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Photon distribution amplitudes in nonlocal chiral quark model

Piotr Kotko* and Michal Praszalowicz†

Marian Smoluchowski Institute of Physics, Jagiellonian University, Reymonta 4, 30-059 Kraków, Poland
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Photon distribution amplitudes up to twist four are calculated within the nonlocal chiral quark model with a simple pole ansatz for momentum dependence of the constituent quark mass. Calculations are performed using a modified electromagnetic vector current in order to satisfy Ward identities. Quark condensate and magnetic susceptibility of the QCD vacuum entering definitions of the distribution amplitudes are computed and compared with existing phenomenological estimates. Both real and off-shell photons are considered and relevant form factors are calculated. Our results are analytical up to the numerical solution of certain algebraic equations.

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I. INTRODUCTION

In the present paper we calculate photon distribution amplitudes (DA) within a low energy nonlocal model based on the instanton model of the QCD vacuum. There are seven different photon DAs corresponding to the Dirac structure probing the photon and to the light-cone twist (here we follow closely the definitions of Ref. [1]). However in reality only one of them, twist two tensor DA, can be accessed experimentally in hard exclusive processes [2–5] (for an experimental overview, see Ref. [6]). Higher twist amplitudes are suppressed in hard processes and vector twist two DA decouples for real photons. However, the interest in the remaining photon DAs is not purely academic. They are normalized through low energy observables such as quark condensate—$\langle\bar{\psi}\psi\rangle$, magnetic susceptibility—χ_m and mixed quark-gluon condensate—$f_{3\gamma}$ that are of importance for our understanding of the properties of the QCD vacuum. Only the calculation of all photon DAs provides a test of the whole approach and may prove its consistency.

In the present work, we employ a nonlocal generalization of the semibosonized Nambu Jona-Lasinio model with momentum dependent constituent quark mass $M(p)$ (which will be denoted as M_p) which follows from the instanton model of the QCD vacuum [7,8]. This model in the present version has been previously used to calculate pion [9–11] and kaon [12–14] distribution amplitudes, two pion DAs [15], pion-to-photon transition distribution amplitudes [16], and also twist two tensor photon DA [17] (see also [9]). However, a complete analysis of all seven photon DAs has been, to the best of our knowledge, conducted only in Ref. [18] in a model similar to ours with, however, results that in some respects are different than the ones obtained in the present paper.

Since instantons do not account for confinement, neither does our model. Therefore some of our results have to be taken with caution since confinement might lead to their modification. We shall come back to this issue in the Summary. It is however interesting to note at this point that the momentum dependent constituent quark mass arises also in the context of deep inelastic scattering. Although deep inelastic scattering is in principle defined for large Q^2 one may try to extrapolate to the low Q^2 region. In this case, in the proton rest frame, the photon dissociates into a quark-antiquark dipole well before the target. The wave function for this process that for large Q^2 can be calculated perturbatively, is matched with the low Q^2 region by introducing a momentum dependent quark mass. Here, however, in contrast to the instanton model of the QCD vacuum, the momentum dependence is attributed to confinement rather than to the chiral symmetry breaking. We refer the reader to Ref. [19] for details.

One of the obvious problems that arises when one considers momentum dependent fermion mass is the nonconservation of the naive vector current containing only the γ^μ Dirac matrix. There are many proposals in the literature about how to extend the vector current to satisfy electromagnetic Ward identities. None of them is unique, since current conservation does not fix the transverse part of the modified vertex. One of the simplest extensions of this type discussed already many years ago in Refs. [20,21] and employed also in Ref. [18], consists in the following substitution

$$\gamma^\mu \longrightarrow \tilde{\gamma}^\mu(k, k-P) = \gamma^\mu - \frac{k^\mu + (k-p)^\mu}{k^2 - (k-p)^2}(M_k - M_{k-p}). \qquad (1)$$

Extension (1) follows from the assumption concerning both Ward identities and analytical structure of the modified vertex that is required to match the perturbative expansion. In principle one could add to (1) terms proportional to r^μ where $r \cdot p = 0$, and the Ward identities would be satisfied. In Refs. [22,23] and also in [24] another modification has been advocated; here the ambiguity is connected to a freedom of choosing the integration path

*kotko@th.if.uj.edu.pl
†michal@if.uj.edu.pl

that defines the nonlocal vertex. In view of this ambiguous situation, we have decided to use the simplest possible extension of Ref. [20] given by Eq. (1).

Unlike the pion or the ρ meson the photon has a dual nature being both pointlike and composite at the same time. Therefore in order to calculate photon DAs that describe nonperturbative quark-antiquark structures, one has to subtract the perturbative part. In order to avoid ambiguities, this procedure has to be well defined. In our case, since we work in the chiral limit, we subtract the perturbative part only from the photon DAs that are nonzero for massless quarks. These are vector and axial DAs which are also UV divergent. Therefore the subtraction of the perturbative part renormalizes these DAs ensuring their finiteness. At the same time, it introduces a term proportional to $\ln(-P^2)$ that develops the imaginary part for positive virtualities. This is the reflection of the fact that the photon can decay into free massless quarks in the chirally even channels. Throughout this paper we plot DAs both for negative and positive photon virtualities; in the latter case we take just the real part if the imaginary part exists. We also display momentum dependence of the pertinent decay constants that are characterized by dimensionless functions $F_{T,V,A}(P^2)$ for brevity referred to as form factors.

Finally, let us make a technical remark concerning loop integrals over d^4k with the $k^+ = uP^+$ component fixed by the δ function. Such integrals, depending on the tensor structure, may contain ''singular'' pieces proportional to $\delta(u)$ and $\delta(u-1)$ or even the derivatives of the δ functions. This is the case for higher twist photon DAs only. We discuss this in more detail in Sec. IV, here we just want to point out that higher twist DAs are in fact distributions rather than ordinary functions. Quite importantly, the δ function contribution is always accompanied by a regular piece that together with the δ piece integrates to zero for any P^2. This allows us to define a properly normalized regular part and a singular part of DA of zero norm.

In the next section, we introduce kinematical variables and define photon distribution amplitudes. In Sec. III, we describe the main features of our model specifying the ansatz for the momentum dependence of the constituent quark mass. We fix model parameters requiring that the experimental value of the pion decay constant $F_\pi = 93$ MeV is reproduced. To this end, we use the model formula given in Eq. (18). Next, in Sec. IV, we describe techniques used to calculate loop integrals with momentum dependent constituent quark mass. We pay special attention to Lorentz invariance and show how the end point singularities proportional to the Dirac δ functions arise. The main results are presented in Sec. V. First we calculate dimensional constants entering definitions of the DAs (7)–(9), namely, quark condensate, magnetic susceptibility, and yet another constant called $f_{3\gamma}$. We obtain numerical results that agree with the ''experimental'' values known from phenomenology. Finally, in Secs. VA and VB we present our main results plotting different DAs and discussing their properties.

Our results can be briefly summarized as follows. Leading twist amplitudes are rather insensitive to model parameters, whereas higher twist amplitudes exhibit much stronger dependence, moreover some of them contain δ functions. We also show the influence of the nonlocal part of the photon vertex (1) on the shape of photon DAs. For some DAs it is rather unimportant, for the other ones it is absolutely crucial. More discussion and comparison with other models is given in Sec. VI. Technical details are collected in the Appendixes.

II. DEFINITIONS AND KINEMATICS

Photon distribution amplitudes are defined through matrix elements of the nonlocal quark-antiquark billinears between the vacuum and one photon state. Quark operators are assumed to be on the light cone separated by the distance 2λ. In the following, we use the light-cone coordinates defined by two lightlike vectors n^μ and $\tilde{n}^\mu$ such that $n^\mu = (1,0,0,-1)$ and $\tilde{n}^\mu = (1,0,0,1)$. In this basis any four-vector v^μ can be decomposed into v^+ and v^- components

$$v^\mu = v^+ \frac{\tilde{n}^\mu}{2} + v^- \frac{n^\mu}{2} + v_\perp^\mu. \tag{2}$$

The scalar product can be written as

$$u \cdot v = \tfrac{1}{2}u^+ v^- + \tfrac{1}{2}u^- v^+ - \vec{u}_\perp \cdot \vec{v}_\perp. \tag{3}$$

We shall work in the system where the photon momentum is expressed as

$$P^\mu = P^+ \frac{\tilde{n}^\mu}{2} + \frac{P^2}{P^+}\frac{n^\mu}{2}. \tag{4}$$

Decomposition of the polarization vector reads

$$\varepsilon^\mu = \varepsilon^+ \frac{\tilde{n}^\mu}{2} + \varepsilon^- \frac{n^\mu}{2} + \varepsilon_\perp^\mu,$$
$$\text{where } \varepsilon_\perp^\mu \varepsilon_{\perp\mu} = -\vec{\varepsilon}_\perp \cdot \vec{\varepsilon}_\perp = -1. \tag{5}$$

Since $P \cdot \varepsilon = 0$ we have the relation

$$\varepsilon^- = -\frac{P^2}{(P^+)^2}\varepsilon^+. \tag{6}$$

For the real photon we obviously have $\varepsilon^+ = \varepsilon \cdot n = 0$ and consequently $\varepsilon^- = 0$ as well.

Depending on the different tensor nature of the bilocal operators, we can define tensor

$$\langle 0|\bar{\psi}(\lambda n)\sigma^{\alpha\beta}\psi(-\lambda n)|\gamma(P,\varepsilon)\rangle = i\frac{e}{2}\langle\bar{\psi}\psi\rangle F_T(P^2)\bigg\{(\varepsilon_\perp^\alpha\tilde{n}^\beta-\varepsilon_\perp^\beta\tilde{n}^\alpha)P^+\chi_m\int_0^1 due^{i\xi\lambda P^+} \times\phi_T(u,P^2)\frac{1}{P^+}(\tilde{n}^\alpha n^\beta-\tilde{n}^\beta n^\alpha)\varepsilon^+\int_0^1 due^{i\xi\lambda P^+} \times\psi_T(u,P^2)\frac{1}{P^+}(\varepsilon_\perp^\alpha n^\beta-\varepsilon_\perp^\beta n^\alpha)\int_0^1 due^{i\xi\lambda P^+}h_T(u,P^2)\bigg\}, \tag{7}$$

vector

$$\langle 0|\bar{\psi}(\lambda n)\gamma^\mu\psi(-\lambda n)|\gamma(P,\varepsilon)\rangle = ef_{3\gamma}F_V(P^2)\bigg\{\frac{1}{2}\tilde{n}^\mu\varepsilon^+\int_0^1 due^{i\xi\lambda P^+}\phi_V(u,P^2) + \varepsilon_\perp^\mu\int_0^1 due^{i\xi\lambda P^+}\psi_V(u,P^2)-\frac{1}{2} \times\frac{P^2}{(P^+)^2}n^\mu\varepsilon^+\int_0^1 due^{i\xi\lambda P^+}h_V(u,P^2)\bigg\}, \tag{8}$$

and axial vector

$$\langle 0|\bar{\psi}(\lambda n)\gamma^\mu\gamma_5\psi(-\lambda n)|\gamma(P,\varepsilon)\rangle = \frac{1}{2}ef_{3\gamma}F_A(P^2)\epsilon_{\mu\nu\alpha\beta}\varepsilon_\perp^\nu\tilde{n}^\alpha n^\beta P^+\lambda \times\int_0^1 due^{i\xi\lambda P^+}\psi_A(u,P^2) \tag{9}$$

distribution amplitudes. For compactness, we used the notation $\xi = 2u-1$ where u is a longitudinal fraction of the quark momentum and dropped Wilson lines $[-\lambda n, \lambda n]$ that ensure gauge invariance of the nonlocal operators. In the light-cone gauge $A\cdot n = 0$ and hence $[-\lambda n, \lambda n] = 1$. Moreover, since we use an effective model where gluonic fields are integrated out, Wilson lines corresponding to gluon fields never appear.

Our definitions follow closely those of Refs. [1,18], however we need only one P^2-dependent dimensionless form factor for each tensor structure: $F_T(P^2)$, $F_V(P^2)$, and $F_A(P^2)$ where subscripts T, V, and A stay for the vector, tensor, and axial vector, respectively. Constant χ_m is the magnetic susceptibility of the quark condensate $\langle\bar{\psi}\psi\rangle$, and $f_{3\gamma}$ corresponds to the axial mixed quark-gluon condensate. They provide natural mass scales for distribution amplitudes. Analytical expressions for $\langle\bar{\psi}\psi\rangle$, χ_m, and $f_{3\gamma}$, and for the form factors can be retrieved from the matrix elements of local operators:

$$\langle 0|\bar{\psi}(0)\sigma^{\alpha\beta}\psi(0)|\gamma(P,\varepsilon)\rangle = ie\langle\bar{\psi}\psi\rangle\chi_m F_T(P^2) \times(\varepsilon^\alpha P^\beta-\varepsilon^\beta P^\alpha), \tag{10}$$

$$\langle 0|\bar{\psi}(0)\gamma^\mu\psi(0)|\gamma(P,\varepsilon)\rangle = ef_{3\gamma}F_V(P^2)\varepsilon^\mu, \tag{11}$$

$$\frac{d}{d\lambda}\langle 0|\bar{\psi}(-\lambda n)\gamma^\mu\gamma_5\psi(\lambda n)|\gamma(P,\varepsilon)\rangle|_{\lambda=0} = ef_{3\gamma}F_A(P^2)\epsilon_{\mu\nu\alpha\beta}\varepsilon^\nu P^\alpha n^\beta. \tag{12}$$

Equations (7)–(9) define photon distribution amplitudes that can be classified according to the kinematical light-cone twist. We have distributions of twist two: ϕ_T, ϕ_V, of twist three: ψ_T, ψ_V, ψ_A, and of twist four: h_T, h_V. This can be easily seen by inspecting Eqs. (7)–(9), since the twist counting actually reduces to counting the powers of P^+. Notice that in the case of axial distribution, the power of P^+ equals 1, which would correspond to twist two, however additionally there is a path stretch λ with inverse mass dimensionality that makes ψ_A to have twist three rather than two.

Constants χ_m, $\langle\bar{\psi}\psi\rangle$, and $f_{3\gamma}$ are chosen in such a way that the following normalization conditions are satisfied:

$$\int_0^1\phi_T(u,P^2)du = 1, \qquad \int_0^1\psi_T(u,P^2)du = \chi_m P^2, \qquad \int_0^1 h_T(u,P^2)du = \chi_m P^2, \tag{13}$$

$$\int_0^1\phi_V(u,P^2)du = 1, \qquad \int_0^1\psi_V(u,P^2)du = 1, \qquad \int_0^1 h_V(u,P^2)du = 1, \tag{14}$$

$$\int_0^1\psi_A(u,P^2)du = 1. \tag{15}$$

Note that due to the conservation of vector current $F_V(0) = 0$. On the other hand $F_T(0) = 1$. Normalization of the axial form factor $F_A(0)$ is arbitrary and depends on the dimensional constant used in definition (9).

III. NONLOCAL CHIRAL QUARK MODEL

In order to calculate relevant matrix elements in the low energy domain, we shall use effective action based on the instanton vacuum theory [7]. Its main feature is momentum dependent constituent quark mass

$$M(k) = MF^2(k) \tag{16}$$

appearing due to the chiral symmetry breaking. This dependence enters not only into propagators, but serves as a nonlocal quark-meson coupling as well. For zero momentum $M(0)\equiv M$ is of order of 350 MeV, while for $k\to\infty$ constituent quark mass vanishes $M(k)\to 0$.

