulk parameters to be greater than 0.5 in our numerical analysis.

The wave function overlap depends on the bulk mass parameters c_{L_i} and c_{R_j} according to the function

$$f(c_{L_i}, c_{R_j}) = \frac{1}{2}\sqrt{\frac{(1-2c_{L_i})(1-2c_{R_j})}{(e^{(1-2c_{L_i})\pi kR}-1)(e^{(1-2c_{R_j})\pi kR}-1)}} \times e^{(1-c_{L_i}-c_{R_j})\pi kR}, \tag{1}$$

where the first factors are from normalization, and the extra $e^{\pi kR}$ is from the canonical normalization of the Higgs kinetic term. For c_{L_i}, $c_{R_j} > 0.5$, the fermion fields are localizated toward the Planck brane and have small wave function overlaps with the Higgs field at the TeV brane. On the other hand, for $c_{L_i}, c_{R_j} < 0.5$, the fields will have large wave function overlaps with the Higgs. This wave function localization mechanism [3–5] can naturally give rise to the observed fermion mass hierarchies.

Even though this is a natural way to generate the mass hierarchy, the nonuniversal bulk mass terms for the three generations of fermions generally lead to tree-level FCNCs. Consider the 4D effective gauge coupling for fermions from the kinetic term after integrating out the fifth coordinate y,

$$\mathcal{L}^{4D}_{\mathrm{Kin}} \supset \int dy e^{-4k|y|} i\bar{\Psi}\gamma^M D_M \Psi \rightarrow gG\bar{\psi}\begin{pmatrix} f(c_1,c_1)^2 & 0 & 0 \\ 0 & f(c_2,c_2)^2 & 0 \\ 0 & 0 & f(c_3,c_3)^2 \end{pmatrix}\psi \tag{2}$$

where $M = \{\mu, 5\}$ labels the coordinates with μ as the usual 4D Lorentz index. Schematically, Ψ denotes the 5D fermion field, $\psi = (\psi_1 \quad \psi_2 \quad \psi_3)^T$ is the three-generation 4D fermion field in the gauge basis, g is the gauge coupling, and G is the gauge field in the adjoint representation. When the above gauge interactions are rotated to the mass basis by insertion of $V^\dagger V$, where the mass eigenstates are $\psi_m = V\psi$, the 4D effective gauge interactions become

$$\begin{aligned} & gG\bar{\psi}V^\dagger V\begin{pmatrix} f(c_1,c_1)^2 & 0 & 0 \\ 0 & f(c_2,c_2)^2 & 0 \\ 0 & 0 & f(c_3,c_3)^2 \end{pmatrix}V^\dagger V\psi \\ & = gG\bar{\psi}_m V\begin{pmatrix} f(c_1,c_1)^2 & 0 & 0 \\ 0 & f(c_2,c_2)^2 & 0 \\ 0 & 0 & f(c_3,c_3)^2 \end{pmatrix}V^\dagger \psi_m \\ & \equiv gG\bar{\psi}_m M \psi_m. \end{aligned} \tag{3}$$

Thus, if any two bulk mass terms are unequal, then nonzero off-diagonal elements in the gauge coupling matrix, M, in the mass basis can generally be present and hence sizable FCNC transitions may be generated.

There are two distinct ways to alleviate this problem. First, if there is flavor universality, which our model possesses for the left-handed lepton doublets in order to generate TBM neutrino mixing, then $c_1 = c_2 = c_3$ and the V and $V^\dagger$ unitary matrices commute through Eq. (3), leaving $M \propto 1_{3\times3}$ without neutral flavor-changing transitions. On the other hand, if the gauge basis can be freely rotated to coincide with the mass basis (alignment), as will be true for the right-handed leptons in our model, then $V = 1_{3\times3}$ and again, M will contain no off-diagonal entries. We note that a combination of these two ideas is also useful. The three generations of quarks in our model, for example, transform as the $2 \oplus 1$ representations under the double tetrahedral group, T', as in the usual 4D models [26–29]. This representation assignment is required in order to generate realistic quark masses and mixing pattern. The universality that is exhibited among the first two generations forbids the tree-level flavor transitions that cause $K^0 - \bar{K}^0$ mixing. The near-identity CKM matrix mimics alignment, and so the resulting neutral current matrix possesses minimal 1–3, 2–3, $3 - 1$, and $3 - 2$ couplings, which are still allowed by current experimental constraints. Our model's features of flavor universality for left-handed (LH) leptons, alignment for right-handed (RH) leptons, and the $2 \oplus 1$ framework for quarks all result from the T' bulk family symmetry. We summarize the relevant properties of the finite group T' in the Appendix. We comment that since the family group T' is a direct product group with the enlarged $SU(2)_L \times SU(2)_R$ symmetry, assigning the SM fermions in the nonminimal representations under the LR group as required by avoiding the EW precision constraints does not affect our analysis.

We note that the universality in the Z couplings to the fermions may be spoiled by higher-order effects. The leading higher-order effects are dim-6 operators which can in general be induced by (i) the mixing between the fermion zero mode and its KK modes and (ii) the mixing between the zero mode and KK modes of the Z boson. In addition, there can be loop contributions to flavor-violating Z couplings in the presence of brane-localized kinetic terms [20]. These nonuniversal kinetic terms are induced by the brane-localized Yukawa couplings, and they could lead to loop-suppressed nonuniversal shifts in the normalization of the zero mode wave functions. Since these loop contributions correspond to dim-8 operators, these effects are subdominant in the presence of nonvanishing contributions from dim-6 operators. In our model, due to the T' symmetry which leads to universal bulk mass parameters for the lighter quarks, the fermion couplings to the Z boson induced by higher-order effects due to the mixing of the Z boson zero mode and its KK modes are flavor-preserving. As a result the leading contributions to flavor-violating Z couplings are due to the dim-6 operators induced by the mixing of fermion zero mode and its KK modes. An estimate of such higher-order effects in our model is presented in Sec. IV. Our estimate shows that with a low first KK mass scale of 3–4 TeV, these higher-order contributions are suppressed enough to satisfy all experimental constraints [18,20].

