Extra vectorlike matter and the lightest Higgs scalar boson mass in low-energy supersymmetry

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The lightest Higgs scalar boson mass in supersymmetry can be raised significantly by extra vectorlike quark and lepton supermultiplets with large Yukawa couplings but dominantly electroweak-singlet masses. I consider models of this type that maintain perturbative gauge coupling unification. The impact of the new particles on precision electroweak observables is found to be moderate, with the fit to Z-pole data as good or better than that of the standard model even if the new Yukawa couplings are as large as their fixed-point values and the extra vectorlike quark masses are as light as 400 GeV. I study the size of corrections to the lightest Higgs boson mass, taking into account the fixed-point behavior of the scalar trilinear couplings. I also discuss the decay branching ratios of the lightest new quarks and leptons and general features of the resulting collider signatures.

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I. INTRODUCTION

The minimal supersymmetric standard model [1] (MSSM) predicts that the lightest neutral Higgs boson, h^0, has a mass that can only exceed that of the Z^0 boson by virtue of radiative corrections. If the superpartners are not too heavy, then it becomes a challenge to evade the constraints on h^0 set by CERN LEPII e^+e^- collider searches. On the other hand, larger superpartner masses tend to require some tuning in order to accommodate the electroweak symmetry breaking scale. In recent years this has motivated an exploration of models that extend the MSSM and can raise the prediction for m_{h^0}.

In the MSSM, the largest radiative corrections to m_{h^0} come from loop diagrams involving top quarks and squarks, and are proportional to the fourth power of the top Yukawa coupling. This suggests that one can further raise the Higgs mass by introducing new heavy supermultiplets with associated large Yukawa couplings. In recent years there has been renewed interest [2–20] in the possibility of a fourth family of quarks and leptons, which can be reconciled with precision electroweak constraints with or without supersymmetry. However, within the context of supersymmetry, if the new heavy supermultiplets are chiral (e.g. a sequential fourth family), then in order to evade discovery at the Fermilab Tevatron $p\bar{p}$ collider the Yukawa couplings would have to be so large that perturbation theory would break down not far above the electroweak scale. This would negate the success of apparent gauge coupling unification in the MSSM. Furthermore, the corrections to precision electroweak physics would rule out such models without some mild tuning of the fourth family quark and lepton masses.

These problems can be avoided if the extra supermultiplets are instead vectorlike, as proposed in [21–24]. If the scalar members of the new supermultiplets are heavier than the fermions, then there is a positive correction to m_{h^0}. As I will show below, the corrections to precision electroweak parameters decouple fast enough to render them benign.

To illustrate the general structure of such models, suppose that the new left-handed chiral supermultiplets include an $SU(2)_L$ doublet Φ with weak hypercharge Y and an $SU(2)_L$ singlet $\bar{\phi}$ with weak hypercharge $-Y - 1/2$, and $\bar{\Phi}$ and ϕ with the opposite gauge quantum numbers. The fields Φ and ϕ transform as the same representation of $SU(3)_C$ (either a singlet, a fundamental, or an antifundamental), and $\bar{\Phi}$ and $\bar{\phi}$ transform appropriately as the opposite. The superpotential allows the terms:

$$W = M_\Phi \Phi\bar{\Phi} + M_\phi \phi\bar{\phi} + kH_u\Phi\bar{\phi} - hH_d\bar{\Phi}\phi, \quad (1.1)$$

where M_Φ and M_ϕ are vectorlike (gauge-singlet) masses, and k and h are Yukawa couplings to the weak hypercharge $+1/2$ and $-1/2$ MSSM Higgs fields H_u and H_d, respectively. In the following, I will consistently use the letter k for Yukawa couplings of new fields to H_u, and h for couplings to H_d. Products of weak isospin doublet fields implicitly have their $SU(2)_L$ indices contracted with an antisymmetric tensor $\epsilon^{12} = -\epsilon^{21} = 1$, with the first component of every doublet having weak isospin $T_3 = 1/2$ and the second $T_3 = -1/2$. So, for example, $\Phi\bar{\Phi} = \Phi_1\bar{\Phi}_2 - \Phi_2\bar{\Phi}_1$, with the components Φ_1, Φ_2, $\bar{\Phi}_1$, and $\bar{\Phi}_2$ having electric charges $Y + 1/2$, $Y - 1/2$, $-Y + 1/2$, and $-Y - 1/2$ respectively.

The scalar members of the new chiral supermultiplets participate in soft supersymmetry-breaking Lagrangian terms:

$$-\mathcal{L}_{\rm soft} = (b_\Phi \Phi\bar{\Phi} + b_\phi \phi\bar{\phi} + a_k H_u\Phi\bar{\phi} - a_h H_d\bar{\Phi}\phi) + {\rm c.c.} + m_\Phi^2|\Phi|^2 + m_\phi^2|\phi|^2, \quad (1.2)$$

where I use the same name for each chiral superfield and its scalar component.

The fermion content of this model consists of two Dirac fermion-anti-fermion pairs with electric charges $\pm(Y+1/2)$ and one Dirac fermion-anti-fermion pair with electric charges $\pm(Y-1/2)$. The doubly degenerate squared-mass eigenvalues of the fermions with charge $\pm(Y+1/2)$ are obtained at tree-level by diagonalizing the matrix

$$m_F^2 = \begin{pmatrix} \mathcal{M}_F \mathcal{M}_F^\dagger & 0 \\ 0 & \mathcal{M}_F^\dagger \mathcal{M}_F \end{pmatrix} \tag{1.3}$$

with

$$\mathcal{M}_F = \begin{pmatrix} M_\Phi & k v_u \\ h v_d & M_\phi \end{pmatrix}, \tag{1.4}$$

which is assumed to be dominated by the M_Φ and M_ϕ entries on the diagonal. Here $v_u = v\sin\beta$ and $v_d = v\cos\beta$ are the vacuum expectation values (VEVs) of the MSSM Higgs fields H_u and H_d, in a normalization where $v \approx 175$ GeV. The scalar partners of these have a squared-mass matrix given by, in the basis $(\Phi, \phi, \bar{\Phi}^*, \bar{\phi}^*)$:

$$m_S^2 = m_F^2 + \begin{pmatrix} m_\Phi^2 + \Delta_{1/2,Y+1/2} & 0 & b_\Phi^* & a_k^* v_u - k\mu v_d \\ 0 & m_\phi^2 + \Delta_{0,Y+1/2} & a_h^* v_d - h\mu v_u & b_\phi^* \\ b_\Phi & a_h v_d - h\mu^* v_u & m_{\bar{\Phi}}^2 + \Delta_{-1/2,-Y-1/2} & 0 \\ a_k v_u - k\mu^* v_d & b_\phi & 0 & m_{\bar{\phi}}^2 + \Delta_{0,-Y-1/2} \end{pmatrix} \tag{1.5}$$

where the $\Delta_{T_3,q} = [T_3 - q\sin^2\theta_W]\cos(2\beta)m_Z^2$ are electroweak D-terms, with T_3 and q the weak isospin and electric charge. The scalar particle squared-mass eigenvalues of Eq. (1.5) are presumably larger than those of their fermionic partners because of the effects of m_Φ^2, m_ϕ^2, $m_{\bar{\Phi}}^2$ and $m_{\bar{\phi}}^2$, inducing a significant positive 1-loop correction to $m_{h^0}^2$. If $\tan\beta$ is not too small, the corrections to $m_{h^0}^2$ are largest if the k-type Yukawa coupling is as large as possible, i.e. near its infrared quasi-fixed point.

The fermions of charge $\pm(Y-1/2)$ have squared mass M_Φ^2, and their scalar partners have a squared-mass matrix

$$\begin{pmatrix} |M_\Phi|^2 + m_\Phi^2 + \Delta_{-1/2,Y-1/2} & -b_\Phi^* \\ -b_\Phi & |M_\Phi|^2 + m_{\bar{\Phi}}^2 + \Delta_{1/2,-Y+1/2} \end{pmatrix}. \tag{1.6}$$

These particles do not contribute to $m_{h^0}^2$ except through the small electroweak D-terms, since they do not have Yukawa couplings to the neutral Higgs boson. Since that contribution is therefore parametrically suppressed, it will be neglected in the following.

With the phases of H_u and H_d chosen so that their vacuum expectation values (VEVs) are real, then in complete generality only three of the new parameters M_Φ, M_ϕ, k and h can be simultaneously chosen real and positive by convention. Nevertheless, I will take all four to be real and positive below. (I will usually be assuming that the magnitude of at least one of the new Yukawa couplings is small, so that the potential CP-violating effects are negligible anyway.)

In the MSSM, the running gauge couplings extrapolated to very high mass scales appear to approximately unify near $Q = M_{\rm unif} = 2.4 \times 10^{16}$ GeV. In order to maintain this success, it is necessary to include additional chiral supermultiplets, besides the ones just mentioned. These other fields again do not have Yukawa couplings to the Higgs boson, so their contribution to $\Delta m_{h^0}^2$ will be neglected below.

I will be assuming that the superpotential vectorlike mass terms are not much larger than the TeV scale. This can be accomplished by whatever mechanism also generates the μ term in the MSSM. For example, it may be that the terms M_Φ and M_ϕ are forbidden at tree-level in the renormalizable Lagrangian, and arise from nonrenormalizable terms in the superpotential of the form:

$$W = \frac{\lambda}{M_{\rm Pl}} S\bar{S}\Phi\bar{\Phi} + \frac{\lambda'}{M_{\rm Pl}} S\bar{S}\phi\bar{\phi}, \tag{1.7}$$

after the scalar components of singlet supermultiplets S and $\bar{S}$ obtain vacuum expectation values of order the geometric mean of the Planck and soft supersymmetry-breaking scales. Then M_Φ, $M_\phi \lesssim 1$ TeV can be natural, just as for μ in the MSSM.

In the remainder of this paper, I will discuss aspects of the phenomenology of models of this type, concentrating on the particle content and renormalization group running (Sec. II), corrections to m_{h^0} (Sec. III), precision electroweak corrections (Sec. IV), and branching ratios and signatures for the lightest of the new fermions in each model (Sec. V).

II. SUPERSYMMETRIC MODELS WITH NEW VECTORLIKE FIELDS

A. Field and particle content

To construct and describe models, consider the following possible fields defined by their transformation properties under $SU(3)_C \times SU(2)_L \times U(1)_Y$:

$$\begin{aligned} &Q = (\mathbf{3}, \mathbf{2}, 1/6), \qquad \bar{Q} = (\bar{\mathbf{3}}, \mathbf{2}, -1/6),\\ &U = (\mathbf{3}, \mathbf{1}, 2/3), \qquad \bar{U} = (\bar{\mathbf{3}}, \mathbf{1}, -2/3),\\ &D = (\mathbf{3}, \mathbf{1}, -1/3), \qquad \bar{D} = (\bar{\mathbf{3}}, \mathbf{1}, 1/3),\\ &L = (\mathbf{1}, \mathbf{2}, -1/2), \qquad \bar{L} = (\mathbf{1}, \mathbf{2}, 1/2),\\ &E = (\mathbf{1}, \mathbf{1}, -1), \qquad \bar{E} = (\mathbf{1}, \mathbf{1}, 1),\\ &N = (\mathbf{1}, \mathbf{1}, 0), \qquad \bar{N} = (\mathbf{1}, \mathbf{1}, 0). \end{aligned} \tag{2.1}$$

Restricting the new supermultiplets to this list assures that small mixings with the MSSM fields can eliminate stable exotic particles which could be disastrous relics from the early universe. In this paper, I will reserve the above capital letters for new extra chiral supermultiplets, and use lowercase letters for the MSSM quark and lepton supermultiplets:

$$\begin{aligned} &q_i = (\mathbf{3}, \mathbf{2}, 1/6), \qquad \bar{u}_i = (\bar{\mathbf{3}}, \mathbf{1}, -2/3),\\ &\bar{d}_i = (\bar{\mathbf{3}}, \mathbf{1}, 1/3), \qquad \ell_i = (\mathbf{1}, \mathbf{2}, -1/2),\\ &\bar{e}_i = (\mathbf{1}, \mathbf{1}, 1), \qquad H_u = (\mathbf{1}, \mathbf{2}, 1/2),\\ &H_d = (\mathbf{1}, \mathbf{2}, -1/2), \end{aligned} \tag{2.2}$$

with $i = 1, 2, 3$ denoting the three families. So the MSSM superpotential, in the approximation that only third-family Yukawa couplings are included, is

$$W = \mu H_u H_d + y_t H_u q_3 \bar{u}_3 - y_b H_d q_3 \bar{d}_3 - y_\tau H_d \ell_3 \bar{e}_3. \tag{2.3}$$

It is well known that gauge coupling unification is maintained if the new fields taken together transform as complete $SU(5)$ multiplets. However, this is not a necessary condition. There are three types of models that can successfully maintain perturbative gauge coupling unification with the masses of new extra chiral supermultiplets at the TeV scale.

First, there is a model to be called the "LND model" in this paper, consisting of chiral supermultiplets L, $\bar{L}$, N, $\bar{N}$, D, $\bar{D}$, with a superpotential

$$W = M_L L\bar{L} + M_N N\bar{N} + M_D D\bar{D} + k_N H_u L\bar{N} - h_N H_d \bar{L} N. \tag{2.4}$$

Here L, $\bar{L}$ play the role of Φ, $\bar{\Phi}$ and N, $\bar{N}$ the role of ϕ, $\bar{\phi}$ in Eqs. (1.1), (1.2), (1.3), (1.4), (1.5), and (1.6). In most of the following, I will consider only the case that the multiplicity of each of these fields is 1, although 1, 2, or 3 copies of each would be consistent with perturbative gauge coupling unification. These fields consist of a $\mathbf{5} + \bar{\mathbf{5}}$ of $SU(5)$, plus a pair[1] of singlet fields. The non-MSSM mass eigenstate fermions consist of a charged lepton τ', a pair of neutral fermions $\nu'_{1,2}$, and a charge $-1/3$ quark b'. Their superpartners are complex scalars $\tilde{\tau}'_{1,2}$, $\tilde{\nu}'_{1,2,3,4}$, and $\tilde{b}'_{1,2}$. The primes are used to distinguish these states from those of the usual MSSM that have the same charges.

[1]Here I choose the minimal model of this type that includes Yukawa couplings of the kind mentioned in the Introduction while not violating lepton number. It is also possible to identify the fields N and $\bar{N}$, since they are gauge singlets, or to eliminate them (and their Yukawa couplings) entirely.

Second, one has a model consisting of a $\mathbf{10} + \overline{\mathbf{10}}$ of $SU(5)$, to be called the "QUE model" below, consisting of fields Q, $\bar{Q}$, U, $\bar{U}$, E, $\bar{E}$ with a superpotential

$$W = M_Q Q\bar{Q} + M_U U\bar{U} + M_E E\bar{E} + k_U H_u Q\bar{U} - h_U H_d \bar{Q} U. \tag{2.5}$$

The non-MSSM particles in this case consist of charge $+2/3$ quarks $t'_{1,2}$, a charge $-1/3$ quark b', and a charged lepton τ', and their scalar partners $\tilde{t}'_{1,2,3,4}$, $\tilde{b}'_{1,2}$ and $\tilde{\tau}'_{1,2}$.

Third, one has a "QDEE model" consisting of fields Q, $\bar{Q}$, D, $\bar{D}$, E_i, $\bar{E}_i$ $(i = 1, 2)$ with a superpotential

$$W = M_Q Q\bar{Q} + M_U D\bar{D} + M_{E_i} E_i \bar{E}_i + k_D H_u \bar{Q} D - h_D H_d Q\bar{D}. \tag{2.6}$$

Although this particle content does not happen to contain complete multiplets of $SU(5)$, it still gives perturbative gauge coupling unification. The non-MSSM particles in this model consist of charge $-1/3$ quarks $b'_{1,2}$, a charge $+2/3$ quark t', and two charged leptons $\tau'_{1,2}$, and their scalar partners $\tilde{b}'_{1,2,3,4}$, $\tilde{t}'_{1,2}$ and $\tilde{\tau}'_{1,2,3,4}$.

The field and particle content of these three models is summarized in Table I.

In Ref. [24], it is suggested that a model with extra chiral supermultiplets in $\mathbf{5} + \bar{\mathbf{5}} + \mathbf{10} + \overline{\mathbf{10}}$ of $SU(5)$, or equivalently (if a pair of singlets is added) $\mathbf{16} + \overline{\mathbf{16}}$ of $SO(10)$, will also result in gauge coupling unification. However, the multiloop running of gauge couplings actually renders them nonperturbative below the putative unification scale, unless the new particles have masses well above the 1 TeV scale. For example, working to 3-loop order, if one requires that the unified coupling (defined to be the common value of α_1 and α_2 at their meeting point) satisfies the perturbativity condition $\alpha_{\rm unif} < 0.35$, then the average threshold of the new particles must exceed 5 TeV if the MSSM particles are treated as having a common threshold at or below 1 TeV as suggested by naturalness and the little hierarchy problem. In that case, the new particles will certainly decouple from LHC phenomenology. Even if one allows the MSSM soft mass scale to be as heavy as the new particles, treating all non-standard model particles as having a common threshold, I find that this threshold must be at least 2.8 TeV if the new Yukawa couplings vanish and at least 2.1 TeV if the new Yukawa couplings are as large as their fixed-point values. While such heavy mass spectra are possible, they go directly against the motivation provided by the little hierarchy problem. Furthermore, at the scale of apparent unification of α_1 and α_2 in such models, the value of α_3 is considerably smaller, rendering the apparent uni-

TABLE I. The new chiral supermultiplets and the new particle content of the models discussed in this paper. The notation for Φ, $\bar{\Phi}$, ϕ, $\bar{\phi}$ follows that of the Introduction.

Model	New supermultiplets			New particles	
	$\Phi, \bar{\Phi}$	$\phi, \bar{\phi}$	Others	Spin 1/2	Spin 0
LND	$L, \bar{L}$	$N, \bar{N}$	$D, \bar{D}$	$\nu'_{1,2}\ \tau'\ b'$	$\tilde{\nu}'_{1,2,3,4}\ \tilde{\tau}'_{1,2}\ \tilde{b}'_{1,2}$
QUE	$Q, \bar{Q}$	$U, \bar{U}$	$E, \bar{E}$	$t'_{1,2}\ b'\ \tau'$	$\tilde{t}'_{1,2,3,4}\ \tilde{b}'_{1,2}\ \tilde{\tau}'_{1,2}$
QDEE	$\bar{Q}, Q$	$\bar{D}, D$	$E_{1,2}, \bar{E}_{1,2}$	$b'_{1,2}\ t'\ \tau'_{1,2}$	$\tilde{b}'_{1,2,3,4}\ \tilde{t}'_{1,2}\ \tilde{\tau}'_{1,2,3,4}$

fication of gauge couplings at best completely accidental, dependent on the whim of out-of-control high-scale threshold corrections. I will therefore not consider that model further here, although it could be viable if one accepts the loss of perturbative unification and control at high scales. The collider phenomenology should be qualitatively similar to that of the LND and QUE models, since the particle content is just the union of them.

B. Renormalization group running

The unification of running gauge couplings in the MSSM, LND, and QUE models is shown in Fig. 1. In this graph, 3-loop beta functions are used for the MSSM gauge couplings, and $m_t = 173.1$ GeV and $\tan\beta = 10$, and all non-standard-model particles are taken to decouple at $Q = 600$ GeV. (The Yukawa couplings k_N and h_N in the LND model and k_U and h_U in the QUE model are set to 0 here for simplicity; they do not have a dramatic effect on the results as long as they are at or below their fixed-point trajectories.) The running for the QDEE model is not shown, because it is very similar to that for the QUE model. Indeed, it will turn out that many features of the QUE and QDEE models are similar, insofar as the $U + \bar{U}$ fields can be interchanged with the $D + \bar{D} + E + \bar{E}$ fields. This similarity does not extend, however, to the collider phenomenology as discussed in Sec. V. Note that the unification scale, defined as the renormalization scale Q at which $\alpha_1 = \alpha_2$, is somewhat higher with the extra chiral supermultiplets in place; in the MSSM, $M_{\rm unif} \approx 2.4 \times 10^{16}$ GeV, but $M_{\rm unif} \approx 2.65 \times 10^{16}$ GeV in the LND model, and $M_{\rm unif} \approx 8.3 \times 10^{16}$ GeV in the QUE and QDEE models. The strong coupling α_3 misses the unified α_1 and α_2, but by a small amount that can be reasonably ascribed to threshold corrections of whatever new physics occurs at $M_{\rm unif}$.

