Meson emission model of $\Psi \to N\bar{N}m$ charmonium strong decays

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In this paper we consider a sequential ''meson emission'' mechanism for charmonium decays of the type $\Psi \to N\bar{N}m$, where Ψ is a generic charmonium state, N is a nucleon, and m is a light meson. This decay mechanism, which may not be dominant in general, assumes that an $N\bar{N}$ pair is created during decay mechanism, which may not be dominant in general, assumes that an $N\bar{N}$ pair is created during charmonium annihilation, and the light meson m is emitted from the outgoing nucleon or antinucleon line. A straightforward generalization of this model can incorporate intermediate N^* resonances. We derive Dalitz plot event densities for the cases $\Psi = \eta_c$, J/ψ , χ_{c0} , χ_{c1} , and ψ' ; and $m = \pi^0$, f_0 , and ω (and implicitly any 0^{-+} , 0^{++} , or 1^{--} final light meson). It may be possible to separate the implicitly, any 0^{-+} , 0^{++} , or 1^{--} final light meson). It may be possible to separate the contribution of this decay mechanism to the full decay amplitude through characteristic event densities. For the decay subset $\Psi \to p\bar{p}\pi^0$ the two model parameters are known, so we are able to predict absolute numerical partial
widths for $\Gamma(W \to p\bar{p}\pi^0)$. In the specific case $I/\psi \to p\bar{p}\pi^0$ the predicted partial width and M event widths for $\Gamma(\Psi \to p\bar{p}\pi^0)$. In the specific case $J/\psi \to p\bar{p}\pi^0$ the predicted partial width and $M_{p\pi}$ event distribution are intriguingly close to experiment. We also consider the possibility of scalar meson and distribution are intriguingly close to experiment. We also consider the possibility of scalar meson and glueball searches in $\Psi \to p\bar{p}f_0$. If the meson emission contributions to $\Psi \to N\bar{N}m$ decays can be isolated
and quantified, they can be used to estimate meson nucleon strong couplings $f_a = \lambda$ which are typically and quantified, they can be used to estimate meson-nucleon strong couplings $\{g_{NNm}\}\$, which are typically poorly known, and are a crucial input in meson exchange models of the NN interaction. The determination of $g_{NN\pi}$ from $J/\psi \rightarrow p\bar{p}\pi^0$ and the (poorly known) $g_{NN\omega}$ and the anomalous "strong magnetic" coupling $\kappa_{NN\omega}$ from $J/\psi \rightarrow p\bar{p}\omega$ are considered as examples.

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I. INTRODUCTION

Charmonium strong decays of the type $\Psi \to N\bar{N}m$,
here Ψ is a generic charmonium state N is a nucleon where Ψ is a generic charmonium state, N is a nucleon, and *m* is a light meson, have recently attracted interest both as sources of information regarding the N^* spectrum [[1](#page-9-0)[–5\]](#page-9-1) and in searches for a low-energy $N\bar{N}$ enhancement "X(1835)," which has been reported in $J/\psi \rightarrow \gamma p \bar{p}$ [\[6\]](#page-9-2)
and $J/\psi \rightarrow \gamma \pi^+ \pi^- \eta'$ [7.8] but thus far not in $\Psi \rightarrow$ and $J/\psi \rightarrow \gamma \pi^+ \pi^- \eta'$ [[7,](#page-9-3)[8\]](#page-9-4), but thus far not in $\Psi \rightarrow$
p $\bar{p}m$. These decays are also of interest because their partial $p\bar{p}m$. These decays are also of interest because their partial widths can be used to estimate the $p\bar{p} \to m\Psi$ associated
charmonium production cross sections at PANDA [9.10] charmonium production cross sections at PANDA [[9,](#page-9-5)[10\]](#page-9-6). As we shall show here, they may also provide information on NNm meson-nucleon coupling constants, which could be used to identify unusual resonances such as molecule or glueball candidates.

Specific $\Psi \to N\bar{N}m$ reactions that have recently been
idied experimentally include $I/\psi \to n\bar{n}\pi^0$ [1] $n\bar{n}n$ studied experimentally include $J/\psi \rightarrow p\bar{p}\pi^0$ [\[1](#page-9-0)], $p\bar{p}\eta$
and $p\bar{p}\pi^{\prime}$ [4] and $p\bar{p}\omega$ [5]; $\psi' \rightarrow p\bar{p}\pi^0$ [2] $p\bar{p}\eta$ and $p\bar{p}\eta'$ [[4](#page-9-7)], and $p\bar{p}\omega$ [\[5\]](#page-9-1); $\psi' \to p\bar{p}\pi^0$ [[2\]](#page-9-8), $p\bar{p}\eta$
[2.11] $p\bar{p}\pi^- +$ H c [3] $p\bar{p}\omega$ [11] and $p\bar{p}\omega$ [11.12] [\[2,](#page-9-8)[11\]](#page-9-9), $p\bar{n}\pi^{-}$ + H.c. [[3\]](#page-9-10), $p\bar{p}\rho$ [\[11\]](#page-9-9) and $p\bar{p}\omega$ [\[11](#page-9-9)[,12\]](#page-9-11),
and (upper limit) $p\bar{p}\phi$ [11.12]; and $y \to p\bar{p}\pi^{0}$ and and (upper limit) $p\bar{p}\phi$ [[11](#page-9-9),[12](#page-9-11)]; and $\chi_{cJ} \rightarrow p\bar{p}\pi^0$ and $p\bar{p}n$ [13] $p\bar{p}\eta$ [[13](#page-9-12)].

These decays may prove to be complicated processes in which several decay mechanisms contribute significantly. For this reason it will be useful to have predictions for

 $\Psi \rightarrow N\bar{N}m$ Dalitz plot (DP) event densities assuming
various decay mechanisms: this paper provides these revarious decay mechanisms; this paper provides these results for one such mechanism. In particular we derive the DP event densities that follow from sequential meson emission, in which the charmonium state (generically Ψ) decays to an intermediate $N\bar{N}$ state, which radiates the light meson from the N or \overline{N} line, $\Psi \to N\overline{N} \to N\overline{N}m$.
The two Feynman diagrams assumed in this model are The two Feynman diagrams assumed in this model are shown in Fig. [1](#page-0-3).

We emphasize that the actual relative importance of this and other $\Psi \to N\bar{N}m$ decay mechanisms is unclear at present and may depend strongly on the charmonium state present, and may depend strongly on the charmonium state Ψ and the light meson *m*; one purpose of this paper is to determine the rates predicted by this meson emission decay model in isolation for comparison with experiment, so that the importance of this decay mechanism can be estimated.

The predictions of this $\Psi \to N\bar{N} \to N\bar{N}m$ decay model
the parameters since can be given in some cases with no free parameters, since the strengths of the *a priori* unknown couplings $\Psi N\overline{N}$ and NNm can be estimated from other processes. Here we will

FIG. 1. Feynman diagrams of the meson emission model.

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give absolute predictions for the set of partial widths $\{\Gamma(\Psi \to p\bar{p}\pi^0)\}\;$; we use the known partial widths $\{\Gamma(\Psi \to p\bar{p})\}\;$ to estimate the $\{\Psi N\bar{N}\}$ couplings, and the final $NN\pi$ $p\bar{p}$) to estimate the $\{\Psi N\bar{N}\}$ couplings, and the final $NN\pi$
coupling is of course well known coupling is of course well known.

Provided that the contribution of the $\Psi \to N\bar{N} \to N\bar{N}m$
cay mechanism can be isolated and quantified experidecay mechanism can be isolated and quantified experimentally, this information can be used to estimate mesonnucleon strong coupling constants; these are generally poorly known, and play an important role in nuclear physics as input parameters in meson exchange models of the NN force [[14](#page-9-13)–[21](#page-9-14)].

We will also discuss the determination of the (well known) $p\bar{p}\pi^0$ and the (poorly known) $p\bar{p}\omega$ couplings from the decays $J/\psi \rightarrow p\bar{p}\pi^0$, $\psi' \rightarrow p\bar{p}\pi^0$, and $J/\psi \rightarrow p\bar{p}\omega$ as examples. This provides a third motivation for the $p\bar{p}\omega$ as examples. This provides a third motivation for the study of $\Psi \rightarrow N\bar{N}m$ decays; they may prove useful for estimating NNm counting constants in addition to their estimating NNm coupling constants, in addition to their relevance to N^* spectroscopy [[1](#page-9-0)[–5\]](#page-9-1) and low-mass $p\bar{p}$ dynamics [[6](#page-9-2)[–8](#page-9-4)]. Another motivation for studying $\Psi \rightarrow$ $N\bar{N}m$ is the possibility of observing light scalars, including the " σ ," the 980 MeV states, and the scalar glueball, in the decays $\Psi \to p\bar{p}f_0$ (and a_0).