One has to remember that the semibosonized Nambu Jona-Lasinio model, although devised to describe chiral physics of Goldstone bosons, has been widely used to incorporate baryons as chiral solitons both in local (for review see, e.g., Ref. [25]) and nonlocal [26] cases. Generally the results of these studies show that the soliton ceases to exist for too small constituent quark mass M. The critical value of M depends on the details of the given model, however it is of the order of 300 MeV or a bit less. Typical values of M that fit well the hyperon spectrum may be as high as 420 MeV [27]. In order to investigate dependence of photon DAs on M, we use three distinct values of M: 300, 350, and 400 MeV.

Because of the momentum dependence of the quark mass, the naive vector current $j^\mu = \bar{\psi}\gamma^\mu\psi$ violates electromagnetic Ward identities. In order to fix this deficiency, new nonlocal terms have to be added to j^μ. As already discussed in the Introduction, there are several ways of constructing extensions that make the current conserved. In the present paper, we use the simplest possible generalization of the vector current, [18,20] replacing γ^μ by an effective vertex of (1). One can easily check that electromagnetic Ward identities are satisfied when (1) is used instead of γ^μ. Although Eq. (1) introduces an extra pole inside Feynman amplitudes, its residue, as we will explicitly show, is zero due to the mass difference in the numerator. This generalization of the vector current has been widely used in the literature and also in the context of the photon DAs [18].

Expression for the form factor $F_{\rm inst}(k)$ within the instanton vacuum model is known analytically in Euclidean space and is highly nontrivial [7]. Therefore, in order to perform analytical calculations directly in Minkowski space, we use the following formula [9]:

$$F(k) = \left(\frac{-\Lambda_n^2}{k^2 - \Lambda_n^2 + i\epsilon}\right)^n, \tag{17}$$

where Λ_n is cutoff parameter adjusted for each n in such a way that the experimental value of the pion decay constant is reproduced. For transparency we shall skip subscript n and use Λ rather than Λ_n in the following.

Equation (17) reproduces reasonably well original shape $F_{\rm inst}(k)$ when continued to Euclidean momentum. It should be however pointed out that expression (17) does not follow the exponential asymptotics of $F_{\rm inst}(k)$ [7]. Parameter n is introduced in order to check sensitivity of our results to the shape of $F(k)$.

In order to fix the model parameter Λ we use the following Euclidean expression for the weak pion decay constant [22]:

$$F_\pi^2 = \frac{N_c}{4\pi^2}\int_0^\infty dk_E^2 k_E^2 \times \frac{M^2(k_E^2) - k_E^2 M(k_E^2)M'(k_E^2) + k_E^4 M'(k_E^2)^2}{(k_E^2 + M^2(k_E^2))^2} \tag{18}$$

where $M'(k_E^2) = dM(k_E^2)/dk_E^2$. Notice that this formula differs from the Pagels-Stokar formula of Ref. [28]. It has been also obtained in Ref. [10] from partial conservation of axial current in Minkowski space. Using experimental value $F_\pi = 93$ MeV and (17) we obtain the cutoff parameters listed in Table I for several choices of the constituent quark mass M and n. The analytical expression obtained within the present model is given in Appendix B. We remark at this point that the cutoff parameter Λ should not be confused with a typical scale of the model, which for the instanton model is about 600 MeV.

TABLE I. Numerical values of the model parameters obtained using the Birse-Bowler formula for pion decay constant F_π.

$M = 300$ MeV	
$n = 1$	$\Lambda = 1016$ MeV
$n = 5$	$\Lambda = 2385$ MeV
$M = 350$ MeV	
$n = 1$	$\Lambda = 836$ MeV
$n = 5$	$\Lambda = 1970$ MeV
$M = 400$ MeV	
$n = 1$	$\Lambda = 721$ MeV
$n = 5$	$\Lambda = 1704$ MeV

IV. LOOP INTEGRALS WITH MOMENTUM DEPENDENT MASS

In this section, we present a brief sketch of our calculations underlying the most important steps. Further technicalities are relegated to the Appendixes. In order to calculate the DA of interest—denoted generically as $f(u)$—we have to invert formulas (7)–(9) by contracting them with appropriate four-vectors and by performing Fourier transformation in λ. This results in the following formulas:

$$f(u) = \frac{-ie_q 4P^+ N_c}{C}\int\frac{d^Dk}{(2\pi)^D} T_\Gamma(k, k-P)\delta(k^+ - uP^+) \tag{19}$$

where C is the constant obtained by the contraction, Γ is the contracted tensor structure defining given amplitude, and

$$T_\Gamma(k,q) = \frac{1}{4}\,\mathrm{Tr}\Big\{\Gamma\frac{1}{\not{k} - M_k + i\epsilon}\varepsilon_\mu\tilde{\gamma}^\mu(k, k-P) \times \frac{1}{(\not{k} - \not{P}) - M_{k-P} + i\epsilon}\Big\} \tag{20}$$

stands for the Dirac trace. Note that in the case of axial DA, because of λ standing in the left-hand side of (9)

$$f(u) = \tfrac{1}{2}(\psi_A'(u) + \psi_A(0)\delta(u) - \psi_A\delta(u-1)). \tag{21}$$

Since some of the integrals can be UV divergent, we shall work in $D = 4 - 2\epsilon$ dimensions.

Previous calculations using the present nonlocal model were done by integration in the light-cone coordinates, with special care concerning the integration contour to ensure analyticity in Λ. Here we present another method of performing such integrals based on the α representation for the propagators. It is especially useful in the case of integrals appearing in higher twist distributions, because of the end point delta-type singularities, which are cumbersome to treat by integration in the light-cone coordinates.

The above complication can be well illustrated by considering a loop integral of the type (19) for the numerator involving k^μ. If not for the δ function, it would have been proportional to P^μ, but because $k^+ - uP^+ = n\cdot k - un\cdot P$ we have

$$\int \frac{d^Dk}{(2\pi)^D}\frac{k^\mu \mathcal{N}}{(k^2 - M_k^2 + i\epsilon)((k-P)^2 - M_{k-P}^2 + i\epsilon)} \times \delta(k^+ - uP^+) = A(u)P^\mu + B(u)n^\mu \tag{22}$$

where $\mathcal{N}$ is some scalar function involving n, ε, and P. There is an obvious condition following from Lorentz invariance

$$\int_0^1 duB(u) = 0. \tag{23}$$

However, as mentioned above and as shown explicitly in Appendix C, function $B(u)$ contains both the regular piece and the piece with delta functions: $\delta(u)$ and $\delta(u-1)$. Only the sum of both contributions integrates over du to zero. Note that this cancellation occurs for any P^2. Since $n\cdot n = 0$ and $\varepsilon_\perp \cdot n = 0$, the delta functions contribute only to the integrals of the k^- component. The integrals with tensor structure $k^\mu k^\nu$ are even more complicated, since they involve derivatives of δ functions.

As it was already discussed in Ref. [9], momentum mass dependence given by (17) introduces a set of poles, whose positions depend on parameter Λ. To this end it is convenient to introduce dimensionless scaled variables

$$\kappa = k/\Lambda_n, \qquad p = P/\Lambda_n, \qquad r = M/\Lambda_n \tag{24}$$

and to define

$$z_1 = (\kappa - p)^2 - 1 + i\epsilon, \qquad z_2 = \kappa^2 - 1 + i\epsilon. \tag{25}$$

Then the loop integral involving two propagators, like the one in Eq. (22), is transformed into

$$I = \Lambda^{D-5}\int \frac{d^D\kappa}{(2\pi)^D}\delta(\kappa\cdot n - up^+)\frac{z_1^{4n}z_2^{4n}\mathcal{N}}{G(z_1)G(z_2)}, \tag{26}$$

where the numerator $\mathcal{N}(z_1, z_2, \kappa\cdot n, \kappa\cdot\tilde{n}, \kappa\cdot\varepsilon_\perp)$ depends on the DA considered. Here

$$G(z_i) = z_i^{4n+1} + z_i^{4n} - r^2 = \prod_{j=1}^{4n+1}(z_i - \eta_j) \tag{27}$$

corresponds to the propagator with momentum dependent mass (for $n = 0$ it reduces to the ordinary propagator in scaled variables) where the complex numbers η_j are roots of polynomial G to be obtained numerically.

Next we decompose the inverse product of $G(z_1)G(z_2)$ into a sum of simple poles

$$\frac{z_1^M z_2^N}{G(z_1)G(z_2)} = \sum_{i,j=1}^{4n+1} f_i f_j \frac{\eta_i^M \eta_j^N}{(z_1 - \eta_i)(z_2 - \eta_j)} \quad \text{for } M, N \le 4n \tag{28}$$

with

$$f_i = \prod_{k=1,k\neq i}^{4n+1}\frac{1}{\eta_i - \eta_k}. \tag{29}$$

In this way, integral (26) is reduced to the sum of contributions involving two propagators only. It is convenient to use the α representation (exponential Schwinger representation) for the product of propagators since also the δ function in (26) can be written as an exponent. Further calculations are rather standard and are summarized in Appendix C. As a result, we obtain analytical expressions given as sums over roots η_i. Certain simplifications occur when we use the following identity [which is true for any set of complex numbers $\{\eta_i\}$ not only for the solutions of $G(\eta_i) = 0$]:

$$\sum_{i=1}^{4n+1} f_i \eta_i^N = \begin{cases} 0 & N < 4n, \\ 1 & N = 4n. \end{cases} \tag{30}$$

The proof of (30) and other useful identities can be found in Appendix A.

Some of the loop diagrams discussed above are UV divergent and require renormalization. This results in the subtraction of the perturbative part which is uninteresting from the point of view of the hadronic component of the photon. To illustrate this problem, consider loop integral (26) with $\mathcal{N} = 1$. Performing $d^D\kappa$ integration gives

$$\begin{aligned}\mathcal{J} &= \frac{i}{16\pi^2P^+}\left(\frac{4\pi e^{-\gamma}}{\Lambda^2}\right)^\epsilon \frac{1}{\epsilon}\sum_{i,j=1}^{4n+1} f_i\eta_i^{4n} f_j\eta_j^{4n} \\ &\quad\times[1 - \bar{u}\eta_i + u\eta_j + u\bar{u}p^2]^{-\epsilon} \\ &= \frac{i}{16\pi^2P^+}\left(\frac{4\pi e^{-\gamma}}{\Lambda^2}\right)^\epsilon \sum_{i,j=1}^{4n+1} f_i\eta_i^{4n} f_j\eta_j^{4n} \\ &\quad\times\left(\frac{1}{\epsilon} - \ln[1 - \bar{u}\eta_i + u\eta_j + u\bar{u}p^2]\right)\end{aligned} \tag{31}$$

with $\bar{u} = u - 1$. Because of (30) the coefficient of the $1/\epsilon$ pole is equal to 1. For $\mathcal{N}$ involving negative powers of η_i (like $\mathcal{N} = M_k$ for example) the coefficient of the pole is 0 and no subtraction is needed. For constant mass ($n = 0$) $G(z) = z + 1 - r^2$ and for zero mass (current masses are zero in the chiral limit) there is only one solution of $G(z) = 0$, namely, $\eta_1 = -1$. Hence the perturbative part of the loop integral (26) reads

$$\mathcal{J}_{\text{pert}} = \frac{i}{16\pi^2P^+}\left(\frac{4\pi e^{-\gamma}}{\Lambda^2}\right)^\epsilon\left(\frac{1}{\epsilon} - \ln[u\bar{u}p^2]\right). \tag{32}$$

This result can of course be obtained by standard techniques for $M = 0$. Renormalization in the $\overline{\text{MS}}$ scheme proceeds by subtracting the pole only. Here we subtract full perturbative contribution and go back to $D = 4$ ($\epsilon = 0$) dimensions which gives

$$\mathcal{J}_{\rm sub} = -\frac{i}{16\pi^2 P^+}\sum_{i,j=1}^{4n+1} f_i\eta_i^{4n} f_j\eta_j^{4n} \times \ln\left[\frac{1-\bar{u}\eta_i + u\eta_j + u\bar{u}p^2}{u\bar{u}p^2}\right] \qquad (33)$$

where we have again used (30). Note that subtraction occurs only for terms which do not involve M_k or M_{k-P}, and these terms are always UV divergent. In other words, in the chiral limit all perturbative photon DA are either UV divergent or identically zero.

V. PHOTON DAs IN THE NONLOCAL MODEL

Before we proceed with photon DAs and systematically present our results, we have to fix numerical constants appearing in the definitions (7)–(9). We have already introduced the expression for pion decay constant (18) which is used to fix model parameters. Next we consider the quark condensate given as the trace of the quark propagator which reads in Euclidean metric

$$\langle\bar{\psi}\psi\rangle = -\frac{N_c}{4\pi^2}\int dk_{\rm E}^2 \frac{k_{\rm E}^2 M(k_{\rm E}^2)}{k_{\rm E}^2 + M^2(k_{\rm E}^2)}, \qquad (34)$$

which in our model turns out to be simply

$$\langle\bar{\psi}\psi\rangle = -\frac{N_c M\Lambda^2}{4\pi^2}\sum_{i=1}^{4n+1} f_i\eta_i^{2n}(1+\eta_i)\ln(1+\eta_i). \qquad (35)$$

Numerical values obtained from this formula coincide with those of Ref. [9] if we use model parameters corresponding to F_π obtained from the Pagels-Stokar formula [28].

The formula for magnetic susceptibility χ_m in the nonlocal model used in Ref. [18]

$$\chi_m = \frac{N_c}{4\pi^2\langle\bar{\psi}\psi\rangle}\int dk_{\rm E}^2 \frac{k_{\rm E}^2(M(k_{\rm E}^2) - k_{\rm E}^2 M'(k_{\rm E}^2))}{(k_{\rm E}^2 + M^2(k_{\rm E}^2))^2} \qquad (36)$$

[with $M'(k_{\rm E}^2) = dM(k_{\rm E})/dk_{\rm E}^2$] reduces in our case to

$$\chi_m = \frac{N_c M}{4\pi^2\langle\bar{q}q\rangle}\sum_{i,j=1}^{4n+1} f_i f_j \eta_i^{4n}(1+\eta_j)[\eta_j^{2n} + 2n(1+\eta_j) \times \eta_j^{2n-1}]\left\{\frac{\varepsilon_{ij}}{\eta_i - \eta_j}(\log(1+\eta_i) - \log(1+\eta_j)) + \frac{\delta_{ij}}{1+\eta_i}\right\} \qquad (37)$$

where ϵ_{ij} is 0 for $i = j$ and 1 otherwise, while δ_{ij} is the Kronecker delta. Numerical values of $\langle\bar{\psi}\psi\rangle$ and χ_m for the present set of model parameters are listed in Table II. Note that in fact we do not have to use (37) to calculate χ_m since it can be retrieved from the normalization condition of $\phi_T(u)$. Numerical values of χ_m obtained in both ways agree, proving consistency of our calculations and definitions (7).