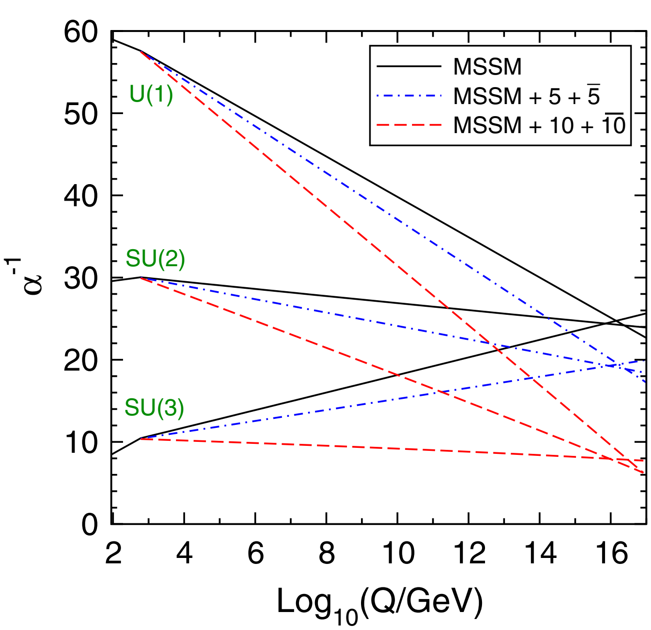

FIG. 1 (color online). Gauge coupling unification in the MSSM, LND and QUE models. The running is performed with 3-loop beta functions, with all particles beyond the standard model taken to decouple at $Q = 600$ GeV, and $m_t = 173.1$ GeV with $\tan\beta = 10$.

The largest corrections to m_{h^0} are obtained when the new Yukawa couplings of the type k_N, k_U, or k_D are as large as possible in the LND, QUE, and QDEE models, respectively. These new Yukawa couplings have infrared quasi-fixed point behavior, which limits how large they can be at the TeV scale while staying consistent with perturbative unification. This is illustrated in Fig. 2, which shows the renormalization group running[2] of the k_N coupling in the LND model and k_U in the QUE model. The running of k_D in the QDEE model is very similar to the latter (and so is not shown). In this paper, I will somewhat arbitrarily define the fixed-point trajectories to be those for which the extreme Yukawa couplings are equal to[3] 3 at the scale $M_{\rm unif}$ where α_1 and α_2 unify. Then, assuming that only one of the new Yukawa couplings is turned on at a time, and that $\tan\beta = 10$ with $m_t = 173.1$ GeV and with all new particle thresholds taken to be at $Q = 600$ GeV, the fixed-point values also evaluated at $Q = 600$ GeV are

[2] In this paper, I use 3-loop beta functions for the gauge couplings and gaugino masses, and 2-loop beta functions for the Yukawa couplings, soft scalar trilinear couplings, and soft scalar squared masses. These can be obtained quite straightforwardly from the general results listed in [25–27], and so are not given explicitly here.

[3] Formally, it turns out that the 2-loop and 3-loop beta functions for these Yukawa couplings have ultraviolet-stable fixed points, although these occur at such large values (> 5) that they cannot be trusted to reflect the true behavior. Simply requiring the high-scale value of the Yukawa couplings to be somewhat smaller avoids this issue.

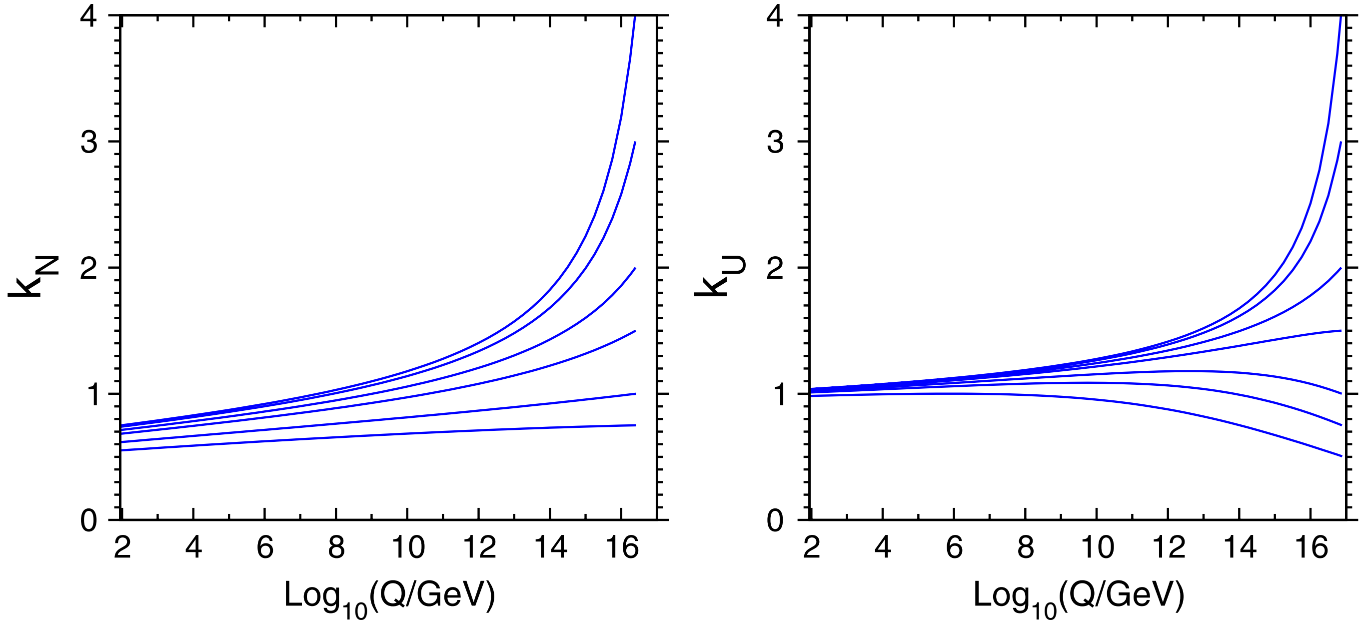

FIG. 2 (color online). Renormalization group trajectories near the fixed point for k_N in the LND model (left panel) and k_U in the QUE model (right panel), showing the infrared-stable quasi-fixed-point behaviors. Here $m_t = 173.1$ GeV and $\tan\beta = 10$ are assumed.

$$\text{LND model: } k_N = 0.765 \quad \text{or} \quad h_N = 0.905, \tag{2.7}$$

$$\text{QUE model: } k_U = 1.050 \quad \text{or} \quad h_U = 1.203, \tag{2.8}$$

$$\text{QDEE model: } k_D = 1.043 \quad \text{or} \quad h_D = 1.196. \tag{2.9}$$

Turning on both Yukawa couplings at the same time in each model hardly affects the results at all, because k_i, h_i decouple from each other's beta functions at 1-loop order for each of $i = N$, U, D. This is illustrated by the very nearly rectangular shape of the fixed-line contours in Fig. 3.

The phenomenology of supersymmetric models is crucially dependent on the ratios of gaugino masses. In the MSSM, it is will known that if the gaugino masses unify at $M_{\rm unif}$, then working to 1-loop order they obey $M_1/\alpha_1 = M_2/\alpha_2 = M_3/\alpha_3 = m_{1/2}/\alpha_{\rm unif}$, and this relation has only moderate corrections from higher-loop contributions to the beta functions. The presence of extra matter particles strongly affects this prediction, however. In Table II, the predictions for M_1, M_2, and M_3 at $Q = 1$ TeV are given for the MSSM, the LND model, the QUE model, and the QDEE model. In the latter two cases, I distinguish between the cases of vanishing extra Yukawa couplings and the fixed-point trajectories with $(k_U, h_U) = (3, 0)$ and $(k_D, h_D) = (3, 0)$, respectively, at $Q = M_{\rm unif}$. As before, I have used $\tan\beta = 10$ and $m_t = 173.1$ GeV, and taken all new particle thresholds to be at $Q = 600$ GeV, and assumed for simplicity that the new scalar trilinear couplings vanish at $Q = M_{\rm unif}$. The results will change slightly if these assumptions are modified, but there are a couple of

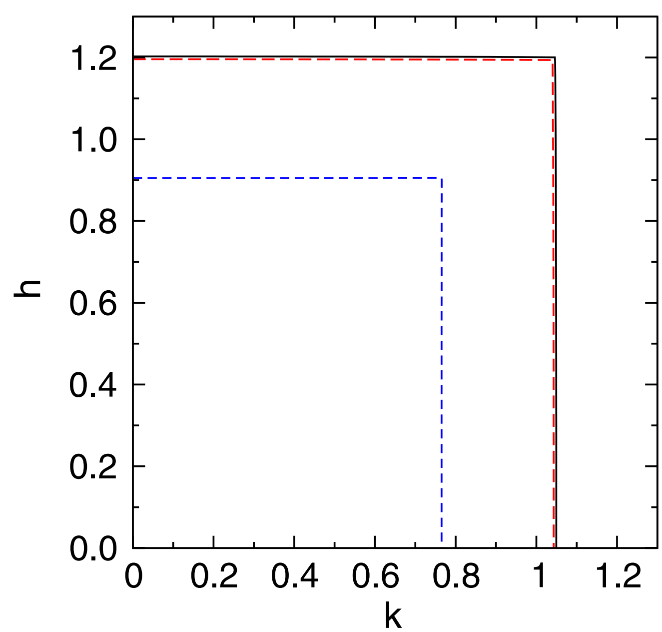

FIG. 3 (color online). The contours represent the infrared-stable quasi-fixed points of the 2-loop renormalization group equations in the plane of Yukawa couplings (k_i, h_i) evaluated at $Q = 500$ GeV. The allowed perturbative regions (defined by k_i, $h_i < 3$ at $Q = M_{\rm unif}$) are to the left and below the contours. The long dashed (blue) line corresponds to k_N, h_N in the LND model. The solid (black) line corresponds to k_U, h_U in the QUE model, and the nearly overlapping short dashed (red) line corresponds to k_D, h_D in the QDEE model. Here $m_t = 173.1$ GeV and $\tan\beta = 10$ are assumed. The very nearly rectangular shape of these contours reflects the absence of direct coupling between the Yukawa couplings in the 1-loop β functions.

TABLE II. Gaugino masses $M_a/m_{1/2}$ for ($a = 1, 2, 3$) and ratios of gaugino masses M_2/M_1 and M_3/M_1, evaluated at $Q = 1$ TeV in the models described in the text, assuming unified gaugino masses $m_{1/2}$ at $M_{\rm unif}$.

	$M_1/m_{1/2}$	$M_2/m_{1/2}$	$M_3/m_{1/2}$	M_2/M_1	M_3/M_1
MSSM	0.41	0.77	2.28	1.88	5.53
LND	0.32	0.59	1.75	1.86	5.52
QUE ($k_U = 0$)	0.097	0.147	0.571	1.52	5.90
($k_U(M_{\rm unif}) = 3$)	0.109	0.176	0.617	1.61	5.66
QDEE ($k_D = 0$)	0.094	0.153	0.572	1.62	6.08
($k_D(M_{\rm unif}) = 3$)	0.107	0.178	0.615	1.66	5.72

striking and robust features to be pointed out about the gaugino masses in these models. First, because the unified gauge couplings are so much larger in the models with extra matter than in the MSSM, the gaugino masses at the TeV scale are suppressed relative to $m_{1/2}$ by a significant amount compared to the MSSM. Second, the 1-loop prediction for the ratios of gaugino masses is very strongly violated by 2-loop effects[4] in the extended models, which were evidently neglected in [24]. For example, in both the QUE and QDEE models, the 1-loop prediction is that $M_3 = m_{1/2}$, independent of Q, since the 1-loop beta function for M_3 happens to vanish because the total number of quark and antiquark chiral supermultiplets is equal to 18, exactly cancelling the $6N_c$ contribution from the gluon/gluino supermultiplet. However, the correct result beyond leading order is that M_3 does run significantly, with $M_3/m_{1/2}$ reduced by some 40% from unity, depending on the Yukawa coupling value. This reflects, in part, the accidental vanishing of the 1-loop beta function; in contrast, the 3-loop contribution to the running is quite small compared to the 2-loop one, which is over a factor of 3 larger than in the MSSM near the TeV scale. This is illustrated for the QUE model in Fig. 4, which shows the renormalization-scale dependence of the running gaugino mass parameters M_1, M_2, and M_3 in the QUE model [with $(k_U, h_U) = (3, 0)$ at $Q = M_{\rm unif}$], evolved according to the 1, 2, and 3-loop beta functions.

Another notable feature of the extended models is that they permit gaugino mass domination for the soft supersymmetry-breaking terms at the unification scale, according to which all soft scalar masses and scalar trilinear couplings are assumed negligible compared to the gaugino masses, or $A_0 = 0$, $m_0^2 = 0$ in the usual mSUGRA language. In the MSSM, this "no-scale" boundary condition is problematic if applied strictly, because it predicts that the lightest supersymmetric particle (LSP) is not a neutralino. However, in the QUE and QDEE models, the increased size of the gauge couplings at high scales gives extra gaugino-mediated renormalization group contributions to the scalar squared masses, so that they are safely heavier than the binolike LSP. For the squarks and sleptons of the first two families, this is illustrated in Table III (for the same models as in Table II), by giving the ratios of the running masses to the unified gaugino-mass parameter $m_{1/2}$. The contributions of the gaugino masses to the new extra squarks and sleptons in the LND, QUE, and QDEE models are listed below:

$$\text{LND}: (m_D, m_{\bar D}, m_L, m_{\bar L}, m_N, m_{\bar N}) = (1.80, 1.80, 0.63, 0.63, 0, 0), \quad (2.10)$$

$$\text{QUE}: (m_Q, m_{\bar Q}, m_U, m_{\bar U}, m_E, m_{\bar E}) = (1.17, 1.29, 1.25, 0.94, 0.267, 0.299), \quad (2.11)$$

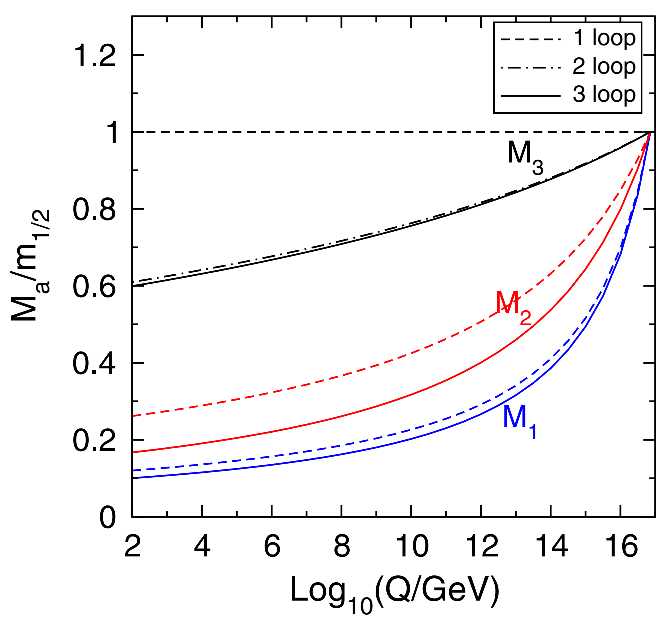

FIG. 4 (color online). Running of gaugino masses in the QUE model, assuming a unified value $m_{1/2}$ at $M_{\rm unif}$. The gluino mass parameter M_3 is evolved according to the 1-, 2-, and 3-loop beta functions in the top three lines. The 1- and 3-loop beta functions are shown for the parameters M_1 (bottom two lines) and M_2, for which the 2-loop and 3-loop results are not visually distinguishable. Note the significant running of M_3 due to multiloop effects.

[4]Similar effects have been noted long ago in the context of "semiperturbative unification" [28].

TABLE III. Ratios of first- and second-family MSSM squark and slepton mass parameters to $m_{1/2}$, evaluated at $Q = 1$ TeV, assuming unified gaugino-mass dominance at $Q = M_{\rm unif}$ ($m_0^2 = 0$ and $A_0 = 0$).

	$m_{\tilde{q}}$	$m_{\tilde{\bar{u}}}$	$m_{\tilde{\bar{d}}}$	$m_{\tilde{\ell}}$	$m_{\tilde{\bar{e}}}$
MSSM	2.08	2.01	2.00	0.67	0.37
LND	1.89	1.82	1.81	0.63	0.35
QUE ($k_U = 0$)	1.24	1.20	1.19	0.45	0.28
($k_U(M_{\rm unif}) = 3$)	1.29	1.24	1.24	0.47	0.30
QDEE ($k_D = 0$)	1.24	1.20	1.20	0.45	0.28
($k_D(M_{\rm unif}) = 3$)	1.30	1.25	1.24	0.47	0.30

$$\text{QDEE}: (m_Q, m_{\bar{Q}}, m_D, m_{\bar{D}}, m_E, m_{\bar{E}}) = (1.30, 1.18, 0.94, 1.24, 0.266, 0.304). \quad (2.12)$$

Here I have chosen to display the results for boundary conditions at $M_{\rm unif}$ of $k_N = h_N = 0$ and for $k_U = 3$, $h_U = 0$ and for $k_D = 3$, $h_D = 0$, respectively. It should be noted that these are all running mass parameters, and the physical mass parameters will be different. Also, if there are nonzero contributions to the running scalar squared masses and scalar trilinear couplings at $M_{\rm unif}$, the results will of course change. For example, including a nonzero common m_0^2, as in mSUGRA, will raise all of the scalar squared masses, yielding a more degenerate scalar mass spectrum.

For the QUE and QDEE models, we see from Tables II and III and Eqs. (2.11) and (2.12) that the bino mass parameter is well over a factor of 2 smaller than the lightest slepton mass, for unified, dominant gaugino masses. Since neutralino mixing only decreases the LSP mass compared to the bino mass parameter, the LSP will be a neutralino. In contrast, for the LND models, the gaugino mass dominance boundary condition would predict that the scalar component of N or $\bar{N}$ (a non-MSSM sneutrino) should be the LSP, and should be nearly massless. In fact, including a nonzero Yukawa coupling h_N or k_N would give the corresponding scalar a negative squared mass. If there is an additional positive contribution to that sneutrino mass, then it can be the LSP, and it might be interesting to consider it as a possible dark matter candidate.