II. FORMULAS

Here we will usually specialize to charmonium decays to a $p\bar{p}$ pair and a neutral meson, $\Psi \to p\bar{p}m^0$; these decays are reasonably well studied and enjoy the simplification of are reasonably well studied, and enjoy the simplification of equal baryon and antibaryon masses. Our results employ conventions for kinematic variables, meson-baryon couplings, and masses that were used in Ref. [\[22\]](#page-9-15). In particular, *M* is the mass of the initial charmonium state, m_p is the proton mass, m_m is the mass of meson (subscript) m, and dimensionless mass ratios $R \equiv M/m_p$ and $r \equiv m_m/m_p$ are defined relative to the proton mass. (Hence the numerical values of R and r depend on the decay process.) We use scaled dimensionless variables $x = M_{p_m}^2 / m_p^2 - 1$ and $y = M^2 / m^2 - 1$ and their inverses $y = 1 / x$ and $y = 1 / y$ to $M_{\bar{p}m}^2/m_p^2 - 1$ and their inverses $u = 1/x$ and $v = 1/y$ to describe Dalitz plots: these greatly simplify our results describe Dalitz plots; these greatly simplify our results. The DP event densities we present here are formally partial width densities in x and y , which are related to the more familiar differential partial widths by a trivial overall constant,

$$
\frac{d^2\Gamma(\Psi \to p\bar{p}m)}{dxdy} = m_p^4 \frac{d^2\Gamma(\Psi \to p\bar{p}m)}{dM_{pm}^2 dM_{\bar{p}m}^2}.
$$
 (1)

Before we give our results for these event densities, it is useful to recall some general properties of a $\Psi \rightarrow p\bar{p}m$
Dalitz plot. The boundary in the dimensionless variables Dalitz plot. The boundary in the dimensionless variables (x, y) is specified by the curves

$$
y_{\pm} = \frac{r^2 R^2 + (r^2 + R^2 - 2)x - x^2 \pm F_m F_{\Psi}}{2(1+x)},
$$
 (2)

where $F_m = F(r, x)$ and $F_{\Psi} = F(R, x)$, with

$$
F(a, x) \equiv (a^2(a^2 - 4) - 2a^2x + x^2)^{1/2}.
$$
 (3)

The range of values of x (and y) in the physical region is

$$
r(r+2) \le x \le R(R-2). \tag{4}
$$

The areas $\{A_D\}$ of these Dalitz plots are useful for estimating $\Gamma(\Psi \to p\bar{p}m)$ partial widths [[9](#page-9-5)]. Although A_D can be evaluated in closed form for $\Psi \to p\bar{p}m$ with general mass evaluated in closed form for $\Psi \to p\bar{p}m$ with general mass
ratios r and R, the resulting expression is quite lengthy so ratios r and R , the resulting expression is quite lengthy, so when required we will simply evaluate each A_D numerically.

In deriving the DP event densities we have usually assumed that the $\Psi p \bar{p}$ coupling is a constant $g_{\Psi p \bar{p}}$ times the simplest relevant Dirac matrix for the given Ψ quantum numbers; for example, for the J/ψ we use a pure vector $J/\psi p \bar{p}$ vertex, $-i g_{J/\psi p \bar{p}} \gamma_{\mu}$. The order of the hadron
labels in g_{μ} , is meant to reflect the fact that the numerical labels in g_{abc} is meant to reflect the fact that the numerical value of this coupling constant is taken from an $a \rightarrow bc$ transition, here $J/\psi \rightarrow p\bar{p}$. This could be a significant concern if form factor effects are large concern if form factor effects are large.

We proceed similarly for the light mesons π^0 and f_0 ; for the pion we use a pure pseudoscalar $NN\pi$ coupling, with vertex $g_{NN\pi}\gamma_5$, and $-ig_{NNf_0}I$ for the NNf_0 vertex. Since light vector mesons (generically represented by the ω) have two interesting strong couplings, Dirac (vector) and Pauli (anomalous magnetic), for this special case we assume a vertex with two interactions,

$$
\Gamma_{\mu}^{(\omega)} = -ig_{NN\omega}(\gamma_{\mu} + i(\kappa_{NN\omega}/2m_p)\sigma_{\mu\nu}q_{\nu}).
$$
 (5)

Reference [[23](#page-10-0)] assumed a similar $J/\psi p\bar{p}$ vertex; see Ref. [\[22\]](#page-9-15) for additional details regarding the couplings assumed here. We generally abbreviate these coupling constants as $g_{\Psi} \equiv g_{\Psi N\bar{N}}$ and $g_m \equiv g_{NNm}$; rationalized
squared countings $g_{\Psi} \equiv \sigma^2 / 4\pi$ and $g_{\Psi} \equiv \sigma^2 / 4\pi$ are squared couplings $\alpha_{\Psi} \equiv g_{\Psi}^2/4\pi$ and $\alpha_m \equiv g_m^2/4\pi$ are also used.

For the special case of $p\bar{p}\pi^0$ final states, these event densities can be obtained by applying crossing relations to our previously published results for the unpolarized differential cross sections for the $2 \rightarrow 2$ processes $p\bar{p}$
[22] the other (f_2 and ω) cases have not been co $\to \pi^0\Psi$ nsidered [\[22\]](#page-9-15); the other (f_0 and ω) cases have not been considered previously. The results for all cases considered here are given below.

$$
\frac{d^2\Gamma(\eta_c \to p\bar{p}\pi^0)}{dxdy} = \alpha_{\eta_c} \alpha_{\pi} \frac{m_p}{8\pi R^3} \times \left\{ (u-v)^2 \cdot \left(\frac{1}{uv} - r^2 R^2 \right) \right\}
$$
(6)

$$
\frac{d^2\Gamma(J/\psi \to p\bar{p}\pi^0)}{dx dy} = \alpha_{J/\psi} \alpha_{\pi} \frac{m_p}{12\pi R^3} \left\{ \frac{(u+v)^2}{uv} - 2(u+v)(u+v+1)r^2 + 2uvr^4 - (u^2+v^2) \cdot r^2R^2 \right\}
$$
(7)

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$$
\frac{d^2\Gamma(\chi_{c0}\to p\bar{p}\pi^0)}{dxdy} = \alpha_{\chi_{c0}}\alpha_\pi \frac{m_p}{8\pi R^3} \Big\{ (u+v)^2
$$

$$
\cdot \left(\frac{1}{uv} + 4r^2 - r^2R^2\right) \Big\} \tag{8}
$$

$$
\frac{d^2\Gamma(\chi_{c1} \to p\bar{p}\pi^0)}{dx dy} = \alpha_{\chi_{c1}} \alpha_{\pi} \frac{m_p}{6\pi R^5} \left\{ -\frac{(u+v)}{uv} + (u+v+1) + r^2 + \frac{(u^2+v^2)}{2uv}R^2 + (2(u^2+v^2)+u+v)r^2R^2 - uvr^4R^2 - \frac{(u^2+v^2)}{2}r^2R^4 \right\}
$$
(9)

$$
\frac{d^2\Gamma(\eta_c \to p\bar{p}f_0)}{dxdy} = \alpha_{\eta_c} \alpha_{f_0} \frac{m_p}{8\pi R^3} \left\{ (u+v)^2 \cdot \left(\frac{1}{uv} + 4R^2 - r^2 R^2 \right) \right\}
$$
(10)

$$
\bar{\mathsf{V}}m \ldots \qquad \qquad \mathsf{PHYSICAL REVIEW\ D\ 81, 034025 (2010)}
$$

$$
\frac{d^2\Gamma(J/\psi \to p\bar{p}f_0)}{dxdy} = \alpha_{J/\psi} \alpha_{f_0} \frac{m_p}{12\pi R^3} \left\{ \frac{(u+v)^2}{uv} + 8(u+v)(u+v+1) - 2(u(u+1)+v(v+1)) + 6uv)r^2 + 4(u+v)^2R^2 + 2uvr^4 - (u^2+v^2) \cdot r^2R^2 \right\}
$$
(11)

(9)
\n
$$
\frac{d^2\Gamma(\chi_{c0} \to p\bar{p}f_0)}{dx dy} = \alpha_{\chi_{c0}} \alpha_{f0} \frac{m_p}{8\pi R^3} \left\{ \frac{(u-v)^2}{uv} + 10(u+v)(u+v+1) + 4(u+v)^2(r^2+R^2) - (u-v)^2r^2R^2 \right\}
$$
\n(12)

$$
\frac{d^2\Gamma(\chi_{c1} \to p\bar{p}f_0)}{dxdy} = \alpha_{\chi_{c1}}\alpha_{f_0}\frac{m_p}{6\pi R^5} \left\{ -\frac{(u+v)}{uv} \cdot (u+v+1) + r^2 + \left(\frac{(u^2+v^2)}{2uv} - 8(u+v) \cdot (u+v+1)\right) R^2 + (2(u^2+v^2) + 8uv + u+v)r^2R^2 + 2(u+v)^2R^4 - uvr^4R^2 - \frac{(u^2+v^2)}{2}r^2R^4 \right\}
$$
(13)

$$
\frac{d^2\Gamma(\eta_c \to p\bar{p}\omega)}{dx dy} = \alpha_{\eta_c} \alpha_{\omega} \frac{m_p}{4\pi R^3} \left\{ \left[\frac{(u+v)^2}{uv} - 2(u+v)(u+v+1)R^2 - (u^2+v^2)r^2R^2 + 2uvR^4 \right] \right. \\ \left. + \kappa_{\omega} \left[-\frac{2(u+v)^2}{uv} + (3(u^2+v^2)+2uv)r^2R^2 \right] + \kappa_{\omega}^2 \left[\frac{(u+v)(u+v-1)}{2uv} + \frac{(u+v)^2}{8uv}r^2 + \frac{1}{2}R^2 \right. \\ \left. + \left(\frac{(u+v)}{2} - (u^2+v^2) \right) r^2R^2 - \frac{(u+v)^2}{8}r^4R^2 - \frac{uv}{2}r^2R^4 \right] \right\} \tag{14}
$$