III. THE MODEL

In our model, we impose the discrete T' symmetry as a flavor symmetry for SM fermion fields placed in the bulk. As mentioned previously, the SM Higgs is confined to the TeV brane, while the SM gauge and fermion fields are allowed to propagate in the bulk. The three generations of LH lepton doublets L and three generations of RH neutrinos N are unified into triplet representations, and the RH charged leptons e, μ, τ transform as inequivalent one-dimensional representations. The first two generations of LH quarks Q_{12}, RH up-type quarks U, and RH down-type quarks D are each in T' doublet representations, while the third-generation LH quark doublet Q_3, RH top quark T, and RH bottom quark B, and the SM Higgs field, H, are all pure singlets under T'. To break the T' symmetry, we need a set of flavons, which are all singlets under the SM gauge group. The representation assignments for the SM fermions and T' flavons are summarized in Table I.

A. The lepton sector

The 5D Lagrangian involving leptons, including the canonical 5D kinetic term, $\mathcal{L}^{\rm lep}_{\rm Kin}$, the 5D bulk mass term, $\mathcal{L}^{\rm lep}_{\rm Bulk}$, and the 5D Yukawa interactions for charged leptons, $\mathcal{L}^{\rm lep}_{{\rm Yuk},\ell}$ and neutrinos $\mathcal{L}^{\rm lep}_{{\rm Yuk},\nu}$, is

$$\mathcal{L}^{\rm lep}_{\rm 5D} \supset \mathcal{L}^{\rm lep}_{\rm Kin} + \mathcal{L}^{\rm lep}_{\rm Bulk} + \mathcal{L}^{\rm lep}_{{\rm Yuk},\ell} + \mathcal{L}^{\rm lep}_{{\rm Yuk},\nu}. \tag{4}$$

The bulk mass terms in our model are given by

$$\mathcal{L}^{\rm lep}_{\rm Bulk} = k(\bar{L}c_L L + \bar{e}c_e e + \bar{\mu}c_\mu \mu + \bar{\tau}c_\tau \tau + \bar{N}c_N N), \tag{5}$$

where the bulk mass terms c_i are dimensionless. Because of the T' symmetry, the number of bulk mass terms is significantly reduced when compared to the generic case without a family symmetry.

We now distinguish the charged lepton Yukawa interactions from the neutrino Yukawa interactions because in our model, neutrinos can be treated in two ways: (i) a

TABLE I. Representation assignments for SM fermion and T' flavon fields. The field definitions are given the main text.

	L	N	e	μ	τ	ϕ	ϕ'	σ	Q_{12}	Q_3	U	T	D	B	α	β	ζ	ξ	χ_U	χ_D	η_U	η_D
T'	3	3	1	$1''$	$1'$	3	3	1	2	1	2	1	2	1	3	3	1	1	2	2	2	2

purely Dirac mass structure or (ii) a type I seesaw realization. For the Dirac (Dc) neutrino case, all 5D Yukawa interactions take place on the TeV brane, where the SM Higgs and the flavon fields are confined. The charged lepton interactions are then

$$\mathcal{L}^{\rm lep}_{{\rm Yuk},\ell} = \delta(y-\pi R)\left[\frac{1}{k}\bar{H}(x)\left(y_e^{5D}\bar{L}(x,y)e(x,y)\frac{\phi(x)}{\Lambda} + y_\mu^{5D}\bar{L}(x,y)\mu(x,y)\frac{\phi(x)}{\Lambda} + y_\tau^{5D}\bar{L}(x,y)\tau(x,y)\frac{\phi(x)}{\Lambda}\right)\right] + {\rm H.c.} \tag{6}$$

and the purely Dirac neutrino mass structure is

$$\mathcal{L}^{\rm lep}_{{\rm Yuk},\nu,{\rm Dc}} = \delta(y-\pi R)\left[\frac{1}{k}H(x)\bar{L}(x,y)N(x,y) \times\left(y^{5D}_{\nu,{\rm Dc,a}}\frac{\phi'_{\rm Dc}(x)}{\Lambda} + y^{5D}_{\nu,{\rm Dc,b}}\frac{\sigma_{\rm Dc}(x)}{\Lambda}\right)\right] + {\rm H.c.}, \tag{7}$$

where the 5D Yukawa coupling constants y_i^{5D} are dimensionless. For this Dirac neutrino case, the flavon fields ϕ, $\phi'_{\rm Dc}$ and $\sigma_{\rm Dc}$ are scalar fields confined to the TeV brane, and therefore the cutoff scale Λ of the higher-dimensional operators in the above equation is $\Lambda \sim \mathcal{O}(1\ {\rm TeV})$.