To calculate $f_{3\gamma}$ we have used the Euclidean formula from Ref. [18]:

$$f_{3\gamma} = -\frac{N_c}{4\pi^2}\int dk_{\rm E}^2 \frac{M^2(k_{\rm E}^2)}{k_{\rm E}^2 + M^2(k_{\rm E}^2)}, \qquad (38)$$

which in our model transforms into

$$f_{3\gamma} = \frac{N_c M^2}{4\pi^2}\sum_{i=1}^{4n+1} f_i \ln(1+\eta_i). \qquad (39)$$

Numerical values of $f_{3\gamma}$ are listed in Table III.

Phenomenological values of $\langle\bar{\psi}\psi\rangle$, χ_m, and $f_{3\gamma}$ are well known only for the quark condensate: approximately $-(250\ \text{MeV})^3$ [29] at low momentum scale. This value is still used in more recent phenomenological applications [1]. Magnetic susceptibility is still a subject of large phenomenological uncertainties. Different estimates are nicely summarized in Ref. [30] where it is shown that $\chi_m \simeq 2.5 \div 5.5\ \text{GeV}^{-2}$ with some preference to the values around $4.3\ \text{GeV}^{-2}$. Finally the value of $f_{3\gamma}$ obtained in different low energy models, as discussed in Ref. [18], is negative and of the order of $-0.004\ \text{GeV}^2$. Our values are here a factor of 2 smaller ($-0.0094\ \text{GeV}^2$), however they are almost insensitive to actual model parameters. On the other hand, magnetic susceptibility is quite sensitive to M and n [see, Eqs. (16) and (17)] remaining, however, within the range of acceptable phenomenological values discussed in Ref. [30]. Similarly, $\langle\bar{\psi}\psi\rangle$ varies with M and n, however, for the preferred value of the constituent quark mass $M = 350$ MeV it is quite close to the phenomenological estimates. From this point of view, our model satisfactorily describes low energy observables relevant for photon DAs.

TABLE II. Numerical values of the quark condensate $\langle\bar{\psi}\psi\rangle$ obtained using model parameters from Table I and magnetic susceptibility χ_m used in the calculations.

$M = 300$ MeV		
$n = 1$	$\langle\bar{\psi}\psi\rangle = -(277\ \text{MeV})^3$	$\chi_m = 2.30\ \text{GeV}^{-2}$
$n = 5$	$\langle\bar{\psi}\psi\rangle = -(230\ \text{MeV})^3$	$\chi_m = 3.75\ \text{GeV}^{-2}$
$M = 350$ MeV		
$n = 1$	$\langle\bar{\psi}\psi\rangle = -(253\ \text{MeV})^3$	$\chi_m = 2.85\ \text{GeV}^{-2}$
$n = 5$	$\langle\bar{\psi}\psi\rangle = -(208\ \text{MeV})^3$	$\chi_m = 4.71\ \text{GeV}^{-2}$
$M = 400$ MeV		
$n = 1$	$\langle\bar{\psi}\psi\rangle = -(236\ \text{MeV})^3$	$\chi_m = 3.34\ \text{GeV}^{-2}$
$n = 5$	$\langle\bar{\psi}\psi\rangle = -(192\ \text{MeV})^3$	$\chi_m = 5.58\ \text{GeV}^{-2}$

TABLE III. Numerical values of $f_{3\gamma}$ obtained using model parameters from Table I.

$M = 300$ MeV	
$n = 1$	$f_{3\gamma} = -0.0095\ \text{GeV}^2$
$n = 5$	$f_{3\gamma} = -0.0093\ \text{GeV}^2$
$M = 350$ MeV	
$n = 1$	$f_{3\gamma} = -0.0095\ \text{GeV}^2$
$n = 5$	$f_{3\gamma} = -0.0092\ \text{GeV}^2$
$M = 400$ MeV	
$n = 1$	$f_{3\gamma} = -0.0094\ \text{GeV}^2$
$n = 5$	$f_{3\gamma} = -0.0091\ \text{GeV}^2$

A. Leading twist distributions

1. Tensor photon DA

Tensor twist two amplitude has already been discussed in Refs. [17] and also [9] in a model with the local vertex only. Here we extend the discussion to off-shell photons and calculate the correction appearing due to the modified vertex (1). In the case of the leading twist tensor DA, we obtain the following expression using local current:

$$\phi_T^{(0)}(u,P^2) = \frac{i4N_cP^+}{\langle\bar\psi\psi\rangle\chi_mF_T(P^2)}\int\frac{d^Dk}{(2\pi)^D}\delta(k\cdot n - un\cdot P) \times \frac{\bar uM_k - uM_{k-P}}{(k^2-M_k^2+i\epsilon)((k-P)^2-M_{k-P}^2+i\epsilon)}, \tag{40}$$

where $\bar u = u - 1$. Notice, that the special choice of the contour described in [9] allows for passing to Euclidean space. Therefore we can use the Schwinger representation for scalar propagators and proceed in the spirit of [18]. The result reads:

$$\phi_T^{(0)}(u,P^2) = \frac{-N_cM}{4\pi^2\langle\bar\psi\psi\rangle\chi_mF_T(P^2)}\sum_{i,j=1}^{4n+1} f_if_j(\bar u\eta_i^{2n}\eta_j^{4n} - u\eta_i^{4n}\eta_j^{2n})\ln(1+u\bar up^2-\bar u\eta_i+u\eta_j) \tag{41}$$

for $0\le u\le 1$.

Now let us consider the part coming from the nonlocal part of the vertex (1). It is given by the integral

$$\phi_T^{(1)}(u,P^2) = \frac{-i8N_cP^+}{\langle\bar\psi\psi\rangle\chi_mF_T(P^2)}\int\frac{d^Dk}{(2\pi)^D}\delta(k\cdot n - un\cdot P)\frac{(M_k-M_{k-P})(\varepsilon_\perp\cdot k_\perp)^2}{(k^2-M_k^2+i\epsilon)((k-P)^2-M_{k-P}^2+i\epsilon)(2k\cdot P - P^2 + i\mu)}. \tag{42}$$

Notice that additional denominator appears $2k\cdot P - P^2 + i\mu$, where the $+i\mu$ prescription introduced at this stage is completely arbitrary. However, as already explained in Sec. III, the residue of this pole is zero so that it does not contribute to the amplitude irrespectively of the sign of μ. Therefore in the following we shall always omit contribution of this spurious pole. After performing the integration as described in Appendix C, we finally obtain

$$\phi_T^{(1)}(u,P^2) = \frac{MN_c}{4\pi^2\langle\bar\psi\psi\rangle\chi_mF_T(P^2)} \times\sum_{i,j}^{4n+1} f_if_j\frac{\eta_i^{2n}\eta_j^{2n}(\eta_i^{2n}-\eta_j^{2n})}{\eta_i-\eta_j}(1+u\bar up^2 - \bar u\eta_i + u\eta_j)\ln(1+u\bar up^2-\bar u\eta_i+u\eta_j) \tag{43}$$

for $0\le u\le 1$. The full twist two tensor photon DA is given by

$$\phi_T = \phi_T^{(0)} + \phi_T^{(1)}. \tag{44}$$

We plot this function in Fig. 1 for several values of photon virtuality and constituent quark mass. Notice that the nonlocal part of the quark-photon vertex is small and the full amplitude is almost equal to the local one. The resulting DA is almost flat for real photons and does not vanish at the end points.

The tensor form factor is shown in Fig. 2. It can be in principle calculated by analytical integration, which has to be performed carefully because of the complex numbers under logarithms.

2. Vector photon DA

Calculation of the vector twist two amplitude proceeds in a similar way. After performing the traces we get

$$\psi_V(u) = \frac{-i4P^+N_c}{f_{3\gamma}F_V(P^2)\varepsilon^+}\int\frac{d^Dk}{(2\pi)^D} \times\frac{(T_V^{(0)}+T_V^{(1)})\delta(k\cdot n - un\cdot P)}{(k^2-M_k^2+i\epsilon)((k-P)^2-M_{k-P}^2+i\epsilon)}, \tag{45}$$

where $T_V^{(0)}$ and $T_V^{(1)}$ stand for traces corresponding to local and nonlocal parts of the photon vertex, respectively:

$$T_V^{(0)} = \varepsilon^+(M_kM_{k-P} + \vec k_\perp^2 - P^2u\bar u) - (\vec\varepsilon_\perp\cdot\vec k_\perp)P^+(\bar u + u),$$
$$T_V^{(1)} = -\frac{(M_k-M_{k-P})(\bar uM_k + uM_{k-P})}{k^- + \bar u\frac{P^2}{P^+}} \times\left[\varepsilon^+\left(k^- - u\frac{P^2}{P^+}\right) - 2(\vec\varepsilon_\perp\cdot\vec k_\perp)\right]. \tag{46}$$

Note that single powers of $(\vec\varepsilon_\perp\cdot\vec k_\perp)$ integrate to zero.

In the case of the twist two vector DA, we have to subtract the perturbative piece corresponding to $M(k) = 0$. Then, the contribution to the vector photon DA coming from the local part of the vertex consists of two parts

$$\phi_V^{(0,a)}(u,P^2) = \frac{N_c}{4\pi^2f_{3\gamma}F_V(P^2)}\sum_{i,j=1}^{4n+1} f_if_j(\Lambda^2\eta_i^{4n}\eta_j^{4n} \times(\bar u\eta_i - u\eta_j - 1) + M^2\eta_i^{2n}\eta_j^{2n}) \times\ln(1+u\bar up^2-\bar u\eta_i+u\eta_j) \tag{47}$$

and

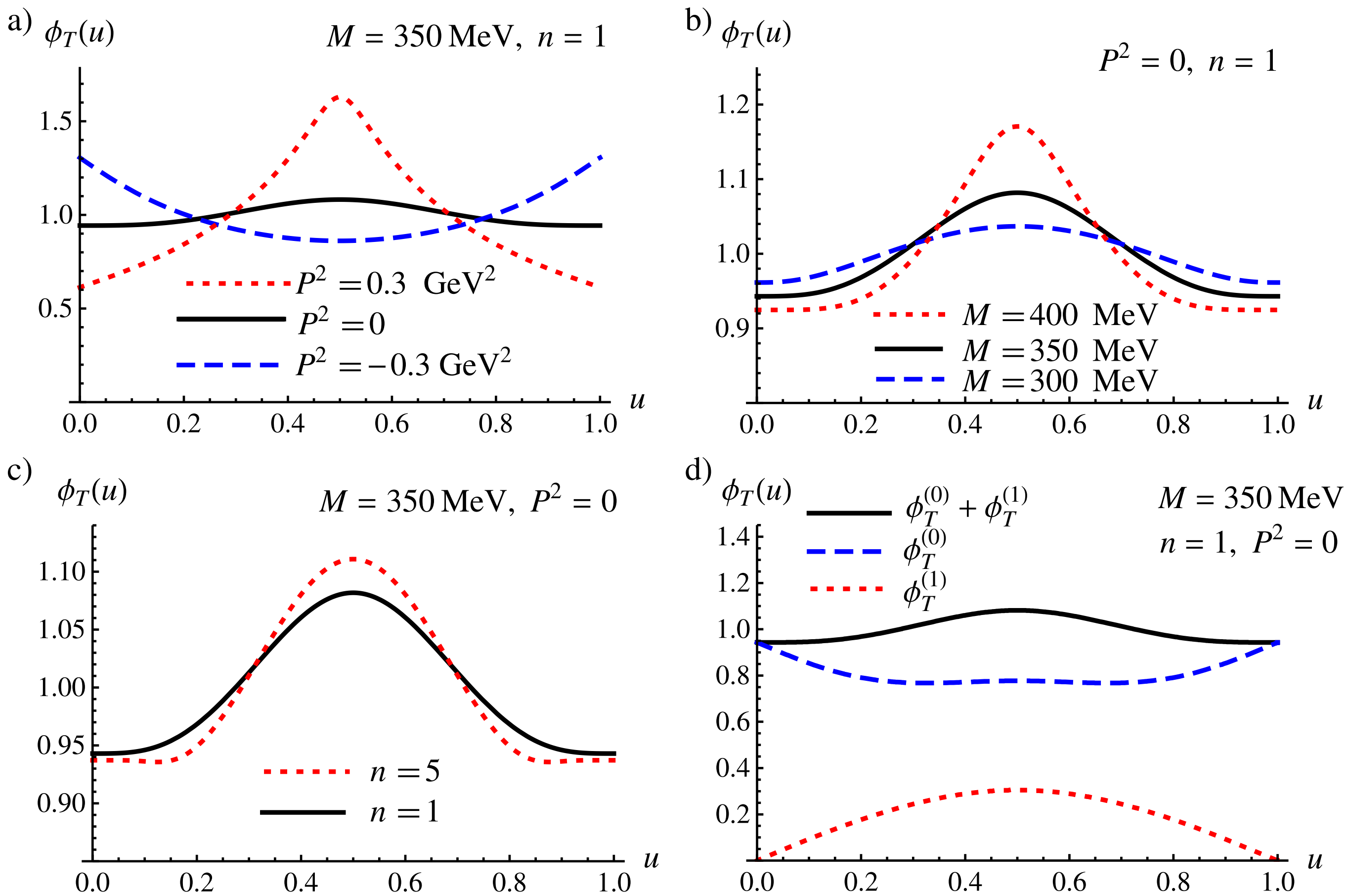

FIG. 1 (color online). Leading twist tensor photon DA for (a) different photon virtualities and fixed $M = 350$ MeV and $n = 1$, (b) different M and fixed $n = 1$ and $P^2 = 0$, (c) different n and fixed $M = 350$ MeV and $P^2 = 0$, (d) decomposition into contributions corresponding to local (dashed lines) and nonlocal (dotted lines) parts of the vector vertex for $M = 350$ MeV, $n = 1$, and $P^2 = 0$.

$$\phi_V^{(0,b)}(u, P^2) = \frac{N_c}{4\pi^2 f_{3\gamma} F_V(P^2)}(-2u\bar{u}P^2) \sum_{i,j=1}^{4n+1} f_i f_j \eta_i^{4n} \eta_j^{4n} \times \ln\left(\frac{1 + u\bar{u}p^2 - \bar{u}\eta_i + u\eta_j}{u\bar{u}p^2}\right). \tag{48}$$

The addition coming from the nonlocal part of the current can be conveniently split into a sum of two contributions

$$\phi_V^{(1,a)}(u) = \frac{-N_c}{4\pi^2 f_{3\gamma} F_V(P^2)} M^2 \sum_{i,j=1}^{4n+1} f_i f_j (\eta_j^{2n} - \eta_i^{2n}) \times (\bar{u}\eta_j^{2n} + u\eta_i^{2n}) \ln(1 + u\bar{u}p^2 - \bar{u}\eta_i + u\eta_j), \tag{49}$$

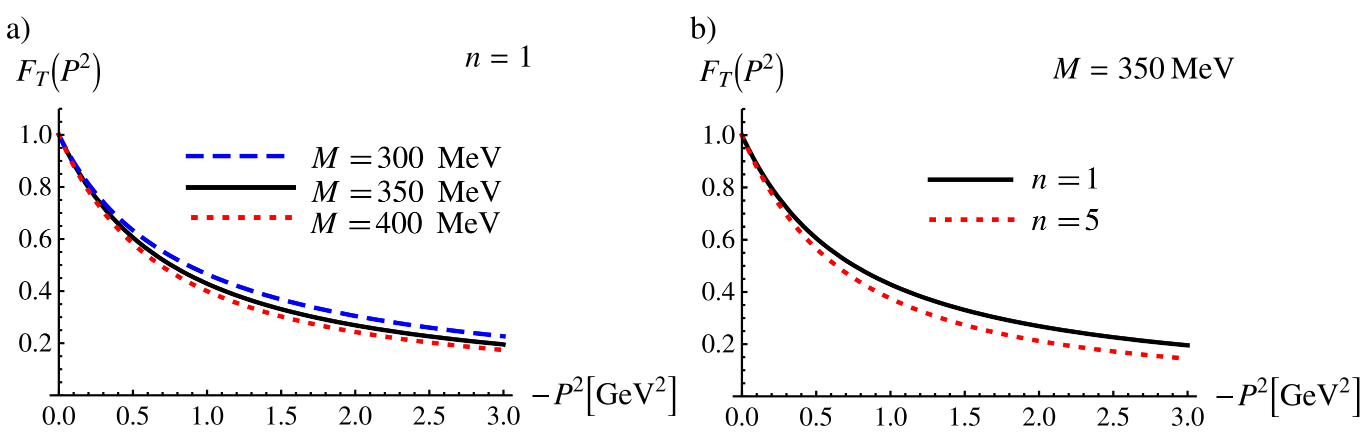

FIG. 2 (color online). Tensor form factor for (a) fixed $n = 1$ and different M, (b) fixed $M = 350$ MeV and two different $n = 1, 5$.

$$\phi_V^{(1,b)}(u) = \frac{N_c}{4\pi^2 f_{3\gamma} F_V(P^2)}(1-2u)\frac{M^2P^2}{\Lambda^2}\sum_{i,j=1}^{4n+1} f_i f_j \times \frac{(\eta_i^{2n}-\eta_j^{2n})(\bar{u}\eta_j^{2n}+u\eta_i^{2n})}{(\eta_i-\eta_j)} \times \ln(1+u\bar{u}p^2-\bar{u}\eta_i+u\eta_j). \quad (50)$$

Notice that subtraction concerns only the $\phi_V^{(0,b)}$ part since $\phi_V^{(1)}$ is always proportional to the mass.

