

Single spin asymmetries in $lp \rightarrow hX$ processes: A test of factorization

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Predictions for the transverse single spin asymmetry (SSA), A_N , are given for the inclusive processes $lp^\uparrow \rightarrow hX$ and $lp^\uparrow \rightarrow \text{jet} + X$, which could be measured in operating or future experiments. These estimates are based on the Sivvers distributions and the Collins fragmentation functions which fit the azimuthal asymmetries measured in semi-inclusive deep inelastic scattering (SIDIS) processes ($lp^\uparrow \rightarrow l'hX$). The factorization in terms of transverse momentum dependent distribution and fragmentation functions (TMD factorization)—which supplies the theoretical framework in which SIDIS azimuthal asymmetries are analyzed—is assumed to hold also for the $lp \rightarrow hX$ inclusive process at large P_T . A measurement of A_N would then provide a direct test of the validity of the TMD factorization in this case and would have important consequences for the study and understanding of SSAs in $pp^\uparrow \rightarrow hX$ processes.

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I. INTRODUCTION

Transverse single spin asymmetries (SSAs) in semi-inclusive deep inelastic scattering (SIDIS), $lN \rightarrow l'hX$, have been measured by HERMES [1–4] and COMPASS [5–8]. A large amount of data is still being analyzed by these collaborations and new results are expected soon from the JLab experiments at 6 GeV. A rich program focused on azimuthal asymmetries, as a way of probing the internal nucleon structure, is planned for JLab operating at an upgraded energy of 12 GeV and for the future electron-ion (EIC) or electron-nucleon (ENC) colliders, which are under active consideration within the hadron physics scientific community (see e.g. Ref. [9] for a short up-to-date overview).

These SIDIS SSAs are interpreted and discussed in terms of unintegrated, transverse momentum dependent, distribution and fragmentation functions (shortly, TMDs). In particular, the Sivvers distributions [10,11] and the Collins fragmentation functions [12] have been extracted [13–18] from SIDIS data, and, thanks to complementary information from Belle on the Collins function [19,20], a first extraction of the transversity distribution has been possible [21,22].

All these analyses have been performed in the $\gamma^* - p$ c.m. frame, within a QCD factorization scheme, according to which the SIDIS cross section is written as a convolution of TMDs and elementary interactions:

$$d\sigma^{lp \rightarrow l'hX} = \sum_q \hat{f}_{q/p}(x, \mathbf{k}_\perp; Q^2) \otimes d\hat{\sigma}^{lq \rightarrow l'q} \otimes \hat{D}_{h/q}(z, \mathbf{p}_\perp; Q^2), \quad (1)$$

where \mathbf{k}_\perp and \mathbf{p}_\perp are, respectively, the transverse momentum of the quark in the proton and of the final hadron with respect to the fragmenting quark. At order k_\perp/Q the observed transverse momentum, \mathbf{P}_T , of the hadron is given by

$$\mathbf{P}_T = \mathbf{k}_\perp + z\mathbf{p}_\perp. \quad (2)$$

There is a general consensus [23–27] that such a scheme holds in the kinematical region defined by

$$P_T \simeq k_\perp \simeq \Lambda_{\text{QCD}} \ll Q. \quad (3)$$

The presence of the two scales, small P_T and large Q , allows one to identify the contribution from the unintegrated partonic distribution ($P_T \simeq k_\perp$), while remaining in the region of validity of the QCD parton model. At larger values of P_T other mechanisms, like quark-gluon correlations and higher order pQCD contributions, become important [27–29]. A similar situation [24,26,30–35] holds for Drell-Yan processes, $AB \rightarrow l^+l^-X$, where the two scales are the small transverse momentum, q_T , and the large invariant mass, M , of the dilepton pair.

The situation is not so clear for processes in which only one large scale is detected, like the inclusive production, at large P_T , of a single particle in hadronic interactions, $AB \rightarrow CX$. However, the most striking and large SSAs have been [36–39] and keep being measured [40–44] in these cases. The TMD factorization for these processes

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was first suggested in Refs. [10,11] and adopted in Refs. [45–47] to explain the large single spin asymmetries observed by the E704 Collaboration [37,39]. The same approach led to successful predictions [48,49] for the values of A_N measured at RHIC [50].

Alternative approaches to explain the origin of SSAs, linking collinear partonic dynamics to higher-twist quark-gluon correlations, were originally proposed in Refs. [51–55] and phenomenologically adopted in Refs. [56–59]. These two approaches, the TMD factorization and the higher-twist correlations, have been shown to be somewhat related [60,61] and consistent with each other [32,33,62].