On the other hand, in our seesaw (SS) realization, the heavy Majorana RH neutrino mass term arises from flavon fields confined to the Planck brane and a Dirac mass contribution from the SM Higgs confined to the TeV brane. While the charged lepton Yukawa interactions are unchanged from Eq. (6), the neutrino Yukawa terms are now replaced by

$$\mathcal{L}^{\rm lep}_{{\rm Yuk},\nu,{\rm SS}} = \left\{\delta(y)\left[\frac{1}{k}N^T(x,y)N(x,y)(y^{5D}_{\nu,{\rm SS,a}}\phi'_{\rm SS}(x') + y^{5D}_{\nu,{\rm SS,b}}\sigma_{\rm SS}(x'))\right] + \delta(y-\pi R)\left[\frac{1}{k}H(x)y^{5D}_{\nu,{\rm SS,c}}\bar{L}(x,y)N(x,y)\right]\right\} + {\rm H.c.}, \tag{8}$$

where x' is the 4D spacetime coordinate on the Planck brane, and the flavon fields $\phi'_{\rm SS}$ and $\sigma_{\rm SS}$ have the same T' representations as their pure Dirac counterparts in Eq. (7) and hence we use the same notation in both equations. The relevant T' breaking scale for these flavons, however, is $\Lambda_{\rm UV} \sim 10^{19}$ GeV, not the TeV scale as before.

We emphasize for both cases that, in order to generate TBM mixing pattern for the neutrinos, the L and N transform as triplets under T'. (We note that in the Dirac neutrino case, we choose the coefficient of the contraction of L and N that gives the antisymmetric triplet 3_A to be zero.) This choice of T' representations also ensures that the bulk mass matrices c_L and c_N each become universal among the three generations, and hence our model avoids tree-level FCNCs in the lepton sector.

The flavon fields in Eq. (6) and Eq. (7) acquire vacuum expectation values (VEVs) along the following directions:

$$\langle\phi\rangle = \phi_0\Lambda\begin{pmatrix}1\\0\\0\end{pmatrix} \qquad \langle\phi'_{\rm Dc}\rangle = \phi'_{0,{\rm Dc}}\Lambda\begin{pmatrix}1\\1\\1\end{pmatrix} \qquad \langle\sigma_{\rm Dc}\rangle = \sigma_{0,{\rm Dc}}\Lambda. \tag{9}$$

For this Dirac neutrino case, we explicitly factor out the T' breaking scale Λ in order to leave the coefficients, ϕ_0, $\phi'_{0{\rm Dc}}$, and $\sigma_{0{\rm Dc}}$, dimensionless. For the seesaw case, the VEV of the ϕ field is unchanged, but the VEVs of the ϕ' and σ fields become

$$\langle\phi'_{\rm SS}\rangle = \phi'_{0,{\rm SS}}\Lambda_{\rm UV}\begin{pmatrix}1\\1\\1\end{pmatrix} \qquad \langle\sigma_{\rm SS}\rangle = \sigma_{0,{\rm SS}}\Lambda_{\rm UV}, \tag{10}$$

where $\Lambda_{\rm UV}$ is the relevant T' breaking scale for these flavons.

Upon T' symmetry breaking due to $\langle\phi\rangle$ in Eq. (9), the charged lepton mass matrix becomes

$$M_e = v\phi_0\begin{pmatrix}y_e & 0 & 0\\ 0 & y_\mu & 0\\ 0 & 0 & y_\tau\end{pmatrix}, \tag{11}$$

where $v = 246$ GeV is the SM Higgs VEV, and the effective 4D Yukawa coupling constants

$$y_\ell = y_\ell^{5D}f(c_L, c_\ell), \tag{12}$$

for $\ell = e, \mu, \tau$, depend on the wave function profiles of the fermions as characterized by the overlap function $f(c_L, c_\ell)$ defined in Eq. (1). We remark that even with universal c_L for the lepton doublets, it is clear that the observed charged lepton mass hierarchy can be obtained via the nonuniversal values for the bulk mass parameters for the RH charged leptons, c_ℓ.

For the case of pure Dirac neutrinos, the Lagrangian in Eq. (7) and VEVs in Eq. (9) lead to the 4D effective neutrino mass matrix

$$M_\nu = f(c_L, c_N)v\begin{pmatrix}2A+B & -A & -A\\ -A & 2A & B-A\\ -A & B-A & 2A\end{pmatrix}, \tag{13}$$

where $f(c_i, c_j)$ is defined in Eq. (1) and the parameters A and B are

$$A = \frac{1}{3}y^{5D}_{\nu,{\rm Dc,a}}\phi'_{0,{\rm Dc}} \qquad B = y^{5D}_{\nu,{\rm Dc,b}}\sigma_{0,{\rm Dc}}. \tag{14}$$

This mass matrix is form-diagonalizable by the TBM mixing matrix,

$$U_{\mathrm{TBM}} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}, \qquad (15)$$

independent of the values of A and B. The three neutrino mass eigenvalues depend on A and B, and the wave function overlap sets the overall scale of the neutrino masses. This leads to the following predictions for the absolute masses of the three neutrinos:

$$M_\nu^D = f(c_L, c_N) v \mathrm{diag}(3A + B, B, 3A - B), \qquad (16)$$

which obey the sum rule $m_1 - m_3 = 2m_2$ [32]. As a consequence of the fact that the solar mixing angle $\Delta m^2_{\mathrm{sol}} = \Delta m^2_{21}$ is known to be positive, this model predicts a normal hierarchy ordering [32]. Correspondingly, the expressions for neutrino mass squared differences are

$$\Delta m^2_{\mathrm{sol}} = \Delta m^2_{21,\mathrm{Dc}} = -(f(c_L, c_N)v)^2(9A^2 + 6AB) \qquad (17)$$

and

$$\Delta m^2_{\mathrm{atm}} = |\Delta m^2_{31,\mathrm{Dc}}| = |-(f(c_L, c_N)v)^2 12AB|. \qquad (18)$$