In the tensor case, the only effect due to the local vertex is a small change in the shape of the distribution. The situation is different for vector DA. When we use local current only, the vector distribution alternates in sign (recall that leading twist DAs have probabilistic interpretation). Only when we include the nonlocal part of the vertex, contributions $\phi_V^{(0,a)}$ and $\phi_V^{(1,a)}$ cancel exactly for any P^2. This is explicitly shown in Fig. 3(d). As a consequence, ϕ_V is effectively the sum of $\phi_V^{(0,b)}$ and $\phi_V^{(1,b)}$. We plot this function in Fig. 3 for different sets of model parameters.

Furthermore, since both $\phi_V^{(0,b)}$ and $\phi_V^{(1,b)}$ are explicitly proportional to P^2, normalization conditions (14) require that $F_V(0)=0$ as it should be in accordance with the conservation of the vector current (Fig. 4). This condition would be violated if not for the nonlocal part of the photon vertex.

B. Higher twist distributions

For higher twist distributions, we encounter an additional difficulty. As shown in Appendix C, it turns out that they are in fact generalized functions. Because of $k^- = k\cdot\tilde{n}$ occurring in the numerator [$k^+ = k\cdot n$ is fixed—see the delta function in (40)], additional end point delta functions appear. They are crucial for Lorentz invariance of the integrals and in consequence for the correct normalization of the distributions. These singularities were already discussed in Ref. [18].

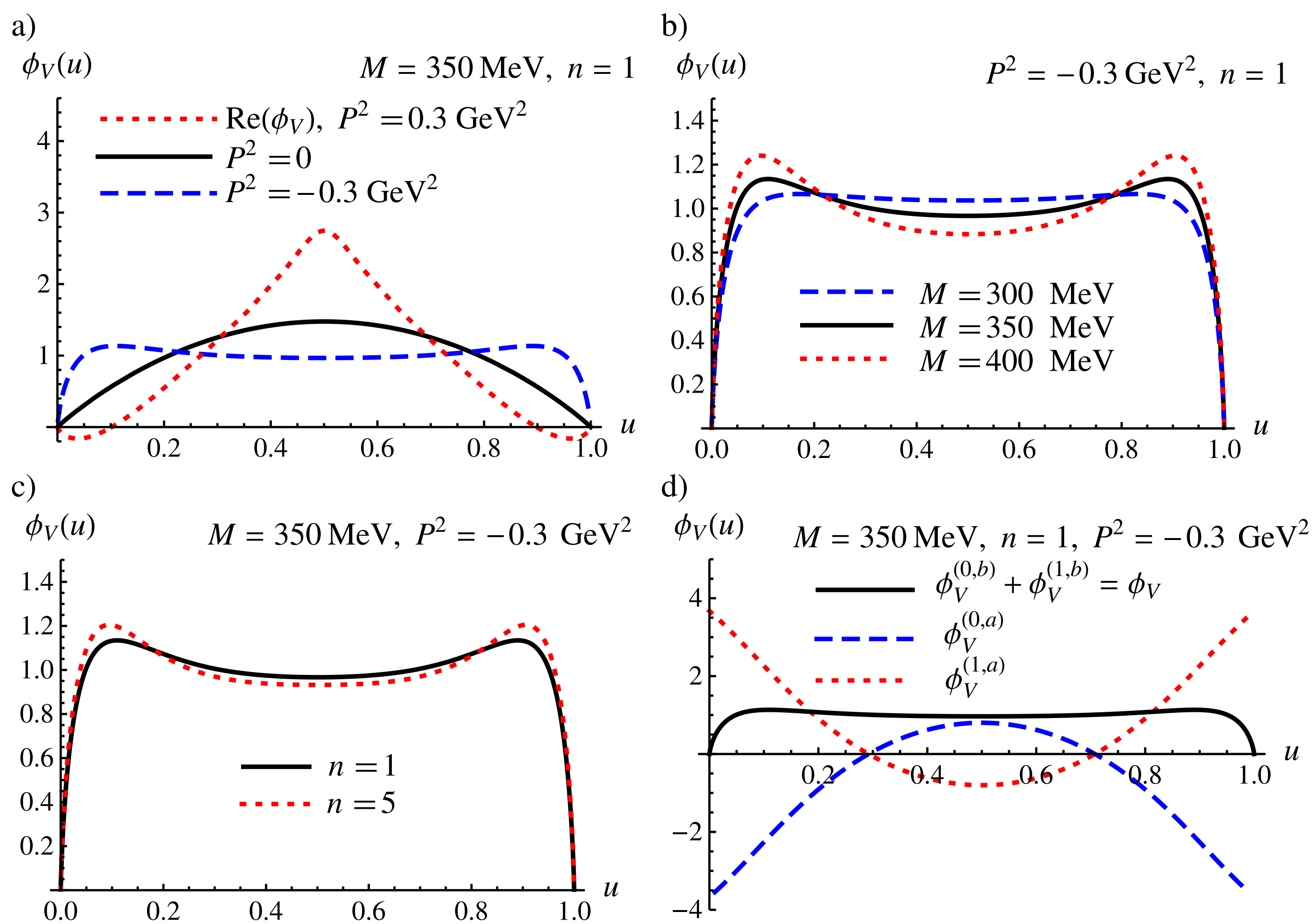

FIG. 3 (color online). Leading twist vector photon DA for (a) different photon virtualities and fixed $M = 350$ MeV and $n = 1$, (b) different M and fixed $n = 1$ and $P^2 = -0.3$ GeV^2, (c) different n and fixed $M = 350$ MeV and $P^2 = -0.3$ GeV^2, (d) decomposition into different contributions, as described in the main text; notice the exact cancellation of $\phi_V^{(0,a)}$ and $\phi_V^{(1,a)}$ following from the gauge invariance. For positive P^2 we show only the real part of DA.

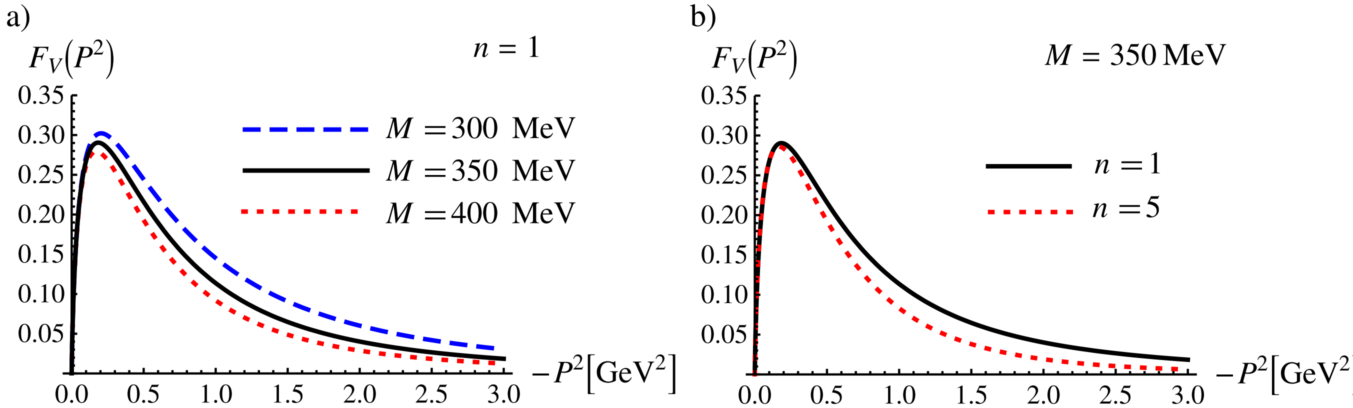

FIG. 4 (color online). Vector form factor for (a) fixed $n = 1$ and different M, (b) fixed $M = 350$ MeV and two different choices of $n = 1, 5$. Notice that the form factor vanishes for zero virtuality as required by vector current conservation.

1. Tensor DAs

We start with tensor DAs. For twist three tensor amplitude, we have

$$\psi_T(u) = \frac{4N_c}{\langle\bar\psi\psi\rangle F_T(P^2)}\frac{P^+}{\varepsilon^+}\int\frac{d^4k}{(2\pi)^4} \times \frac{(T_T^{(0)} + T_T^{(1)})\delta(k\cdot n - uP^+)}{(k^2 - M_k^2 + i\epsilon)((k-P)^2 - M_{k-P}^2 + i\epsilon)} \tag{51}$$

where $T_T^{(0)}$ and $T_T^{(1)}$ stand for traces corresponding to local and nonlocal parts of the photon vertex, respectively:

$$T_T^{(0)} = -\frac{i}{2}\varepsilon^+\left[P^+(M_k - M_{k-P})\left(k^- + \bar u\frac{P^2}{P^+}\right) - P^2(M_k + M_{k-P})\right], \tag{52}$$

$$T_T^{(1)} = \frac{i}{2}\frac{P^+(M_k - M_{k-P})}{k^- + \bar u\frac{P^2}{P^+}}\left(k^- - u\frac{P^2}{P^+}\right) \times\left[\varepsilon^+\left(k^- - u\frac{P^2}{P^+}\right) - 2\vec k_\perp\cdot\vec\varepsilon_\perp\right]. \tag{53}$$

Note that even powers of $\vec k_\perp\cdot\vec\varepsilon_\perp$ integrate to zero under $d^2\vec k_\perp$.

Luckily, in the case of $\psi_T(u)$, contributions involving k^- cancel out in the sum of (52) and (53) and the final result is the sum of two pieces

$$\psi_T^{(a)}(u) = \frac{N_c M}{8\pi^2\langle\bar\psi\psi\rangle\chi_m F_T(P^2)}(\chi_m P^2) \times\sum_{i,j=1}^{4n+1} f_i f_j \eta_i^{2n}\eta_j^{2n}[(\eta_j^{2n} + \eta_i^{2n}) + 2(1-2u) \times(\eta_j^{2n} - \eta_i^{2n})]\ln(1 + u\bar u p^2 - \bar u\eta_i + u\eta_j) \tag{54}$$

and

$$\psi_T^{(b)}(u) = \frac{-N_c M}{8\pi^2\langle\bar\psi\psi\rangle\chi_m F_T(P^2)}(\chi_m P^2)\frac{P^2}{\Lambda^2}(1-2u)^2 \times\sum_{i,j=1}^{4n+1} f_i f_j\eta_i^{2n}\eta_j^{2n}\frac{\eta_j^{2n} - \eta_i^{2n}}{\eta_j - \eta_i} \times\ln(1 + u\bar u p^2 - \bar u\eta_i + u\eta_j). \tag{55}$$

Note that ψ_T is proportional to the same normalization constant as ϕ_T times $(\chi_m P^2)$ which means that it decouples for real photons.

In the case of twist four tensor amplitude, the δ function contributions do not cancel out. Performing Dirac traces, we have

$$h_T(u) = \frac{4N_c P^{+2}}{\langle\bar\psi\psi\rangle F_T(P^2)}\int\frac{d^4k}{(2\pi)^4} \times\frac{(R_T^{(0)} + R_T^{(1)})\delta(k\cdot n - un\cdot P)}{(k^2 - M_k^2 + i\epsilon)((k-P)^2 - M_{k-P}^2 + i\epsilon)} \tag{56}$$

where again the contributions of local and nonlocal parts of the vector current have been singled out:

$$R_T^{(0)} = -i\frac{P^2}{P^{+2}}\varepsilon^+(M_k - M_{k-P})(\vec\varepsilon_\perp\cdot\vec k_\perp) - i\left[(M_k - M_{k-P})k^- - M_k\frac{P^2}{P^+}\right], \tag{57}$$

$$R_T^{(1)} = -i\frac{P^2}{P^{+2}}\frac{M_k - M_{k-P}}{k^- + \bar u\frac{P^2}{P^+}}(\vec\varepsilon_\perp\cdot\vec k_\perp) \times\left[\varepsilon^+\left(k^- - u\frac{P^2}{P^+}\right) - 2(\vec\varepsilon_\perp\cdot\vec k_\perp)\right]. \tag{58}$$

Note that significant simplifications occur since the single power of $\vec k_\perp\cdot\vec\varepsilon$ integrates to zero. Following the steps described in Appendix C, we finally arrive at the final