However, a definite proof of the validity of the TMD factorization for hadronic inclusive processes with one large scale only is still lacking. Because of this, the study of dijet production at large P_T in hadronic processes was proposed [63–66], where the second small scale is the total q_T of the two jets, which is of the order of the intrinsic partonic momentum k_\perp . This approach leads to a modified TMD factorization approach, with the inclusion in the elementary processes of gauge link color factors [67–69].

In this paper we propose a phenomenological test of the validity of the TMD factorization in cases in which only one large scale is detected, by considering SSAs for the $lp^\dagger \rightarrow hX$ process, with the detection, in the lepton-proton c.m. frame, of a single large P_T final particle, typically a pion. The final lepton is not observed; notice, however, that a large value of P_T implies, at leading perturbative order, large values of Q^2 . Such a measurement is the exact analogue of the SSAs observed in the $pp^\dagger \rightarrow hX$ processes, the well-known and large left-right asymmetries A_N [36–44]. We compute these SSAs assuming the TMD factorization and using the relevant TMDs (Sivers and Collins functions) as extracted from SIDIS data.

Such a choice is natural for the Collins function, which is expected to be universal [70,71]. The Sivers distribution, instead, is expected to be process dependent as it is originated by final (or initial, depending on the process considered) state interactions, which also model the gauge links necessary for its correct gauge invariant definition [23,72,73]. However, these final state interactions should be the same in usual SIDIS processes and in the process considered here.

A similar idea of computing left-right asymmetries in SIDIS processes, although with different motivations and still demanding the observation of the final lepton, has been discussed in Ref. [74]. A first simplified study of A_N in $lp^\dagger \rightarrow hX$ processes was performed in Ref. [75]. The process was also considered in Refs. [76,77] in the framework of collinear factorization with twist-three correlation functions, obtaining anomalously large asymmetries with a sign opposite to that of the corresponding asymmetries in pp processes.

The plan of the paper is the following: in Sec. II we present the formalism for the study of SSAs in a TMD

approach for both the $p^\dagger l \rightarrow hX$ and the $p^\dagger l \rightarrow \text{jet} + X$ processes; in Sec. III we show our numerical estimates of the contributions of the Sivers and Collins effects to A_N , based on the present knowledge of TMDs, for several different kinematical setups and discuss their phenomenological aspects; finally, in Sec. IV we give some comments and conclusions. Technical details on the full noncollinear kinematics are given in Appendix A, while the calculation of the helicity amplitudes is worked out in Appendix B. The complete expression of A_N for the process $p^\dagger l \rightarrow hX$, including all TMD contributions at leading twist, can be found in Appendix C.

II. FORMALISM

A. Large P_T hadron production

We propose to study single spin asymmetries for the process $p^\dagger l \rightarrow hX$ in close analogy to the study of the SSAs for the process $p^\dagger p \rightarrow hX$, assuming the validity of the TMD factorization. The cross section for this process can then be written as a particular case of the general treatment, in a factorized scheme, of the $(A, S_A) + (B, S_B) \rightarrow C + X$ large P_T inclusive polarized process [48,78–80]:

$$\begin{aligned} \frac{E_h d\sigma^{(p,S)+l \rightarrow h+X}}{d^3 P_h} &= \sum_{q,\{\lambda\}} \int \frac{dx dz}{16\pi^2 x z^2 s} d^2 \mathbf{k}_\perp d^3 \mathbf{p}_\perp \delta(\mathbf{p}_\perp \cdot \hat{\mathbf{p}}'_q) \\ &\times J(\mathbf{p}_\perp) \delta(\hat{s} + \hat{t} + \hat{u}) \left\{ \rho_{\lambda_q, \lambda'_q}^{q/p,S} \right. \\ &\times \hat{f}_{q/p,S}(x, \mathbf{k}_\perp) \frac{1}{2} \hat{M}_{\lambda_q, \lambda_l; \lambda_q, \lambda_l} \hat{M}_{\lambda'_q, \lambda_l; \lambda'_q, \lambda_l}^* \\ &\left. \times \hat{D}_{\lambda_q, \lambda'_q}^{\lambda_h, \lambda_h}(z, \mathbf{p}_\perp) \right\}, \end{aligned} \quad (4)$$