The corrections to the lightest Higgs squared mass considered in the next section depend on the scalar trilinear coupling a_{k_N}, a_{k_U}, or a_{k_D} of the type appearing in Eq. (1.2). It is therefore useful to note that these couplings have a strongly attractive fixed-point behavior in the infrared when the corresponding superpotential couplings k_N, k_U and k_D are near their fixed points. To illustrate this, consider the quantities

$$A_{k_N} \equiv a_{k_N}/k_N, \qquad A_{k_U} \equiv a_{k_U}/k_U, \qquad A_{k_D} \equiv a_{k_D}/k_D, \quad (2.13)$$

for the LND, QUE, and QDEE models, respectively. The renormalization group runnings of A_{k_N} and A_{k_U} (each normalized to $m_{1/2}$) are shown in Fig. 5, for various input values at the unification scale. The running of A_{k_N} in the LND model is seen to have a mild focusing behavior,

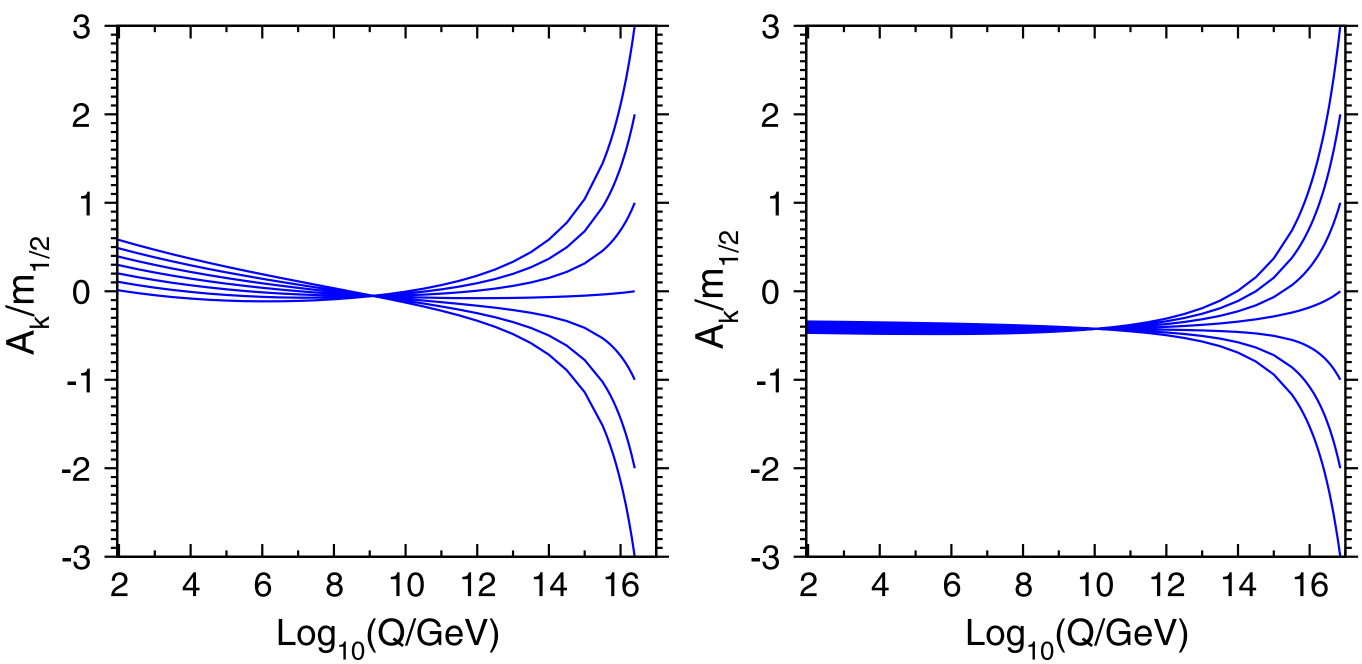

FIG. 5 (color online). Renormalization group running of scalar trilinear couplings A_{k_N} in the LND model (left panel) and A_{k_U} in the QUE model (right panel), normalized to $m_{1/2}$, the common gaugino-mass parameter at the unification scale $M_{\rm unif}$. The different lines correspond to different boundary conditions at $M_{\rm unif}$. The corresponding Yukawa couplings k_N and k_U are taken to be near their fixed-point trajectories, with $k_N = 3$ and $k_U = 3$ at $M_{\rm unif}$. The running of A_{k_D} in the QDEE model is very similar to that shown here for A_{k_U}.

leading to values at the weak scale of $-0.1 \lesssim A_{k_N}/m_{1/2} \lesssim 0.6$ for input values at $M_{\rm unif}$ in the range $-3 \lesssim A_{k_N}/m_{1/2} \lesssim 3$. In the case of A_{k_U} in the QUE model, one finds an even stronger focusing behavior leading to $-0.5 \lesssim A_{k_U}/m_{1/2} \lesssim -0.3$ at the weak scale. The running of A_{k_D} in the QDEE model is very similar (and so is not shown). It is useful to note that in the cases of A_{k_U} in the QUE model and A_{k_D} in the QDEE model, most of the contribution to the running comes from the gluino mass parameter. This will still be true if one does not assume gaugino mass unification, provided only that the gluino mass parameter M_3 is not very small compared to the bino and wino mass parameters M_1 and M_2. Therefore, the previous results concerning the fixed-point behavior of A_{k_D} and A_{k_U} remain approximately valid if $m_{1/2}$ is replaced by the value of M_3 at the unification scale.

C. Fine-tuning considerations

One of the primary model-building motivations in recent years is the supersymmetric little hierarchy problem, which concerns the tuning required to obtain the electroweak scale, given the large supersymmetry-breaking effects needed to avoid a light Higgs boson that should have been seen at LEP and to evade direct searches for superpartners at LEP and the Tevatron. One way to express this problem is to note that the Z boson mass is related to the parameters $|\mu|$ and $m^2_{H_u}$ near the weak scale by

$$-\frac{1}{2}m_Z^2 = |\mu|^2 + m^2_{H_u} + \frac{1}{2v_u}\frac{\partial}{\partial v_u}\Delta V + \mathcal{O}(1/\tan^2\beta), \tag{2.14}$$

where ΔV is the radiative part of the effective potential. Although there can be no such thing as an objective measure of fine-tuning in parameter space, the cancellation needed between $|\mu|^2$ and $m^2_{H_u}$ can be taken as an indication of how "difficult" it is to achieve the observed weak scale. Large values of $-m^2_{H_u}$ require more tuning in this sense.

In the MSSM, with the gauge and Yukawa couplings taken to be the values of the infamous benchmark point SPS1a′ [29] for a concrete example, one finds

$$\begin{aligned}-m^2_{H_u} = {}& 1.82\hat{M}_3^2 - 0.212\hat{M}_2^2 + 0.156\hat{M}_3\hat{M}_2 \\ & + 0.023\hat{M}_1\hat{M}_3 - 0.32\hat{A}_t\hat{M}_3 - 0.07\hat{A}_t\hat{M}_2 \\ & + 0.11\hat{A}_t^2 - 0.64\hat{m}^2_{H_u} + 0.36\hat{m}^2_{\tilde{q}_3} + 0.28\hat{m}^2_{\tilde{u}_3} \\ & + \ldots .\end{aligned} \tag{2.15}$$

Here $m^2_{H_u}$ on the left side is evaluated at the scale $Q = 600$ GeV, where corrections to ΔV are presumably not too large. The non-MSSM particle thresholds are also taken to be at the same scale. The hats on the parameters on the right side denote that they are inputs at the apparent unification scale $M_{\rm unif} = 2.4 \times 10^{16}$ GeV. They consist of gaugino masses $\hat{M}_{1,2,3}$, scalar squared masses $\hat{m}^2_{H_u}$, $\hat{m}^2_{\tilde{q}_3}$ and $\hat{m}^2_{\tilde{u}_3}$, and $\hat{A}_t \equiv a_t/y_t$. I have neglected to write other contributions with small coefficients. Note that the gaugino masses and scalar squared masses are not assumed to be unified here. The essence of the supersymmetric little hierarchy problem is that after constraints from nonobservation of the lightest Higgs boson, the charged supersymmetric particles, and from the relic abundance of dark matter are taken into account, the remaining parameter space tends to yield $-m^2_{H_u} \gg m_Z^2/2$, so that some fine adjustment is needed between $-m^2_{H_u}$ and $|\mu|^2$. It was noted long ago in Ref. [30] that the gluino mass parameter M_3 is actually mostly responsible for the tuning needed in $m^2_{H_u}$, because of its large coefficient as seen in Eq. (2.15), and this problem can be ameliorated significantly by taking $|\hat{M}_3/\hat{M}_2|$ smaller than unity at $M_{\rm unif}$. This can easily be achieved in non-mSUGRA models. For example, taking $|\hat{M}_3/\hat{M}_2| \sim 1/3$ produces near cancellation between the $\hat{M}_3^2$ and $\hat{M}_2^2$ terms with opposite signs in Eq. (2.15), yielding a smaller value for $m^2_{H_u}$.

Now let us compare to the corresponding formulas in the LND, QUE and QDEE models under study here. For the QUE model, I find near the fixed point $k_U = 1.05$ with $h_U = 0$ that the most significant contributions are approximately

$$\begin{aligned}-m^2_{H_u} = {}& 2.10\hat{M}_3^2 + 0.035\hat{M}_2^2 + 0.019\hat{M}_1^2 - 0.014\hat{M}_3\hat{M}_2 \\ & - 0.075\hat{A}_t\hat{M}_3 - 0.016\hat{A}_t\hat{M}_2 + 0.022\hat{A}_{k_U}\hat{M}_3 \\ & + 0.014\hat{A}_{k_U}\hat{M}_2 + 0.057\hat{A}_t^2 - 0.015\hat{A}_t\hat{A}_{k_U} \\ & + 0.25\hat{A}_{k_U}^2 - 0.17\hat{m}^2_{H_u} + 0.34\hat{m}^2_{\tilde{q}_3} + 0.27\hat{m}^2_{\tilde{u}_3} \\ & + 0.47m^2_Q + 0.40m^2_{\bar{U}} + \ldots .\end{aligned} \tag{2.16}$$

Again the hats on parameters on the right side denote their status as input values at $M_{\rm unif}$, and $m^2_{H_u}$ on the left side is evaluated at $Q = 600$ GeV, which is also where the new particle thresholds are placed, and $\tan\beta = 10$. The result of Eq. (2.16) seems to reflect a worsening of the little hierarchy problem, since the contribution to $-m^2_{H_u}$ proportional to $\hat{M}_3^2$ is even larger than in the MSSM case, while the physical MSSM superpartner masses are actually lower for fixed values of the input soft parameters, as can be seen from Tables II and III. This implies that for a given scale of physical superpartner masses, including notably the top squarks that contribute strongly to $m^2_{h^0}$, one will need larger $-m^2_{H_u}$, and thus larger $|\mu|^2$, and so a more delicate cancellation between the two. Note also that since the contribution proportional to $\hat{M}_2^2$ is positive (and quite small), there cannot be a cancellation as in the MSSM for large $|\hat{M}_2/\hat{M}_3|$. Counteracting these considerations, there is the fact that there are large positive corrections to $m^2_{h^0}$ from the new particles, as discussed in the following section, so that the top squark masses need not be so large.

It is interesting to compare with the corresponding result when the new Yukawa coupling k_U is instead taken to vanish:

$$-m_{H_u}^2 = 1.14\hat{M}_3^2 - 0.107\hat{M}_2^2 + 0.153\hat{M}_3\hat{M}_2 + 0.022\hat{M}_1\hat{M}_3 - 0.436\hat{A}_t\hat{M}_3 - 0.090\hat{A}_t\hat{M}_2 + 0.125\hat{A}_t^2 - 0.70\hat{m}_{H_u}^2 + 0.30\hat{m}_{\tilde{q}_3}^2 + 0.21\hat{m}_{\tilde{u}_3}^2 + \ldots \tag{2.17}$$

for $k_U = 0$. Here the impact on fine-tuning is not as bad as in Eq. (2.16) because the coefficient of $\hat{M}_3^2$ is reduced, there is no large positive contribution from the new scalar soft masses, and the possibility of significant cancellation between the gluino and wino mass contributions (if $|\hat{M}_2/\hat{M}_3| > 1$) is restored. But, counteracting this, there is no large positive contribution to $m_{h^0}^2$ from the extra vectorlike sector when $k_U = 0$.

Results for the QDEE model are quite similar. At the fixed point with $k_D = 1.043$, I find

$$-m_{H_u}^2 = 2.12\hat{M}_3^2 + 0.034\hat{M}_2^2 + 0.006\hat{M}_1^2 - 0.013\hat{M}_3\hat{M}_2 - 0.085\hat{A}_t\hat{M}_3 - 0.017\hat{A}_t\hat{M}_2 + 0.029\hat{A}_{k_D}\hat{M}_3 + 0.014\hat{A}_{k_D}\hat{M}_2 + 0.054\hat{A}_t^2 - 0.027\hat{A}_t\hat{A}_{k_D} + 0.12\hat{A}_{k_D}^2 - 0.22\hat{m}_{H_u}^2 + 0.33\hat{m}_{\tilde{q}_3}^2 + 0.26\hat{m}_{\tilde{u}_3}^2 + 0.37m_{\bar{Q}}^2 + 0.39m_D^2 + \ldots, \tag{2.18}$$

and for $k_D = 0$,

$$-m_{H_u}^2 = 1.15\hat{M}_3^2 - 0.106\hat{M}_2^2 + 0.154\hat{M}_3\hat{M}_2 + 0.024\hat{M}_1\hat{M}_3 - 0.439\hat{A}_t\hat{M}_3 - 0.090\hat{A}_t\hat{M}_2 + 0.125\hat{A}_t^2 - 0.70\hat{m}_{H_u}^2 + 0.30\hat{m}_{\tilde{q}_3}^2 + 0.21\hat{m}_{\tilde{u}_3}^2 + \ldots. \tag{2.19}$$

The same general comments therefore apply for the QDEE model as for the QUE model.

Treating the LND model in the same way, I find for $k_N = 0.765$:

$$-m_{H_u}^2 = 1.74\hat{M}_3^2 - 0.166\hat{M}_2^2 + 0.131\hat{M}_3\hat{M}_2 + 0.020\hat{M}_3\hat{M}_1 - 0.33\hat{A}_t\hat{M}_3 - 0.06\hat{A}_t\hat{M}_2 + 0.07\hat{A}_{k_N}\hat{M}_3 + 0.11\hat{A}_t^2 - 0.04\hat{A}_t\hat{A}_{k_N} + 0.05\hat{A}_{k_N}^2 - 0.62\hat{m}_{H_u}^2 + 0.38\hat{m}_{\tilde{q}_3}^2 + 0.30\hat{m}_{\tilde{u}_3}^2 + \ldots. \tag{2.20}$$

The dependence on the soft parameters in the sector of new extra particles is very slight, due to the fact that the fixed-point Yukawa coupling is not too large. Here we see that even at its fixed point the LND model is qualitatively quite similar to the MSSM, in that a ratio of $|\hat{M}_2/\hat{M}_3|$ larger than 1 at the unification scale can reduce $-m_{H_u}^2$ and therefore mitigate the amount of tuning required with $|\mu|^2$. For comparison, the result with $k_N = 0$ is

$$-m_{H_u}^2 = 1.68\hat{M}_3^2 - 0.178\hat{M}_2^2 + 0.164\hat{M}_3\hat{M}_2 + 0.020\hat{M}_3\hat{M}_1 - 0.36\hat{A}_t\hat{M}_3 - 0.08\hat{A}_t\hat{M}_2 + 0.12\hat{A}_t^2 - 0.66\hat{m}_{H_u}^2 + 0.34\hat{m}_{\tilde{q}_3}^2 + 0.26\hat{m}_{\tilde{u}_3}^2 + \ldots, \tag{2.21}$$

which shows quite similar characteristics.

Summarizing the preceding discussion, there are two general counteracting effects on the little hierarchy problem from introducing vectorlike supermultiplets with large Yukawa couplings. The impact of contributions to $-m_{H_u}^2$ generally tends to worsen the problem, but the additional correction to $m_{h^0}^2$ discussed in the next section works to mitigate the problem. (Ref. [24] obtained qualitatively similar results, but with quite different numerical details, presumably due to neglect of higher-loop contributions to the running of gaugino masses, as noted above.) I will make no attempt to further quantify the competition between these two competing and opposite impacts on the little hierarchy problem, because there is simply no such thing as an objective measure on parameter space, and because there is great latitude in choosing the remaining parameters anyway.

III. CORRECTIONS TO THE LIGHTEST HIGGS SCALAR BOSON MASS

The contributions of the new supermultiplets to the lightest Higgs scalar boson mass can be computed using the effective potential approximation, which amounts to neglecting nonzero external momentum effects in h^0 self-energy diagrams. Since $m_{h^0}^2$ is much smaller than any of the new particle masses, this approximation is quite good for these contributions. The 1-loop contribution to the effective potential due to the supermultiplets in Eqs. (1.1), (1.2), (1.3), (1.4), and (1.5) is

$$\Delta V = 2N_c \sum_{i=1}^{4} [F(M_{S_i}^2) - F(M_{F_i}^2)], \tag{3.1}$$

where N_c is the number of colors of Φ, and $M_{S_i}^2$ and $M_{F_i}^2$ are the squared-mass eigenvalues of Eqs. (1.3) and (1.5), and $F(x) = x^2[\ln(x/Q^2) - 3/2]/64\pi^2$. Here Q is the renormalization scale. I will assume the decoupling approximation that the neutral Higgs mixing angle is $\alpha \approx \beta - \pi/2$, which is valid if $m_{A^0}^2 \gg m_{h^0}^2$. Then the correction to $m_{h^0}^2$ is

$$\Delta m_{h^0}^2 = \left\{ \frac{\sin^2\beta}{2}\left[\frac{\partial^2}{\partial v_u^2} - \frac{1}{v_u}\frac{\partial}{\partial v_u}\right] + \frac{\cos^2\beta}{2}\left[\frac{\partial^2}{\partial v_d^2} - \frac{1}{v_d}\frac{\partial}{\partial v_d}\right] + \sin\beta\cos\beta\frac{\partial^2}{\partial v_u \partial v_d}\right\} \times \Delta V. \tag{3.2}$$

Before presenting some numerical results, it is useful to note a relatively simple analytical result that can be obtained if the superpotential vectorlike fermion masses are taken to be equal ($M_\Phi = M_\phi \equiv M_F$) and the soft supersymmetry-breaking nonholomorphic masses are equal ($m^2_\Phi = m^2_{\bar\Phi} = m^2_\phi = m^2_{\bar\phi} \equiv m^2$), and the small electroweak D-terms and the holomorphic soft mass terms b_Φ and b_ϕ are neglected. Then, writing

$$M_S^2 = M_F^2 + m^2 = \text{average scalar mass} \tag{3.3}$$

$$x \equiv M_S^2/M_F^2 \tag{3.4}$$

$$\bar k \equiv k \sin\beta, \qquad \bar h \equiv h \cos\beta, \tag{3.5}$$

$$X_k \equiv A_k - \mu \cot\beta, \qquad X_h \equiv A_h - \mu \tan\beta, \tag{3.6}$$

and expanding to leading order in the normalized Yukawa couplings $\bar k$ and $\bar h$, one obtains

$$\Delta m^2_{h^0} = \frac{N_c v^2}{4\pi^2}\Bigg(\bar k^4\Big[f(x) + \frac{X_k^2}{xM^2}\Big(1-\frac{1}{3x}\Big) - \frac{X_k^4}{12x^2M^4}\Big] + \bar k^3\bar h\Big[-\frac{2}{3}\Big(2-\frac{1}{x}\Big)\Big(1-\frac{1}{x}\Big) - X_k(2X_k+X_h)/(3x^2M^2)\Big]$$
$$+ \bar k^2\bar h^2\Big[-\Big(1-\frac{1}{x}\Big)^2 - (X_k+X_h)^2/(3x^2M^2)\Big] + \bar k\bar h^3\Big[-\frac{2}{3}\Big(2-\frac{1}{x}\Big)\Big(1-\frac{1}{x}\Big) - X_h(2X_h+X_k)/(3x^2M^2)\Big]$$
$$+ \bar h^4\Big[f(x) + \frac{X_h^2}{xM^2}\Big(1-\frac{1}{3x}\Big) - \frac{X_h^4}{12x^2M^4}\Big]\Bigg), \tag{3.7}$$

where

$$f(x) \equiv \ln(x) - \frac{1}{6}\Big(5-\frac{1}{x}\Big)\Big(1-\frac{1}{x}\Big). \tag{3.8}$$

It is often a good approximation to keep only the contribution proportional to $\bar k^4$, corresponding to the case where $k\tan\beta \gg h$. In that limit, Eq. (3.7) agrees with the result given in [24], which can be rewritten as simply

$$\Delta m^2_{h^0} = \frac{N_c}{4\pi^2} k^4 v^2 \sin^4\beta\Big[f(x) + \frac{X_k^2}{xM^2}\Big(1-\frac{1}{3x}\Big) - \frac{X_k^4}{12x^2M^4}\Big]. \tag{3.9}$$

Note that x is, to first approximation, the ratio of the mean squared masses of the scalars to the fermions. A key feature of the result for $\Delta m^2_{h^0}$ is that the contribution of the vectorlike particles does not decouple with the overall extra particle mass scale, provided that there is a hierarchy x maintained between the scalar and fermion squared masses. To get an idea of the impact of this hierarchy, the function $f(x)$ is depicted in Fig. 6. In the limit of unbroken supersymmetry, $f(1) = 0$, and $f(x)$ monotonically increases for scalars heavier than fermions ($x > 1$). The other significant feature that could lead to enhanced $\Delta m^2_{h^0}$ is the mixing parametrized by X_k. The maximum possible value of the X_k contribution in Eq. (3.9) is obtained when $X_k^2 = 2M^2(3x-1)$, leading to a "maximal mixing" result $\Delta m^2_{h^0} = \frac{N_c}{4\pi^2} k^4 v^2 \sin^4\beta f_{\max}(x)$ where $f_{\max} = f(x) + (3-1/x)^2/3$. This function is also graphed in Fig. 6 to show the maximal effects of mixing from the new fermion sector. In Fig. 7, I show an estimate of the corresponding corrections to Δm_{h^0}, taking $N_c = 3$ and $k^4 v^2 \sin^4\beta = (190\ \text{GeV})^2$ (corresponding roughly to the QUE or QDEE model near the fixed point with reasonably large $\tan\beta$) and assuming that the predicted Higgs mass before the correction is 110 GeV, so that $\Delta m_{h^0} = \sqrt{(110\ \text{GeV})^2 + \Delta m^2_{h^0}} - 110$ GeV.