$$
\frac{d^2\Gamma(J/\psi \to p\bar{p}\omega)}{dx dy} = \alpha_{J/\psi}\alpha_{\omega}\frac{m_p}{6\pi R^3} \Biggl\{ \Biggl[\frac{(u^2 + v^2)}{uv} - 4(u + v)(u + v + 1) - 2(u(u + 1) + v(v + 1)) \cdot (r^2 + R^2) \Biggr] \Biggr\} + 2uvr^4 - (u^2 + v^2 - 4uv)r^2R^2 + 2uvR^4 \Biggr] + \kappa_{\omega} \Biggl[-\frac{(u + v)^2}{uv} + 6(u + v)(u + v + 1)r^2 - 6uvr^4 \Biggr] + (3(u^2 + v^2) - 8uv)r^2R^2 \Biggr] + \kappa_{\omega}^2 \Biggl[-\frac{(u + v)}{2uv} + \left(\frac{(u + v)^2}{8uv} - 2(u + v)(u + v + 1) \right) r^2 + \frac{1}{2}R^2 \Biggr] - \frac{1}{4}(u(u + 1) + v(v + 1) - 6uv)r^4 - (u - v)^2r^2R^2 + \frac{1}{4}uvr^6 - \frac{1}{8}(u^2 + v^2 - 4uv)r^4R^2 \Biggr] \Biggr\} \tag{15}
$$

$$
\frac{d^2\Gamma(\chi_{c0} \to p\bar{p}\omega)}{dx dy} = \alpha_{\chi_{c0}} \alpha_{\omega} \frac{m_p}{4\pi R^3} \left[\frac{(u+v)^2}{uv} + 8(u+v)(u+v+1) + 4(u+v)^2 r^2 - 2(u(u+1) + v(v+1)) + 6uv)R^2 - (u^2+v^2)r^2R^2 + 2uvR^4 \right] + \kappa_{\omega} \left[-12(u+v)\left(u+v+\frac{1}{2}\right)r^2 + 3(u+v)^2r^2R^2 \right] + \kappa_{\omega}^2 \left[-\frac{(u+v)(u+v+1)}{2uv} + \left(\frac{(u^2+6uv+v^2)}{8uv} + 4(u+v)(u+v+1)\right)r^2 + \frac{1}{2}R^2 + \frac{(u+v)^2}{2}r^4 - \left(u^2+4uv+v^2+\frac{(u+v)}{2}\right)r^2R^2 - \frac{(u-v)^2}{8}r^4R^2 + \frac{uv}{2}r^2R^4 \right] \right]
$$
(16)

$$
\frac{d^2\Gamma(\chi_{c1} \to p\bar{p}\omega)}{dx dy} = \alpha_{\chi_{c1}} \alpha_{\omega} \frac{m_p}{3\pi R^5} \Biggl[\frac{(u+v)^2}{uv} + \left(\frac{(u^2+v^2)}{2uv} + 4(u+v)(u+v+1) \right) R^2 + (2(u-v)^2 - u-v)r^2 R^2
$$

\n
$$
- (u^2+v^2+6uv+u+v)R^4 + uvr^4 R^2 + \left(2uv - \frac{(u^2+v^2)}{2} \right) r^2 R^4 + uvR^6 \Biggr]
$$

\n
$$
+ \kappa_{\omega} \Biggl[-\frac{(u+v)^2}{uv} - r^2 - \frac{(u^2+v^2-4uv)}{2uv} \cdot R^2 - (6(u^2+v^2)+u+v)r^2 R^2 - uvr^4 R^2
$$

\n
$$
+ \frac{3}{2} (u^2+v^2)r^2 R^4 \Biggr] + \kappa_{\omega}^2 \Biggl[\frac{(1+2(u+v)+2(u+v)^2)}{4uv} - \frac{(u+v+(u-v)^2)}{8uv}r^2 - \left(\frac{3}{2} + \frac{(u+v)}{4uv} \right) R^2
$$

\n
$$
+ \frac{1}{8} r^4 + \left(2(u^2+v^2-uv) + \frac{(u+v)}{2} + \frac{(u^2+v^2+4uv)}{16uv} \right) \cdot r^2 R^2 + \frac{1}{4} R^4 + \frac{1}{8} (u+v+2(u^2+v^2+4uv)) r^4 R^2 - \frac{1}{2} (u^2+v^2-uv) r^2 R^4 - \frac{1}{8} u v r^6 R^2 - \frac{1}{16} (u^2+v^2+4uv) r^4 R^4 \Biggr].
$$
 (17)

The symmetry of these event densities under (x, y) and hence (u, v) interchange is a consequence of C-parity invariance. There are singularities in these events densities along the lines $x = 0$ ($u = \infty$) and $y = 0$ ($v = \infty$), corresponding to $M_{pm}^2 = m_p^2$ and $M_{pm}^2 = m_p^2$. These are due to the poles of the n and \bar{p} proposators in the Esympan the poles of the p and \bar{p} propagators in the Feynman diagrams for the decay process $\Psi \to p\bar{p} \to p\bar{p}m$ (Fig. [1\)](#page-0-3)
and lie outside the physical regions of the Dalitz plots and lie outside the physical regions of the Dalitz plots.

III. APPLICATIONS

A. The $J/\psi \rightarrow p\bar{p}\pi^0$ partial width

As a first application we will evaluate the partial width for $J/\psi \rightarrow p\bar{p}\pi^0$ assuming this meson emission decay
mechanism. The Particle Data Group (PDG) [24] quotes mechanism. The Particle Data Group (PDG) [[24](#page-10-1)] quotes a branching fraction of

$$
B(J/\psi \to p\bar{p}\pi^0) = 1.09 \pm 0.09 \cdot 10^{-3},\qquad(18)
$$

which is the average of early measurements by Mark-I [\[25\]](#page-10-2), DASP [\[26\]](#page-10-3), and Mark-II [\[27\]](#page-10-4). There are also more recent experimental results on this decay from BES-II [[1\]](#page-9-0). Using the current PDG value for the J/ψ total width of 93.2 ± 2.1 keV, this branching fraction corresponds to a partial width of

$$
\Gamma(J/\psi \to p\bar{p}\pi^0) = 102 \pm 9 \text{ eV}.
$$
 (19)

To evaluate this partial width in the meson emission model we simply integrate the theoretical event density [\(7\)](#page-1-0) over the Dalitz plot. This event density is given by

$$
\frac{d^2\Gamma(J/\psi \to p\bar{p}\pi^0)}{dx dy} = \alpha_{J/\psi} \alpha_{\pi} m_p \rho(x, y),\tag{20}
$$

where the dimensionless density function ρ is

$$
\rho(x, y) = \frac{1}{12\pi R^3} \left\{ \frac{(u+v)^2}{uv} - 2(u+v)(u+v+1)r^2 + 2uvr^4 - (u^2+v^2)r^2R^2 \right\}.
$$
 (21)

This (parameter-free) relative event density, scaled to the

maximum value in the physical region, is shown in Fig. [2.](#page-3-0) Integrating [\(20\)](#page-3-1) over the Dalitz plot gives

$$
\Gamma(J/\psi \to p\bar{p}\pi^0) = \alpha_{J/\psi} \alpha_{\pi} m_p \cdot \langle \rho \rangle \cdot A_D/m_p^4, \quad (22)
$$

where $\langle \rho \rangle$ is the mean value of $\rho(x, y)$ in the Dalitz plot, which has (physical) area A_D ;

$$
\iint_{\text{DP}} \rho \, dx \, dy = \langle \rho \rangle \cdot A_D / m_P^4. \tag{23}
$$

We evaluate $\langle \rho \rangle$ and A_D numerically, assuming physical hadron masses; we use PDG masses rounded to 0.1 MeV; $m_{\pi^0} = 0.1350 \text{ GeV}, \quad m_p = 0.9383 \text{ GeV}, \quad \text{and} \quad m_{J/\psi} =$ 3.0969 GeV, which leads to $\langle \rho \rangle = 3.070 \cdot 10^{-3}$ and $A_D = 9.265$ GeV⁴ and a partial width of 9.265 $GeV⁴$, and a partial width of

FIG. 2. The $J/\psi \rightarrow p\bar{p}\pi^0$ DP event density predicted by the meson emission decay mechanism $I/\psi \rightarrow p\bar{p} \rightarrow p\bar{p}\pi^0$ Eq. (7) meson emission decay mechanism $J/\psi \to p\bar{p} \to p\bar{p}\pi^0$, Eq. ([7\)](#page-1-0).
Contours of equal density are shown Contours of equal density are shown.