which can be shortened, with obvious notations, as

$$\begin{aligned} d\sigma^S &= \sum_{q,\{\lambda\}} \int \frac{dx dz}{16\pi^2 x z^2 s} d^2 \mathbf{k}_\perp d^3 \mathbf{p}_\perp \delta(\mathbf{p}_\perp \cdot \hat{\mathbf{p}}'_q) J(\mathbf{p}_\perp) \\ &\times \delta(\hat{s} + \hat{t} + \hat{u}) \Sigma(S)^{q l \rightarrow q l}(x, z, \mathbf{k}_\perp, \mathbf{p}_\perp), \end{aligned} \quad (5)$$

where $\Sigma(S)$ is the term in curly brackets of Eq. (4).

Let us recall the main features of these equations.

- We consider the collision of a polarized proton (or, in general, a nucleon) in a pure transverse spin state S with an unpolarized lepton, *in the proton-lepton center of mass frame*. The proton p moves along the positive $Z_{\text{c.m.}}$ axis and hadron h is produced in the $(XZ)_{\text{c.m.}}$ plane. We define as transverse polarization for the proton the $Y_{\text{c.m.}}$ direction, often using the notation \uparrow and \downarrow , respectively, for protons polarized along or opposite to $Y_{\text{c.m.}}$. The $X_{\text{c.m.}}$ axis is defined in such a way that a hadron h with $(P_h)_{X_{\text{c.m.}}} > 0$ is produced *to the left* of the incoming proton. The transverse momentum is denoted as \mathbf{P}_T . This kinematical configuration is shown in Fig. 1. Results for

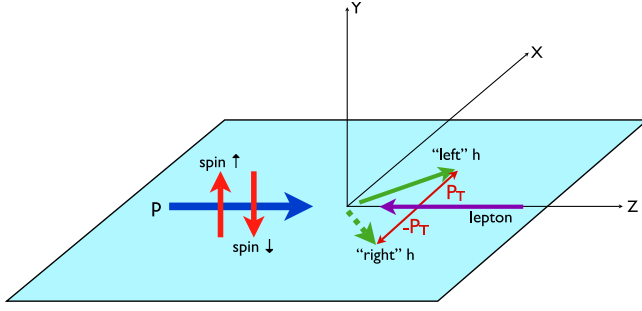


FIG. 1 (color online). Kinematical configuration and conventions for the $p^l l \rightarrow hX$ process.

the case of leptons moving along the positive $Z_{\text{c.m.}}$ axis ($lp^l \rightarrow hX$) will also be discussed in the paper.

- The notation $\{\lambda\}$ implies a sum over *all* helicity indices. x and z are the usual light-cone momentum fractions, of partons in hadrons (x) and hadrons in partons (z). \mathbf{k}_\perp and \mathbf{p}_\perp are, respectively, the transverse momentum of the parton q with respect to its parent nucleon p , and of hadron h with respect to its parent parton q . \mathbf{p}'_q is the three-momentum of the final fragmenting parton; it can be expressed in terms of the integration variables and the observed final hadron momentum. We consider all partons as massless, neglecting heavy quark contributions. Full details can be found in Ref. [80] and useful expressions are given in Appendix A.
- With massless partons, the function J is given by [48]

$$J(p_\perp) = \frac{(E_h + \sqrt{\mathbf{P}_h^2 - \mathbf{p}_\perp^2})^2}{4(\mathbf{P}_h^2 - \mathbf{p}_\perp^2)}. \quad (6)$$

In the kinematical regions which we shall consider J is close to 1.

- $\rho_{\lambda_q, \lambda'_q}^{q/p, S}$ is the helicity density matrix of parton q inside the polarized proton p , with spin state S . $\hat{f}_{q/p, S}(x, \mathbf{k}_\perp)$ is the distribution function of the unpolarized parton q inside the polarized proton p . The products $\rho_{\lambda_q, \lambda'_q}^{q/p, S} \hat{f}_{q/p, S}(x, \mathbf{k}_\perp)$ are directly related to the leading-twist TMDs, with a dependence on ϕ , the azimuthal angle of \mathbf{k}_\perp [80].
- The $\hat{M}_{\lambda_q, \lambda_l; \lambda_q, \lambda_l}$'s are the helicity amplitudes for the elementary process $ql \rightarrow ql$, normalized so that the unpolarized cross section, for a collinear collision, is given by

$$\frac{d\hat{\sigma}^{ql \rightarrow ql}}{d\hat{t}} = \frac{1}{16\pi\hat{s}^2} \frac{1}{4} \sum_{\lambda_q, \lambda_l} |\hat{M}_{\lambda_q, \lambda_l; \lambda_q, \lambda_l}|^2. \quad (7)$$