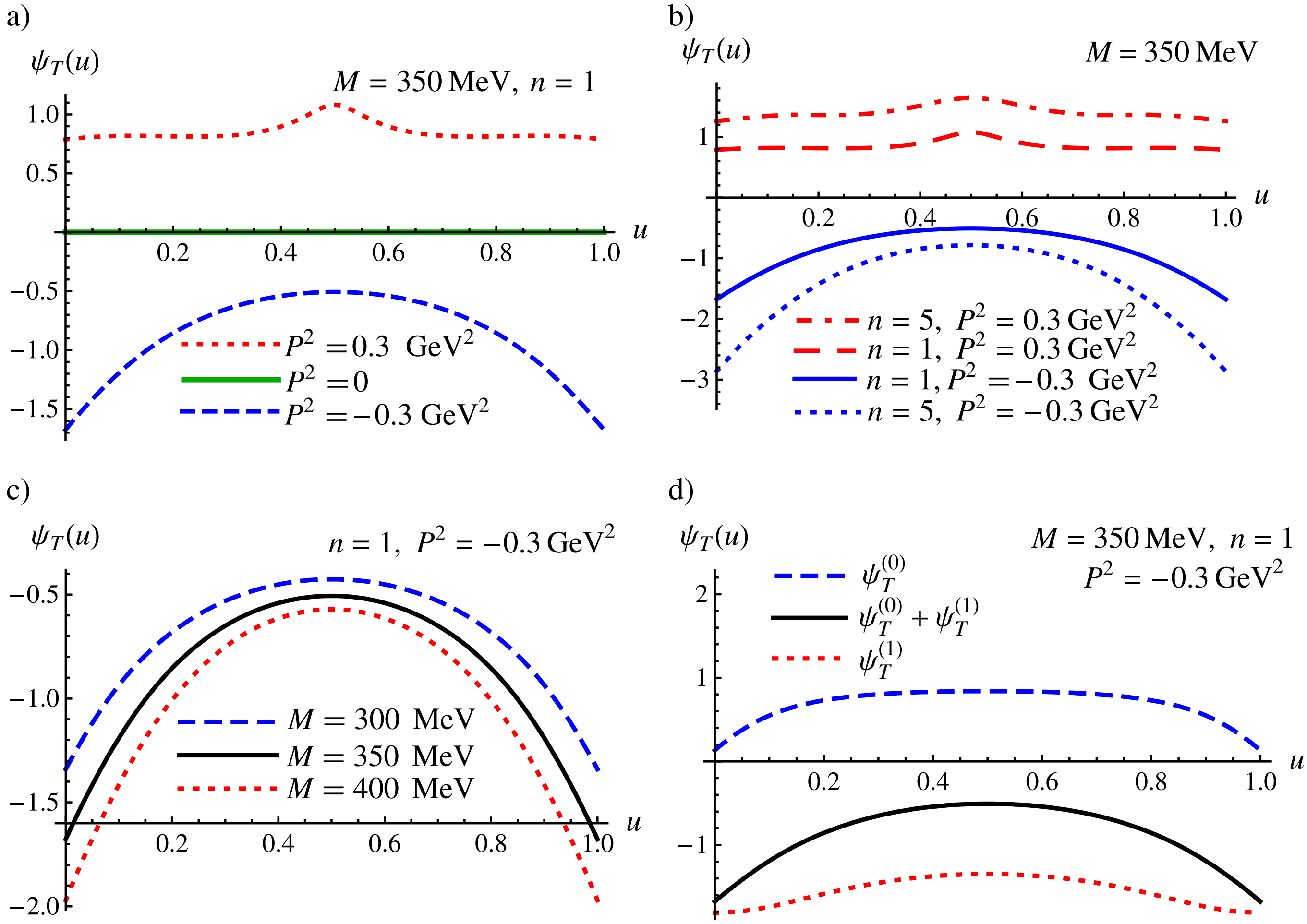

FIG. 5 (color online). Tensor twist three photon DA for (a) different photon virtualities and fixed $M = 350$ MeV and $n = 1$ (for $P^2 = 0$ it is identically zero), (b) different n and P^2 for fixed $M = 350$ MeV, (c) different M and fixed $n = 1$ and $P^2 = -0.3$ GeV^2, (d) decomposition into contributions corresponding to local (dashed lines) and nonlocal (dotted lines) parts of the vector vertex.

formula for $h_T(u)$. It is convenient to split it into 4 different pieces—regular local vertex contribution:

$$h_T^{(0,a)}(u) = \frac{N_c M}{4\pi^2\langle\bar\psi\psi\rangle\chi_m F_T(P^2)}\chi_m P^2 \sum_{i,j=1}^{4n+1} f_i f_j \eta_i^{2n}\eta_j^{2n} \times (u\eta_i^{2n} - \bar u\eta_j^{2n})\ln(1 + u\bar u p^2 - \bar u\eta_i + u\eta_j), \quad (59)$$

second local vertex contribution:

$$h_T^{(0,b)}(u) = \frac{-N_c M}{4\pi^2\langle\bar\psi\psi\rangle\chi_m F_T(P^2)}\chi_m \Lambda^2 \sum_{i,j=1}^{4n+1} f_i f_j \eta_i^{2n}\eta_j^{2n} \times (\eta_j^{2n} - \eta_i^{2n})((\eta_i - \eta_j) + (1-2u)p^2) \times \ln(1 + u\bar u p^2 - \bar u\eta_i + u\eta_j) \quad (60)$$

that integrates to zero with δ-function contribution:

$$h_T^{(0,\mathrm{delta})}(u) = \frac{N_c M}{4\pi^2\langle\bar\psi\psi\rangle\chi_m F_T(P^2)}\chi_m \Lambda^2 \sum_{i,j=1}^{4n+1} f_i f_j \eta_i^{2n}\eta_j^{2n} \times (\eta_i^{2n} - \eta_j^{2n})[(1+\eta_j)\ln(1+\eta_j)\delta(u-1) - (1+\eta_i)\ln(1+\eta_i)\delta(u)], \quad (61)$$

and hence does not contribute to the normalization, and the contribution corresponding to the nonlocal part of the photon vertex:

$$h_T^{(1)}(u) = \frac{N_c M}{4\pi^2\langle\bar\psi\psi\rangle\chi_m F_T(P^2)}\chi_m P^2 \sum_{i,j=1}^{4n+1} f_i f_j \eta_i^{2n}\eta_j^{2n} \times (1 + u\bar u p^2 - \bar u\eta_i + u\eta_j)\frac{\eta_i^{2n} - \eta_j^{2n}}{\eta_i - \eta_j} \times \ln(1 + u\bar u p^2 - \bar u\eta_i + u\eta_j). \quad (62)$$

The delta contribution $h_T^{(0,\mathrm{delta})}$ can be rewritten using the expression (39) for $f_{3\gamma}$ and the identities given in Appendix A:

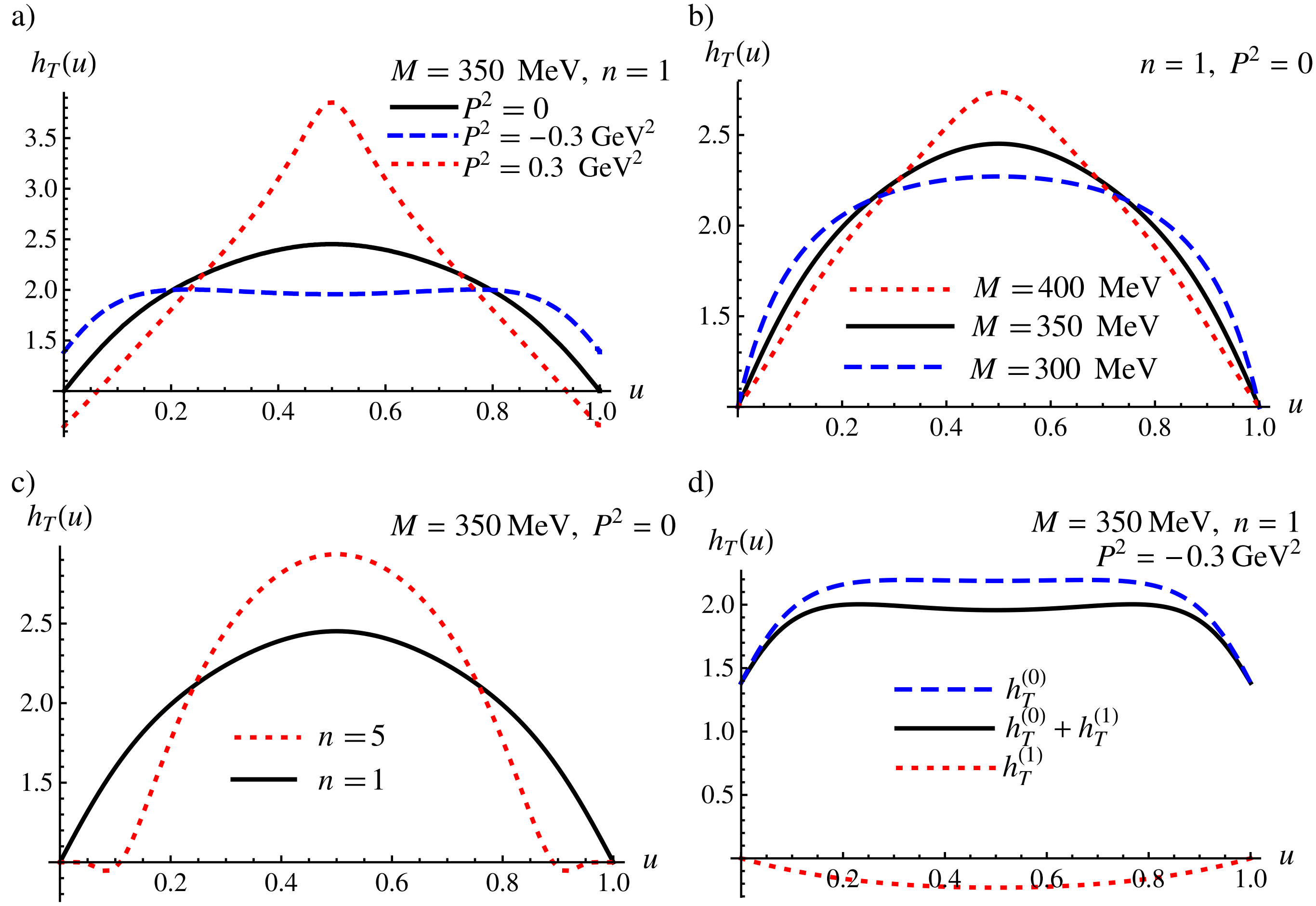

FIG. 6 (color online). Tensor twist four photon DA (without end point delta functions) for (a) different photon virtualities and fixed $M = 350$ MeV and $n = 1$, (b) various M and fixed $n = 1$ and $P^2 = 0$, (c) two choices of $n = 1$, 5, and fixed $M = 350$ MeV and $P^2 = 0$, d) decomposition into contributions corresponding to local (dashed lines) and nonlocal (dotted lines) parts of the vector vertex.

$$h_T^{(0,\mathrm{delta})}(u) = \frac{-1}{F_T(P^2)}[\delta(u-1) + \delta(u)]. \tag{63}$$

Note that the fact that the sum of (60) and (61) integrates over du to zero is a consequence of the Lorentz invariance discussed in Sec. IV [see Eq. (23)]. Therefore only $h_T^{(0,a)}$ and $h_T^{(1)}(u)$ contribute to the normalization condition (13) given by $(\chi_m P^2)$. If not for the δ-term $h_T^{(0,b)}$ would also contribute to the norm spoiling the normalization condition.

Full results (with nonlocal current) for twist three ψ_T and twist four h_T tensor distributions are show in Figs. 5 and 6, respectively.

2. Vector DAs

In the case of higher twist vector DAs, ψ_V and h_V, calculations are basically the same, with the restriction that we have to perform subtractions similarly to the twist two case. For twist three amplitude, we obtain

$$\psi_V(u) = \frac{i4N_c P^+}{f_{3\gamma}F_V(P^2)} \int \frac{d^4k}{(2\pi)^4} \times \frac{(T_V^{(0)} + T_V^{(1)})\delta(k\cdot n - un\cdot P)}{(k^2 - M_k^2)((k-P)^2 - M_{k-P}^2)} \tag{64}$$

with

$$T_V^{(0)} = [2(\vec{\varepsilon}_\perp \cdot \vec{k}_\perp)^2 - \vec{k}_\perp^2] - \varepsilon^+(\vec{\varepsilon}_\perp \cdot \vec{k}_\perp)\left(k^- - u\frac{P^2}{P^+}\right) - \frac{1}{2}(1-2u)P^+k^- - \frac{1}{2}uP^2 - M_k M_{k-P}. \tag{65}$$

In fact, after integrating over the transverse angle the terms in the first line vanish. Next,

$$T_V^{(1)} = \frac{(\vec{\varepsilon}_\perp \cdot \vec{k}_\perp)}{P^+}\frac{(M_k^2 - M_{k-P}^2)}{k^- + \bar{u}\frac{P^2}{P^+}} \times \left[\varepsilon^+\left(k^- - u\frac{P^2}{P^+}\right) - 2(\vec{k}_\perp \cdot \vec{\varepsilon}_\perp)\right]. \tag{66}$$

Again only the quadratic term $(\vec{\varepsilon}_\perp \cdot \vec{k}_\perp)^2$ survives integration over the transverse angle. The result can be split into a regular part coming from the local part of the vertex:

$$\psi_V^{(0,\mathrm{reg})}(u) = \frac{N_c}{8\pi^2 f_{3\gamma} F_V(P^2)} \sum_{i,j=1}^{4n+1} f_i f_j \eta_i^{2n} \eta_j^{2n} \times \ln\left(\frac{1 + u\bar{u}p^2 - \bar{u}\eta_i + u\eta_j}{u\bar{u}p^2}\right)((1 + 2u\bar{u})\eta_i^{2n} \times \eta_j^{2n} P^2 + 2M^2 + (1 - 2u)(\eta_i - \eta_j) \times \eta_i^{2n}\eta_j^{2n}\Lambda^2), \tag{67}$$

the part with delta functions also coming from the local part of the vertex:

$$\psi_V^{(0,\mathrm{delta})}(u) = \frac{-N_c}{8\pi^2 f_{3\gamma} F_V(P^2)} \Lambda^2 \sum_{i=1}^{4n+1} f_i \eta_i^{4n}(1 + \eta_i) \times \ln(1 + \eta_i)[\delta(u - 1) + \delta(u)], \tag{68}$$

and the nonlocal part:

$$\psi_V^{(1)}(u) = \frac{N_c}{8\pi^2 f_{3\gamma} F_V(P^2)} 2M^2 \sum_{i,j=1}^{4n+1} f_i f_j (\eta_j^{2n} + \eta_i^{2n}) \times (1 - \eta_i\bar{u} + \eta_j u + u\bar{u}r^2) \frac{\eta_i^{2n} - \eta_j^{2n}}{\eta_i - \eta_j} \times \ln(1 + u\bar{u}p^2 - \bar{u}\eta_i + u\eta_j) \tag{69}$$

The part with delta functions can be rewritten as

$$\psi_V^{(0,\mathrm{delta})}(u) = \frac{-1}{2F_V(P^2)}[\delta(u - 1) + \delta(u)], \tag{70}$$

where we used (39) and the identity $\Lambda^2\eta_i^{4n}(1 + \eta_i) = M^2$ following from Eq. (27) for zeros of $G(z)$.