The previous depiction may be too simplistic, since the superpotential and soft supersymmetry-breaking masses need not have the simple degeneracies that were assumed. Also, as found in the previous section, the scalar trilinear

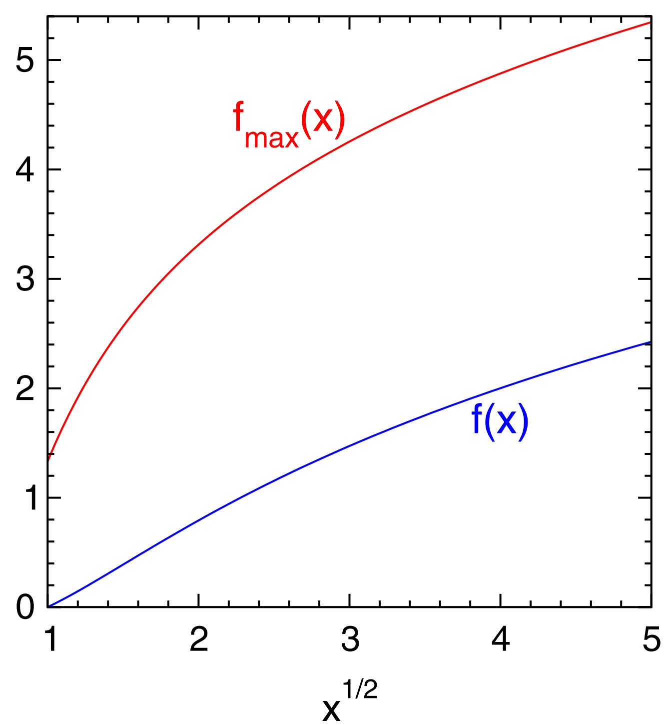

FIG. 6 (color online). The functions $f(x)$ and $f_{\max}(x)$ described in the text, graphed as a function of $\sqrt{x}$ = the average ratio of scalar to fermion masses.

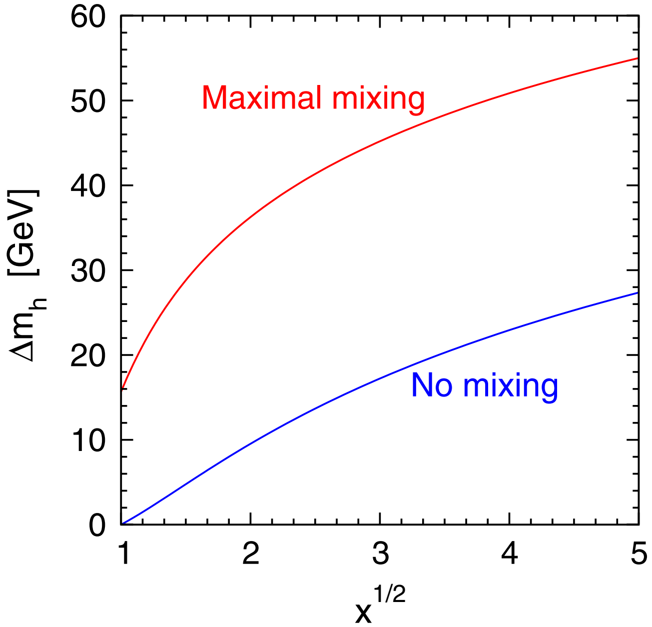

FIG. 7 (color online). Estimates for the corrections to the Higgs mass as a function of $\sqrt{x}$, where $x = (M^2 + m^2)/M^2$ is the ratio of the mean scalar squared mass to the mean fermion squared mass, in the simplified model framework used in Eq. (3.9) of the text, using $N_c = 3$ and $k^4 v^2 \sin^4\beta = (190\ \mathrm{GeV})^2$, corresponding roughly to the QUE or QDEE model near the fixed point with reasonably large $\tan\beta$. The lower line is the no-mixing case $X_k = 0$, and the upper line is the maximal mixing case $X_k = 2M^2(3x - 1)$. The Higgs mass before the correction is taken to be 110 GeV.

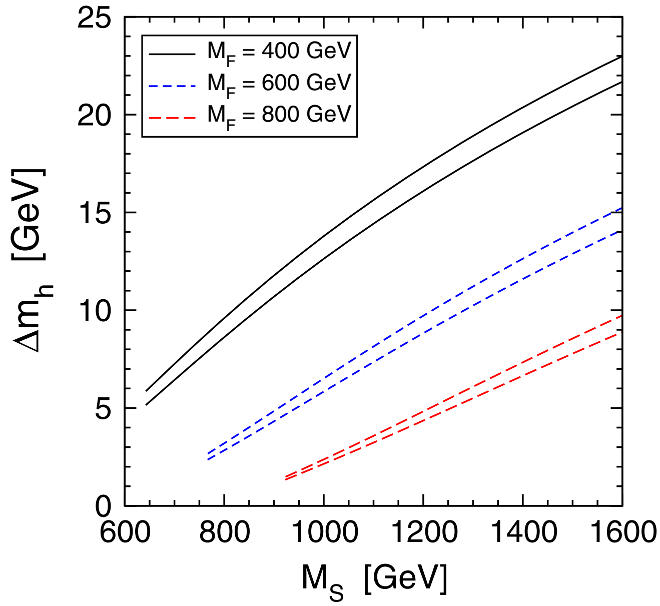

FIG. 8 (color online). Corrections to m_{h^0} in the QUE model with $k_U = 1.05$, for varying $m_{1/2}$ with other parameters described in the text. Here $M_F = 400$, 600, and 800 GeV is the vectorlike superpotential fermion mass term, and M_S is the geometric mean of the new up-type scalar masses. The upper and lower lines in each case correspond to $A_k = -0.5m_{1/2}$ and $A_k = -0.3m_{1/2}$, respectively, at the TeV scale. The value of m_{h^0} before these corrections is assumed to be 110 GeV.

coupling has a fixed-point behavior that implies that the mixing is neither maximal nor zero (but closer to the latter). A more realistic estimate is therefore as depicted in Fig. 8. Here, I take scalar masses inspired by the renormalization group solutions of the previous section for the QUE model. In particular, I take three cases for the vectorlike superpotential masses at the TeV scale, $M_Q = M_U \equiv M_F = 400$, 600, and 800 GeV. The Yukawa couplings are taken to be at the fixed-point values $k_U = 1.05$ and $h_U = 0$. The soft supersymmetry-breaking terms are parametrized by $(m_Q, m_{\bar{Q}}, m_U, m_{\bar{U}}) = (1.17, 1.29, 1.25, 0.94)m_{1/2}$ and $(b_Q, b_U) = -(M_Q, M_U)m_{1/2}$, and $A_k = -0.3m_{1/2}$ and $-0.5m_{1/2}$, all at a renormalization scale of 1 TeV. The results turn out to be not very sensitive to b_Q or b_U, or to the MSSM supersymmetric Higgs mass parameter μ (taken to be 800 GeV here), or to $\tan\beta$ as long as it is not too small ($\tan\beta = 10$ was used here). Figure 8 shows the results for $\Delta m_{h^0} \equiv \sqrt{(110\ \mathrm{GeV})^2 + \Delta m^2_{h^0}} - 110$ GeV, as a function of M_S, the geometric mean of the scalar masses. Quite similar results obtain for the QDEE model at the fixed point with $k_D = 1.043$, $h_D = 0$.

Figure 8 illustrates that the contribution of the new extra particles to m_{h^0} is probably much less than the "maximal mixing" scenario, if one assumes that the TeV-scale parameters (particularly the scalar trilinear coupling a_k) can be obtained by renormalization group running from the unification scale. Note that the models illustrated in Fig. 8 represent gaugino-mass dominated examples. If one assumes that the soft scalar squared masses at M_{unif} actually have significant positive values (from, for example, running between M_{unif} and M_{Planck}), then the low-scale model will be even closer to the no-mixing scenario, since the diagonal entries in the scalar mass matrix will be enhanced, while the mixing terms are still subject to the strong focusing behavior seen in Fig. 5.

In the case of the minimal LND model, one expects the maximum contributions to $\Delta m^2_{h^0}$ to be suppressed by a factor of roughly $(k_N/k_U)^4/N_c \approx (0.765/1.05)^4/3 \approx 0.094$. This leads to corrections that are typically not large compared to the inherent uncertainties in the total prediction. This counts against the minimal LND model as a way of significantly increasing the Higgs mass. One can also consider $n > 1$ copies of the LND model, with each k_N Yukawa coupling near a common fixed point to maximize $\Delta m^2_{h^0}$. However, then the common fixed-point value is even smaller, with $k_N = 0.695$ for $n = 2$, and $k_N = 0.650$ for $n = 3$. (A much more significant correction to $m^2_{h^0}$ can occur if one enhances the model with several copies of the extra fields connected by the "lateral" gauge group idea of [23].)

IV. PRECISION ELECTROWEAK EFFECTS

Because the Yukawa couplings responsible for large effects on $m^2_{h^0}$ break the custodial symmetry of the Higgs sector, it is necessary to consider the possibility of constraints due to precision electroweak observables arising from virtual corrections to electroweak vector boson self-energies. In this section, I will show that these corrections are actually benign (and much smaller than previously estimated), at least if one uses only M_t, M_W, and Z-peak observables as in the LEP Electroweak Working Group analyses [31,32] rather than including also low-energy observables as in [33]. The essential reason for this is that the corrections decouple with larger vectorlike masses, even if the Yukawa couplings are large and soft supersymmetry-breaking effects produce a large scalar-fermion hierarchy. Indeed, they decouple even when the corrections to $m^2_{h^0}$ do not.

The most important new physics contributions to precision electroweak observables can be summarized in terms of the Peskin-Takeuchi S and T parameters [34] (similar parametrizations of oblique electroweak observables were discussed in [35–39]). For the measurements of standard model observables, I use the updated values:

$$s^2_{\rm eff} = 0.23153 \pm 0.00016 \tag{4.1}$$

Reference [31];

$$M_W = 80.399 \pm 0.025\ \text{GeV} \tag{4.2}$$

References [32,40];

$$\Gamma_\ell = 83.985 \pm 0.086\ \text{MeV} \tag{4.3}$$

Reference [31];

$$\Delta\alpha_h^{(5)}(M_Z) = 0.02758 \pm 0.00035 \tag{4.4}$$

Reference [31];

$$M_t = 173.1 \pm 1.3\ \text{GeV} \tag{4.5}$$

Reference [41];

$$\alpha_s(M_Z) = 0.1187 \pm 0.0020 \tag{4.6}$$

Reference [33]; with $M_Z = 91.1875$ GeV held fixed. For the standard model predictions for $s^2_{\rm eff}$, M_W, and Γ_ℓ in terms of the other parameters, I use Refs. [42–44], respectively. These values are then used to determine the best experimental fit values and the 68% and 95% confidence level (CL) ellipses for S and T, relative to a standard model template with $M_t = 173.1$ GeV and $M_h = 115$ GeV, using

$$\frac{s^2_{\rm eff}}{(s^2_{\rm eff})_{\rm SM}} = 1 + \frac{\alpha}{4 s^2_W c_{2W}} S - \frac{\alpha c^2_W}{c_{2W}} T, \tag{4.7}$$

$$\frac{M^2_W}{(M^2_W)_{\rm SM}} = 1 - \frac{\alpha}{2 c_{2W}} S + \frac{\alpha c^2_W}{c_{2W}} T, \tag{4.8}$$

$$\frac{\Gamma_\ell}{(\Gamma_\ell)_{\rm SM}} = 1 - \alpha d_W S + \alpha(1 + s^2_{2W} d_W) T, \tag{4.9}$$

where $s_W = \sin\theta_W$, $c_W = \cos\theta_W$, $s_{2W} = \sin(2\theta_W)$, $c_{2W} = \cos(2\theta_W)$, and $d_W = (1 - 4s^2_W)/[(1 - 4s^2_W + 8s^4_W)c_{2W}]$. The best fit turns out to be $S = 0.057$ and $T = 0.080$.

The new physics contributions to S and T are given in terms of 1-loop corrections to the electroweak vector boson self-energies Π_{WW}, Π_{ZZ}, $\Pi_{Z\gamma}$ and $\Pi_{\gamma\gamma}$, which are computed for each of the LND, QUE and QDEE models in Appendix A. They are dominated by the contributions from the fermions when the soft supersymmetry-breaking scalar masses are large. It is useful and instructive to consider the simplified example that occurs when, in the notation of the Introduction, $M_\Phi = M_\phi = M_F$ with an expansion in small $m_u \equiv k v_u$, $m_d \equiv h v_d$, and M_W. Then one finds for the new fermion contributions

$$\Delta T = \frac{N_c}{480\pi s^2_W M^2_W M^2_F}[13(m^4_u + m^4_d) + 2(m^3_u m_d + m^3_d m_u) + 18 m^2_u m^2_d] \tag{4.10}$$

$$\Delta S = \frac{N_c}{30\pi M^2_F}[4(m^2_u + m^2_d) + m_u m_d(3 + 20 Y_\Phi)], \tag{4.11}$$

where Y_Φ is the weak hypercharge of the left-handed fermion doublet, denoted Φ in the Introduction, that has a Yukawa coupling to H_u (so that $Y_\Phi = -1/2$, $1/6$, and $-1/6$ for the LND, QUE, and QDEE models, respectively). Equations (4.10) and (4.11) agree[5] with the results found in [45,46]. An important feature of this is that the corrections decouple quadratically with increasing M_F, regardless of the soft supersymmetry-breaking terms. This is in contrast to the contributions to $\Delta m^2_{h^0}$, which do not decouple as long as there is a hierarchy between the scalar and fermion masses within a heavy supermultiplet. It also contrasts with the situation for chiral fermions (as in a sequential fourth family), which yields much larger ΔS, ΔT.

If $h = 0$, then the results of Eqs. (4.10) and (4.11) from the fermions become, numerically,

$$\Delta T = 0.54 N_c k^4 \sin^4(\beta)\left(\frac{100\ \text{GeV}}{M_F}\right)^2, \tag{4.12}$$

$$\Delta S = 0.13 N_c k^2 \sin^2(\beta)\left(\frac{100\ \text{GeV}}{M_F}\right)^2. \tag{4.13}$$

These rough formulas show that it is not too hard to obtain

[5]However, note that the result for ΔT quoted in Ref. [24] actually corresponds to the improbable case $h v_d = k v_u$, rather than $h = 0$. So, for small h, the actual correction to ΔT is almost a factor of 4 smaller than their estimate. As a result, much smaller values for M_F are admissible than would be indicated by Ref. [24].

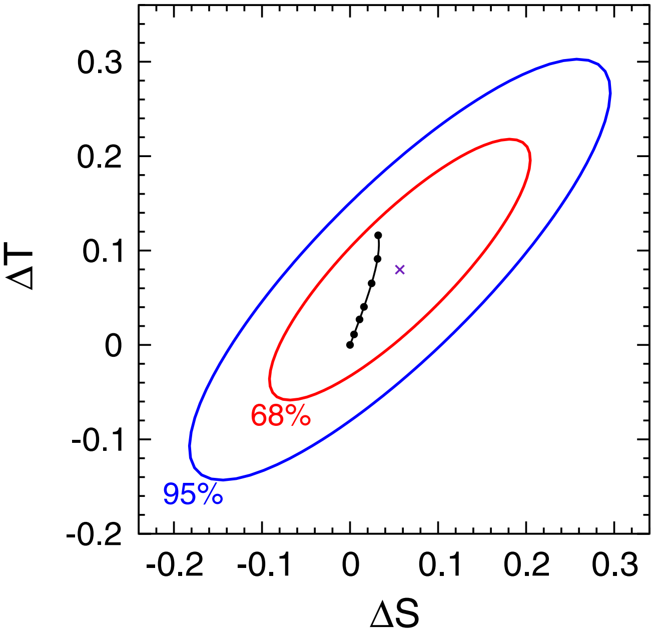

FIG. 9 (color online). Corrections to electroweak precision observables S, T from the LND model at the fixed point $(k_N, h_N) = (0.765, 0)$, for varying $M_L = M_N = m_{\tau'} >$ 100 GeV, in the limit of heavy scalar superpartners. The seven dots on the line segment correspond to $m_{\tau'} = 100$, 120, 150, 200, 250, 400 GeV and ∞, from top to bottom. The experimental best fit is shown as the $\times$ at $(\Delta S, \Delta T) = (0.057, 0.080)$. Also shown are the 68% and 95% CL ellipses, obtained as described in the text. The point $\Delta S = \Delta T = 0$ is defined to be the standard model prediction for $m_t = 173.1$ GeV and $m_{h^0} = 115$ GeV.

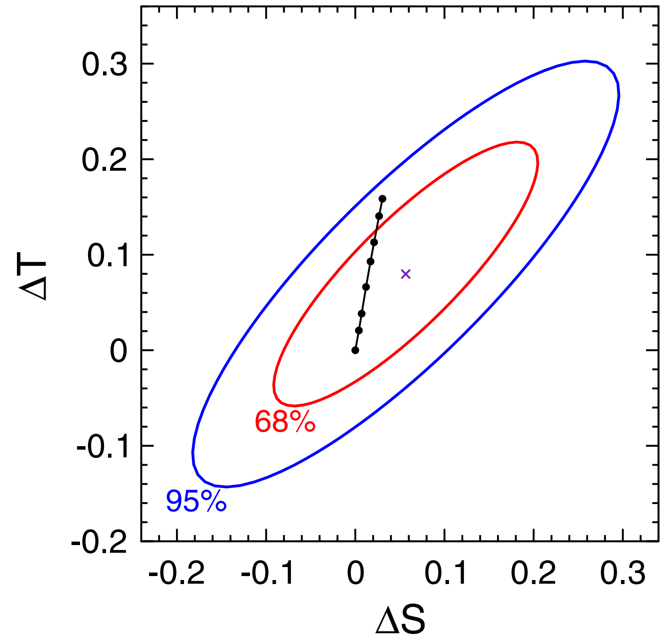

FIG. 10 (color online). Corrections to electroweak precision observables S, T from the QUE model at the fixed point $(k_U, h_U) = (1.050, 0)$, for varying $M_Q = M_U = m_{b'}$, with $m_{1/2} = 600$ GeV and $A_{k_U} = -0.4 m_{1/2}$ and $m_0 = 0$ and $b_Q = b_U = -m_{1/2} M_Q$, using Eqs. (2.11) and (2.13). The eight dots on the line segment correspond to $m_{t'_1} = 275$, 300, 350, 400, 500, 700, 1000 GeV and ∞, from top to bottom. The experimental best fit is shown as the $\times$ at $(\Delta S, \Delta T) = (0.057, 0.080)$. Also shown are the 68% and 95% CL ellipses, obtained as described in the text. The point $\Delta S = \Delta T = 0$ is defined to be the standard model prediction for $m_t = 173.1$ GeV and $m_{h^0} = 115$ GeV. Results for the QDEE model are very similar.

agreement with the precision electroweak data, provided that M_F is not too small, but it should be noted that especially for light new fermions with mass of order 100 GeV, the expansion in large M_F is not very accurate, with Eqs. (4.12) and (4.13) overestimating the actual corrections.