MESON EMISSION MODEL OF $\Psi \rightarrow N\bar{N}m$...

$$
\Gamma(J/\psi \to p\bar{p}\pi^0) = 34.44 \cdot \alpha_{J/\psi} \alpha_{\pi} \text{ MeV.}
$$
 (24)

To complete this estimate we require numerical values for the $NN\pi$ and $J/\psi N\bar{N}$ coupling constants. For $NN\pi$ there is general agreement from meson exchange models of NN scattering that $g_{NN\pi} \approx 13$ (see for example [\[14–](#page-9-13)[21](#page-9-14)]); we accordingly set $g_{NN\pi} = 13.0$. The value of the $J/\psi N\bar{N}$
coupling constant (here $g_{\lambda}(t, z)$) can be estimated from the coupling constant (here $g_{J/\psi p\bar{p}}$) can be estimated from the measured partial width to $p\bar{p}$, which is (again using PDG numbers) $\Gamma(J/\psi \to p\bar{p}) = 202 \pm 8$ eV. Equating this to the theoretical decay rate the theoretical decay rate

$$
\Gamma(J/\psi \to p\bar{p}) = \alpha_{J/\psi} \beta_p (1 + 2/R^2) M/3 \tag{25}
$$

gives a value of $g_{J/\psi p\bar{p}} = 1.62 \cdot 10^{-3}$, as was quoted
previously in Ref. [22] Hsing these couplings in Eq. (24) previously in Ref. [[22](#page-9-15)]. Using these couplings in Eq. [\(24\)](#page-4-0) gives our meson emission model prediction

$$
\Gamma(J/\psi \to p\bar{p}\pi^0) = 97 \text{ eV}.
$$
 (26)

This is consistent with the experimental value of $102 \pm$ 9 eV quoted in Eq. [\(19\)](#page-3-2). This excellent agreement is somewhat fortuitous, since this version of the model does not include the N^* contributions evident in the $J/\psi \rightarrow p\bar{p}\pi^0$
Dalitz plot [1] (see also Fig. 3) Dalitz plot [[1\]](#page-9-0) (see also Fig. [3\)](#page-4-1).

We note in passing that the charged-pion cases $J/\psi \rightarrow$ $p\bar{n}\pi^-$ and $n\bar{p}\pi^+$ should have branching fractions close to twice $B(J/\psi \to p\bar{p}\pi^0)$, reflecting an isospin factor of two.
Experimentally this is indeed the case; the ratio of the PDG Experimentally this is indeed the case; the ratio of the PDG J/ψ branching fractions to $p\bar{n}\pi^-$ and $p\bar{p}\pi^0$ is $B(J/\psi \rightarrow p\bar{n}\pi^-)/B(J/\psi \rightarrow p\bar{n}\pi^0) = 1.94 \pm 0.18$ $p\bar{n}\pi^{-}$)/ $B(J/\psi \to p\bar{p}\pi^{0}) = 1.94 \pm 0.18.$

B. Projected $J/\psi \rightarrow p\bar{p}\pi^0$ event densities

Projections of DP event densities onto the invariant mass of one pair of particles are useful in searches for intermediate resonances. For $J/\psi \rightarrow p\bar{p}\pi^0$, the BES
Collaboration has published acceptance-corrected event Collaboration has published acceptance-corrected event

FIG. 3. $J/\psi \rightarrow p\bar{p}\pi^0$ experimental (BES) and theoretical (meson emission model Fig. 2) Dalitz plot (DP) event densities (meson emission model, Fig. [2](#page-3-0)) Dalitz plot (DP) event densities, projected onto $M_{p\pi}$. This is not a fit; see text for discussion.

densities in $M_{p\pi}$ and $M_{\bar{p}\pi}$ invariant mass (Fig. 6 of Ref. [\[1](#page-9-0)]), which show clear evidence for N^* resonances. Here we will generate the corresponding theoretical $M_{p\pi}$ -projected event distributions in the meson emission model for comparison with experiment. Although N^* resonances are not incorporated in our calculation, this comparison will test the relative importance of the meson emission decay mechanism in this decay, and establish whether the model predicts a non- N^* "background" invariant mass distribution that is similar to the data in form and magnitude.

The full two-dimensional DP event density $d^2\Gamma/dxdy$ predicted by the meson emission model is given by Eq. ([7\)](#page-1-0). Projecting this onto $M_{p\pi}$ is straightforward; first one integrates over $y = M_{\tilde{p}\pi}^2/m_p^2 - 1$ between the DP bounda-
rise y (x) of Eq. (2) which gives $d\Gamma/dx$ Converting this ries $y_{\pm}(x)$ of Eq. ([2](#page-1-1)), which gives $d\Gamma/dx$. Converting this into a distribution in $M_{p\pi}$ introduces a Jacobean, which is specified by the definition $x = M_{p\pi}^2/m_p^2 - 1$. This gives

$$
\frac{d\Gamma(J/\psi \to p\bar{p}\pi^0)}{dM_{p\pi}} = \frac{2M_{p\pi}}{m_p^2} \int_{y_-(x)}^{y_+(x)} dy
$$

$$
\times \frac{d^2\Gamma(J/\psi \to p\bar{p}\pi^0)}{dxdy}.
$$
 (27)

We have evaluated this distribution numerically, given the $d^2\Gamma(J/\psi \to p\bar{p}\pi^0)/dxdy$ of Eq. [\(7](#page-1-0)) and the masses and
couplings α_{max} and α_{max} used in Sec. III A. The result is couplings $\alpha_{J/\psi}$ and α_{π} used in Sec. III A. The result is shown in Fig. [3](#page-4-1), together with an experimental distribution provided by BES (reported in Ref. [[1\]](#page-9-0)), using a common scale. The data are the combined acceptance-corrected $M_{p\pi}$ and $M_{\bar{p}\pi}$ distribution, scaled to give their reported $B(J/\psi \to p\bar{p}\pi^0) = 1.33 \cdot 10^{-3}$ rather than the PDG value
of $1.09 \cdot 10^{-3}$ of $1.09 \cdot 10^{-3}$.
Clearly then

Clearly there is a close resemblance between the meson emission model prediction for the $J/\psi \rightarrow p\bar{p}\pi^0$ event
distribution in M and the observed BES distribution distribution in $M_{p\pi}$ and the observed BES distribution, both in form and magnitude. This suggests that a study of this reaction assuming this model for the experimental background combined with N^* resonance contributions would be an interesting exercise. Although BES [\[1](#page-9-0)] recently reported a similar study, they introduced an ad hoc $s_{\pi N} (= M_{p\pi}^2)$ -dependent form factor that suppressed this
background meson emission amplitude relative to N^* conbackground meson emission amplitude relative to N^* contributions. The similarity to experiment evident in Fig. [3](#page-4-1) suggests that this mechanism merits additional consideration.

C. Other $\Psi \to p\bar{p}\pi^0$ partial widths

Since the two meson emission model parameters $g_{\Psi p\bar{p}}$ and g_{NNm} are both known for several $\Psi \to p\bar{p}\pi^0$ decays,
we are able to give absolute predictions for these partial we are able to give absolute predictions for these partial widths. (We previously used $\Gamma(\Psi \to p\bar{p})$ to estimate $g_{\Psi p\bar{p}}$
[22] there we again use these values, and set $g = 13$) [\[22\]](#page-9-15); here we again use these values, and set $g_{NN\pi} = 13.$) These $\Psi \to p\bar{p}\pi^0$ partial widths are given in Table [I](#page-5-0),

TABLE I. Comparison of theory (meson emission model) and experiment for $\Gamma(\Psi \to p \bar{p} \pi^0)$ (see text).

Ψ	$10^3 g_{\Psi p\bar{p}}$		$10^3 \langle \rho \rangle$ A_D [GeV ⁴]	Γ^{thy} $\n p\bar{\nabla}\pi^0$	Γ^{expt} $\n p\bar{\nabla}\pi^0$
η_c	19.0	0.530	6.862	1.7 keV	
J/ψ	1.62	3.070	9.265	97 eV	102 ± 9 eV
χ_0	5.42	3.691	18.605	2.6 keV	$6.0 \pm 1.3 \text{ keV}$
χ_1	1.03	0.554	22.351	17 eV	103 ± 43 eV
ψ'	0.97	2.010	30.501	75 eV	41 ± 5 eV

together with some intermediate theoretical quantities and the experimental widths. (These experimental values are the PDG total widths times branching fractions, with errors added in quadrature.)

These rates were derived using Eq. [\(22\)](#page-3-3), with the appropriate density function ρ chosen from the set Eqs. (6)–([9\)](#page-2-0). In addition to the rates, Table [I](#page-5-0) also gives the coupling constants assumed, the average of the density function ρ over the Dalitz plot, and the DP area A_D in physical units.

It is clear from the table that the wide variation in the absolute scale of partial widths observed experimentally is approximately reproduced by the model, at least at a ''factor-of-two'' level of accuracy. This suggests that the meson emission decay mechanism may indeed be an important component of the decay amplitude in all these decays; a more definitive test would involve a direct comparison of the DP event densities or their two-body projections, as in Fig. [3.](#page-4-1)

The χ_{c1} case appears to be an exception to this approximate agreement, however in view of the large experimental error it is not clear if this is a real discrepancy; theory and experiment only differ by 2σ .

Although the single experimentally unobserved decay $\eta_c \rightarrow p\bar{p}\pi^0$ is predicted by the meson emission model to have a relatively large partial width of 1.7 keV it is actually have a relatively large partial width of 1.7 keV, it is actually considerably suppressed by the presence of an on-diagonal node in the DP event distribution. An experimental study of $\eta_c \rightarrow p\bar{p}\pi^0$ would accordingly be very interesting,
since the contributions of other decay mechanisms may since the contributions of other decay mechanisms may be easier to identify in the region of the DP where the meson emission model gives a zero or weak contribution.