For our seesaw realization, the neutrino sector has the block mass matrix form

$$M_\nu = \begin{pmatrix} 0 & M^T_{\mathrm{Dc}} \\ M_{\mathrm{Dc}} & M_{\mathrm{RR}} \end{pmatrix}, \qquad (19)$$

where each entry is understood to be a 3×3 matrix. With the Lagrangian in Eq. (8) and VEVs in Eq. (10), the 4D effective matrices in Eq. (19) are

$$M_{\mathrm{RR}} = \frac{1 - 2c_N}{2(e^{(1-2c_N)\pi kR} - 1)} \times \Lambda_{\mathrm{UV}} \begin{pmatrix} 2A + B & -A & -A \\ -A & 2A & B - A \\ -A & B - A & 2A \end{pmatrix}, \qquad (20)$$

where

$$A = \frac{1}{3} y^{5\mathrm{D}}_{\nu,\mathrm{SS,a}} \phi'_{0,\mathrm{SS}} \qquad B = y^{5\mathrm{D}}_{\nu,\mathrm{SS,b}} \sigma_{0,\mathrm{SS}}, \qquad (21)$$

and

$$M_{\mathrm{Dc}} = y^{5\mathrm{D}}_{\nu,\mathrm{SS,c}} f(c_L, c_N) v \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}. \qquad (22)$$

Given this seesaw structure, the resulting light effective LH neutrino mass matrix is thus [33]

$$M_{\nu,\mathrm{eff}} = M_{\mathrm{Dc}} M^{-1}_{\mathrm{RR}} M^T_{\mathrm{Dc}} \qquad (23)$$

which is diagonalized by the TBM mixing matrix [34] to give eigenvalues

$$M^D_{\nu,\mathrm{eff}} = \frac{(y^{5\mathrm{D}}_{\nu,\mathrm{SS,c}} v)^2}{2\Lambda_{\mathrm{UV}}} \frac{1 - 2c_L}{e^{(1-2c_L)\pi kR} - 1} e^{2(1-c_L-c_N)\pi kR} \times \mathrm{diag}\left(\frac{1}{3A + B}, \frac{1}{B}, \frac{1}{3A - B}\right). \qquad (24)$$

Unlike the Dirac neutrino case where the mass eigenvalues predicted a normal hierarchy, the seesaw realization can accommodate either a normal or inverted hierarchy. For either hierarchy, we have

$$\Delta m^2_{\mathrm{sol}} = \Delta m^2_{21,\mathrm{SS}} = \kappa^2\left[\frac{1}{A^2} - \frac{1}{(3A + B)^2}\right] \qquad (25)$$

and

$$\Delta m^2_{\mathrm{atm}} = |\Delta m^2_{31,\mathrm{SS}}| = \left|\kappa^2\left[\frac{1}{(3A - B)^2} - \frac{1}{(3A + B)^2}\right]\right|, \qquad (26)$$

with

$$\kappa = \frac{(y^{5\mathrm{D}}_{\nu,\mathrm{SS,c}} v)^2}{2\Lambda_{\mathrm{UV}}} \frac{1 - 2c_L}{e^{(1-2c_L)\pi kR} - 1} e^{2(1-c_L-c_N)\pi kR}. \qquad (27)$$

B. The quark sector

In the quark sector, since the top quark is heavy, it suggests that its mass is allowed by the T' symmetry and thus can be generated at the renormalizable level. The lighter generations, on the other hand, have mass terms which are generated after the breaking of the T' symmetry. Their mass terms hence are generated by higher-dimensional operators and are suppressed by the (IR) T' breaking scale, Λ. These considerations therefore suggest the $2 \oplus 1$ representation assignment, whereby the first two generations of quarks form a doublet of T' and the third generation transforms as a pure singlet. The 5D Lagrangian involving quarks is given by

$$\mathcal{L}^{\mathrm{qrk}}_{5\mathrm{D}} \supset \mathcal{L}^{\mathrm{qrk}}_{\mathrm{Kin}} + \mathcal{L}^{\mathrm{qrk}}_{\mathrm{Bulk}} + \mathcal{L}^{\mathrm{qrk}}_{\mathrm{Yuk}}, \qquad (28)$$

where $\mathcal{L}^{\mathrm{qrk}}_{\mathrm{Kin}}$ is the canonical 5D kinetic term and the 5D bulk mass terms are

$$\mathcal{L}^{\mathrm{qrk}}_{\mathrm{Bulk}} = k(\bar{Q}_{12} c_{Q_{12}} Q_{12} + \bar{Q}_3 c_{Q_3} Q_3 + \bar{U} c_U U + \bar{T} c_T T + \bar{D} c_D D + \bar{B} c_B B). \qquad (29)$$

Similar to the lepton sector, due to the T' family symmetry and the $2 \oplus 1$ structure, the number of bulk parameters is greatly reduced in our model.