Next we calculate the twist four vector distribution amplitude $h_V(u)$:

$$h_V(u) = \frac{i4N_c P^+}{f_{3\gamma} F_V(P^2)} \frac{(P^+)}{2P^2} \int \frac{d^4k}{(2\pi)^4} \times \frac{(R_V^{(0)} + R_V^{(1)})\delta(k \cdot n - un \cdot P)}{(k^2 - M_k^2)((k - P)^2 - M_{k-P}^2)} \tag{71}$$

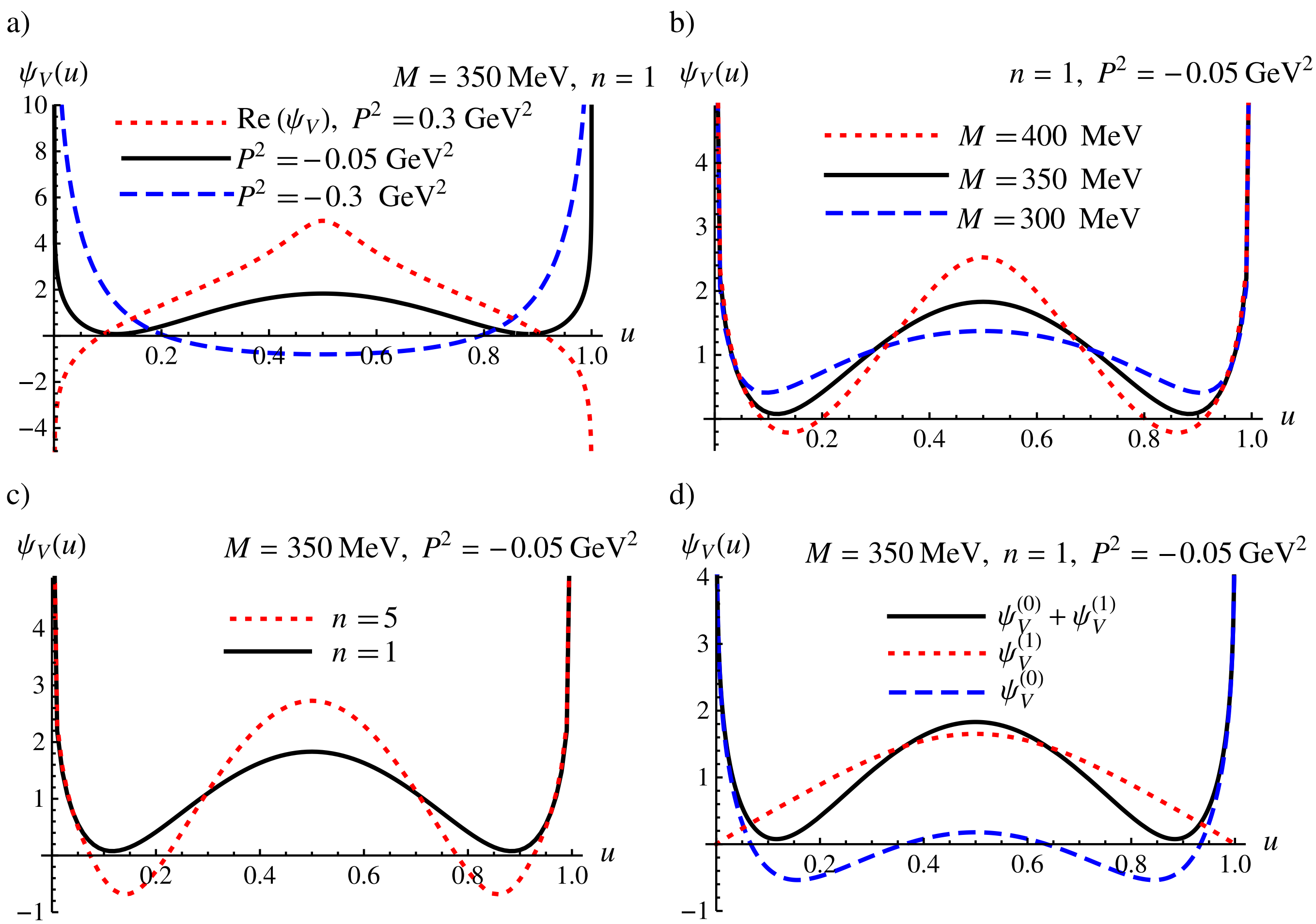

FIG. 7 (color online). Vector twist three photon DA for (a) different photon virtualities and fixed $M = 350$ MeV and $n = 1$, (b) various M and fixed $n = 1$ and $P^2 = -0.05$ GeV^2, (c) two choices of $n = 1, 5$ and fixed $M = 350$ MeV and $P^2 = -0.05$ GeV^2, (d) decomposition into contributions corresponding to local (dashed lines) and nonlocal (dotted lines) parts of the vector vertex.

where the traces read

$$R_V^{(0)} = (k^-)^2 - \frac{P^2}{P^+}k^- - \frac{P^2}{(P^+)^2}k_T^2 - M_k M_{k-P} \tag{72}$$

$$R_V^{(1)} = \frac{k^- - \frac{P^2}{P^+}u}{k^- + \bar{u}\frac{P^2}{P^+}}[k^-(M_k^2 - M_{k-P}^2) - M_k(M_k - M_{k-P})]. \tag{73}$$

The only additional complication is due to the second derivative of the delta function—the details can be found in Appendix C, Eqs. (C20)–(C22). Results for the local part read

$$h_V^{(0,\mathrm{reg})}(u) = \frac{-N_c}{8\pi^2 f_{3\gamma}F_V(P^2)} 2\sum_{i,j=1}^{4n+1} f_i f_j \eta_i^{2n}\eta_j^{2n} \times \ln\left(\frac{1 + u\bar{u}p^2 - \bar{u}\eta_i + u\eta_j}{u\bar{u}p^2}\right)\Big(2u\bar{u}\eta_i^{2n}\eta_j^{2n}P^2 - 2M^2 + \eta_i^{2n}\eta_j^{2n}(2\eta_i - \eta_j + 1 - 3u(\eta_i - \eta_j))\Lambda^2 \tag{74}$$

$$+ \frac{(\eta_i - \eta_j)^2\Lambda^4}{P^2}\Big), \tag{75}$$

and

$$h_V^{(0,\mathrm{delta})}(u) = \frac{-N_c}{8\pi^2 f_{3\gamma}F_V(P^2)}\frac{2\Lambda^2}{P^2}\Big\{-2M^2\sum_{i=1}^{4n+1} f_i \times \ln(1+\eta_i)[(1+\eta_i)[\delta(u) + \delta(u-1)] + p^2\delta(u)] + \Lambda^2 \sum_{i,j=1}^{4n+1} f_i f_j \eta_i^{4n}\eta_j^{4n}\frac{1}{2} \times \ln\left(\frac{1 + u\bar{u}p^2 - \bar{u}\eta_i + u\eta_j}{u\bar{u}p^2}\right)(1 + u\bar{u}p^2 - \bar{u}\eta_i + u\eta_j)^2[\delta'(u) + \delta'(u-1)]\Big\}, \tag{76}$$

and for the contribution coming from the nonlocal part of the photon vertex:

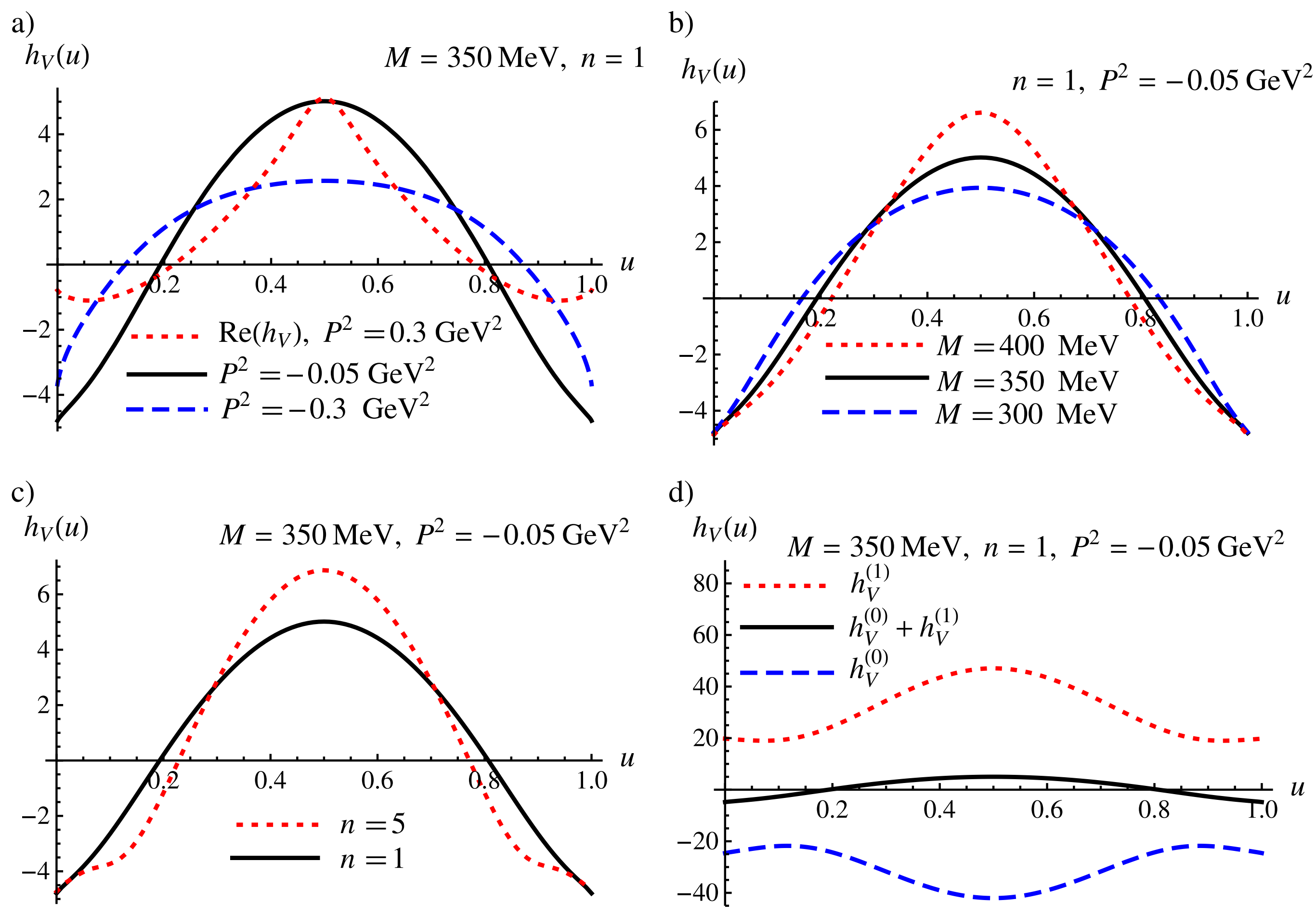

FIG. 8 (color online). Vector twist four photon DA for (a) different photon virtualities and fixed $M = 350$ MeV and $n = 1$, (b) various M and fixed $n = 1$ and $P^2 = -0.05$ GeV^2, (c) two choices of $n = 1, 5$ and fixed $M = 350$ MeV and $P^2 = -0.05$ GeV^2, (d) decomposition into contributions corresponding to local (dashed lines) and nonlocal (dotted lines) parts of the vector vertex.

$$h_V^{(1,\mathrm{reg})}(u) = \frac{-N_c}{8\pi^2 f_{3\gamma} F_V(P^2)} \frac{2M^2\Lambda^2}{P^2} \sum_{i,j=1}^{4n+1} f_i f_j \times \ln(1 + u\bar{u}p^2 - \bar{u}\eta_i + u\eta_j) \frac{\eta_i^{2n} - \eta_j^{2n}}{\eta_i - \eta_j} \times (\eta_i - \eta_j - (2u-1)p^2) \times ((\eta_i^{2n} + \eta_j^{2n}) - p^2\eta_j^{2n}), \tag{77}$$

$$h_V^{(1,\mathrm{delta})}(u) = \frac{-N_c}{8\pi^2 f_{3\gamma} F_V(P^2)} \frac{2M^2\Lambda^2}{P^2} \sum_{i=1}^{4n+1} f_i(1+\eta_i) \times \ln(1+\eta_i)[\delta(u) + \delta(u-1)]. \tag{78}$$

We note that

$$\int_0^1 h_V^{(0,\mathrm{delta})}(u)du = \frac{-N_c}{8\pi^2 f_{3\gamma} F_V(P^2)} \frac{-4M^2\Lambda^2}{P^2} \times \sum_{i=1}^{4n+1} f_i(1+\eta_i)\ln(1+\eta_i), \tag{79}$$

thus it cancels with $\int_0^1 h_V^{(1,\mathrm{delta})}(u)du$ as can be easily seen.

Our results are shown in Figs. 7 and 8 for twist three and twist four, respectively. The magnitude of both distributions is growing unlimitedly when the photon becomes softer (obviously distributions multiplied by the vector form factor remain finite). Notice however that for the real photon h_V decouples.

3. Axial DA

We have only one distribution in the axial vector channel which is of twist three. When inverting the definition (9), due to the presence of λ on the right-hand side, we obtain the expression for the derivative of DA rather then for DA itself

$$\tilde{\psi}'_A(u) = \psi'_A(u) + \psi_A(0)\delta(u) - \psi_A(1)\delta(\bar{u}) = -i\frac{8N_c}{f_{3\gamma}F_A(P^2)}\int\frac{d^Dk}{(2\pi)^D}\frac{T_A^{(0)}}{(k^2-M_k^2)((k-P)^2-M_{k-P}^2)} \times \delta(k\cdot n - uP^+), \tag{80}$$

with

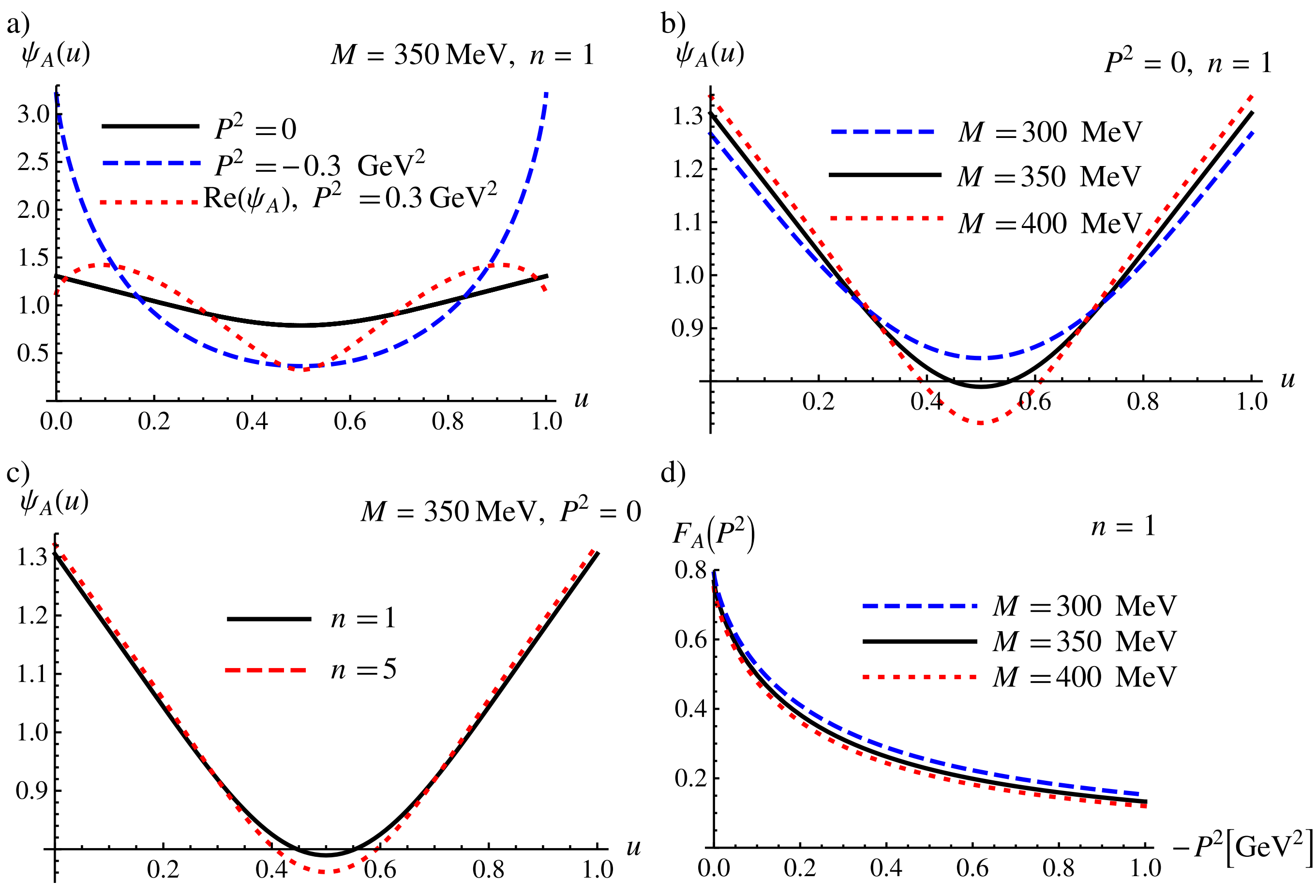

FIG. 9 (color online). Axial photon DA for (a) different photon virtualities and fixed $M = 350$ MeV and $n = 1$, (b) various M and fixed $n = 1$ and $P^2 = 0$, (c) two choices of $n = 1, 5$ and fixed $M = 350$ MeV and $P^2 = 0$. (d) Axial form factor for different choices of M (there is almost no n dependence).