D. $g_{NN\pi}$ from $B(J/\psi \to p\bar{p}\pi^0)/B(J/\psi \to p\bar{p})$

Previously we noted that $\Psi \to p\bar{p}m$ decays can be used
estimate N/m couplings, provided that the contribution to estimate NNm couplings, provided that the contribution of the meson emission decay mechanism to the decay amplitude can be quantified experimentally. In the following we will use the decay $J/\psi \rightarrow p\bar{p}\pi^0$ as an illustration
of this approach, since the agreement between the experiof this approach, since the agreement between the experimental and theoretical partial widths suggests that domination of this decay by meson emission is a reasonable first approximation.

Since the *a priori* unknown coupling $\alpha_{J/\psi}$ cancels in the theoretical branching fraction ratio $B(J/\psi \rightarrow$

 $p\bar{p}\pi^{0}/B(J/\psi \to p\bar{p})$, we can use it to estimate $g_{NN\pi}$
directly. The meson emission model decay width for directly. The meson emission model decay width for $\Gamma(J/\psi \to p\bar{p}\pi^0)$ [\(7\)](#page-1-0) and the two-body decay width [\(25\)](#page-4-2) imply the following relation between this ratio and the imply the following relation between this ratio and the coupling $\alpha_{\pi} \equiv g_{NN\pi}^2/4\pi$:

$$
\alpha_{\pi} = (1 - 4/R^2)^{1/2} \frac{(R + 2/R)}{3\langle \rho \rangle A_D / m_p^4} \cdot \frac{B(J/\psi \to p\bar{p}\pi^0)}{B(J/\psi \to p\bar{p})}.
$$
\n(28)

Substitution of the experimental PDG numbers $B(J/\psi \rightarrow$ $p\bar{p}\pi^0$ = $(1.09 \pm 0.09) \cdot 10^{-3}$ and $B(J/\psi \rightarrow p\bar{p})$
(2.17 + 0.07) · 10⁻³ for these branching fractions le $(2.17 \pm 0.07) \cdot 10^{-3}$ for these branching fractions leads
to the estimate to the estimate

$$
g_{NN\pi}|_{J/\psi \to p\bar{p}\pi^0} = 13.3 \pm 0.6,
$$
 (29)

which is consistent with NN meson exchange model values.

We expect to find approximately equal $g_{NN_{\pi}}$ estimates from other $\Psi \to p\bar{p}$ and $p\bar{p}m$ decay pairs if the meson
emission decay mechanism $\Psi \to p\bar{p} \to p\bar{p}\pi^0$ is indeed emission decay mechanism $\Psi \to p\bar{p} \to p\bar{p}\pi^0$ is indeed
dominant. A second state Ψ that can be use to estimate dominant. A second state Ψ that can be use to estimate $g_{NN\pi}$ is the $\psi'(3686)$. Since the ψ' has the same quantum
numbers as the I/ψ . Eq. (28) is again appropriate for our numbers as the J/ψ , Eq. [\(28\)](#page-5-1) is again appropriate for our coupling constant estimate. This ψ' case has a much larger $p\bar{p}\pi^0$ DP area A_D than the J/ψ , which is partially compensated by a smaller mean event density $\langle \rho \rangle$. (These quantities are given in Table [I](#page-5-0).) Using $M = 3.6861$ GeV and the PDG branching fractions $B(\psi' \rightarrow p\bar{p}\pi^{0})$
(1.33 + 0.17) · 10⁻⁴ and $B(\psi' \rightarrow p\bar{p}) = (2.75 \pm 0.1)$ $(1.33 \pm 0.17) \cdot 10^{-4}$ and $B(\psi' \to p\bar{p}) = (2.75 \pm 0.12) \cdot 10^{-4}$ we find the *W*-based *g_{NM}* estimate 10^{-4} , we find the ψ' -based $g_{NN\pi}$ estimate

$$
g_{NN\pi}|_{\psi'\to p\bar{p}\pi^0} = 9.9 \pm 0.7. \tag{30}
$$

This is similar to but somewhat smaller than the estimate obtained above from J/ψ decays [\(29\)](#page-5-2), and may give an indication of the accuracy of this approach for estimating NNm coupling constants.

Of course the other relations for α_m analogous to [\(28\)](#page-5-1) will only be useful if the meson emission decay mechanism is dominant in those decays as well. Otherwise the contribution of this mechanism to the decay must be identified and quantified, for example, through a detailed study of the DP event density.

E. Scalar mesons in $\Psi \to N\bar{N}m$

The long-standing interest in the light scalars makes the possibility of studying them using these decays an attractive prospect. This motivated our inclusion of decay formulas for the processes $\Psi \to p\bar{p}f_0$ in our set of theoretical
DP event densities DP event densities.

Here we will give meson emission model predictions for the branching fractions $B(\Psi \to p\bar{p}f_0)$, where $\Psi = \eta_c$,
 $I/d\psi \times \rho \times \chi$, and $d\psi'$ for a light "sigma" meson with J/ψ , χ_{c0} , χ_{c1} , and ψ' , for a light "sigma" meson with $m_{f_0} = 0.5$ GeV. To evaluate these partial widths we proceed as in Sec. III C, and integrate the appropriate decay width formula from the set Eqs. (10) – (13) (13) (13) over the Dalitz plot. We again use the $\Psi p\bar{p}$ coupling constants of Sec. [I](#page-5-0)II C, as given in Table I. The total widths used to convert the calculated partial widths to branching fractions are the current PDG values, $\Gamma(\eta_c) = 27.4 \text{ MeV}$, $\Gamma(J/\psi) = 93.2 \text{ keV}, \quad \Gamma(\chi_{c0}) = 10.4 \text{ MeV}, \quad \Gamma(\chi_{c0}) = 0.86 \text{ MeV}$ and $\Gamma(\psi) = 309 \text{ keV}$ Our results are $\Gamma(\gamma/\psi) = 93.2 \text{ keV}, \quad \Gamma(\chi_{c0}) = 10.4 \text{ MeV}, \quad \Gamma(\chi_{c1}) = 0.86 \text{ MeV}, \text{ and } \Gamma(\psi') = 309 \text{ keV}.$ Our results are given in Table II Since there is no general agreement regarding in Table [II](#page-6-0). Since there is no general agreement regarding an $NNf_0(500)$ coupling constant, in the table we first give the predicted branching fraction relative to $B(J/\psi \rightarrow$ $p\bar{p}f_0(500) \equiv B_0$, which is numerically $0.338 \cdot 10^{-4}$ $g_{NNf_0}^2$. (The unknown g_{NNf_0} cancels in these ratios.) The next column gives absolute $B(\Psi \to p\bar{p}f_0(500))$ branching
fractions for a rather arbitrarily chosen $g_{\mu\nu\kappa} = 10$ fractions for a rather arbitrarily chosen $g_{NNf_0} = 10$. Finally, the table quotes experimental branching fractions for the related processes $\Psi \to p\bar{p}\pi^+\pi^-$ for comparison.
The relative theoretical branching fractions (Table II)

The relative theoretical branching fractions (Table [II](#page-6-0), column 2) suggest that the best channel for identifying a light scalar in $\Psi \to p\bar{p}f_0$ is $J/\psi \to p\bar{p}f_0$ itself (assuming that the meson emission model is a reasonable quide) that the meson emission model is a reasonable guide). Given a somewhat larger event sample, $\psi' \to p \bar{p} f_0$ should
be competitive with I/ψ and has the advantage of more be competitive with J/ψ , and has the advantage of more phase space, so the scalars near 1 GeV and the $f_0(1500)$ could also be investigated. $\eta_c \rightarrow p\bar{p}f_0$ has a comparable
theoretical branching fraction, but the difficulty of productheoretical branching fraction, but the difficulty of producing the η_c makes this a less attractive channel. Finally, the χ_{cJ} states are predicted to have much smaller $p\bar{p}f_0$ branching fractions than $J/\psi \rightarrow p\bar{p}f_0$, and accordingly
are less attractive experimentally if this decay model is are less attractive experimentally if this decay model is accurate.

We have also estimated the effect of an $f_0(500)$ width on these results. Imposing a Breit-Wigner f_0 mass profile with $\Gamma_{f_0} = 0.5$ GeV, truncated at $2m_{\pi}$, decreases all the absolute theoretical $\Psi \to p\bar{p}f_0(500)$ branching fractions in
Table II (column 3) by $\approx 10\%$ The *relative* theoretical Table [II](#page-6-0) (column 3) by $\approx 10\%$. The *relative* theoretical branching fractions (column 2) are even less sensitive to the $f_0(500)$ width, and become 0.41, 0.048, 0.017, and 0.21.

A light scalar f_0 meson would presumably decay strongly and perhaps dominantly to $\pi\pi$, so decays of the type $\Psi \to p\bar{p}\pi\pi$ are of special interest, notably $J/\psi \to p\bar{p}\pi\pi$ (in view of our large theoretical $R(J/\psi \to p\bar{p}\pi\pi)$ $p\bar{p}\pi\pi$ (in view of our large theoretical $B(J/\psi \rightarrow$

TABLE II. Theoretical (meson emission model) branching fractions for light scalar meson production. The numerical columns are (1) the ratio $B(\Psi \to p\bar{p}f_0(500))/B(J/\psi \to$
 $p\bar{p}f_0(500))$: (2) $10^3 \cdot B(\Psi \to p\bar{p}f_0(500))$ for $g_{\text{avg}} = 10$; $p\bar{p}f_0(500)$; (2) $10^3 \cdot B(\Psi \to p\bar{p}f_0(500))$ for $g_{NNf_0} = 10$;
(3) $10^3 \cdot B^{\exp(j\Psi)} \to p\bar{p}\pi^+\pi^-$ for comparison with (2) (See (3) $10^3 \tcdot B^{\text{expt}}(\Psi \to p\bar{p}\pi^+\pi^-)$, for comparison with (2). (See text.)