Without further symmetries or assumptions, the most general Yukawa interactions within the $2 \oplus 1$ framework have at most eight flavon fields, which we will now demonstrate. The most general T' invariant quark Yukawa terms can be written as

$$\mathcal{L}_{\rm Yuk}^{\rm qrk} = \delta(y-\pi R)\bigg\{\frac{1}{k}H(x)\bigg[y_{12}^U\bar{Q}_{12}(x,y)U(x,y)\bigg(\frac{\alpha(x)+\zeta(x)}{\Lambda}\bigg) + y_3^U\bar{Q}_3(x,y)U(x,y)\frac{\chi_U(x)}{\Lambda} + y_{12}^T\bar{Q}_{12}(x,y)T(x,y)\frac{\eta_U(x)}{\Lambda} + y_3^T\bar{Q}_3(x,y)T(x,y)\bigg] + \frac{1}{k}\bar{H}(x)\bigg[y_{12}^D\bar{Q}_{12}(x,y)D(x,y)\bigg(\frac{\beta(x)+\xi(x)}{\Lambda}\bigg) + y_3^D\bar{Q}_3(x,y)D(x,y)\frac{\chi_D(x)}{\Lambda} + y_{12}^B\bar{Q}_{12}(x,y)B(x,y)\frac{\eta_D(x)}{\Lambda} + y_3^B\bar{Q}_3(x,y)B(x,y)\bigg]\bigg\}. \quad (30)$$

In principle, there are two separate Yukawa couplings for the triplet and singlet flavon fields, α and ζ (and similarly for β and ξ). Nevertheless, the difference between the two couplings can be absorbed into the values of the VEVs of α and ζ. With this rescaling freedom, we can assume the same coupling constant for both terms. The flavon fields α, ζ, χ_U, η_U (as well as their down-type counterparts β, ξ, χ_D, η_D) transform under T' as 3, 1, 2, and 2, respectively, as reflected in Table I. Since every possible SM field contraction is exercised, introducing new flavon fields in the same representations as above would be redundant.

We continue our derivation of the most general $2 \oplus 1$ quark mass matrix and present the up-type quarks, seeing that the down-type matrix will be exactly analogous. Given the most general T' field content in Eq. (30), we allow a fully general VEV structure,

$$\langle\alpha\rangle = \Lambda\begin{pmatrix}\alpha_1\\ \alpha_2\\ \alpha_3\end{pmatrix}, \qquad \langle\zeta\rangle = \Lambda\zeta_0, \qquad \langle\chi_U\rangle = \Lambda\begin{pmatrix}\chi_{U1}\\ \chi_{U2}\end{pmatrix}, \qquad \langle\eta_U\rangle = \Lambda\begin{pmatrix}\eta_{U1}\\ \eta_{U2}\end{pmatrix}, \quad (31)$$

which completely illustrates the maximum number of new parameters. This allows us to write down the resulting 4D up-type mass matrix[1]

$$M_U = v\begin{pmatrix} i\alpha_3 y_{12}^U f(c_{Q_{12}},c_U) & [(\frac{1-i}{2})\alpha_1 - \zeta_0]y_{12}^U f(c_{Q_{12}},c_U) & \chi_{U2}y_3^U f(c_{Q_3},c_U)\\ [(\frac{1-i}{2})\alpha_1 + \zeta_0]y_{12}^U f(c_{Q_{12}},c_U) & \alpha_2 y_{12}^U f(c_{Q_{12}},c_U) & -\chi_{U1}y_3^U f(c_{Q_3},c_U)\\ \eta_{U2}y_{12}^T f(c_{Q_{12}},c_T) & -\eta_{U1}y_{12}^T f(c_{Q_{12}},c_T) & y_3^T f(c_{Q_3},c_T)\end{pmatrix}. \quad (32)$$

When performing the fit to the SM, we find that the general mass matrix in Eq. (32) has an overabundance of parameters. To simplify the presentation, we assume the following, more restricted VEV structure for the up-type and down-type flavons:

$$\langle\alpha\rangle = \langle\beta\rangle = \Lambda\alpha_0\begin{pmatrix}1\\1\\1\end{pmatrix}, \qquad \langle\zeta\rangle = \langle\xi\rangle = \Lambda\zeta_0, \qquad \langle\chi_r\rangle = \Lambda\chi_0^r\begin{pmatrix}\cos\theta_r\\ \sin\theta_r\end{pmatrix}, \qquad \langle\eta_r\rangle = \Lambda\eta_0^r\begin{pmatrix}1\\0\end{pmatrix}, \quad (33)$$

where $r = U, D$. This VEV pattern leads to a 4D effective up-type mass matrix

$$M_U = v\begin{pmatrix} i\alpha_0 f(c_{Q_{12}},c_U) & [(\frac{1-i}{2})\alpha_0 - \zeta_0]f(c_{Q_{12}},c_U) & \chi_0^U\sin\theta_U f(c_{Q_3},c_U)\\ [(\frac{1-i}{2})\alpha_0 + \zeta_0]f(c_{Q_{12}},c_U) & \alpha_0 f(c_{Q_{12}},c_U) & -\chi_0^U\cos\theta_U f(c_{Q_3},c_U)\\ 0 & -\eta_0^U f(c_{Q_{12}},c_T) & y_3^T f(c_{Q_3},c_T)\end{pmatrix} \quad (34)$$

and a 4D effective down-type mass matrix

$$M_D = v\begin{pmatrix} i\alpha_0 f(c_{Q_{12}},c_D) & [(\frac{1-i}{2})\alpha_0 - \zeta_0]f(c_{Q_{12}},c_D) & \chi_0^D\sin\theta_D f(c_{Q_3},c_D)\\ [(\frac{1-i}{2})\alpha_0 + \zeta_0]f(c_{Q_{12}},c_D) & \alpha_0 f(c_{Q_{12}},c_D) & -\chi_0^D\cos\theta_D f(c_{Q_3},c_D)\\ 0 & -\eta_0^D f(c_{Q_{12}},c_B) & y_3^B f(c_{Q_3},c_B)\end{pmatrix}, \quad (35)$$

where all other Yukawas have been set to 1.