A more precise evaluation, using the formulas of Appendix A, is shown in Figs. 9 and 10, which compare the experimental best fit and 68% and 95% CL ellipses to the predictions from the models. Note that in these figures I do not include the contributions from the ordinary MSSM superpartners, which are typically not very large and which become small quadratically with large soft supersymmetry-breaking masses. Figure 9 shows the corrections for the LND model at the Yukawa coupling fixed point $(k_N, h_N) = (0.765, 0)$, for varying $M_N = M_L = m_{\tau'} > 100$ GeV as a line segment with dots at $m_{\tau'} = 100$, 120, 150, 200, 250, 400 GeV and ∞. These contributions are due to the fermions $\nu'_{1,2}$, τ', with their scalar superpartners assumed heavy enough to decouple. Note that in the LND model b' and $\tilde{b}'_{1,2}$ do not contribute to S, T as defined above, since they do not have Yukawa couplings to the Higgs sector. Figure 9 shows that even for $m_{\tau'}$ as small as 100 GeV, the S and T parameters remain within the 68% CL ellipse, and can even give a slightly better fit to the experimental results provided that $m_{\tau'} \gtrsim 120$ GeV. If the Yukawa coupling k_N is less than the fixed-point value, or if $M_L < M_N$, then the corrections to S and T are smaller, for a given $m_{\tau'}$.

Figure 10 shows the corrections for the QUE model at the Yukawa coupling fixed point $(k_U, h_U) = (1.050, 0)$, for varying $M_U = M_Q = m_{b'}$ as a line segment with dots at $m_{t'_1} = 275$, 300, 350, 400, 500, 700, 1000 GeV and ∞. [For a comparison to the approximate formulas (4.12) and (4.13), the appropriate values are $M_F = m_{b'} \approx 355$, 381, 432, 483, 584, 786, 1088 GeV and ∞, respectively.] Here, I have included the contributions from the scalar states $\tilde{t}_{1,2,3,4}$ and $\tilde{b}_{1,2}$, obtained for $m_{1/2} = 600$ GeV and $A_{k_U} = -0.4 m_{1/2}$ and $m_0 = 0$ and $b_Q = b_U = -m_{1/2} M_Q$, using Eqs. (2.11) and (2.13). Smaller values of $m_{1/2}$ would imply a chargino lighter than the LEP2 bound; see Table II. From Fig. 10 we see that a slightly better fit than the standard model can be obtained for $m_{t'_1} \gtrsim 400$ GeV, but even for $m_{t'_1}$ as light as 275 GeV, the corrections remain within the 95% CL ellipse. The corrections to S and T for a given $m_{t'_1}$ are even smaller (and so the fit is even better) if any of the following conditions apply: the Yukawa coupling k_U is below its fixed-point value, $m_{1/2}$ or m_0 is larger so that the new squarks are heavier, or $M_Q \neq M_U$. For t'_1 masses less than about 400 GeV, the fit also improves slightly if m_{h^0} is larger than 115 GeV.

I have also looked at the QDEE model at its fixed point $(k_D, h_D) = (1.043, 0)$, with scalar squared-mass soft terms given by Eq. (2.12) with $m_{1/2} = 600$ GeV. The results are nearly identical to those found in the QUE model in Fig. 10 with the values for $m_{t'_1}$ replaced by $m_{b'_1}$, and so are not depicted.

If one also included lower energy data as used in [33], the fits to S and T would be somewhat worse, so it is important to keep in mind that the results above are sensitive to the choice of following the LEP Electroweak Working Group [31,32] in using fits based on the Z-pole data and m_t and m_W. With this caveat, one may conclude that the models considered here fit at least as well as the standard model, provided that the new quarks with large Yukawa couplings are heavier than roughly 400 GeV, and can do even better if the new squarks are heavy enough to decouple.

V. COLLIDER PHENOMENOLOGY OF THE EXTRA FERMIONS

The extra particles in the models discussed above will add considerable richness to the already complicated LHC phenomenology of the MSSM. A full discussion of the different signals, and how to disentangle them, is beyond the scope of the present paper, but it is likely that the most important distinguishing collider signals will arise from production of the new fermions, especially the new quarks. This is simply because of the relatively large production cross section compared to the scalars, which are presumably much heavier due to the effects of soft supersymmetry-breaking masses. One can therefore expect signals from direction pair production of the lightest new quark, and possibly also from cascade decays of somewhat heavier fermions down to them. For concreteness, I will concentrate on only the final states from decays of the lightest new quark in each model.

In general, the lightest new quark and the lightest new lepton would be stable, were it not for mixing with the standard model fermions. At least some small such mixing is necessary to avoid a cosmological disaster from unwanted heavy relics. If the mixing is very small, then the new fermions could be quasistable, with decay lengths on the scale of collider detectors. Then the collider signatures will involve particles that leave highly-ionizing and slow muonlike tracks in the detectors, or feature macroscopic decay kinks or charge-changing tracks. These can be either the new charged leptons or hadronic bound states of the new quarks. The hadron collider signals have been discussed before in a variety of different model-building contexts, see for example [47–59]. The LHC signals depend sensitively on the detector characteristics for the ionization loss dE/dx and time-of-flight measurements, so it is difficult to make a definitive estimate of the ATLAS and CMS capabilities, but conservatively the LHC in early running should discover new quasistable quarks up to at least 1 TeV [55,56,58] and leptons up to several hundred GeV [51,53]. The mass for a 500 GeV quasistable quark should be measurable to within 20 GeV or so using time-of-flight [57]. Measurements of angular distributions of quasistable fermion pair production will enable them to be distinguished from spin-0 squarks and sleptons [54].

In the following, I will instead assume that the mixing of the new fermions with standard model fermions is large enough to provide for prompt decays. Mixing of the new fermions with the first- and second-family standard model fermions is highly constrained by flavor-changing neutral currents, since the vectorlike gauge quantum number assignments eliminate the GIM-type suppression. Therefore, I will assume that the mixing is with the third standard model family, for which the constraints are much easier to satisfy. Then the final states of the decays will always involve a single third-family quark or lepton, together with a W, Z, or h^0 boson. Below, I will discuss the possibilities for the branching ratios of the new quarks and leptons, and their dependence on the type of mixing.

There are existing limits on the extra quarks coming from Tevatron, although these have mostly been found with assumed 100% branching ratios for particular decay modes (which as we will see below is not necessarily likely). The current limits are, for prompt decays:

(i) $m_{t'} > 311$ GeV for $\mathrm{BR}(t' \to Wq) = 1$, based on 2.8 fb^{-1} [60]
(ii) $m_{b'} > 325$ GeV for $\mathrm{BR}(b' \to Wt) = 1$, based on 2.7 fb^{-1} [61]
(iii) $m_{b'} > 268$ GeV for $\mathrm{BR}(b' \to Zb) = 1$, based on 1.06 fb^{-1} [62]
(iv) $m_{b'} > 295$ GeV for $\mathrm{BR}(b' \to Wt, Zb, h^0 b) = 0.5$, 0.25, 0.25, based on 1.2 fb^{-1} [63]

and for quasistable quarks:

(i) $m_{t'} > 220$ GeV, based on dE/dx for 90 pb^{-1} at $\sqrt{s} = 1.8$ TeV [64]
(ii) $m_{b'} > 190$ GeV, based on dE/dx for 90 pb^{-1} at $\sqrt{s} = 1.8$ TeV [64]
(iii) $m_{b'} > 170$ GeV for 3 mm $< c\tau_{b'} <$ 20 mm, based on 163 pb^{-1} [65]

Also, if the cross-section upper bound found from time-of-flight measurements with 1.0 fb^{-1} in Ref. [66] for stable top squarks also applies to stable t' quarks with no change in efficiency, then I estimate a bound $m_{t'} \gtrsim 360$ GeV should be obtainable, with a somewhat weaker bound for stable b' due to a lower detector efficiency.

At hadron colliders, the production cross section of the new quarks is due to gg and $q\bar{q}$ initial states and is mediated by the strong interactions, and so is nearly

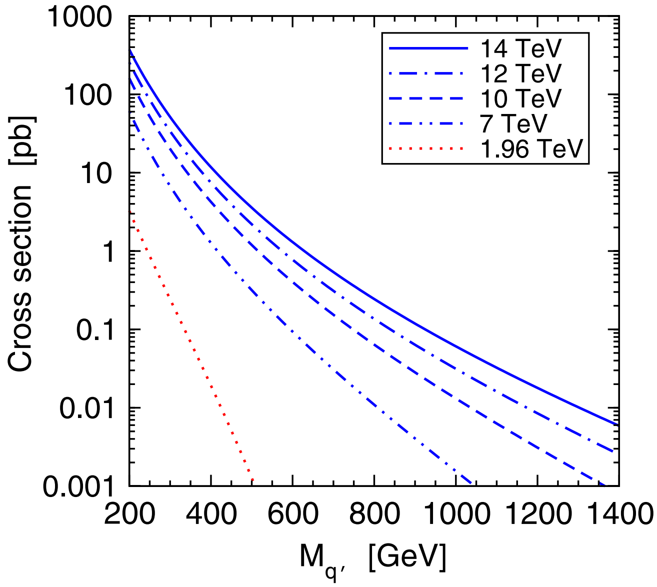

FIG. 11 (color online). Production cross section for new quarks as a function of the mass, for the Tevatron $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV, and for the LHC pp collisions with $\sqrt{s} = 7$, 10, 12, and 14 TeV. The graph was made at leading order using CTEQ5LO parton distribution functions [82] with $Q = m_{q'}$ and applying a K factor of 1.5 for LHC and 1.25 for Tevatron.

model-independent when expressed as a function of the mass. The leading-order cross section is shown in Fig. 11 for the Tevatron $p\bar{p}$ collider at $\sqrt{s} = 1.96$ TeV and for the LHC pp collider with $\sqrt{s} = 7$, 10, 12, and 14 TeV. Note the Tevatron will probably be unable to strengthen the existing constraints very significantly, at least for promptly decaying new quarks, due to the rather steep fall of the production cross-section with mass. At the LHC pair production of mostly vectorlike quarks should provide a robust signal; see, for example, studies (in diverse other model contexts) in Refs. [6,67–71]. (Note that in the models under study here, there is no reason that the flavor-violating charged-current couplings should be large enough to enable a viable signal from single q' production in association with a standard model fermion through t-channel W exchange, unlike in other model contexts as studied in Refs. [72–78].) The branching ratios and possible signals for the LND, QUE, and QDEE models are examined below.

A. The LND model

In the LND model, the fermions consist of a b', τ', and two neutral fermions ν'_1 and ν'_2. The ν'_1 is always lighter than the τ'. The fermions b' and ν'_1 can therefore decay only through their mixing with the standard model fermions from the superpotential

$$W = -\epsilon_D H_d q_3 \bar{D} + \epsilon_N H_u \ell_3 \bar{N} - \epsilon_E H_d L \bar{e}_3, \qquad (5.1)$$

where ϵ_D, ϵ_N, and ϵ_E are new Yukawa couplings that are assumed here to be small enough to provide mass mixings that can be treated as perturbations compared to the other entries in the mass matrices.

First consider the decays of b'. The mass matrix for the down-type quarks resulting from Eqs. (2.3), (2.4), and (5.1) is

$$\mathcal{M}_d = \begin{pmatrix} M_D & 0 \\ \epsilon_D v_d & y_b v_d \end{pmatrix}, \qquad (5.2)$$

with eigenstates b and b'. The b' decay can take place only through the ϵ_D coupling, to final states Wt, Zb, and $h^0 b$. Formulas for these decay widths are given in Appendix B. To leading order, the branching ratios only depend on the mass of the b', and the results are graphed in Fig. 12. Note that in the limit of large $m_{b'}$, the branching ratios are "democratic" between charged and neutral currents, approaching 0.5, 0.25, and 0.25 for Wt, Zb, and $h^0 b$ respectively, in accord with the Goldstone boson equivalence theorem. However, for smaller masses, kinematic suppression reduces the Wt branching ratio, so that, for example, the three final states have comparable branching ratios for $m_{b'}$ in the vicinity of 300 to 400 GeV.

The LHC signals include $pp \to b'_1 \bar{b}'_1 \to W^+ W^- t\bar{t} \to W^+ W^- W^+ W^- b\bar{b}$. When two same-charge W's decay leptonically and the other two W's decay hadronically, this leads to a same-charge dilepton plus multijets (including two b jets) plus missing transverse energy signal, with a total branching ratio as high as 25%. This signal is also the basis for the current Tevatron bound $m_{b'} > 325$ GeV, but this assumes $\mathrm{BR}(b' \to Wt) = 100\%$; since the actual branching ratio predicted by the LND model for that mass range is more than a factor of 3 smaller, the model prediction for the signal in the channel that was searched is more than an order of magnitude smaller, and decreases sharply for lower $m_{b'_1}$. In over half of the other $b'_1 \bar{b}'_1$ production events, there will be four or more b jets, coming mostly from events with $h^0 b \to b b \bar{b}$ decays but also from $Zb \to b b \bar{b}$. The Tevatron limit [62] of $m_{b'} > 268$ GeV from assuming $\mathrm{BR}(b' \to Zb) = 100\%$ is in a mass range where the actual branching ratio is about 0.55, so the actual predicted signal from the LND model is more than a factor of 3 smaller. The limit of $m_{b'} > 295$ GeV from [63], a search which is motivated in part by [79,80], is based on the idealized large mass limit democratic branching, but in the relevant mass range the model prediction has $\mathrm{BR}(b' \to Wt)$ more than a factor of 2 smaller, and decreasing very rapidly for smaller $m_{b'}$, due to the kinematic suppression. The neutral-current decays, including $Z \to \ell^+ \ell^-$, could also play an important role at the LHC, see for example [71] for a similar case.

The decay of ν'_1 in the LND model is dependent on two different mixing Yukawa couplings ϵ_N and ϵ_E. The mass matrix for the neutral leptons in the $(L, N, \ell_3, \bar{L}, \bar{N})$ basis resulting from Eqs. (2.3), (2.4), and (5.1) is

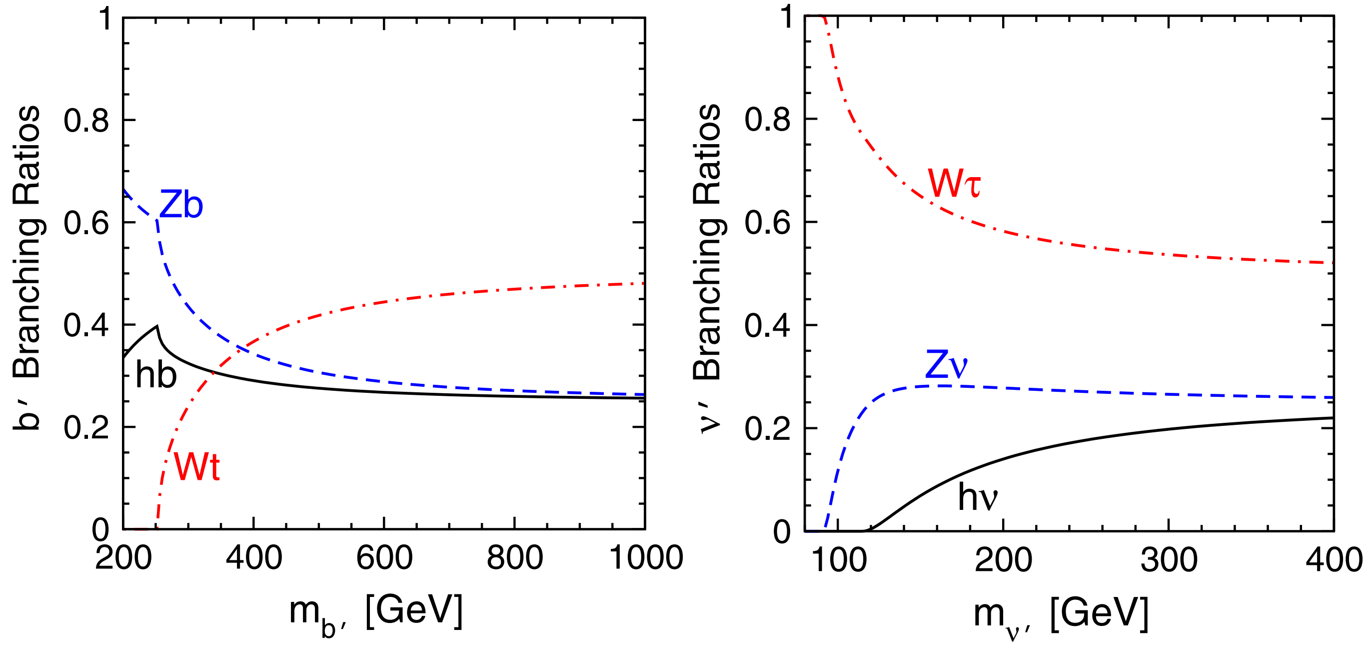

FIG. 12 (color online). The branching ratios of the lightest new quark b' (left panel) and the lightest new lepton ν_1' (right panel) in the LND model. The ν_1' results assume that $\epsilon_N \gg \epsilon_E$; if instead $\epsilon_E \gg \epsilon_N$ then $\mathrm{BR}(\nu_1' \to W\tau) = 1$ (not shown).

$$\begin{pmatrix} 0 & \mathcal{M}_\nu \\ \mathcal{M}_\nu^T & 0 \end{pmatrix}, \quad \text{where } \mathcal{M}_\nu^T = \begin{pmatrix} M_L & h_N v_d & 0 \\ k_N v_u & M_N & \epsilon_N v_U \end{pmatrix}, \tag{5.3}$$

with the masses of the standard model neutrinos neglected. The corresponding mass eigenstates are a standard model neutrino ν and two extra massive neutrino states ν_1' and ν_2'. The mass matrix for the charged leptons is

$$\mathcal{M}_e = \begin{pmatrix} -M_L & \epsilon_E v_d \\ 0 & y_\tau v_d \end{pmatrix}. \tag{5.4}$$

Formulas for the resulting decay widths for $\nu_1' \to W\tau$ and $Z\nu$ and $h^0\nu$ are given in Appendix B. If one assumes that $\epsilon_E \gg \epsilon_N$, then the decay $\nu_1' \to W\tau$ has a nearly 100% branching ratio. If the opposite limit applies, $\epsilon_N \gg \epsilon_E$, then the branching ratios as a function of $m_{\nu_1'}$ are as shown in the right panel of Fig. 12. Note that in the limit of large $m_{\nu_1'}$, the branching ratios for $W\tau$, $Z\nu$, and $h^0\nu$ asymptote to 0.5, 0.25, 0.25, respectively, when ϵ_N dominates, again in accordance with Goldstone boson equivalence with equal charged and neutral currents. So, depending on which Yukawa coupling dominates, one could have interesting hadron collider signatures from $\nu_1'\bar{\nu}_1'$ production, such as $W^+W^-\tau^+\tau^-$, and $h^0h^0 + E_T^{\mathrm{miss}}$, and $ZZ + E_T^{\mathrm{miss}}$, and $Wh^0 + E_T^{\mathrm{miss}}$ and $Zh^0 + E_T^{\mathrm{miss}}$. So far, there are no published limits specifically on $m_{\nu'}$ based on collider pair production with these final states. If $M_L \lesssim M_N$ in this model, then τ' will be not much heavier than ν_1', and so there will be additional contributions to the signal from $\tau'\nu_1'$ production and $\tau'^+\tau'^-$ production, followed by $\tau' \to W^{(*)}\nu'$. It should also be noted that production of $\nu_{1,2}'$ and τ_1' might well be dominated by cascade decays from heavier strongly interacting superpartners.