Ψ	$B^\mathrm{thy}_{\underline{\bar{\rho}}\bar{\bar{p}}f_0}$, $/B_0$	$10^3 B_{p\bar{p}f_0}^{\text{thy}(g=10)}$	$10^3 B_{p\bar{p}\pi^+\pi^-}^{\rm expt}$
η_c	0.40	1.4	$<$ 12 (90% C.L.)
J/ψ	\equiv 1	3.4	6.0 ± 0.5
χ_0	0.045	0.15	2.1 ± 0.7
χ_1	0.016	0.054	0.50 ± 0.19
ψ'	0.21	0.72	0.60 ± 0.04

 $p\bar{p}f_0(500))$. The charged case $J/\psi \rightarrow p\bar{p}\pi^+\pi^-$ has
been studied by Mark-I [25] DESY [28] and Mark-II been studied by Mark-I [\[25\]](#page-10-2), DESY [\[28\]](#page-10-5), and Mark-II [\[27\]](#page-10-4). Although no light scalar mesons have yet been identified in this decay, it is suggestive that $J/\psi \rightarrow p\bar{p}\pi^+\pi^-$
is the largest known exclusive $J/\psi \rightarrow p\bar{p}X$ mode, with a is the largest known exclusive $J/\psi \rightarrow p\bar{p}X$ mode, with a
branching fraction of $B(J/\psi \rightarrow p\bar{p}\pi^{+}\pi^{-}) = (6.0 +$ branching fraction of $B(J/\psi \to p\bar{p}\pi^+\pi^-) = (6.0 \pm 0.5) \cdot 10^{-3}$ $(0.5) \cdot 10^{-3}$.
In additi

In addition to the $p\bar{p}f_0$ intermediate state of interest here, this decay may also receive important contributions from NN^* , N^*N^* , and $\Delta\Delta$, as well as other two-baryon and NNm states; this may complicate the comparison with experiment considerably. Reference [\[27\]](#page-10-4), which has the largest event sample, finds a large but not dominant $\Delta\Delta$ contribution, $B(J/\psi \to \Delta^{++} \bar{\Delta}^{--}) = (1.10 \pm 0.29) \cdot 10^{-3}$, and gives a rather tight upper limit of $\approx 5\%$ on the contributing subprocess $I/\psi \to p\bar{p}q^0$. $B(I/\psi \to p\bar{p}q^0)$ $(\Delta^{-1}) = (1.10 \pm 0.29) \cdot$
r limit of \approx 5% on the contributing subprocess $J/\psi \rightarrow p\bar{p}\rho^0$; $B(J/\psi \rightarrow p\bar{p}\rho^0) < 3.1 \cdot 10^{-4}$ (90% CT) As Ref [27] shows in $p\bar{p}\rho^0$ < 3.1 · 10⁻⁴ (90% C.L.). As Ref. [\[27\]](#page-10-4) shows in their Fig. 31, that the $\pi^+\pi^-$ invariant mass distribution their Fig. 31, that the $\pi^+\pi^-$ invariant mass distribution from non- $\Delta\Delta$ events is a broad sigmalike distribution, there may well be a large $J/\psi \rightarrow p\bar{p}f_0(\sim 500) \rightarrow$
 $p\bar{p}\pi^+\pi^-$ contribution to this decay with a branching there may well be a large $J/\psi \rightarrow ppj_0(\infty,00) \rightarrow$
 $p\bar{p}\pi^+\pi^-$ contribution to this decay, with a branching fraction comparable to the theoretical $3.4 \cdot 10^{-3}$ predicted
for $g_{\text{true}} = 10$ (see Table II). It will be very interesting to for $g_{NNf_0} = 10$ (see Table [II](#page-6-0)). It will be very interesting to investigate this possible light f_0 contribution in a future experimental study, as well as to search for the $f_0(980)$ and $a₀(980)$ scalar states and the scalar glueball candidate $f₀(1500)$ in (higher-mass) charmonium decays, notably of the ψ' .

F. Decays to $NN\omega$ and NNV

Charmonium decays to $N\bar{N}\omega$ are especially interesting, since the ω plays a crucial role in meson exchange models of the NN force, as the dominant origin of the short-ranged "hard core repulsion," through *t*-channel ω exchange. Conceptual problems with this ω -exchange mechanism include (1) the very small NN separation implied by this mechanism $(R_{NN} \approx 1/m_V \approx 0.3$ fm), at which quarkgluon dynamics may be a more appropriate description of the interaction; and (2) the prediction of a corresponding short-ranged $N\bar{N}$ attraction and deeply bound $N\bar{N}$ states, which are not observed. (See, for example, Refs. [[29](#page-10-6)–[31\]](#page-10-7), and references cited therein.)

There are two strong coupling constants in the $NN\omega$ vertex, as summarized by Eq. ([5](#page-1-2)), the overall strength $g_{NN\omega}$ of the Dirac (γ_{μ}) coupling, and the relative strong magnetic Pauli coupling $\kappa_{NN\omega}$ (here abbreviated g_{ω} and κ_{ω} , with $\alpha_{\omega} = g_{\omega}^2/4\pi$). Unfortunately the NNV couplings in the meson exchange models are not *a priori* well estabthe meson exchange models are not a priori well established experimentally, and are therefore usually fitted directly to NN scattering data. These NN scattering studies are thus fits to the data rather than predictions that test the assumed vector-meson-exchange scattering mechanism. These NN fits typically find $g_{\omega} \approx 10-15$ for the Dirac $NN\omega$ coupling, whereas the $NN\omega$ Pauli coupling has remained poorly determined; examples of $NN\omega$ parameter sets from the NN scattering literature include $(g_{\omega}, \kappa_{\omega})$ = $(12.2, -0.12)$ (Paris), $(12.5, +0.66)$ (Nijmegen), and $(15.9, 0)$ (CD-Bonn) (these are cited in Ref. [321) (15.9, 0) (CD-Bonn) (these are cited in Ref. [\[32](#page-10-8)]). Independent estimates of the $NN\omega$ coupling from experiment have been reported by Sato and Lee [[33](#page-10-9)] (from pion photoproduction) and by Mergell, Meissner, and Drechsel [\[34\]](#page-10-10) (from nucleon electromagnetic form factors). Sato *et al.* assumed $\kappa_{\omega} = 0$, and quoted the range $g_{\omega} =$ 7–10:5 for experimentally favored values of the Dirac coupling. Mergell *et al.* found a small κ_{ω} but a much larger Dirac coupling, $(g_{\omega}, \kappa_{\omega}) = (20.86 \pm 0.25, -0.16 \pm 0.01)$.
Theoretical calculations include a OCD sum rule result of Theoretical calculations include a QCD sum rule result of Zhu [\[35\]](#page-10-11), who finds $(g_{\omega}, \kappa_{\omega}) = (18 \pm 8, 0.8 \pm 0.4)$, and a recent ${}^{3}P_{0}$ quark model calculation of *NNm* couplings [\[32\]](#page-10-8) which found the analytic result $\kappa_{\omega} = -3/2$.
Charmonium decays to $n\bar{p}\omega$ final state

Charmonium decays to $p\bar{p}\omega$ final states (and $N\bar{N}V$ more generally) may allow independent estimates of these $NN\omega$ and NNV couplings, again provided that they are dominated by the meson emission decay mechanism, or at least that this contribution to the decay amplitude can be isolated and quantified. In the following discussion we will consider the decay $J/\psi \rightarrow p\bar{p}\omega$ as an example.
Results from experimental studies of the decay

Results from experimental studies of the decay $J/\psi \rightarrow$ $p\bar{p}\omega$ have been published by Mark-I [[25](#page-10-2)], Mark-II [\[27\]](#page-10-4), and most recently BES-II [[5\]](#page-9-1). The PDG value of the $J/\psi \rightarrow p\bar{p}\omega$ branching fraction, estimated from these
results is $B(J/\psi \rightarrow p\bar{p}\omega) = (1.10 \pm 0.15) \cdot 10^{-3}$ which results, is $B(J/\psi \to p\bar{p}\omega) = (1.10 \pm 0.15) \cdot 10^{-3}$, which
combined with the PDG J/ψ total width gives an expericombined with the PDG J/ψ total width gives an experimental partial width of

$$
\Gamma(J/\psi \to p\bar{p}\omega) = 103 \pm 14 \text{ eV}.
$$
 (31)

The fact that this is approximately equal to the $p\bar{p}\pi^0$ partial width [\(19\)](#page-3-2) despite the much smaller phase space suggests a robust $NN\omega$ coupling.