$$T_A^{(0)} = -\frac{P^{+2}}{2}\left(k^- - \frac{P^2}{P^+}u\right) - \varepsilon^+ P^2(\vec{\varepsilon}_\perp \cdot \vec{k}_\perp). \qquad (81)$$

In the case of axial DA, the nonlocal part of the vertex does not give any contribution. This is simply because the Dirac trace is equal to zero. To obtain $\psi_A(u)$, one has to integrate (80) over du' from 0 to u:

$$\psi_A(u) = \int_0^u \tilde{\psi}'_A(u')du'. \qquad (82)$$

Notice that the end point contributions cancel out and one might get an impression that $\psi_A(u)$ is determined up to a constant. Fortunately, we have at our disposal an independent formula for $F_A(P^2)$ given by Eq. (12) [see, also, Eq. (D8) in Appendix D] and the normalization condition (15) that fix the value of $\psi_A(0) = \psi_A(1) \neq 0$ at nonzero value.

As in the case of vector twist two DA, $\psi_A(u)$ is UV divergent and requires subtraction of the perturbative part. The result splits into a regular part

$$\tilde{\psi}_A^{(a)\prime}(u) = \frac{N_c}{4\pi^2 f_{3\gamma} F_A(P^2)} \sum_{i,j=1}^{4n+1} f_i f_j \eta_i^{4n} \eta_j^{4n} ((\eta_i - \eta_j)\Lambda^2 + (1-2u)P^2) \ln\left(\frac{1 + u\bar{u}p^2 - \bar{u}\eta_i + u\eta_j}{u\bar{u}p^2}\right) \qquad (83)$$

and the piece involving δ functions:

$$\tilde{\psi}_A^{(a)\prime}(u) = \frac{N_c}{4\pi^2 f_{3\gamma} F_A(P^2)} \Lambda^2 \sum_{i=1}^{4n+1} f_i \eta_i^{4n}(1+\eta_i) \times \ln(1+\eta_i)[\delta(u) - \delta(u-1)]. \qquad (84)$$

Note that $\eta_i^{4n}(1+\eta_i) = r^2$, and in virtue of (39)

$$\tilde{\psi}_A^{(a)\prime}(u) = \frac{1}{F_A(P^2)}[\delta(u) - \delta(u-1)]. \qquad (85)$$

The form factor $F_A(P^2)$ and $\psi_A(u)$ itself is shown in Fig. 9. We obtain the following values for the axial form factor at zero momenta: $F_A(0) \approx 0.77$ for $M = 350$ MeV, $F_A(0) \approx 0.79$ for $M = 300$ MeV, and $F_A(0) \approx 0.75$ for $M = 400$ MeV.

VI. SUMMARY

In this work, we calculated analytically a set of photon distribution amplitudes up to twist four in tensor, vector, and axial vector channels. We used a nonlocal chiral quark model with momentum dependent quark mass. In order to get a correct behavior of low energy matrix elements, we modified vector vertices (making them nonlocal) in such a way that Ward-Takahashi identities were fulfilled (1). Similar, although the numerical calculation was already done in Ref. [18]. They also used an instanton motivated nonlocal model with dressed vertices, taking into account rescattering in the ρ meson channel. The shape of the mass dependence on momentum was chosen as an exponent decreasing with k^2. Here we use $F(k)$ as given by (17) and neglect rescattering which turns out to be small.

First we obtained numerical estimates for quark condensate $\langle\bar{\psi}\psi\rangle$, magnetic susceptibility χ_m, and decay constant $f_{3\gamma}$ in our model. For larger values of constituent quark mass M or power n, our results are getting close to the ones of Ref. [18]. Unlike $\langle\bar{\psi}\psi\rangle$ and χ_m, the value of $f_{3\gamma}$ is rather stable as far as model parameters are concerned. Using evolution equations (following Refs. [1,18]) we find that for $M = 350$ MeV and $n = 5$, the scale of our model is about $\mu \approx 500$ MeV (this estimation was done using $\langle\bar{\psi}\psi\rangle$, χ_m, $f_{3\gamma}$ as given by sum rules at 1 GeV scale and evolving them backwards down to the model values). This is in rough agreement with the scale of the instanton liquid model which is believed to be of the order of 600 MeV [7].

Next, let us discuss the properties of the distribution amplitudes obtained within the present approach. Leading twist amplitudes are not very sensitive to the value of power n. However, it seems that higher twist DAs are rather strongly model dependent.

Comparing our results with those of Ref. [18], we find some similarities, but also some discrepancies. Tensor leading twist DAs are in fact the same. For real photons they are almost constant with small maximum at $u = 1/2$ and they do not vanish at the end points. The contribution of the nonlocal part of the vertex is rather small; it is however producing the small maximum in the middle. For $P^2 \neq 0$, the end points move up for negative P^2 and down for positive P^2, whereas the middle value behaves in the opposite way.

Twist two vector DA (ϕ_V) should decouple for $P^2 = 0$. Here the importance of gauge invariance shows up. We find cancellation of two contributions to ϕ_V coming from the local and nonlocal parts of the photon vertex which are not proportional to P^2. The remaining part is therefore proportional to P^2 and decouples as required by the gauge invariance. We find that ϕ_V vanishes at the end points and develops minimum in the middle for $P^2 \ll 0$, whereas for $P^2 \gg 0$ it has a bell-like shape with a small dip in the middle. This behavior is very different from the one obtained in Ref. [18], where ϕ_V is almost flat and does not vanish at the end points. However both vector and also tensor form factors are quite similar in both cases.

One has to note that because of the subtraction of the perturbative part that is required in this case, ϕ_V develops an imaginary part for positive photon virtualities, so in this case we only discuss the real part.

As far as higher twist DAs are concerned, the situation is as follows.

Our tensor twist three DA (ψ_T) is identically zero for $P^2 = 0$, because it is simply proportional to P^2. For $P^2 < 0$ it is negative and has the shape of an inverted "U," similar to the one of Ref. [18]. In this case, DA is a regular function without δ-type singularities. Vector twist three DA (ψ_V) in our case blows up at the end points; such behavior is not seen in Ref. [18]. However, similar to Ref. [18], we also obtained delta-type singularities at the edges of the physical support. Twist three axial DAs (ψ_A) in both cases show similar behavior: they do not vanish at the end points and have a minimum for $u = 1/2$. Despite the fact that for $P^2 = 0$ axial vector DAs, both in our case and in the case of Ref. [18] look similar, the axial form factors behave differently for $P^2 < 0$. In our case, $F_A(P^2)$ vanishes at large negative momenta, contrary to the one of Ref. [18] that tends to unity in the same limit.

A regular part (without delta-type singularities) of twist four tensor DA (h_T) is in our case positive and vanishes at the end points for $P^2 = 0$ whereas in Ref. [18] it is negative and does not vanish at the end points. For spacelike photon momentum $P^2 < 0$, we see some similarity in shape between our h_T and $-h_T$ of Ref. [18]. Vector twist four DA (h_V) is in our case a result of large cancellation of the positive nonlocal piece and the negative local piece. Its properties are not discussed in detail in Ref. [18].

The only phenomenologically accessible photon distribution amplitude is the leading twist tensor DA—ϕ_T. It is almost flat and does not vanish at the end points. This behavior is seen in our model and in other models discussed in Ref. [18] and also in Refs. [9,17]. Flat DA is characteristic for the elementary pointlike particle, however, it is violating factorization theorems of QCD that require the DAs to vanish for $u = 0, 1$. Formal evolution of such an amplitude is questionable, not only because the Gegenbauer series is not convergent at the end points, but also, because potentially large contributions coming from the vicinity of $u = 0, 1$ are not summed by the Efremov-Radyushkin-Brodsky-Lepage evolution equations [31].

Let us finish by a remark, that the light-cone wave functions that do not vanish at the end points are unnatural in the models with confinement. Indeed, the configurations in which one of the quarks takes all of the longitudinal momentum are expected to be suppressed, as discussed in Ref. [32].

ACKNOWLEDGMENTS

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APPENDIX A: IDENTITIES

In this appendix, we summarize some of the identities used in this paper that deal with the sums of factors f_i and powers of η_i. Some of them have been already introduced in Ref. [9], but the general proofs have not been given. For definiteness, let us recall that η_i, $i = 1, \ldots, 4n+1$ are the solutions of the algebraic equation $G(z) = z^{4n+1} + z^{4n} - r^2 = 0$. We denote

$$f_i = \prod_{j\neq i} (\eta_i - \eta_j)^{-1}.$$

For *any* set of $4n + 1$ complex numbers η_i [not necessarily satisfying $G(z) = 0$] we have

$$\sum_{i=1}^{4n+1} f_i \eta_i^N = \begin{cases} 1 & \text{for } N = 4n, \\ 0 & \text{for } N < 4n. \end{cases} \tag{A1}$$

To prove (A1), let us define the following function

$$f(z) = \frac{z^M}{\prod_{i=1}^{4n+1}(z - \eta_i)}, \tag{A2}$$

where $M \leq 4n$ and integrate it over a circle with infinite radius. On one hand, we can use residue technique to get the sum entering (A1), on the other hand, direct integration over the large circle gives the right-hand side of (A1).

If, in addition, η_i satisfies $G(\eta_i) = 0$, then

$$\sum_{i=1}^{4n+1} f_i \eta_i^P = (-1)^P \tag{A3}$$

for $4n \leq P \leq 8n$. This can be proven in the following way. Notice that for $P = 4n$ equality (A3) is satisfied due to (A1). Let us move to $P = 4n + 1$, that is we want to calculate

$$x = \sum_{i=1}^{4n+1} f_i \eta_i^{4n+1}. \tag{A4}$$

Adding to this equation the result for $P = 4n$ we get

$$\sum_{i=1}^{4n+1} f_i (\eta_i^{4n+1} + \eta_i^{4n}) = x + 1. \tag{A5}$$

Using the fact that $G(\eta_i) = \eta_i^{4n+1} + \eta_i^{4n} - r^2 = 0$ and (A1) we have

$$x + 1 = r^2 \sum_{i=1}^{4n+1} f_i = 0, \tag{A6}$$

and $x = -1$. This procedure can be repeated several times to prove (A3) until $P = 8n$.

For any set of $4n + 1$ complex numbers η_i we have the following identity

$$\sum_{i=1}^{4n+1} f_i \eta_i^{4n+1} = \sum_{i=1}^{4n+1} \eta_i. \tag{A7}$$

Imposing in addition the constraint $G(\eta_i) = 0$ we get $\sum_i^{4n+1} \eta_i = -1$. The proof is similar to the one of (A1), however we integrate the following function

$$g(z) = \frac{z^{4n+1}}{(z-\eta_1)(z-\eta_2)\ldots(z-\eta_i)^2\ldots(z-\eta_{4n+1})}. \tag{A8}$$

After several algebraic steps we arrive at (A7).

APPENDIX B: PION DECAY CONSTANT

In our model Birse-Bowler formula (18) for pion decay the constant reduces to the following form

$$F_\pi^2 = -\frac{N_c M^2}{4\pi^2}\sum_{i,j=1}^{4n+1} f_i f_j \eta_i^{2n}((1+2n(1+2n))\eta_j^{2n+1} + (1+4n(1+3n))\eta_j^{2n} + 2n(1+6n)\eta_j^{2n-1} + 4n^2\eta_j^{2n-2})\left(\frac{\epsilon_{ij}}{\eta_i-\eta_j}(\ln(1+\eta_i)-\ln(1+\eta_j)) + \frac{\delta_{ij}}{1+\eta_i}\right) \tag{B1}$$

where ϵ_{ij} is 0 for $i=j$ and 1 otherwise, while δ_{ij} is the Kronecker delta.

APPENDIX C: LIGHT-CONE INTEGRALS IN SCHWINGER REPRESENTATION

Here we summarize formulas used to perform $d^D k$ loop integration in the presence of $\delta(n\cdot\kappa - un\cdot p)$. We will consider three cases when the numerator contains no k^- at all and one or two powers of k^-. We follow closely the method of Ref. [33].