[1] When performing the $2\otimes 2\otimes 3$ contraction in the upper 2×2 block of M_U, it may seem that one could contract in two ways, $(2\otimes 2)\otimes 3$ or $2\otimes(2\otimes 3)$, whereby the different Clebsh-Gordan coefficients of the two contractions would lead to different contributions. This is not the case, however, since the first contraction can be rescaled by $(1+i)$ to then become identical to the second contraction.

IV. RESULTS

In this section, we present our numerical fits for the SM observed values of fermion masses and mixings using the parameters of our T' in RS model. A naive counting for our model gives 8 ($=4\,\text{bulk}+3\,\text{Yukawa}+1\,\text{flavon}$) parameters for the charged leptons, 6[7] ($=2\,\text{bulk}+2[3]\,\text{Yukawa}+2\,\text{flavon}$) parameters for the Dirac [seesaw realization] neutrinos, and 24 ($=6\,\text{bulk}+8\,\text{Yukawa}+10\,\text{flavon}$) parameters for the quarks. This is contrasted with the general anarchy case, which has 45[39] parameters for the charged leptons and Dirac [Majorana] neutrinos and 45 for the quarks. In actuality, the number of independent parameters for our model is much smaller: we have 3 for the charged leptons, 2[2] for the neutrinos, and 11 for the quarks, which compares to 36[30] for the anarchic leptons and 36 for the anarchic quarks.

We briefly comment on the renormalization group (RG) effects to our fit. Our mass matrices for the SM fields are given at the (IR) T' breaking scale, which we take to be $\sim$3 TeV. For the charged leptons and neutrinos, RG effects are negligible since the running of Yukawa couplings from 3 TeV down to the m_Z scale is demonstrably small (cf. Table IV of Ref. [35], where charged lepton Yukawa coupling running from 10^9 GeV to m_Z is less than a 10% effect). The mixing of neutrinos is also negligibly affected by RG running since the neutrino masses at the T' scale are not sufficiently degenerate to enhance mixing [36]. Quark masses, however, acquire non-negligible corrections from running, while the corresponding corrections to quark mixings are expected to be small [35]. Thus, we will fit to the charged leptons masses at m_Z [37], the low-energy neutrino mixing data [38], the quark masses at $\sim$3 TeV (cf. Table 1 of Ref. [39]), and the CKM matrix at m_Z [40].

We now fit the entire SM using our 16 independent parameters. In the charged lepton sector, all 5D Yukawas are set to 1, and we have $c_L = 0.40000$, $c_e = 0.82925$, $c_\mu = 0.66496$, $c_\tau = 0.57126$, and $\phi_0 = 1$ as our input parameters. These values give, at m_Z, an electron mass of 511.1 keV, muon of 105.7 MeV, and a tau of 1.777 GeV, which are consistent with the experimental values [37] of $m_e = 510.998$ keV, $m_\mu = 105.658$ MeV, $m_\tau = 1.77684 \pm 0.00017$ GeV.

In the neutrino sector, for the Dirac case, we use the value of c_L above, $c_N = 1.27000$, $\phi'_{0,\mathrm{Dc}} = -0.1768$, $\sigma_{0,\mathrm{Dc}} = 0.0944$, and we set both 5D Yukawas set to 1. These parameters give absolute neutrino masses of $m_1 = -0.01563$ eV, $m_2 = 0.01791$ eV, and $m_3 = -0.05145$ eV. These correspond to mass squared differences of $\Delta m^2_{21} = 7.6370 \times 10^{-5}$ eV^2 and $\Delta m^2_{31} = 2.4031 \times 10^{-3}$ eV^2, which are in good agreement with the experimental results [38], $\Delta m^2_{\mathrm{sol}} = 7.65^{+0.23}_{-0.20} \times 10^{-5}$ eV^2 for solar neutrino oscillation and $|\Delta m^2_{\mathrm{atm}}| = 2.40^{+0.12}_{-0.11} \times 10^{-3}$ eV^2 from atmospheric neutrinos.

In the seesaw realization of our model, to produce a normal hierarchy, we use the value of c_L above, $c_N = 0.40000$, $\phi'_{0,\mathrm{SS}} = 0.07427$, $\sigma_{0,\mathrm{SS}} = 0.06191$, and we set all three 5D Yukawas to 1. This gives $m_1 = 0.004465$ eV, $m_2 = 0.009821$ eV, and $m_3 = 0.04919$ eV, and also we get $\Delta m^2_{21} = 7.652 \times 10^{-5}$ eV^2 and $\Delta m^2_{31} = 2.4001 \times 10^{-3}$ eV^2. An inverted hierarchy solution arises if we use $c_N = 0.40000$, $\phi'_{0,\mathrm{SS}} = 0.02321$, $\sigma_{0,\mathrm{SS}} = -0.0115241$, and again assign all Yukawas to be 1. The absolute masses are now $m_1 = 0.05203$ eV, $m_2 = -0.05276$ eV, and $m_3 = 0.01751$, and the mass squared differences become $\Delta m^2_{21} = 7.656 \times 10^{-5}$ eV^2 and $\Delta m^2_{31} = -2.4009 \times 10^{-3}$ eV^2. Both seesaw solutions satisfy the current experimental bounds quoted above.