B. The QUE model

In the QUE model, the lightest of the new quarks is always the charge 2/3 quark t_1'. After being pair-produced at hadron colliders, it can decay due to mixing with the standard model fermions through the superpotential

$$W = \epsilon_U H_u q_3 \bar{U} + \epsilon_U' H_u Q \bar{u}_3 - \epsilon_D H_d Q \bar{d}_3, \tag{5.5}$$

where ϵ_U, ϵ_U', and ϵ_D are new Yukawa couplings that are assumed here to be small enough to treat as perturbations compared to other entries in the mass matrices. The resulting mass matrices for the up-type quarks and down-type quarks are

$$\mathcal{M}_u = \begin{pmatrix} M_Q & k_U v_u & \epsilon_U' v_u \\ h_U v_d & M_U & 0 \\ 0 & \epsilon_U v_u & y_t v_u \end{pmatrix}, \quad \mathcal{M}_d = \begin{pmatrix} -M_Q & \epsilon_D v_d \\ 0 & y_b v_d \end{pmatrix}, \tag{5.6}$$

with mass eigenstates t, t_1', t_2' and b, b' respectively. Formulas for the resulting decay widths for t_1' to Wb, Zt, and $h^0 t$ are presented in Appendix B. I will concentrate on the three cases where one of the mixing Yukawa couplings in Eq. (5.5) dominates over the other two. The branching ratios depend on the mass of t_1' and on the type of mixing. If ϵ_D provides the dominant effect, then the decays are dominantly charged-current, or "W-philic," with $\mathrm{BR}(t_1' \to Wb) = 1$. This is the scenario for which the Tevatron limit is now $m_{t'} > 311$ GeV [60]. If instead ϵ'_U dominates, then the decays are dominantly neutral-current, or "W-phobic"; in the limit of large $m_{t'}$, the branching ratios asymptote to $\mathrm{BR}(t'_1 \to Wb) = 0$ and $\mathrm{BR}(t'_1 \to Zt) = \mathrm{BR}(t'_1 \to h^0 t) = 0.5$. Finally, if ϵ_U dominates, the decays are democratic, with branching ratios for Wb, Zt, and h^0 approaching 0.5, 0.25, and 0.25, respectively, in the large $m_{t'_1}$ limit. Numerical results are shown in Fig. 13 as a function of $m_{t'_1}$, for the case that k_U is at its fixed-point value, and $h_U = 0$, and $M_Q = M_U$. (The results are only mildly sensitive to the last two assumptions.) By taking the different mixing Yukawa couplings ϵ_U, ϵ'_U, and ϵ_D to be comparable, one can get essentially any result one wants for the branching ratios, but it seems reasonable to assume that one of the individual mixing Yukawa couplings dominates in the absence of some organizing principle. So the possible signatures will include $W^+W^-b\bar{b}$, (similar to the standard model $t\bar{t}$ signature, but with larger invariant masses; see [6,67,68,70,71] for recent studies of comparable signals), and $ZZt\bar{t}$ and $h^0h^0t\bar{t}$, etc. If $M_Q \lesssim M_U$ in this model, then the b' will be not much heavier than the t', and one should expect an additional component of the signal from $b'\bar{t}'$ and $b'\bar{b}'$ production, followed by $b' \to W^{(*)}t'$.

The τ' in the QUE model mixes with the standard model τ lepton through a superpotential term:

$$W = -\epsilon_E H_d \ell_3 \bar{E}. \tag{5.7}$$

The mass matrix for the charged leptons resulting from this and Eqs. (2.3) and (2.5) is

$$\mathcal{M}_e = \begin{pmatrix} M_E & 0 \\ \epsilon_E v_d & y_\tau v_d \end{pmatrix}, \tag{5.8}$$

with mass eigenstates τ and τ'. It follows that τ' can decay to $W\nu$, $Z\tau$, and $h^0\tau$, with decay widths that are computed in Appendix B. Because there is only one relevant Yukawa mixing term, the branching ratios depend only on $m_{\tau'}$. They are shown in Fig. 14, assuming $m_{h^0} = 115$ GeV. The largest branching ratio for τ' is always to $W\nu$, and in the large $m_{\tau'}$ limit, Goldstone boson equivalence provides that the $W\nu$, $Z\tau$, and $h^0\tau$ branching ratios approach 0.5, 0.25, and 0.25, respectively. The most immediately relevant searches at hadron colliders will be in the mass range of $m_{\tau'}$ just above 100 GeV, where the electroweak pair-production cross section can be sufficiently large, and limits do not presently exist. However, note that the appearance of τ'_1 could easily be dominated by cascade decays from heavier strongly interacting superpartners.

C The QDEE model

In the QDEE model, the new fermions consist of a b'_1, b'_2, t', and τ'_1, τ'_2. In this model, the lighter charge $-1/3$ quark b'_1 is always lighter than the t'. The decays of b'_1 in the QDEE model are brought about by superpotential mixing terms with third-family quarks:

$$W = -\epsilon_D H_d q_3 \bar{D} - \epsilon'_D H_d Q \bar{d}_3 + \epsilon_U H_u Q \bar{u}_3. \tag{5.9}$$

In the gauge eigenstate basis, the resulting mass matrices for the down-type quarks and up-type quarks are

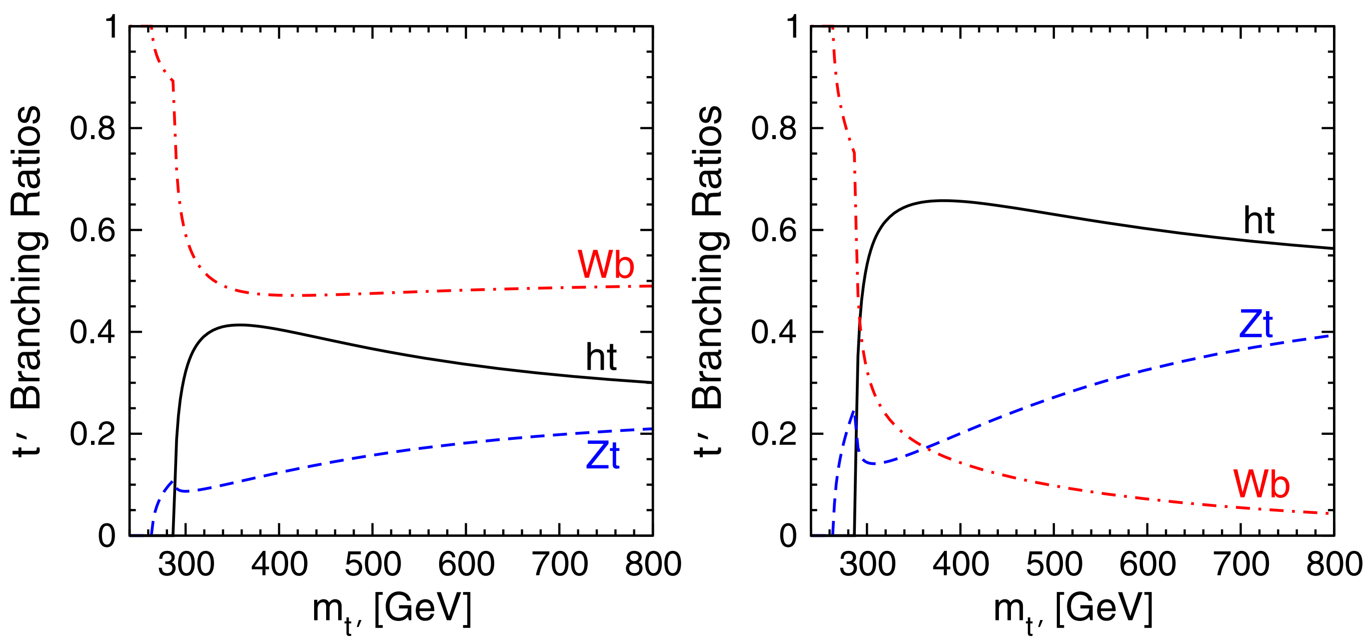

FIG. 13 (color online). Branching ratios for the lightest extra quark, t'_1, in the QUE model with $M_Q = M_U$, to final states Wb, Zt, and h^0t, as a function of $m_{\tilde{t}_1}$. The left panel shows the democratic case that arises when ϵ_U dominates (with equal charged and neutral currents), and the right panel shows the "W-phobic" (mostly neutral current) case that arises when ϵ'_U dominates. In the "W-philic" case that arises when ϵ_D dominates, then $\mathrm{BR}(t' \to Wb) = 1$ (not shown).

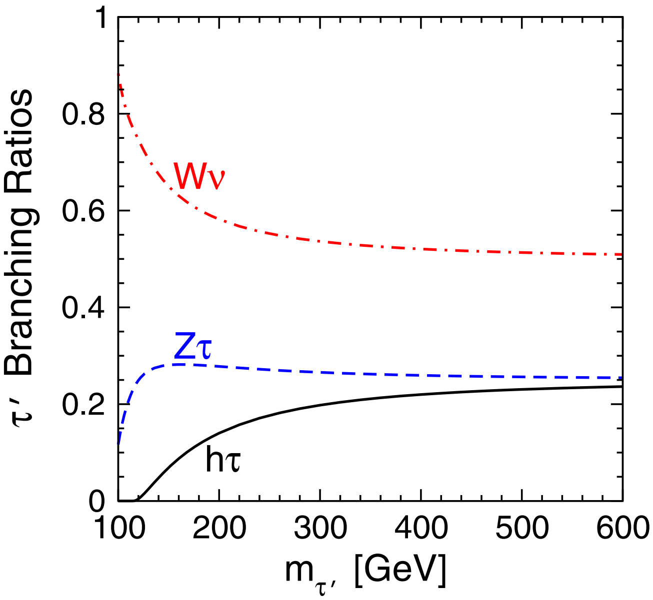

FIG. 14 (color online). Branching ratios for τ' decays to $W\nu$, $Z\tau$, and $h^0\tau$ in the QUE and QDEE models, as a function of $m_{\tau'}$.

$$\mathcal{M}_d = \begin{pmatrix} M_Q & k_D v_u & 0 \\ h_D v_d & M_D & \epsilon_D v_d \\ \epsilon'_D v_d & 0 & y_b v_d \end{pmatrix},$$
$$\mathcal{M}_u = \begin{pmatrix} -M_Q & 0 \\ \epsilon_U v_u & y_t v_u \end{pmatrix}, \tag{5.10}$$

with mass eigenstates b, b'_1, b'_2 and t, t' respectively. Formulas for the resulting decay widths for b'_1 to Wt, Zb, and $h^0 b$ are given in Appendix B. As in the case of the QUE model, I will consider the three cases where one of the mixing Yukawa couplings in Eq. (5.9) dominates over the other two. Then the branching ratios depend on the mass of b'_1 and on the type of mixing. If ϵ_U provides the dominant effect, then the decays are dominantly charged-current, or "W-philic," with $\mathrm{BR}(b'_1 \to Wt) = 1$ provided that it is kinematically allowed. The resulting signal at hadron colliders will be $b'_1\bar{b}'_1 \to W^+W^-t\bar{t} \to W^+W^+W^-W^-b\bar{b}$. This is the scenario for which the Tevatron limit is presently $m_{b'} > 325$ GeV [61], based on the same-charge dilepton plus b-jets signal already mentioned above for the LND model. If instead ϵ'_D is dominant, then the decays are dominantly neutral-current, or "W-phobic," with $\mathrm{BR}(b'_1 \to Wt) = 0$; in the limit of large $m_{b'}$, the branching ratios slowly approach $\mathrm{BR}(b'_1 \to Zb) = \mathrm{BR}(b'_1 \to h^0 b) = 0.5$, but with $h^0 b$ larger for finite masses. Finally, if ϵ_D is dominant, the decays are democratic, with branching ratios for Wt, Zb, and $h^0 b$ approaching 0.5, 0.25, and 0.25, respectively, in the large $m_{b'_1}$ limit. The predicted branching ratios are shown in Fig. 15 as a function of $m_{b'_1}$ for the latter two cases, assuming k_D is at its fixed-point value, and $h_D = 0$ and $M_Q = M_D$. (However, it should be noted that, unlike in the QUE model case, the results shown are somewhat sensitive to the last of these assumptions.) Note that in the democratic case, the branching ratios are similar to what one obtains for the b' of the LND model. The CDF limit $m_{b'} > 295$ GeV was obtained in the idealized case of branching ratios obtained in the high mass

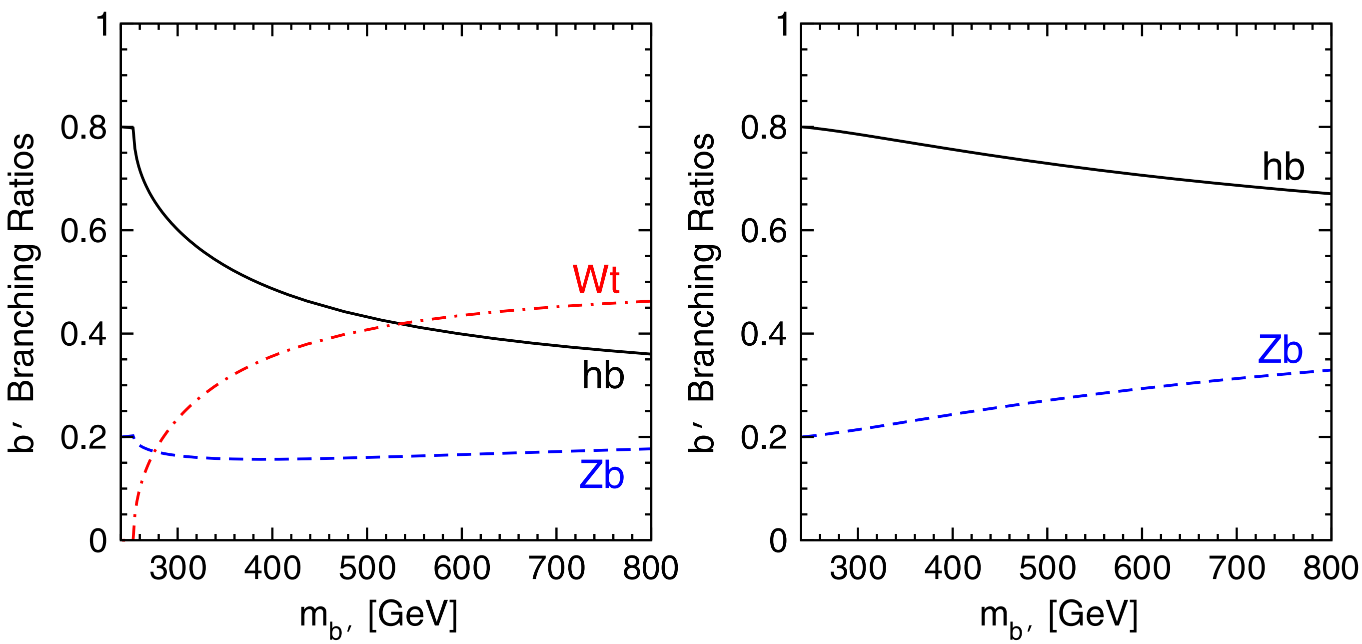

FIG. 15 (color online). Branching ratios for the lightest extra quark, b'_1, in the QDEE model with $M_Q = M_D$, to final states Wt, Zb, and $h^0 b$. The left panel shows the democratic case that ϵ_D dominates, and the right panel shows the "W-phobic" case that ϵ'_D dominates, leading to mostly neutral-current decays. In the "W-philic" case that ϵ_U dominates leading to mostly charged-current decays, then $\mathrm{BR}(b'_1 \to Wt) = 1$ (not shown).

limit, but for finite $m_{b'}$, the actual BR($b' \to Wt$) is much smaller and BR($b' \to h^0 b$) is larger. In contrast, the same-charge dilepton signal from $b'_1 b'_1 \to W^+ W^- t\bar{t}$ is turned off in the "W-phobic" case, where the largest overall branching ratio is typically to six b quarks, yielding the interesting possible signal $b'_1 b'_1 \to h^0 h^0 b\bar{b} \to bbb\bar{b}\,\bar{b}\,\bar{b}$. Decays to leptons through Z bosons are unfortunately suppressed by both small BR($Z \to \ell^+\ell^-$) and small BR($b'_1 \to Zb$) in this case. If $M_Q \lesssim M_D$ in this model, then the t' will be not much heavier than the b', and one should expect an additional component of the signal from $t'\bar{b}'$ and $t'\bar{t}'$ production, followed by $t' \to W^{(*)} b'$.

For τ'_1 in the QDEE model, the branching ratio situation is essentially the same as for the QUE model as discussed above.

VI. OUTLOOK

In this paper, I have studied supersymmetric models that have vectorlike fermions that are consistent with perturbative gauge coupling unification and have large Yukawa couplings that can significantly raise the Higgs mass in supersymmetry. Some of the more important features found for these models are

(i) There are three types of models consistent with perturbative gauge coupling unification and all new particles near the TeV scale. The first type (LND) contains up to three copies of the $\mathbf{5} + \bar{\mathbf{5}}$ of $SU(5)$. The second type (QUE) contains a $\mathbf{10} + \overline{\mathbf{10}}$ of $SU(5)$. The third type (QDEE) is not classifiable in terms of complete representations of $SU(5)$, but consists of the fields Q, D, E, E and their conjugates.

(ii) A complete vectorlike family (i.e. a $\mathbf{16} + \overline{\mathbf{16}}$ of $SO(10)$) could also be entertained, but was not considered here because a multiloop renormalization group analysis shows that this would forfeit perturbative unification and high-scale control unless (at least some of) the new particles are much heavier than 1 TeV.

(iii) The constraints imposed by oblique corrections to electroweak observables are rather mild, especially in comparison to the corresponding constraints on a chiral fourth family, and are easily accommodated by present data as long as the new quarks with Yukawa couplings are heavier than about 400 GeV, and perhaps considerably lower.

(iv) The model framework is consistent with the hypothesis that gaugino masses dominate soft supersymmetry breaking near the unification scale, without problems from sleptons being too light as is the case in so-called mSUGRA models.

(v) The lightest Higgs mass can be substantially raised in the QUE and QDEE models if the Yukawa couplings are near their fixed points. However, the extent of this is limited if one takes seriously the prediction for the fixed-point behavior of the scalar trilinear couplings, which limits the mixing in the new squark sector. For example, if the new quarks are at $M_F = 400$ GeV, and their scalar partners have an average mass of $M_S = 1000$ GeV, then one finds an increase in m_{h^0} of up to about 15 GeV (see Fig. 8). For larger M_S, this contribution increases, but at the expense of apparently more severe fine-tuning of the electroweak scale.

(vi) Despite the sizable positive contribution to the lightest Higgs, the contributions to the μ parameter are also raised, so it is difficult to make any unambiguous claim for an improvement in the supersymmetric little hierarchy problem.