Consider loop integral (26) and apply to it (28):

$$\mathcal{J} = \mathcal{A}\Lambda^{D-5}\int\frac{d^D\kappa}{(2\pi)^D}\delta(n\cdot\kappa - un\cdot p) \times \sum_{i,j=1}^{4n+1} f_i f_j \frac{\eta_i^{4n}\eta_j^{4n}\mathcal{N}}{(z_1-\eta_i)(z_2-\eta_j)} \tag{C1}$$

expressed in terms of scaled variables (24). Here $\mathcal{N}$ is the numerator to be specified later. Recall that

$$z_1 = (\kappa-p)^2 - 1 + i\epsilon, \qquad z_2 = \kappa^2 - 1 + i\epsilon. \tag{C2}$$

We shall now make continuation to the Euclidean metric:

$$\kappa^0 = i\kappa^4 \tag{C3}$$

with

$$\kappa^2 \to -\vec{\kappa}^2, \qquad \kappa\cdot p \to -\vec{\kappa}\cdot\vec{p}, \qquad n\cdot\kappa \to -\vec{n}\cdot\vec{\kappa} \tag{C4}$$

where the arrows denote D dimensional Euclidean vectors. Therefore

$$\mathcal{J} = i\mathcal{A}\Lambda^{D-5}\int\frac{d^D\vec{\kappa}}{(2\pi)^D}\delta(\vec{n}\cdot\vec{\kappa} + up^+) \times \sum_{i,j=1}^{4n+1} f_i f_j \frac{\eta_i^{4n}\eta_j^{4n}\mathcal{N}}{(\vec{\kappa}^2+1+\eta_i)((\vec{\kappa}-\vec{p})^2+1+\eta_j)}. \tag{C5}$$

We shall parametrize now

$$\frac{1}{(\vec{\kappa}^2+1+\eta_i)((\vec{\kappa}-\vec{p})^2+1+\eta_j)} = \int_0^\infty d\alpha\int_0^\infty d\beta e^{-\alpha(\vec{\kappa}^2+1+\eta_i)-\beta((\vec{\kappa}-\vec{p})^2+1+\eta_j)} \tag{C6}$$

and

$$\delta(\vec{n}\cdot\vec{\kappa}+up^+) = \int_{-\infty}^{\infty}\frac{d\lambda}{2\pi}e^{-i\lambda(\vec{n}\cdot\vec{\kappa}+up^+)}. \tag{C7}$$

It is convenient to introduce new variables:

$$\alpha+\beta = s, \qquad \beta = ys, \qquad \alpha = (1-y)s = -\bar{y}s. \tag{C8}$$

Integration measure then reads

$$\int_0^\infty d\alpha\int_0^\infty d\beta = \int_0^\infty sds\int_0^1 dy. \tag{C9}$$

Finally, we will shift momentum

$$\vec{\kappa} = \vec{\kappa}' + \left(y\vec{p} - i\frac{\lambda}{2s}\vec{n}\right). \tag{C10}$$

In these new variables we have

$$\mathcal{J} = i\mathcal{A}\Lambda^{D-5}\sum_{i,j=1}^{4n+1} f_i f_j\eta_i^{4n}\eta_j^{4n}\int_0^1 dy\int\frac{d\lambda}{2\pi}e^{-i\lambda p^+(u-y)} \times\int_0^\infty sdse^{-s[1-\bar{y}\eta_i+y\eta_j+y\bar{y}p^2]}\int\frac{d^D\vec{\kappa}'}{(2\pi)^D}\mathcal{N}e^{-s\vec{\kappa}'^2}. \tag{C11}$$

Further calculations depend on the nature of $\mathcal{N}$. If $\mathcal{N}$ can be expressed entirely in terms of $z_{1,2}$ then, in virtue of (28), it is enough to replace pertinent powers of $z_{1,2}^N \to \eta_{i,j}^N$ and perform Gaussian integration over κ'. Let us denote such an integral as $\mathcal{J}_0$. Also the integral over $d\lambda$ is trivial. In the following we shall need also integrals with λ and λ^2 which read

$$\int\frac{d\lambda}{2\pi}\{1,\lambda,\lambda^2\}e^{-i\lambda p^+(u-y)} = \frac{1}{p^+}\left\{1, -\frac{i}{p^+}\partial_y, -\frac{1}{p^{+2}}\partial_y^2\right\} \times\delta(u-y). \tag{C12}$$

Hence (for $D = 4-2\varepsilon$) we get

$$\mathcal{J}_0 = i\mathcal{A}\left(\frac{1}{4\pi}\right)^{2-\varepsilon}\frac{\Lambda^{-1-2\varepsilon}}{p^+}\sum_{i,j=1}^{4n+1} f_i\eta_i^{4n} f_j\eta_j^{4n}\mathcal{N}(\eta_i,\eta_j) \times \int_0^\infty dss^{\varepsilon-1}e^{-s[1-\bar{u}\eta_i+u\eta_j+u\bar{u}p^2]}. \quad (C13)$$

In order to perform the integral over ds we shall use

$$\int_0^\infty dss^{\varepsilon-1-n}e^{-s[\cdots]} = [\cdots]^{n-\varepsilon}\Gamma(\varepsilon-n) \simeq \frac{[\cdots]^{n-\varepsilon}}{\varepsilon(\varepsilon-1)\ldots(\varepsilon-n)}e^{-\gamma\varepsilon}+\ldots \quad (C14)$$

arriving at

$$\mathcal{J}_0 = i\frac{\mathcal{A}}{16\pi^2P^+}\left(\frac{4\pi e^{-\gamma}}{\Lambda^2}\right)^{\varepsilon}\frac{1}{\varepsilon}\sum_{i,j=1}^{4n+1} f_i\eta_i^{4n} f_j\eta_j^{4n} \times \frac{\mathcal{N}}{[1-\bar{u}\eta_i+u\eta_j+u\bar{u}p^2]^{\varepsilon}} = i\frac{\mathcal{A}}{16\pi^2P^+}\left(\frac{4\pi e^{-\gamma}}{\Lambda^2}\right)^{\varepsilon}\sum_{i,j=1}^{4n+1} f_i\eta_i^{4n} f_j\eta_j^{4n} \times \mathcal{N}\left(\frac{1}{\varepsilon}-\ln[1-\bar{u}\eta_i+u\eta_j+u\bar{u}p^2]\right). \quad (C15)$$

If numerator $\mathcal{N}$ involves additionally one power of κ^μ, we have then

$$\mathcal{N} \to \mathcal{N}(\eta_i,\eta_j)(w\cdot\kappa) = -\mathcal{N}(\eta_i,\eta_j)(\vec{w}\cdot\vec{\kappa}) \quad (C16)$$

where w is a constant four-vector. Let us denote such an integral by $\mathcal{J}_1$. Here the only difference from the previous case comes from the integration over κ'. Since

$$\vec{w}\cdot\vec{\kappa} = \vec{w}\cdot\vec{\kappa}' + \left(y\vec{w}\cdot\vec{p} - i\frac{\lambda}{2s}\vec{w}\cdot\vec{n}\right) \quad (C17)$$

only the terms in parenthesis survive. After integrating over $d\lambda$ with the help of (C12) and over dy (in the case of δ' we have to integrate by parts) we arrive, back in the Minkowski metric, at

$$\mathcal{J}_1 = i\frac{\mathcal{A}}{16\pi^2P^+}\left(\frac{4\pi e^{-\gamma}}{\Lambda^2}\right)^{\varepsilon}\frac{1}{\varepsilon} \times \sum_{i,j=1}^{4n+1} f_if_j\frac{z_i^{4n}z_j^{4n}\mathcal{N}}{[1-\bar{u}z_i+uz_j+u\bar{u}p^2]^{\varepsilon}}\left\{u(w\cdot P) + \frac{(w\cdot n)}{2P^+}((\eta_i-\eta_j)\Lambda^2+(1-2u)P^2) - \frac{(w\cdot n)}{2P^+} \times \frac{\Lambda^2}{\varepsilon-1}[(1+\eta_j)\delta(u-1)-(1+\eta_i)\delta(u)]\right\} \quad (C18)$$

where $p^2 = P^2/\Lambda^2$. Note that if $w = n$ then $w\cdot n = 0$ and we get $\mathcal{J}_0$ of Eq. (C15) multiplied by uP^+ as it should be, since we could have used $\delta(k\cdot n - un\cdot P)$ in the first place. Similarly, if $w = \varepsilon_\perp$ we have $\mathcal{J}_1 = 0$ which means that a single power of $\kappa_\perp$ integrates to zero. Note that due to Lorentz invariance after du integration, the coefficient in front of $w\cdot n$ should vanish in accordance with (23).

Finally, if the numerator contains $\kappa^\mu\kappa^\nu$, let us call such an integral $\mathcal{J}_2$, we have

$$\mathcal{N} \to \mathcal{N}(\eta_i,\eta_j)(w\cdot\kappa)(v\cdot\kappa) = \mathcal{N}(\eta_i,\eta_j)(\vec{w}\cdot\vec{\kappa})(\vec{v}\cdot\vec{\kappa}). \quad (C19)$$

Using (C12) and integrating over dy, we get three different contributions to $\mathcal{J}_2$ depending on the tensor structure:

$$\mathcal{J}_2^{(0)} = i\frac{\mathcal{A}}{16\pi^2}\frac{\Lambda^2}{P^+}\left(\frac{e^{-\gamma}}{4\pi\Lambda^2}\right)^{-\varepsilon}\frac{1}{\varepsilon}\sum_{i,j=1}^{4n+1} f_if_j \times \frac{\eta_i^{4n}\eta_j^{4n}\mathcal{N}}{[1-\bar{u}\eta_i+u\eta_j+u\bar{u}p^2]^{\varepsilon}} \times \left\{-\frac{1}{2}\frac{[1-\bar{u}\eta_i+u\eta_j+u\bar{u}p^2]}{(\varepsilon-1)}(w\cdot v) + \frac{u^2}{\Lambda^2}(v\cdot P)(w\cdot P)\right\}, \quad (C20)$$

$$\mathcal{J}_2^{(1)} = i\frac{\mathcal{A}}{16\pi^2}\frac{\Lambda^2}{2P^{+2}}\left(\frac{e^{-\gamma}}{4\pi\Lambda^2}\right)^{-\varepsilon}\frac{1}{\varepsilon}\{(w\cdot P)(v\cdot n) + (w\cdot n)(v\cdot P)\}\sum_{i,j=1}^{4n+1} f_if_j\frac{\eta_i^{4n}\eta_j^{4n}\mathcal{N}}{[1-\bar{u}\eta_i+u\eta_j+u\bar{u}p^2]^{\varepsilon}} \times\{(1+\eta_j)\delta(u-1)-(1-\bar{u}\eta_i+u\eta_j+u\bar{u}p^2) + u[(\eta_i-\eta_j)+(1-2u)p^2]\}, \quad (C21)$$

and finally

$$\mathcal{J}_2^{(2)} = i\frac{\mathcal{A}}{16\pi^2}\frac{\Lambda^4}{4P^{+3}}\left(\frac{e^{-\gamma}}{4\pi\Lambda^2}\right)^{-\varepsilon}\frac{1}{\varepsilon}(w\cdot n)(v\cdot n) \times \sum_{i,j=1}^{4n+1} f_if_j\frac{\eta_i^{4n}\eta_j^{4n}\mathcal{N}}{[1-\bar{u}\eta_i+u\eta_j+u\bar{u}p^2]^{\varepsilon}} \times\left\{\frac{1}{2}[[1+\eta_j]^2\delta'(u-1)-[1+\eta_i]^2\delta'(u)] + [(\eta_i-\eta_j)+(1-2u)p^2][[1+\eta_j]\delta(u-1) - [1+\eta_i]\delta(u)] + [1-\bar{u}\eta_i+u\eta_j+u\bar{u}p^2]up^2 + [(\eta_i-\eta_j)+(1-2u)p^2]^2\right\}. \quad (C22)$$

In Eq. (C22) we encounter derivatives of δ functions; it is here implicitly assumed that the coefficient $\mathcal{N}[1-\bar{u}\eta_i+u\eta_j+u\bar{u}p^2]^{-\varepsilon}$ when multiplied by $\delta'(u-1)$ or $\delta'(u)$ is taken at the corresponding value of u. Note that Lorentz invariance requires that

$$\int_0^1 du \mathcal{J}_2^{(1,2)} = 0 \tag{C23}$$

(modulo possible subtraction of the perturbative part).

Finally, let us remark that if we need an integral of $k_\perp^2$ we may use the following trick in two dimensional transverse plane:

$$\int d^2\vec{k}_\perp \vec{k}_\perp^2 = 2\int d^2\vec{k}_\perp (\vec{\varepsilon}_\perp \cdot \vec{k}_\perp)^2 \tag{C24}$$

if there is no other dependence on the transverse angle, as it indeed happens in our case. We can then evaluate the right-hand side of Eq. (C24) using the formulas from the present appendix.

APPENDIX D: AXIAL FORM FACTOR

In this appendix we show, as an example, simple calculation of the axial form factor. We start from the matrix element on the left-hand side in Eq. (12):

$$\begin{aligned}\langle 0|\bar{\psi}(-\lambda n)\gamma^\mu\gamma_5\psi(\lambda n)|\gamma(P,\varepsilon)\rangle &= -eN_c\varepsilon_\nu \int \frac{d^Dk}{(2\pi)^D} e^{i(2k\cdot n - P\cdot n)\lambda} \\ &\times \mathrm{Tr}\left\{\gamma^\mu\gamma_5 \frac{1}{\not{k} - M_k}\gamma^\nu \frac{1}{(\not{k}-\not{P}) - M_{k-P}}\right\}. \end{aligned}\tag{D1}$$

Calculating the trace and taking the derivative with respect to λ as in the definition (12) we obtain

$$\begin{aligned}\mathcal{M} &\equiv \frac{d}{d\lambda}\langle 0|\bar{\psi}(-\lambda n)\gamma^\mu\gamma_5\psi(\lambda n)|\gamma(P,\varepsilon)\rangle|_{\lambda=0} \\ &= 4eN_c\varepsilon_\nu P_\beta \varepsilon^{\mu\nu\alpha\beta}\int \frac{d^Dk}{(2\pi)^D}\frac{k_\alpha(2k\cdot n - P\cdot n)}{D(k)D(k-P)}.\end{aligned}$$

Using Lorentz invariance and some simple algebra we get

$$\begin{aligned}\mathcal{M} &= -2eN_c\varepsilon_\nu P_\alpha n_\beta \varepsilon^{\mu\nu\alpha\beta}\int\frac{d^Dk}{(2\pi)^D} \\ &\times \frac{(2k\cdot n - P\cdot n)(k\cdot\tilde{n} - \bar{p}k\cdot n)}{D(k)D(k-P)}\end{aligned}\tag{D2}$$

with $\bar{p} = P^2/P^{+2}$. Comparing this with the right-hand side of (12) we get the following expression:

$$F_A(P^2) = \frac{2iN_c}{f_{3\gamma}}\mathcal{J}, \tag{D3}$$

where $\mathcal{J}$ denotes the integral in (D2). However, it can be easily shown that Lorentz invariance requires that

$$\begin{aligned}&\int \frac{d^Dk}{(2\pi)^D}\frac{k\cdot nk\cdot\tilde{n} - \bar{p}(k\cdot n)^2}{D(k)D(k-P)} \\ &= \frac{2}{2-D}\int\frac{d^Dk}{(2\pi)^D}\frac{k_T^2}{D(k)D(k-P)}\end{aligned}\tag{D4}$$

and

$$\int \frac{d^Dk}{(2\pi)^D}\frac{k\cdot\tilde{n} - \bar{p}k\cdot n}{D(k)D(k-P)} = 0. \tag{D5}$$

Using this, $\mathcal{J}$ reduces to

$$\mathcal{J} = \frac{4}{2-D}\int\frac{d^Dk}{(2\pi)^D}\frac{k_T^2}{D(k)D(k-P)}. \tag{D6}$$

This integral reads

$$\begin{aligned}\mathcal{J} &= \frac{\Lambda^{D-2}}{p^+}\frac{4i}{D-2}\left(\frac{1}{4\pi}\right)^{D/2}\sum_{i,j=1}^{4n+1} f_if_j\eta_i^{4n}\eta_j^{4n} \\ &\times \int_0^1 du(1-\bar{u}\eta_i + u\eta_j + u\bar{u}p^2)^{1-\epsilon}\Gamma(\epsilon).\end{aligned}\tag{D7}$$

Expanding in ϵ and subtracting the perturbative part we obtain the final expression

$$\begin{aligned}F_A(P^2) &= \frac{1}{4\pi^2}\frac{\Lambda^2N_c}{f_{3\gamma}}\sum_{i,j=1}^{4n+1} f_if_j\eta_i^{4n}\eta_j^{4n}\int_0^1 du(1-\bar{u}\eta_i \\ &+ u\eta_j + u\bar{u}p^2)\ln\left(\frac{1-\bar{u}\eta_i + u\eta_j + u\bar{u}p^2}{u\bar{u}p^2}\right).\end{aligned}\tag{D8}$$

Integration over du can, in principle, be done analytically (taking into account remarks given in the main text), however here we just plot $F_A(P^2)$ in Sec. V B 3.

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