At lowest perturbative order $ql \rightarrow ql$ is the only elementary interaction which contributes; notice that, in the presence of parton intrinsic motion, it is not a planar process in our chosen frame and depends

on the intrinsic momenta, including their phases. Neglecting lepton and quark masses there are two independent helicity amplitudes:

$$\begin{aligned} \hat{M}_{++++}(\hat{s}, \hat{t}, \hat{u}, \mathbf{k}_\perp) &= \hat{M}_{----}^* = -8\pi e_q \alpha \frac{\hat{s}}{\hat{t}} e^{i\varphi_1} \\ &\equiv \hat{M}_1^0 e^{i\varphi_1} \end{aligned} \quad (8)$$

$$\begin{aligned} \hat{M}_{+-;+-}(\hat{s}, \hat{t}, \hat{u}, \mathbf{k}_\perp) &= \hat{M}_{-+;-+}^* = 8\pi e_q \alpha \frac{\hat{u}}{\hat{t}} e^{i\varphi_2} \\ &\equiv \hat{M}_2^0 e^{i\varphi_2}, \end{aligned} \quad (9)$$

where $\varphi_{1,2}$ are phases explicitly given in Appendix B, Eqs. (B8) and (B9).

- $\hat{D}_{\lambda_q, \lambda'_q}^{\lambda_h, \lambda'_h}(z, \mathbf{p}_\perp)$ is the product of *fragmentation amplitudes* for the $q \rightarrow h + X$ process

$$\hat{D}_{\lambda_q, \lambda'_q}^{\lambda_h, \lambda'_h} = \sum_{X, \lambda_X} \hat{D}_{\lambda_h, \lambda_X; \lambda_q} \hat{D}_{\lambda'_h, \lambda_X; \lambda'_q}^* \quad (10)$$

where the \sum_{X, λ_X} stands for a spin sum and phase space integration over all undetected particles, considered as a system X . The usual unpolarized fragmentation function $D_{h/q}(z)$, i.e. the number density of hadrons h resulting from the fragmentation of an unpolarized parton q and carrying a light-cone momentum fraction z , is given by

$$D_{h/q}(z) = \frac{1}{2} \sum_{\lambda_q, \lambda_h} \int d^2 \mathbf{p}_\perp \hat{D}_{\lambda_q, \lambda_q}^{\lambda_h, \lambda_h}(z, \mathbf{p}_\perp). \quad (11)$$

We shall only consider the case of spinless final particles ($\lambda_h = 0$), in particular, pions. In general $\hat{D}_{\lambda_q, \lambda'_q}^{\lambda_h, \lambda'_h}(z, \mathbf{p}_\perp)$ depends on the azimuthal angle of h around the direction of motion of the fragmenting polarized parton [80].

We compute the SSA:

$$A_N = \frac{d\sigma^\uparrow(\mathbf{P}_T) - d\sigma^\downarrow(\mathbf{P}_T)}{d\sigma^\uparrow(\mathbf{P}_T) + d\sigma^\downarrow(\mathbf{P}_T)} = \frac{d\sigma^\uparrow(\mathbf{P}_T) - d\sigma^\uparrow(-\mathbf{P}_T)}{2d\sigma^{\text{unp}}(\mathbf{P}_T)}, \quad (12)$$

which can be measured either by looking at the production of hadrons at a fixed transverse momentum \mathbf{P}_T , changing the incoming proton polarization from \uparrow to \downarrow , or keeping a fixed proton polarization and looking at the hadron production to the left and the right of the $Z_{\text{c.m.}}$ axis, see Fig. 1. A_N is defined (and computed) for a proton polarization normal (N) to the production plane and a pure spin state (a pseudovector polarization \mathbf{S}_T with $|\mathbf{S}_T| = S_T = 1$). For a generic transverse polarization along an azimuthal direction ϕ_S (in our chosen reference frame) and a polarization $S_T \neq 1$, one has

$$A(\phi_S, S_T) = \mathbf{S}_T \cdot (\hat{\mathbf{p}} \times \hat{\mathbf{P}}_T) A_N = S_T \sin \phi_S A_N. \quad (13)$$