For the quarks, we have the following input values for the flavon VEVs and Yukawa couplings: $\alpha_0 = -0.00143 + 0.00104i$, $\zeta_0 = 0.00200$, $\chi_0^U = \eta_0^U = -0.448$, $\theta_U = 0.181\pi$, $\chi_0^D = -0.00230$, $\theta_D = 0.1135\pi$, $\eta_0^D = -0.540 - 0.540i$, $y_3^T = 1.00$, and $y_3^B = 0.060$, with all other Yukawa coupling constants set to 1. In addition, the bulk mass terms are $c_{Q_{12}} = 0.503$, $c_{Q3} = 0.150$, $c_U = 0.512$, $c_T = -0.350$, $c_D = 0.503$, and $c_B = 0.508$. These input parameters give, at the (IR) T' breaking scale of 3 TeV, an up quark mass of 1.49 MeV, a charm mass of 0.541 GeV, and a top mass of 134.8 GeV. The down-type quark masses are predicted to be 2.92 MeV, 36.6 MeV, and 2.41 GeV. These masses are within the bounds of Table 1 of Ref. [39]: $m_u = 0.75$–1.5 MeV, $m_c = 0.56 \pm 0.04$ GeV, $m_t = 136.2 \pm 3.1$ GeV, $m_d = 2$–4 MeV, $m_s = 47 \pm 12$ MeV, $m_b = 2.4 \pm 0.04$ GeV.

The resulting CKM matrix from these input parameters is given by

$$V_{\mathrm{CKM,th}} = \begin{pmatrix} 0.974282e^{-0.0558i} & 0.225305e^{-0.381i} & 0.003464e^{1.31i} \\ 0.225147e^{-2.76i} & 0.973485e^{0.0557i} & 0.040450e^{3.13i} \\ 0.00910164e^{-3.12i} & 0.0395649e^{0.0865i} & 0.999176e^{0.0000095i} \end{pmatrix}. \tag{36}$$

The absolute values of the CKM matrix elements agree with experimental values at m_Z within 3σ [40]:

$$|V_{\mathrm{CKM,ex}}| = \begin{pmatrix} 0.97433^{+0.00052}_{-0.00052} & 0.2251^{+0.0022}_{-0.0022} & 0.00351^{+0.00044}_{-0.00032} \\ 0.2250^{+0.0022}_{-0.0022} & 0.97349^{+0.00053}_{-0.00052} & 0.0412^{+0.0011}_{-0.0019} \\ 0.00859^{+0.00057}_{-0.00064} & 0.0404^{+0.0011}_{-0.0020} & 0.999146^{+0.000078}_{-0.000047} \end{pmatrix}. \tag{37}$$

In addition, we have a predictions for CP violation in the quark and lepton sectors. For the quark sector, our model predicts the following value for the Jarlskog invariant,

$$J_{\mathrm{th}} \equiv \mathrm{Im} V_{ud} V_{cs} V_{us}^* V_{cd}^* = 3.02 \times 10^{-5}, \tag{38}$$

which is within the 3σ uncertainty of the experimental value [40],

$$J_{\mathrm{ex}} = 2.93^{+0.45}_{-0.25} \times 10^{-5}. \tag{39}$$

We remark that, in our model, this value arises from a combination of both complex Clebsch-Gordan coefficients [29] and complex VEVs of T' flavon fields (which are indistinguishable from complex Yukawa coefficients). For the leptons, our Dirac mass matrices are completely real and diagonal, giving a prediction of a vanishing leptonic Jarlskog.

In the absence of the FCNCs at tree level at the renormalizable level due to the T' family symmetry, the leading contributions to flavor-violating Z couplings are due to the dim-6 operators induced by the mixing of fermion zero mode and its KK modes. For the first KK mode, which gives the least suppressed contributions, these dim-6 operators lead to flavor-violating Z couplings, $Z\psi_j^{(0)}\psi_k^{(0)}$. Normalized to the SM Z coupling, these higher-order effects contribute the following factor,

$$(\langle\alpha_0\rangle + \langle\zeta_0\rangle)^2 \frac{(f_i^{(1)})^2 f_j^{(0)} f_k^{(0)}}{4\pi^2 k^2 R^2} \exp(2\pi k R) \frac{v^4}{M_{\mathrm{KK}}^2}, \tag{40}$$

where $f_i^{(1)}$ and $f_j^{(0)}$ are the wave function profiles of the first KK mode and the zero mode of the fermion. Numerically, for the $u - c$ transition, the contribution is 2.965×10^{-6} times the regular Z coupling. The $d - s$ flavor-violating transition contributes 4.156×10^{-6} times the regular Z coupling, assuming the first KK mass scale $\sim$3 TeV. These higher-order effects are thus highly suppressed and are allowed by the experiments.

V. CONCLUSION

We have proposed a Randall-Sundrum Model with a bulk T' family symmetry. The T' symmetry gives rise to a TBM mixing matrix for the neutrinos and a realistic quark CKM matrix. In the lepton sector, exact neutrino tribimaximal mixing is generated due to the group theoretical CG coefficients of T'. Since the neutrino mass matrix is form diagonal, the neutrino mass eigenvalues are decoupled from its mixing. This thus alleviates the tension generally present in the anarchical scenarios between generating large neutrino mixing angles and their hierarchical masses (the hierarchy among the masses are determined by the flavon VEVs.) For the charged leptons, even though all three left-handed doublets have common bulk mass terms, the mass hierarchy among them is generated due to the wave function profiles of the right-handed charged leptons. In the quark sector, the mass hierarchy between the first and second generations is due to the structure of the T' flavon VEVs, and the realistic CKM mixing arises due to both the flavon VEV pattern and the wave function profiles.