On evaluating this theoretical decay rate by integrating Eq. ([15](#page-2-2)) numerically with physical masses, both $NN\omega$ couplings free, and (as used previously) $g_{J/\psi p\bar{p}} = 1.62 \cdot 10^{-3}$ we find 10^{-3} , we find

$$
\Gamma(J/\psi \to p\bar{p}\omega) = \alpha_{\omega} \cdot (2.468 - 1.101\kappa_{\omega} + 0.886\kappa_{\omega}^{2}) \text{ eV}.
$$
\n(32)

The single number $\Gamma(J/\psi \to p\bar{p}\omega)$ alone does not suf-
e to determine the $NN\omega$ strong couplings since there are fice to determine the $NN\omega$ strong couplings since there are two free parameters, g_{ω} and κ_{ω} . If we set $\kappa_{\omega} = 0$, following CD-Bonn [[15](#page-9-16)] and the Sato-Lee photoproduction study [\[33\]](#page-10-9), the measured partial width ([31](#page-7-0)) and the theoretical decay rate ([32\)](#page-7-1) imply $g_{\omega} = 23 \pm 3$ (provided that meson emission dominates this decay). This g_{ω} is rather larger than these references prefer for g_{ω} ; it is consistent however with the electromagnetic form factor fit of Mergell et al. [\[34\]](#page-10-10) and the range 18 ± 8 reported by Zhu [[35](#page-10-11)] from QCD sum rules. If we instead assume the ${}^{3}P_{0}$ quark model value $\kappa_{\omega} = -3/2$ for the Pauli term [\[32\]](#page-10-8), we find $g_{\omega} = 14.6 \pm 2.0$ which is consistent with the values quoted by NN 2:0, which is consistent with the values quoted by NN scattering studies, and is somewhat larger than the photoproduction value.

It is possible to estimate the two parameters g_{ω} and κ_{ω} independently through a more detailed comparison between $J/\psi \rightarrow p\bar{p}\omega$ data and the theoretical DP event
density Eq. (15) This theoretical event density is strongly density, Eq. [\(15\)](#page-2-2). This theoretical event density is strongly dependent on the Pauli coefficient κ_{ω} ; with $\kappa_{\omega} = 0$ the event density at lower $M_{p\bar{p}}$ is strongly suppressed (see Fig. [4\)](#page-7-2).

In contrast, for moderately large $|\kappa_{\omega}|$, such as the quark model value $\kappa_{\omega} = -3/2$, the theoretical DP event density
is closer to uniform. This is illustrated in Fig. 5, which is closer to uniform. This is illustrated in Fig. [5,](#page-8-0) which shows this event density along the diagonal $M_{p\omega}^2 = M_{\bar{p}\omega}^2$
(relative to the maximum value on diagonal) for various (relative to the maximum value on diagonal) for various $\kappa_{\omega} \leq 0$. If the meson emission model does give a good description of this decay, evidently it may be possible to determine κ_{ω} by comparing the $J/\psi \rightarrow p\bar{p}\omega$ DP event density on diagonal to the prediction in Fig. 5 density on diagonal to the prediction in Fig. [5.](#page-8-0)

G. Other $\Psi \to p\bar{p}V$ decays

Other decays to $p\bar{p}V$ final states that are closely related to $J/\psi \to p\bar{p}\omega$ include $J/\psi \to p\bar{p}\rho^0$ and $J/\psi \to p\bar{p}\phi$.
In the meson emission model these are both described by In the meson emission model these are both described by the decay formula ([15](#page-2-2)), albeit with different vector meson masses and NNV couplings. Given the rounded PDG masses $m_{\rho^0} = 0.7755$ GeV and $m_{\phi} = 1.0195$ GeV, we predict the numerical decay widths

$$
\Gamma(J/\psi \to p\bar{p}\rho^0) = \alpha_\rho \cdot (2.614 - 1.155\kappa_\rho + 0.930\kappa_\rho^2) \text{ eV}
$$
\n(33)

FIG. 4. Theoretical relative $J/\psi \rightarrow p\bar{p}\omega$ DP event density for $\epsilon = 0$ from Eq. (15). Note the suppression near $p\bar{p}$ threshold $\kappa_{\omega} = 0$, from Eq. ([15](#page-2-2)). Note the suppression near $p\bar{p}$ threshold (upper right) (upper right).

FIG. 5. Theoretical $J/\psi \rightarrow p\bar{p}\omega$ DP event density along the diagonal $M^2 = M^2$ showing the strong κ dependence diagonal $M_{p\omega}^2 = M_{\bar{p}\omega}^2$, showing the strong κ_{ω} dependence.

and

$$
\Gamma(J/\psi \to p\bar{p}\phi) = \alpha_{\phi} \cdot (0.184 - 0.109\kappa_{\phi} + 0.087\kappa_{\phi}^{2}) \text{ eV}.
$$
\n(34)

The ρ^0 case is especially interesting due to the range of values reported for κ_{ρ} , as discussed by Brown and Machleidt [\[36\]](#page-10-12). Although vector dominance predicts κ_0 = 3.7 "weak ρ ," and some data have been interpreted as supporting this, fits to $\pi \pi \rightarrow N\bar{N}$ and S-D mixing in NN
scattering prefer a larger value "strong a": the Bonn scattering prefer a larger value "strong ρ "; the Bonn potential model, for example, uses $(\alpha_{\rho}, \kappa_{\rho}) = (0.84, 6.1)$ [\[14\]](#page-9-13). A QCD sum rule calculation by Zhu [\[37\]](#page-10-13) finds $(g_{\rho}, \kappa_{\rho}) = (2.5 \pm 0.2, 8.0 \pm 2.0)$, comparable to the fitted Bonn values. In contrast, the valence quark model with a ³P₀ NN ρ coupling predicts a much smaller $\kappa_{\rho} = -\kappa_{\omega} = +3/2$ [32] $+3/2$ [[32](#page-10-8)].

Using Eq. ([33](#page-7-3)) we can give meson emission model predictions for $\Gamma(J/\psi \to p\bar{p}\rho^0)$ that follow from these
various ($\epsilon \times \rho$) parameters. The Bonn parameters cited various $(g_{\rho}, \kappa_{\rho})$ parameters. The Bonn parameters cited above give $\Gamma(J/\psi \to p\bar{p}\rho^0) = 25$ eV and $B(J/\psi \to p\bar{p}\rho^0) = 27 \cdot 10^{-4}$, this is essentially equal to the current $p\bar{p}\rho^0$ = 2.7 \cdot 10⁻⁴; this is essentially equal to the current
experimental upper limit [24.27] of 3.1 \cdot 10⁻⁴ (90% C L) experimental upper limit $[24,27]$ $[24,27]$ $[24,27]$ $[24,27]$ $[24,27]$ of $3.1 \cdot 10^{-4}$ (90% C.L.),
which is a Mark-II result dating from 1984. The Zhu OCD which is a Mark-II result dating from 1984. The Zhu QCD sum rule central values for $(g_{\rho}, \kappa_{\rho})$ give essentially identical results. In contrast the valence quark model with ${}^{3}P_{0}$ couplings $g_{\rho} = g_{\omega}/3$ and $\kappa_{\rho} = +3/2$ (and using $g_{\rho} =$ 14.6 from the $J/\psi \rightarrow p\bar{p}\omega$ discussion above) gives a
much lower $\Gamma (J/\psi \rightarrow p\bar{p}\omega^0) = 5.6 \text{ eV}$ and hence much lower $\Gamma(J/\psi \to p\bar{p}\rho^0) = 5.6$ eV and hence $R(J/\psi \to p\bar{p}\rho^0) = 6.0 \cdot 10^{-5}$ which is about a factor of $B(J/\psi \to p\bar{p}\rho^0) = 6.0 \cdot 10^{-5}$, which is about a factor of 5 below the current experimental limit. The proximity of 5 below the current experimental limit. The proximity of the Bonn and QCD sum rule parameter predictions for $B(J/\psi \to p\bar{p}\rho^0)$ to the current limit suggests that an experimental study with significantly improved sensitivity experimental study with significantly improved sensitivity could make a useful contribution to establishing $NN\rho$ couplings.

The decay $J/\psi \rightarrow p\bar{p}\phi$ in contrast has been observed,
d has an experimental (PDG) branching fraction of and has an experimental (PDG) branching fraction of $B^{\text{expt}}(J/\psi \to p\bar{p}\phi)$
to $\Gamma^{\text{expt}}(J/\psi \to p\bar{p})$ $\overline{p}\phi$) = $(4.5 \pm 1.5) \cdot 10^{-5}$, corresponding
 $\overline{p} \cdot \overline{p} \cdot \overline{p}$ = $4.2 + 1.4$ eV Unfortunately in to $\Gamma^{\text{expt}}(J/\psi \to p\bar{p}\phi)$
this case we do not h $\bar{p}\phi$) = 4.2 ± 1.4 eV. Unfortunately in
t have a theoretical estimate for either this case we do not have a theoretical estimate for either α_{ϕ} or κ_{ϕ} , since the (valence level, leading-order) ³P₀ model predicts no $NN\phi$ coupling. Clearly it would be very interesting to obtain experimental values for these couplings, since little is known about the properties of Zweig-suppressed vertices. Again, if the meson emission model gives a good description of this decay, a comparison of Eq. ([15](#page-2-2)) to the experimental $J/\psi \rightarrow p\bar{p}\phi$ DP event
distribution should allow an experimental determination of distribution should allow an experimental determination of the $NN\phi$ couplings.