(vii) The new fermions can decay through any mixture of neutral and charged currents to third-family fermions and W, Z, h^0 weak bosons, but with different combinations correlated to the possible superpotential couplings that mix the new fermions with the standard model ones.

(viii) Existing bounds from direct searches at the Tevatron do not significantly constrain the parameter space of these models after precision electroweak constraints are taken into account.

The collider phenomenology of the MSSM augmented by the new particles in these models should be both rich and confusing, leading to a difficult challenge at the LHC and beyond in deciphering the new discoveries. The new quarks and leptons can also impact the production and decay rates for the Higgs boson, through virtual corrections to the hgg, $h\gamma\gamma$, hW^+W^-, and hZZ effective couplings, which should be studied. The implications of these models for dark matter are also well worthy of future study.

ACKNOWLEDGMENTS

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Note added.—Shortly after the present paper, one with some related subject matter appeared [83].

APPENDIX A: CONTRIBUTIONS TO PRECISION ELECTROWEAK PARAMETERS

This appendix gives formulas for the contributions of the new chiral supermultiplets to the Peskin-Takeuchi precision electroweak parameters [34]. For convenience I will follow the notations and conventions of [81]. The oblique parameters S and T are defined in terms of electroweak vector boson self-energies by

$$\frac{\alpha S}{4 s_W^2 c_W^2} = \Big[\Pi_{ZZ}(M_Z^2) - \Pi_{ZZ}(0) - \frac{c_{2W}}{c_W s_W} \Pi_{Z\gamma}(M_Z^2) - \Pi_{\gamma\gamma}(M_Z^2) \Big] / M_Z^2, \tag{A1}$$

$$\alpha T = \Pi_{WW}(0)/M_W^2 - \Pi_{ZZ}(0)/M_Z^2. \tag{A2}$$

In the following, the 1-loop integral functions $G(x)$, $H(x, y)$, $B(x, y)$, and $F(x, y)$ are as defined in Ref. [81], and particle names should be understood to stand for the squared mass when used as an argument of one of these functions, which also have an implicit argument s, which is identified with the invariant mass of the self-energy function in which they appear.

1. Corrections to electroweak vector boson self-energies in the LND model

For the LND model, define the gauge eigenstate new neutral lepton mass matrix by

$$\mathcal{M}_\nu = \begin{pmatrix} M_L & k_N v_u \\ h_N v_d & M_N \end{pmatrix}, \tag{A3}$$

and unitary mixing matrices L and R by

$$L^* \mathcal{M}_\nu R^\dagger = \mathrm{diag}(m_{\nu'_1}, m_{\nu'_2}), \tag{A4}$$

and note $m_{\tau'} = M_L$. Then the (ν'_1, ν'_2, τ') fermion contributions to the electroweak vector boson self-energies are:

$$\Delta\Pi_{\gamma\gamma} = -\frac{N_c}{16\pi^2} 2g^2 s_W^2 e_e^2 G(\tau'), \tag{A5}$$

$$\Delta\Pi_{Z\gamma} = -\frac{N_c}{16\pi^2} g s_W e_e (g^Z_{\tau'\tau'^\dagger} - g^Z_{\bar\tau'\bar\tau'^\dagger}) G(\tau'), \tag{A6}$$

$$\Delta\Pi_{ZZ} = -\frac{N_c}{16\pi^2}\Big[(|g^Z_{\tau'\tau'^\dagger}|^2 + |g^Z_{\bar\tau'\bar\tau'^\dagger}|^2)G(\tau') + \sum_{i,j=1}^{2}(|g^Z_{\nu'_i\nu'^\dagger_j}|^2 + |g^Z_{\bar\nu'_i\bar\nu'^\dagger_j}|^2)H(\nu'_i, \nu'_j) - 4\,\mathrm{Re}(g^Z_{\nu'_i\nu'^\dagger_j} g^Z_{\bar\nu'_i\bar\nu'^\dagger_j}) m_{\nu'_i} m_{\nu'_j} B(\nu'_i, \nu'_j)\Big], \tag{A7}$$

$$\Delta\Pi_{WW} = -\frac{N_c}{16\pi^2}\sum_{i=1,2}[(|g^W_{\nu'_i\tau'^\dagger}|^2 + |g^W_{\bar\nu'_i\bar\tau'^\dagger}|^2)H(\tau', \nu'_i) - 4\,\mathrm{Re}(g^W_{\nu'_i\tau'^\dagger} g^W_{\bar\nu'_i\bar\tau'^\dagger}) m_{\tau'} m_{\nu'_i} B(\tau', \nu'_i)], \tag{A8}$$

where $N_c = 1$ and $e_e = -1$ and the massive vector boson couplings with the new leptons are

$$g^Z_{\nu'_i\nu'^\dagger_j} = \frac{g}{2c_W} L^*_{i1} L_{j1}, \qquad g^Z_{\bar\nu'_i\bar\nu'^\dagger_j} = -\frac{g}{2c_W} R^*_{i1} R_{j1}, \tag{A9}$$

$$g^Z_{\tau'\tau'^\dagger} = -g^Z_{\bar\tau'\bar\tau'^\dagger} = \frac{g}{c_W}\left(-\frac{1}{2} - e_e s_W^2\right), \tag{A10}$$

$$g^W_{\nu'_i\tau'^\dagger} = g L^*_{i1}/\sqrt{2}, \qquad g^W_{\bar\nu'_i\bar\tau'^\dagger} = -g R^*_{i1}/\sqrt{2}. \tag{A11}$$

To obtain the $(\tilde\nu'_{1,2,3,4}, \tilde\tau'_{1,2})$ scalar contribution, consider the new sneutrino squared-mass matrix:

$$M^2_{\tilde\nu} = M^2_\nu + \begin{pmatrix} m^2_L + \Delta_{(1/2),0} & 0 & b^*_L & a^*_{k_N} v_u - \mu k_N v_d \\ 0 & m^2_N & a^*_{h_N} v_d - \mu h_N v_u & b^*_N \\ b_L & a_{h_N} v_d - \mu^* h_N v_u & m^2_{\bar L} + \Delta_{-(1/2),0} & 0 \\ a_{k_N} v_u - \mu^* k_N v_d & b_N & 0 & m^2_{\bar N} \end{pmatrix}, \tag{A12}$$

where the supersymmetric part (also equal to the fermion squared-mass matrix) is

$$M^2_\nu = \begin{pmatrix} \mathcal{M}_\nu \mathcal{M}^\dagger_\nu & 0 \\ 0 & \mathcal{M}^\dagger_\nu \mathcal{M}_\nu \end{pmatrix}. \tag{A13}$$

Also, the new charged slepton squared-mass matrix is given by

$$M^2_{\tilde e} = \begin{pmatrix} M^2_L + m^2_L + \Delta_{-(1/2),-1} & -b^*_L \\ -b_L & M^2_L + m^2_{\bar L} + \Delta_{(1/2),1} \end{pmatrix}. \tag{A14}$$

Now define unitary scalar mixing matrices U and V by

$$U M^2_{\tilde\nu} U^\dagger = \mathrm{diag}(m^2_{\tilde\nu'_1}, m^2_{\tilde\nu'_2}, m^2_{\tilde\nu'_3}, m^2_{\tilde\nu'_4}), \quad V M^2_{\tilde e} V^\dagger = \mathrm{diag}(m^2_{\tilde\tau'_1}, m^2_{\tilde\tau'_2}). \tag{A15}$$

Then the scalar contributions to the vector boson self-energies are

$$\Delta\Pi_{\gamma\gamma} = \frac{N_c}{16\pi^2} g^2 s_W^2 e_e^2 \sum_{i=1}^{2} F(\tilde\tau'_i, \tilde\tau'_i), \tag{A16}$$

$$\Delta\Pi_{Z\gamma} = \frac{N_c}{16\pi^2} g s_W e_e \sum_{i=1}^{2} g^Z_{\tilde\tau'_i\tilde\tau'^*_i} F(\tilde\tau'_i, \tilde\tau'_i), \tag{A17}$$

$$\Delta\Pi_{ZZ} = \frac{N_c}{16\pi^2}\Big[\sum_{i,j=1}^{4} |g^Z_{\tilde\nu'_i\tilde\nu'^*_j}|^2 F(\tilde\nu'_i, \tilde\nu'_j) + \sum_{i,j=1}^{2} |g^Z_{\tilde\tau'_i\tilde\tau'^*_j}|^2 F(\tilde\tau'_i, \tilde\tau'_j)\Big], \tag{A18}$$

$$\Delta\Pi_{WW} = \frac{N_c}{16\pi^2}\sum_{i=1}^{2}\sum_{j=1}^{4} |g^W_{\tilde\tau'_i\tilde\nu'^*_j}|^2 F(\tilde\tau'_i, \tilde\nu'_j), \tag{A19}$$

where the vector boson couplings with the new sleptons are

$$g^Z_{\tilde{\nu}'_i\tilde{\nu}'^*_j} = \frac{g}{2c_W}(U^*_{i1}U_{j1} + U^*_{i3}U_{j3}),$$
$$g^Z_{\tilde{\tau}'_i\tilde{\tau}'^*_j} = \frac{g}{c_W}\left(-\frac{1}{2} - e_e s^2_W\right)\delta_{ij}, \tag{A20}$$

$$g^W_{\tilde{\tau}'_i\tilde{\nu}'^*_j} = g(V^*_{i1}U_{j1} - V^*_{i2}U_{j3})/\sqrt{2}. \tag{A21}$$

2. Corrections to electroweak vector boson self-energies in the QUE model

For the QUE model, the gauge eigenstate new up-type quark mass matrix is

$$\mathcal{M}_u = \begin{pmatrix} M_Q & k_U v_u \\ h_U v_d & M_U \end{pmatrix}, \tag{A22}$$

with unitary mixing matrices L and R defined by

$$L^*\mathcal{M}_u R^\dagger = \mathrm{diag}(m_{t'_1}, m_{t'_2}), \tag{A23}$$

and $m_{b'} = M_Q$. Then the (t'_1, t'_2, b') fermion contributions to the electroweak vector boson self-energies are

$$\Delta\Pi_{\gamma\gamma} = -\frac{N_c}{16\pi^2}2g^2 s^2_W\left[e^2_u\sum_{i=1,2}G(t'_i) + e^2_d G(b')\right], \tag{A24}$$

$$\Delta\Pi_{Z\gamma} = -\frac{N_c}{16\pi^2}g s_W\left[e_u\sum_{i=1,2}(g^Z_{t'_i t'^\dagger_i} - g^Z_{\bar{t}'_i\bar{t}'^\dagger_i})G(t'_i) + e_d(g^Z_{b'b'^\dagger} - g^Z_{\bar{b}'\bar{b}'^\dagger})G(b')\right], \tag{A25}$$

$$\Delta\Pi_{ZZ} = -\frac{N_c}{16\pi^2}\left[(|g^Z_{b'b'^\dagger}|^2 + |g^Z_{\bar{b}'\bar{b}'^\dagger}|^2)G(b') + \sum_{i,j=1}^{2}(|g^Z_{t'_i t'^\dagger_j}|^2 + |g^Z_{\bar{t}'_i\bar{t}'^\dagger_j}|^2)H(t'_i, t'_j) - 4\,\mathrm{Re}(g^Z_{t'_i t'^\dagger_j}g^Z_{\bar{t}'_i\bar{t}'^\dagger_j})m_{t'_i}m_{t'_j}B(t'_i, t'_j)\right], \tag{A26}$$

$$\Delta\Pi_{WW} = -\frac{N_c}{16\pi^2}\sum_{i=1,2}[(|g^W_{t'_i b'^\dagger}|^2 + |g^W_{\bar{t}'_i\bar{b}'^\dagger}|^2)H(b', t'_i) - 4\,\mathrm{Re}(g^W_{t'_i b'^\dagger}g^W_{\bar{t}'_i\bar{b}'^\dagger})m_{b'}m_{t'_i}B(b', t'_i)], \tag{A27}$$

where $N_c = 3$ and $e_u = 2/3$ and $e_d = -1/3$ and the massive vector boson couplings with the new quarks are

$$g^Z_{t'_i t'^\dagger_j} = \frac{g}{c_W}\left(\frac{1}{2}L^*_{i1}L_{j1} - e_u s^2_W\delta_{ij}\right),$$
$$g^Z_{\bar{t}'_i\bar{t}'^\dagger_j} = \frac{g}{c_W}\left(-\frac{1}{2}R^*_{i1}R_{j1} + e_u s^2_W\delta_{ij}\right), \tag{A28}$$

$$g^Z_{b'b'^\dagger} = -g^Z_{\bar{b}'\bar{b}'^\dagger} = \frac{g}{c_W}\left(-\frac{1}{2} - e_d s^2_W\right), \tag{A29}$$

$$g^W_{t'_i b'^\dagger} = gL^*_{i1}/\sqrt{2}, \qquad g^W_{\bar{t}'_i\bar{b}'^\dagger} = -gR^*_{i1}/\sqrt{2}. \tag{A30}$$

To obtain the $(\tilde{t}'_{1,2,3,4}, \tilde{b}'_{1,2})$ scalar contribution, consider the up-type squark squared-mass matrix:

$$M^2_{\tilde{u}} = M^2_u + \begin{pmatrix} m^2_Q + \Delta_{(1/2),(2/3)} & 0 & b^*_Q & a^*_{k_U}v_u - \mu k_U v_d \\ 0 & m^2_U + \Delta_{0,(2/3)} & a^*_{h_U}v_d - \mu h_U v_u & b^*_U \\ b_Q & a_{h_U}v_d - \mu^* h_U v_u & m^2_{\bar{Q}} + \Delta_{-(1/2),-(2/3)} & 0 \\ a_{k_U}v_u - \mu^* k_U v_d & b_U & 0 & m^2_{\bar{U}} + \Delta_{0,-(2/3)} \end{pmatrix}, \tag{A31}$$

where the supersymmetric part (also equal to the fermion squared-mass matrix) is

$$M^2_u = \begin{pmatrix} \mathcal{M}_u\mathcal{M}^\dagger_u & 0 \\ 0 & \mathcal{M}^\dagger_u\mathcal{M}_u \end{pmatrix}. \tag{A32}$$

Also, the down-type squark mass matrix is

$$M^2_{\tilde{d}} = \begin{pmatrix} M^2_Q + m^2_Q + \Delta_{-(1/2),-(1/3)} & -b^*_Q \\ -b_Q & M^2_Q + m^2_{\bar{Q}} + \Delta_{(1/2),(1/3)} \end{pmatrix}. \tag{A33}$$

Now define unitary scalar mixing matrices U and V by

$$UM^2_{\tilde{u}}U^\dagger = \mathrm{diag}(m^2_{\tilde{t}'_1}, m^2_{\tilde{t}'_2}, m^2_{\tilde{t}'_3}, m^2_{\tilde{t}'_4}), \qquad VM^2_{\tilde{d}}V^\dagger = \mathrm{diag}(m^2_{\tilde{b}'_1}, m^2_{\tilde{b}'_2}). \tag{A34}$$

Then the scalar contributions to the vector boson self-energies are:

$$\Delta\Pi_{\gamma\gamma} = \frac{N_c}{16\pi^2}g^2 s^2_W\left[e^2_u\sum_{i=1}^{4}F(\tilde{t}'_i, \tilde{t}'_i) + e^2_d\sum_{i=1}^{2}F(\tilde{b}'_i, \tilde{b}'_i)\right], \tag{A35}$$

$$\Delta\Pi_{Z\gamma} = \frac{N_c}{16\pi^2} g s_W \Big[e_u \sum_{i=1}^{4} g^Z_{\tilde{t}'_i \tilde{t}'^*_i} F(\tilde{t}'_i, \tilde{t}'_i) + e_d \sum_{i=1}^{2} g^Z_{\tilde{b}'_i \tilde{b}'^*_i} F(\tilde{b}'_i, \tilde{b}'_i)\Big], \tag{A36}$$

$$\Delta\Pi_{ZZ} = \frac{N_c}{16\pi^2} \Big[\sum_{i,j=1}^{4} |g^Z_{\tilde{t}'_i \tilde{t}'^*_j}|^2 F(\tilde{t}'_i, \tilde{t}'_j) + \sum_{i,j=1}^{2} |g^Z_{\tilde{b}'_i \tilde{b}'^*_j}|^2 F(\tilde{b}'_i, \tilde{b}'_j)\Big], \tag{A37}$$

$$\Delta\Pi_{WW} = \frac{N_c}{16\pi^2} \sum_{i=1}^{2}\sum_{j=1}^{4} |g^W_{\tilde{b}'_i \tilde{t}'^*_j}|^2 F(\tilde{b}'_i, \tilde{t}'_j), \tag{A38}$$

where the vector boson couplings with the new squarks are

$$g^Z_{\tilde{t}'_i \tilde{t}'^*_j} = \frac{g}{c_W}\Big[\frac{1}{2}(U^*_{i1}U_{j1} + U^*_{i3}U_{j3}) - e_u s^2_W \delta_{ij}\Big], \quad g^Z_{\tilde{b}'_i \tilde{b}'^*_j} = \frac{g}{c_W}\Big(-\frac{1}{2} - e_d s^2_W\Big)\delta_{ij}, \tag{A39}$$

$$g^W_{\tilde{b}'_i \tilde{t}'^*_j} = g(V^*_{i1}U_{j1} - V^*_{i2}U_{j3})/\sqrt{2}. \tag{A40}$$

3. Corrections to electroweak vector boson self-energies in the QDEE model

For the QDEE model, define the gauge eigenstate new down-type quark mass matrix by

$$\mathcal{M}_d = \begin{pmatrix} M_Q & k_D v_u \\ h_D v_d & M_D \end{pmatrix} \tag{A41}$$

and unitary mixing matrices L and R by

$$R^* \mathcal{M}_d L^\dagger = \mathrm{diag}(m_{b'_1}, m_{b'_2}), \tag{A42}$$

and note $m_{t'} = M_Q$. Then the (b'_1, b'_2, t') fermion contributions to the electroweak vector boson self-energies are

$$\Delta\Pi_{\gamma\gamma} = -\frac{N_c}{16\pi^2} 2g^2 s^2_W \Big[e^2_d \sum_{i=1,2} G(b'_i) + e^2_u G(t')\Big], \tag{A43}$$

$$\Delta\Pi_{Z\gamma} = -\frac{N_c}{16\pi^2} g s_W \Big[e_d \sum_{i=1,2} (g^Z_{b'_i b'^\dagger_i} - g^Z_{\bar{b}'_i \bar{b}'^\dagger_i}) G(b'_i) + e_u (g^Z_{t' t'^\dagger} - g^Z_{\bar{t}' \bar{t}'^\dagger}) G(t')\Big], \tag{A44}$$

$$\begin{aligned}\Delta\Pi_{ZZ} = -\frac{N_c}{16\pi^2} \Big[&(|g^Z_{t' t'^\dagger}|^2 + |g^Z_{\bar{t}' \bar{t}'^\dagger}|^2) G(t') + \sum_{i,j=1}^{2} (|g^Z_{b'_i b'^\dagger_j}|^2 + |g^Z_{\bar{b}'_i \bar{b}'^\dagger_j}|^2) H(b'_i, b'_j) \\ &- 4\,\mathrm{Re}(g^Z_{b'_i b'^\dagger_j} g^Z_{\bar{b}'_i \bar{b}'^\dagger_j}) m_{b'_i} m_{b'_j} B(b'_i, b'_j)\Big],\end{aligned} \tag{A45}$$

$$\Delta\Pi_{WW} = -\frac{N_c}{16\pi^2} \sum_{i=1,2} [(|g^W_{b'_i t'^\dagger}|^2 + |g^W_{\bar{b}'_i \bar{t}'^\dagger}|^2) H(t', b'_i) - 4\,\mathrm{Re}(g^W_{b'_i t'^\dagger} g^W_{\bar{b}'_i \bar{t}'^\dagger}) m_{t'} m_{b'_i} B(t', b'_i)], \tag{A46}$$

where $N_c = 3$ and $e_u = 2/3$ and $e_d = -1/3$ and the massive vector boson couplings with the new quarks are

$$g^Z_{b'_i b'^\dagger_j} = \frac{g}{c_W}\Big(-\frac{1}{2} L^*_{i1} L_{j1} - e_d s^2_W \delta_{ij}\Big), \quad g^Z_{\bar{b}'_i \bar{b}'^\dagger_j} = \frac{g}{c_W}\Big(\frac{1}{2} R^*_{i1} R_{j1} + e_d s^2_W \delta_{ij}\Big), \tag{A47}$$

$$g^Z_{t' t'^\dagger} = -g^Z_{\bar{t}' \bar{t}'^\dagger} = \frac{g}{c_W}\Big(\frac{1}{2} - e_u s^2_W\Big), \tag{A48}$$

$$g^W_{b'_i t'^\dagger} = -g L^*_{i1}/\sqrt{2}, \qquad g^W_{\bar{b}'_i \bar{t}'^\dagger} = g R^*_{i1}/\sqrt{2}. \tag{A49}$$