We emphasize that the T' representation assignments required for giving realistic masses and mixing patterns automatically forbid all leptonic tree-level FCNCs and those involving the first and the second generations of quarks, which are present in generic RS models. As a result, a low scale for the first KK mass scale can be allowed, rendering the RS model a viable solution to the gauge hierarchy problem and making it testable at collider experiments.

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APPENDIX: T' FAMILY SYMMETRY

The T' group is the double covering of the tetrahedral group A_4, in an analogous way that $SU(2)$ is the double covering of $SO(3)$. It has 24 elements and two generators, S and T. It contains three inequivalent, irreducible one-dimensional representations, three two-dimensional representations, and one three-dimensional representation. The generators satisfy the following algebra:

$$S^2 = R, \qquad T^3 = 1, \qquad (ST)^3 = 1, \qquad R^2 = 1 \tag{A1}$$

where $R = 1$ for the one-dimensional and three-dimensional representations, and $R = -1$ for the two-dimensional representations. This can be understood from the nomenclature that the one-dimensional and three-dimensional representations are vectorial representations, while the two-dimensional representations are spinorial. Just like spinors in four dimensions, the two-dimensional representations acquire an extra -1 after rotation in T' space (or, analogously, $SU(2)$ space) by 2π. It is interesting to note that this feature generates imaginary CG coefficients, which can be a source of *CP* violation [29]. Using the conventions from Ref. [27], the generators can be chosen as follows:

$$\begin{array}{lll}
1 & S = 1, & T = 1,\\
1' & S = 1, & T = \omega,\\
1'' & S = 1, & T = \omega^2,\\
2 & S = A_1, & T = \omega A_2,\\
2' & S = A_1, & T = \omega^2 A_2,\\
2'' & S = A_1, & T = A_2,\\
3 & S = \frac{1}{3}\begin{pmatrix} -1 & 2\omega & 2\omega^2 \\ 2\omega^2 & -1 & 2\omega \\ 2\omega & 2\omega^2 & -1 \end{pmatrix}, & T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}
\end{array} \tag{A2}$$

where the matrices A_1 and A_2 are

$$A_1 = -\frac{1}{\sqrt{3}}\begin{pmatrix} i & \sqrt{2}e^{i\pi/12} \\ -\sqrt{2}e^{-i\pi/12} & i \end{pmatrix}, \qquad A_2 = \begin{pmatrix} \omega & 0 \\ 0 & 1 \end{pmatrix}. \tag{A3}$$

We briefly present the product rules relevant for our choice of representation assignments in Table I and the Lagrangian specified in Eq. (4) and (28). In particular, the product of $3 \otimes 3$ appears in both the lepton and quark sectors. In the following, α_i denotes the ith component of the first representation in the product, while β_j denotes the jth component of the second representation in the product. We have

$$3 \otimes 3 = 3_S \oplus 3_A \oplus 1 \oplus 1' \oplus 1'', \tag{A4}$$

where

$$\begin{aligned}
3_S &= \frac{1}{3}\begin{pmatrix} 2\alpha_1\beta_1 - \alpha_2\beta_3 - \alpha_3\beta_2 \\ 2\alpha_3\beta_3 - \alpha_1\beta_2 - \alpha_2\beta_1 \\ 2\alpha_2\beta_2 - \alpha_1\beta_3 - \alpha_3\beta_1 \end{pmatrix},\\
3_A &= \frac{1}{2}\begin{pmatrix} \alpha_2\beta_3 - \alpha_3\beta_2 \\ \alpha_1\beta_2 - \alpha_2\beta_1 \\ \alpha_3\beta_1 - \alpha_1\beta_3 \end{pmatrix},\\
1 &= \alpha_1\beta_1 + \alpha_2\beta_3 + \alpha_3\beta_2,\\
1' &= \alpha_3\beta_3 + \alpha_1\beta_2 + \alpha_2\beta_1,\\
1'' &= \alpha_2\beta_2 + \alpha_1\beta_3 + \alpha_3\beta_1.
\end{aligned} \tag{A5}$$

We remark that the factors of $\frac{1}{3}$ and $\frac{1}{2}$ in the triplet representations of the direct sum are normalization coefficients.

From the quark sector, we also require the products of $2 \otimes 2$ and $2 \otimes 3$. The first product is

$$2 \otimes 2 = 3 \oplus 1 \tag{A6}$$

where the corresponding CG coefficients are

$$3 = \begin{pmatrix} \frac{1-i}{2}(\alpha_1\beta_2 + \alpha_2\beta_1) \\ i\alpha_1\beta_1 \\ \alpha_2\beta_2 \end{pmatrix}, \qquad 1 = \alpha_1\beta_2 - \alpha_2\beta_1, \tag{A7}$$

while the second product is

$$2 \otimes 3 = 2 \oplus 2' \oplus 2'' \tag{A8}$$

where

$$2 = \begin{pmatrix} (1+i)\alpha_2\beta_2 + \alpha_1\beta_1 \\ (1-i)\alpha_1\beta_3 - \alpha_2\beta_1 \end{pmatrix}. \tag{A9}$$

We omit the other terms in the direct sums since they do not contract to give pure singlets for the Lagrangian in Eqs. (4) and (28). The remaining singlet contractions are straightforward and detailed in [27]. From here, some algebra on the Lagrangian gives the mass matrix structure of Eq. (11), (13), (20), (22), and (32).

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