Finally, we note in passing that ψ' decays to NNV are apparently not in agreement with the meson emission model; proceeding as above, we would predict a branching fraction of $B(\psi' \to p\bar{p}\omega) = 9.4 \cdot 10^{-4}$, whereas the PDG experimental value is an order of magnitude smaller experimental value is an order of magnitude smaller, $B^{\text{expt}}(\psi' \to p\bar{p}\omega)$
tions for this disc $\bar{p}\omega$) = $(6.9 \pm 2.1) \cdot 10^{-5}$. Possible explana-
discrepancy including form factors and (detions for this discrepancy, including form factors and (destructive interference with) additional decay mechanisms, are discussed in the next section. Since the total $\psi' \rightarrow$ $p\bar{p}\omega$ data sample reported by CLEO [[11](#page-9-9)] and BES [\[12\]](#page-9-11) comprises only about 35 events, it is not yet possible to establish the reason for this large discrepancy between experiment and the meson emission model. This would ideally involve a comparison between the predicted and observed Dalitz plot event densities. Hopefully this comparison will be possible using the large ψ' data set being accumulated at BES-III.

IV. SUMMARY, CONCLUSIONS, AND FUTURE DEVELOPMENTS

In this paper we have presented and developed a hadronlevel ''meson emission model'' of charmonium decays of the type $\Psi \to N\bar{N}m$, where Ψ is a generic charmonium
resonance N is a nucleon and m is a light meson. The resonance, N is a nucleon, and m is a light meson. The model assumes that the decays take place through meson emission from the nucleon or antinucleon line, as a hadronic "final state radiation" correction to a $\Psi \to N\bar{N}$
transition. As the model is relatively simple, we are able transition. As the model is relatively simple, we are able to evaluate the predicted DP event densities for many experimentally accessible and measured processes; in particular, we give event densities for $\Psi = \eta_c$, J/ψ (and ψ'), χ_{c0} , χ_{c1} , and ψ' ; and $m = \pi^0$, f_0 , and ω ; and implicitly all cases with the same J^{PC} quantum numbers. ticular, we give event densities for $\Psi = \eta_c$, J/ψ (and ψ'),

We used the reaction $J/\psi \rightarrow p\bar{p}\pi^0$ as a test case with
free parameters (the $J/\psi p\bar{p}$ and $NN\pi$ couplings are no free parameters (the $J/\psi p\bar{p}$ and $NN\pi$ couplings are known), and compared the meson emission model predictions for the projected event density in $M_{p\pi}$ and the partial width $\Gamma(J/\psi \to p\bar{p}\pi^0)$ to experiment; the partial width is
in good agreement, and the M event density appears to in good agreement, and the $M_{p\pi}$ event density appears to describe the non- N^* background contribution to this reaction observed experimentally. We also give predictions for

 $\Gamma(\Psi \to p\bar{p}\pi^0)$ for all Ψ cases considered here; the overall
trend of large and small widths, and their approximate trend of large and small widths, and their approximate scale is reproduced by the model.

We also considered scalar and vector meson production. We estimated $\Psi \to p\bar{p}f_0$ branching fractions for a light $f_0(500)$ and noted that the $I/\nu l$ and $\nu l'$ are favored for f_0 (500), and noted that the J/ψ and ψ' are favored for these studies, and the ψ' is favored for glueball searches. In vector production we considered $J/\psi \rightarrow p\bar{p}\omega$ in particu-
lar, and noted that a high-statistics study of this reaction lar, and noted that a high-statistics study of this reaction could be used to estimate the $NN\omega$ couplings (g_{ω} and κ_{ω}), which play a crucial role in meson exchange models of the NN force. We showed that the $J/\psi \rightarrow p\bar{p}\omega$ Dalitz plot event density is rather sensitive to the poorly known $NN\omega$ event density is rather sensitive to the poorly known $NN\omega$ Pauli coupling κ_{ω} . Determination of meson-nucleon strong couplings is a potentially very interesting application of $\Psi \rightarrow p\bar{p}m$ decays.
There are several

There are several theoretical developments that will be very important for future applications of this model. One should incorporate N^* resonances; this is conceptually straightforward but may be technically complicated, as it will introduce many new and poorly known resonance coupling parameters and phases. This development is of course crucial to describe the data in reactions such as $J/\psi \rightarrow p\bar{p}\pi^0$, which clearly shows N^* resonance peaks
(Fig. 3). Another important development is the substitution (Fig. [3](#page-4-1)). Another important development is the substitution of plausible $\Psi N^{(*)}\bar{N}^{(*)}$ and $N^{(*)}Nm$ hadron vertex form factors for the assumed coupling constants; the difficulty here is that hadronic form factors are poorly known, and models such as ${}^{3}P_0$ that predict form factors have not been adequately developed and tested. Another interesting generalization of the strong vertices assumed here would be to include a $J/\psi p\bar{p}$ Pauli coupling; as we noted previously [\[23\]](#page-10-0), this can explain the observed $e^+e^- \rightarrow J/\psi \rightarrow p\bar{p}$
angular distribution. Finally, one should include other sigangular distribution. Finally, one should include other significant decay mechanisms, as they become apparent through high-statistics studies of experimental Dalitz plots. These additional mechanisms might include intermediate meson resonances m' that couple strongly to $N\bar{N}$, as in $\Psi \to m'm \to p\bar{p}m$; if the m' resonances lie in the physical
region, they will give rise to characteristic m' resonance region, they will give rise to characteristic m' resonance bands in $M_{p\bar{p}}$ that could be identified and incorporated in a more complete decay model.

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- [1] M. Ablikim *et al.* (BES Collaboration), Phys. Rev. D **80**, 052004 (2009).
- [2] M. Ablikim et al. (BES Collaboration), Phys. Rev. D 71, 072006 (2005).
- [3] M. Ablikim et al. (BES Collaboration), Phys. Rev. D 74, 012004 (2006).
- [4] M. Ablikim et al. (BES Collaboration), arXiv:0902.3501.
- [5] M. Ablikim et al. (BES Collaboration), Eur. Phys. J. C 53, 15 (2008).
- [6] J.Z. Bai et al. (BES Collaboration), Phys. Rev. Lett. 91, 022001 (2003).
- [7] M. Ablikim et al. (BES Collaboration), Phys. Rev. Lett. 95, 262001 (2005).
- [8] Y. Huang, "Confirmation of proton-antiproton mass threshold enhancement structure and X(1835) at BES-III'' in Proceedings of HADRON 2009, Florida State University, Tallahassee (unpublished).
- [9] A. Lundborg, T. Barnes, and U. Wiedner, Phys. Rev. D 73, 096003 (2006).
- [10] M. Kotulla et al. (PANDA Collaboration), "Technical Progress Report for PANDA, Strong Interaction Studies with Antiprotons," 2005 (unpublished).
- [11] R. A. Briere et al. (CLEO Collaboration), Phys. Rev. Lett.

95, 062001 (2005).

- [12] J. Z. Bai et al. (BES Collaboration), Phys. Rev. D 67, 052002 (2003).
- [13] S. B. Athar et al. (CLEO Collaboration), Phys. Rev. D 75, 032002 (2007).
- [14] R. Machleidt, K. Holinde, and C. Elster, Phys. Rep. 149, 1 (1987).
- [15] R. Machleidt, Phys. Rev. C 63, 024001 (2001).
- [16] M. M. Nagels, T. A. Rijken, and J. J. de Swart, Phys. Rev. D 15, 2547 (1977).
- [17] M. M. Nagels, T. A. Rijken, and J. J. de Swart, Phys. Rev. D 20, 1633 (1979).
- [18] V. G. J. Stoks, R. A. M. Klomp, C. P. F. Terheggen, and J. J. de Swart, Phys. Rev. C 49, 2950 (1994).
- [19] W. N. Cottingham, M. Lacombe, B. Loiseau, J. M. Richard, and R. Vinh Mau, Phys. Rev. D 8, 800 (1973).
- [20] M. Lacombe, B. Loiseau, J. M. Richard, R. Vinh Mau, J. Conte, P. Pires, and R. de Tourreil, Phys. Rev. C 21, 861 (1980).
- [21] C. Downum and D. Phil, Oxford University, 2009 (unpublished).
- [22] T. Barnes and X. Li, Phys. Rev. D 75, 054018 (2007).

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- [23] T. Barnes, X. Li, and W. Roberts, Phys. Rev. D 77, 056001 (2008).
- [24] C. Amsler et al. (Particle Data Group), Phys. Lett. B 667, 1 (2008), and 2009 partial update for the 2010 edition.
- [25] I. Peruzzi et al., Phys. Rev. D 17, 2901 (1978).
- [26] R. Brandelik et al. (DASP Collaboration), Z. Phys. C 1, 233 (1979).
- [27] M. W. Eaton et al., Phys. Rev. D 29, 804 (1984).
- [28] H.J. Besch et al., Z. Phys. C 8, 1 (1981).
- [29] K. Maltman and N. Isgur, Phys. Rev. Lett. 50, 1827 (1983).
- [30] K. Maltman and N. Isgur, Phys. Rev. D **29**, 952 (1984).
- [31] T. Barnes, S. Capstick, M. D. Kovarik, and E. S. Swanson, Phys. Rev. C 48, 539 (1993).
- [32] C. Downum, T. Barnes, J.R. Stone, and E.S. Swanson, Phys. Lett. B 638, 455 (2006).
- [33] T. Sato and T. S. H. Lee, Phys. Rev. C 54, 2660 (1996).
- [34] P. Mergell, U.G. Meissner, and D. Drechsel, Nucl. Phys. A596, 367 (1996).
- [35] Shi-Lin Zhu, Phys. Rev. C **59**, 3455 (1999).
- [36] G.E. Brown and R. Machleidt, Phys. Rev. C 50, 1731 (1994).
- [37] Shi-Lin Zhu, Phys. Rev. C 59, 435 (1999).