To obtain the $(\tilde{b}'_{1,2,3,4}, \tilde{t}'_{1,2})$ scalar contribution, start with the down-type squark squared-mass matrix:

$$M^2_{\tilde{d}} = M^2_d + \begin{pmatrix} m^2_{\tilde{Q}} + \Delta_{(1/2),(1/3)} & 0 & b^*_Q & a^*_{k_D} v_u - \mu k_D v_d \\ 0 & m^2_{\bar{D}} + \Delta_{0,(1/3)} & a^*_{h_D} v_d - \mu h_D v_u & b^*_D \\ b_Q & a_{h_D} v_d - \mu^* h_D v_u & m^2_{\bar{Q}} + \Delta_{-(1/2),-(1/3)} & 0 \\ a_{k_D} v_u - \mu^* k_D v_d & b_D & 0 & m^2_D + \Delta_{0,-(1/3)} \end{pmatrix}, \tag{A50}$$

where the supersymmetric part (also equal to the fermion squared-mass matrix) is

$$M^2_d = \begin{pmatrix} \mathcal{M}_d \mathcal{M}^\dagger_d & 0 \\ 0 & \mathcal{M}^\dagger_d \mathcal{M}_d \end{pmatrix}. \tag{A51}$$

Also, the up-type squark squared-mass matrix is given by

$$M_{\tilde{u}}^2 = \begin{pmatrix} M_Q^2 + m_{\bar{Q}}^2 + \Delta_{-(1/2),-(2/3)} & -b_Q^* \\ -b_Q & M_Q^2 + m_Q^2 + \Delta_{(1/2),(2/3)} \end{pmatrix}. \tag{A52}$$

Now define unitary scalar mixing matrices U and V by

$$UM_{\tilde{d}}^2 U^\dagger = \mathrm{diag}(m_{\tilde{b}_1'}^2, m_{\tilde{b}_2'}^2, m_{\tilde{b}_3'}^2, m_{\tilde{b}_4'}^2), \quad VM_{\tilde{u}}^2 V^\dagger = \mathrm{diag}(m_{\tilde{t}_1'}^2, m_{\tilde{t}_2'}^2). \tag{A53}$$

Then the scalar contributions to the vector boson self-energies are

$$\Delta\Pi_{\gamma\gamma} = \frac{N_c}{16\pi^2} g^2 s_W^2 \bigg[e_d^2 \sum_{i=1}^4 F(\tilde{b}_i', \tilde{b}_i') + e_u^2 \sum_{i=1}^2 F(\tilde{t}_i', \tilde{t}_i') \bigg], \tag{A54}$$

$$\Delta\Pi_{Z\gamma} = \frac{N_c}{16\pi^2} g s_W \bigg[-e_d \sum_{i=1}^4 g^Z_{\tilde{b}_i'^* \tilde{b}_i'} F(\tilde{b}_i', \tilde{b}_i') - e_u \sum_{i=1}^2 g^Z_{\tilde{t}_i'^* \tilde{t}_i'} F(\tilde{t}_i', \tilde{t}_i') \bigg], \tag{A55}$$

$$\Delta\Pi_{ZZ} = \frac{N_c}{16\pi^2} \bigg[\sum_{i,j=1}^4 |g^Z_{\tilde{b}_i'^* \tilde{b}_j'}|^2 F(\tilde{b}_i', \tilde{b}_j') + \sum_{i,j=1}^2 |g^Z_{\tilde{t}_i'^* \tilde{t}_j'}|^2 F(\tilde{t}_i', \tilde{t}_j') \bigg], \tag{A56}$$

$$\Delta\Pi_{WW} = \frac{N_c}{16\pi^2} \sum_{i=1}^2 \sum_{j=1}^4 |g^W_{\tilde{t}_i'^* \tilde{b}_j'}|^2 F(\tilde{t}_i', \tilde{b}_j'), \tag{A57}$$

where the vector boson couplings with the new squarks are

$$g^Z_{\tilde{b}_i'^* \tilde{b}_j'} = \frac{g}{c_W} \bigg[\frac{1}{2}(U_{i1}^* U_{j1} + U_{i3}^* U_{j3}) + e_d s_W^2 \delta_{ij} \bigg], \quad g^Z_{\tilde{t}_i'^* \tilde{t}_j'} = \frac{g}{c_W} \bigg(-\frac{1}{2} + e_u s_W^2 \bigg) \delta_{ij}, \tag{A58}$$

$$g^W_{\tilde{t}_i'^* \tilde{b}_j'} = g(V_{i1}^* U_{j1} - V_{i2}^* U_{j3})/\sqrt{2}. \tag{A59}$$

APPENDIX B: FORMULAS FOR DECAY WIDTHS OF NEW QUARKS AND LEPTONS

This appendix gives formulas for the decay widths of the lightest of the new quarks and leptons to standard model states. These decays are assumed to be mediated by Yukawa couplings that provide small mass mixings that can be treated as perturbations compared to the other entries in the mass matrices. In the following, $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$.

1. Decays of b' in the LND model

In the LND model, the lightest quark b' can decay to standard model states because of the mixing Yukawa parameter ϵ_D in Eq. (5.1). In terms of the mass matrix $\mathcal{M}_d$ in Eq. (5.2), define unitary mixing matrices L and R by

$$L^* \mathcal{M}_d R^\dagger = \mathrm{diag}(m_b, m_{b'}). \tag{B1}$$

The relevant couplings of b' to standard model particles are

$$g^W_{b' t^\dagger} = g L_{22}^*/\sqrt{2}, \qquad g^Z_{b' b^\dagger} = -\frac{g}{2c_W} L_{22}^* L_{12}, \tag{B2}$$

$$y^{h^0}_{b' \bar{b}} = -\sin(\alpha)(y_b R_{12} + \epsilon_D R_{11}) L_{22}/\sqrt{2}, \tag{B3}$$

$$y^{h^0}_{\bar{b}' b} = -\sin(\alpha)(y_b R_{22} + \epsilon_D R_{21}) L_{12}/\sqrt{2}. \tag{B4}$$

It follows that the decay widths of b' are

$$\Gamma(b' \to Wt) = \frac{m_{b'}}{32\pi} |g^W_{b' t^\dagger}|^2 \lambda^{1/2}(1, r_W, r_t) \times (1 + r_t - 2r_W + (1 - r_t)^2/r_W), \tag{B5}$$

$$\Gamma(b' \to Zb) = \frac{m_{b'}}{32\pi} |g^Z_{b' b^\dagger}|^2 (1 - r_Z)^2 (2 + 1/r_Z), \tag{B6}$$

$$\Gamma(b' \to h^0 b) = \frac{m_{b'}}{32\pi} (|y^{h^0}_{b' \bar{b}}|^2 + |y^{h^0}_{\bar{b}' b}|^2)(1 - r_{h^0})^2, \tag{B7}$$

where m_b is neglected for kinematic purposes and $r_i = m_i^2/m_{b'}^2$ for $i = Z, W, h^0$.

2. Decays of ν_1' in the LND model

Consider the decays of ν_1', the lighter new neutral lepton in the LND model, brought about by the superpotential mixing terms ϵ_N and ϵ_E in Eq. (5.1). Define unitary mixing matrices $L(3 \times 3)$ and $R(2 \times 2)$ in terms of the neutral lepton mass matrix in Eq. (5.3) by

$$R^* \mathcal{M}_\nu^T L^\dagger = \begin{pmatrix} 0 & m_{\nu_1'} & 0 \\ 0 & 0 & m_{\nu_2'} \end{pmatrix} \tag{B8}$$

where we are neglecting the tau neutrino mass. Also define unitary matrices L' and R' in terms of the charged lepton mass matrix in Eq. (5.4) by

$$L'^* \mathcal{M}_e R'^\dagger = \mathrm{diag}(m_\tau, m_{\tau'}). \tag{B9}$$

Then the relevant couplings of ν_1' to standard model particles are

$$g^W_{\nu_1' \tau^\dagger} = g(L_{21}^* L_{11}' + L_{23}^* L_{12}')/\sqrt{2}, \quad g^W_{\bar{\nu}_1' \bar{\tau}^\dagger} = g R_{11}^* R_{11}'/\sqrt{2} \tag{B10}$$

$$g^{Z}_{\nu'_1\nu^\dagger} = \frac{g}{2c_W}(L^*_{21}L_{11} + L^*_{23}L_{13}) \tag{B11}$$

$$y^{h^0}_{\bar{\nu}'_1\nu} = \frac{\cos\alpha}{\sqrt{2}}(\epsilon_N L_{13} + k_N L_{11})R_{12} - \frac{\sin\alpha}{\sqrt{2}}h_N L_{12}R_{11}. \tag{B12}$$

It follows that the decay widths of ν'_1 are

$$\Gamma(\nu'_1 \to W\tau) = \frac{m_{\nu'_1}}{32\pi}(1 - r_W)^2(2 + 1/r_W) \times (|g^W_{\nu'_1\tau^\dagger}|^2 + |g^W_{\bar{\nu}'_1\bar{\tau}^\dagger}|^2), \tag{B13}$$

$$\Gamma(\nu'_1 \to Z\nu_\tau) = \frac{m_{\nu'_1}}{32\pi}(1 - r_Z)^2(2 + 1/r_Z)|g^Z_{\nu'_1\nu^\dagger}|^2, \tag{B14}$$

$$\Gamma(\nu'_1 \to h^0\nu_\tau) = \frac{m_{\nu'_1}}{32\pi}(1 - r_{h^0})^2|y^{h^0}_{\bar{\nu}'_1\nu}|^2, \tag{B15}$$

where m_τ and m_{ν_τ} are neglected for kinematic purposes and $r_i = m_i^2/m^2_{\nu'_1}$ for $i = Z, W, h^0$.

3. Decays of t'_1 in the QUE model

Consider the decays of t'_1, the lightest new quark in the QUE model, brought about by the superpotential mixing terms in Eq. (5.5). Define unitary mixing matrices L, R, L', R' in terms of the mass matrices in Eq. (5.6) by

$$L^*\mathcal{M}_u R^\dagger = \mathrm{diag}(m_t, m_{t'_1}, m_{t'_2}), \quad L'^*\mathcal{M}_d R'^\dagger = \mathrm{diag}(m_b, m_{b'}). \tag{B16}$$

Then the relevant couplings of t'_1 to standard model particles are

$$g^W_{t'_1 b^\dagger} = g(L^*_{21}L'_{11} + L^*_{23}L'_{12})/\sqrt{2}, \quad g^W_{\bar{t}'_1\bar{b}^\dagger} = gR^*_{21}R'_{11}/\sqrt{2}, \tag{B17}$$

$$g^Z_{t'_1 t^\dagger} = \frac{g}{2c_W}(L^*_{21}L_{11} + L^*_{23}L_{13}), \quad g^Z_{\bar{t}'_1\bar{t}^\dagger} = -\frac{g}{2c_W}R^*_{21}R_{11}, \tag{B18}$$

$$y^{h^0}_{t'_1\bar{t}} = \frac{\cos\alpha}{\sqrt{2}}(\epsilon_U L_{23}R_{12} + \epsilon'_U L_{21}R_{13} + k_U L_{21}R_{12} + y_t L_{23}R_{13}) - \frac{\sin\alpha}{\sqrt{2}}h_U L_{22}R_{11}, \tag{B19}$$

$$y^{h^0}_{\bar{t}'_1 t} = \frac{\cos\alpha}{\sqrt{2}}(\epsilon_U L_{13}R_{22} + \epsilon'_U L_{11}R_{23} + k_U L_{11}R_{22} + y_t L_{13}R_{23}) - \frac{\sin\alpha}{\sqrt{2}}h_U L_{12}R_{21}. \tag{B20}$$

It follows that the decay widths of t'_1 are

$$\Gamma(t'_1 \to Wb) = \frac{m_{t'_1}}{32\pi}(1 - r_W)^2(2 + 1/r_W) \times (|g^W_{t'_1 b^\dagger}|^2 + |g^W_{\bar{t}'_1\bar{b}^\dagger}|^2), \tag{B21}$$

$$\Gamma(t'_1 \to Zt) = \frac{m_{t'_1}}{32\pi}\lambda^{1/2}(1, r_Z, r_t) \times [(1 + r_t - 2r_Z + (1 - r_t)^2/r_Z) \times (|g^Z_{t'_1 t^\dagger}|^2 + |g^Z_{\bar{t}'_1\bar{t}^\dagger}|^2) + 12\sqrt{r_t}\,\mathrm{Re}(g^Z_{t'_1 t^\dagger}g^Z_{\bar{t}'_1\bar{t}^\dagger})], \tag{B22}$$

$$\Gamma(t'_1 \to h^0 t) = \frac{m_{t'_1}}{32\pi}\lambda^{1/2}(1, r_{h^0}, r_t) \times [(1 + r_t - r_{h^0})(|y^{h^0}_{t'_1\bar{t}}|^2 + |y^{h^0}_{\bar{t}'_1 t}|^2) + 4\sqrt{r_t}\,\mathrm{Re}(y^{h^0}_{\bar{t}'_1 t}y^{h^0}_{t'_1\bar{t}})], \tag{B23}$$

where the bottom quark is treated as massless for purposes of kinematics and $r_i = m_i^2/m^2_{t'_1}$ for $i = t, Z, W, h^0$.

4. Decays of b'_1 in the QDEE model

Consider the decays of b'_1, the lightest new quark in the QDEE model, brought about by the superpotential mixing terms in Eq. (5.9). Define unitary mixing matrices R, L, R', L' in terms of the mass matrices in Eq. (5.10) by

$$R^*\mathcal{M}_d L^\dagger = \mathrm{diag}(m_b, m_{b'_1}, m_{b'_2}), \quad R'^*\mathcal{M}_u L'^\dagger = \mathrm{diag}(m_t, m_{t'}). \tag{B24}$$

Then the relevant couplings of b'_1 to standard model particles are

$$g^W_{b'_1 t^\dagger} = g(L^*_{21}L'_{11} + L^*_{23}L'_{12})/\sqrt{2}, \quad g^W_{\bar{b}'_1\bar{t}^\dagger} = gR^*_{21}R'_{11}/\sqrt{2}, \tag{B25}$$

$$g^Z_{b'_1 b^\dagger} = -\frac{g}{2c_W}(L^*_{21}L_{11} + L^*_{23}L_{13}), \quad g^Z_{\bar{b}'_1\bar{b}^\dagger} = \frac{g}{2c_W}R^*_{21}R_{11}, \tag{B26}$$

$$y^{h^0}_{b'_1\bar{b}} = -\frac{\sin\alpha}{\sqrt{2}}(\epsilon_D L_{23}R_{12} + \epsilon'_D L_{21}R_{13} + h_D L_{21}R_{12} + y_b L_{23}R_{13}) + \frac{\cos\alpha}{\sqrt{2}}k_D L_{22}R_{11}, \tag{B27}$$

$$y^{h^0}_{\bar{b}'_1 b} = -\frac{\sin\alpha}{\sqrt{2}}(\epsilon_D L_{13}R_{22} + \epsilon'_D L_{11}R_{23} + h_D L_{11}R_{22} + y_b L_{13}R_{23}) + \frac{\cos\alpha}{\sqrt{2}}k_D L_{12}R_{21}. \tag{B28}$$

It follows that the decay widths of b'_1 are

$$\Gamma(b_1' \to Wt) = \frac{m_{b_1'}}{32\pi}\lambda^{1/2}(1, r_W, r_t) \times [(1 + r_t - 2r_W + (1 - r_t)^2/r_W) \times (|g^W_{b_1' t^\dagger}|^2 + |g^W_{\bar{b}_1' \bar{t}^\dagger}|^2) + 12\sqrt{r_t}\,\mathrm{Re}(g^W_{b_1' t^\dagger} g^W_{\bar{b}_1' \bar{t}^\dagger})], \qquad \text{(B29)}$$

$$\Gamma(b_1' \to Zb) = \frac{m_{b_1'}}{32\pi}(1 - r_Z)^2(2 + 1/r_Z) (|g^Z_{b_1' b^\dagger}|^2 + |g^Z_{\bar{b}_1' \bar{b}^\dagger}|^2), \qquad \text{(B30)}$$

$$\Gamma(b_1' \to h^0 b) = \frac{m_{b_1'}}{32\pi}(1 - r_{h^0})^2(|y^{h^0}_{b_1' \bar{b}}|^2 + |y^{h^0}_{\bar{b}_1' b}|^2), \qquad \text{(B31)}$$

where the bottom quark is treated as massless for purposes of kinematics and $r_i = m_i^2/m_{b_1'}^2$ for $i = t, Z, W, h^0$.

5. Decays of τ' in the QUE and QDEE models

Consider the decays of τ' in the QUE model, brought about by the superpotential mixing term ϵ_E in Eq. (5.7). In terms of the mass matrix Eq. (5.8), define unitary mixing matrices L and R by

$$L^* \mathcal{M}_e R^\dagger = \mathrm{diag}(m_\tau, m_{\tau'}). \qquad \text{(B32)}$$

Then the relevant couplings of τ' to standard model particles are

$$g^W_{\tau' \nu^\dagger} = g L^*_{22}/\sqrt{2}, \qquad g^Z_{\tau' \tau^\dagger} = -\frac{g}{2c_W} L^*_{22} L_{12}, \qquad \text{(B33)}$$

$$y^{h^0}_{\tau' \bar{\tau}} = -\sin(\alpha) L_{22}(y_\tau R_{12} + \epsilon_E R_{11})/\sqrt{2}, \qquad \text{(B34)}$$

$$y^{h^0}_{\bar{\tau}' \tau} = -\sin(\alpha) L_{12}(y_\tau R_{22} + \epsilon_E R_{21})/\sqrt{2}. \qquad \text{(B35)}$$

It follows that the decay widths of τ' are

$$\Gamma(\tau' \to W\nu) = \frac{m_{\tau'}}{32\pi}(1 - r_W)^2(2 + 1/r_W)|g^W_{\tau' \nu^\dagger}|^2, \qquad \text{(B36)}$$

$$\Gamma(\tau' \to Z\tau) = \frac{m_{\tau'}}{32\pi}(1 - r_Z)^2(2 + 1/r_Z)|g^Z_{\tau' \tau^\dagger}|^2, \qquad \text{(B37)}$$

$$\Gamma(\tau' \to h^0 \tau) = \frac{m_{\tau'}}{32\pi}(1 - r_{h^0})^2(|y^{h^0}_{\tau' \bar{\tau}}|^2 + |y^{h^0}_{\bar{\tau}' \tau}|^2), \qquad \text{(B38)}$$

where $r_i = m_i^2/m_{\tau'}^2$ for $i = Z, W, h^0$, and m_τ is neglected for kinematic purposes. In the QDEE model, the same calculation holds, provided that M_E is replaced by M_{E_1} corresponding to the lighter mass eigenstate $m_{\tau'}$.

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