Notice that if, according to the usual procedure in SIDIS experiments, one defines

$$A_{TU}^{\sin\phi_s} \equiv \frac{2}{S_T} \frac{\int d\phi_s [d\sigma(\phi_s) - d\sigma(\phi_s + \pi)] \sin\phi_s}{\int d\phi_s [d\sigma(\phi_s) + d\sigma(\phi_s + \pi)]}, \quad (14)$$

one simply has

$$A_{TU}^{\sin\phi_s} = A_N. \quad (15)$$

In order to compute A_N , Eq. (12), we need to compute $[\Sigma(\uparrow) - \Sigma(\downarrow)]$ and $[\Sigma(\uparrow) + \Sigma(\downarrow)]$, which can be done by performing the helicity sum in Eqs. (4) and (5). As our process is a simple particular case of $(A, S_A) + (B, S_B) \rightarrow C + X$, the result agrees with Eqs. (82) and (86) of

$$A_N = \frac{\sum_{q,\{\lambda\}} \int \frac{dx dz}{16\pi^2 x z^2 s} d^2 \mathbf{k}_\perp d^3 \mathbf{p}_\perp \delta(\mathbf{p}_\perp \cdot \hat{\mathbf{p}}'_q) J(p_\perp) \delta(\hat{s} + \hat{t} + \hat{u}) [\Sigma(\uparrow) - \Sigma(\downarrow)]^{q \rightarrow ql}}{\sum_{q,\{\lambda\}} \int \frac{dx dz}{16\pi^2 x z^2 s} d^2 \mathbf{k}_\perp d^3 \mathbf{p}_\perp \delta(\mathbf{p}_\perp \cdot \hat{\mathbf{p}}'_q) J(p_\perp) \delta(\hat{s} + \hat{t} + \hat{u}) [\Sigma(\uparrow) + \Sigma(\downarrow)]^{q \rightarrow ql}}, \quad (16)$$

with

$$\sum_{\{\lambda\}} [\Sigma(\uparrow) - \Sigma(\downarrow)]^{q \rightarrow ql} = \frac{1}{2} \Delta^N f_{q/p^i}(x, k_\perp) \cos\phi [|\hat{M}_1^0|^2 + |\hat{M}_2^0|^2] D_{h/q}(z, p_\perp) + h_{1q}(x, k_\perp) \hat{M}_1^0 \hat{M}_2^0 \Delta^N D_{h/q^i}(z, p_\perp) \cos(\phi' + \phi_q^h) \quad (17)$$

and (dropping negligible contributions from other TMDs [80])

$$\begin{aligned} & \sum_{\{\lambda\}} [\Sigma(\uparrow) + \Sigma(\downarrow)]^{q \rightarrow ql} \\ &= f_{q/p}(x, k_\perp) [|\hat{M}_1^0|^2 + |\hat{M}_2^0|^2] D_{h/q}(z, p_\perp). \end{aligned} \quad (18)$$

- The first term on the right-hand side (rhs) of Eq. (17) shows the contribution to A_N of the Siverson function $\Delta^N f_{q/p^i}(x, k_\perp)$ [10,11,81],

$$\begin{aligned} \Delta \hat{f}_{q/p,S}(x, \mathbf{k}_\perp) &= \hat{f}_{q/p,S}(x, \mathbf{k}_\perp) - \hat{f}_{q/p,-S}(x, \mathbf{k}_\perp) \\ &\equiv \Delta^N f_{q/p^i}(x, k_\perp) \mathbf{S}_T \cdot (\hat{\mathbf{p}} \times \hat{\mathbf{k}}_\perp) \\ &= -2 \frac{k_\perp}{M} f_{1T}^{\perp q}(x, k_\perp) \mathbf{S}_T \cdot (\hat{\mathbf{p}} \times \hat{\mathbf{k}}_\perp), \end{aligned} \quad (19)$$

coupled to the unpolarized elementary interaction [$\propto \frac{1}{2} (|\hat{M}_1^0|^2 + |\hat{M}_2^0|^2)$] and the unpolarized fragmentation function $D_{h/q}(z, p_\perp)$; the $\cos\phi$ factor arises from the $\mathbf{S}_T \cdot (\hat{\mathbf{p}} \times \hat{\mathbf{k}}_\perp)$ factor, the spin-transverse motion correlation of the Siverson function in the case of a normal spin direction with $S_T = 1$.

- The second term on the rhs of Eq. (17) shows the contribution to A_N of the unintegrated transversity distribution $h_{1q}(x, k_\perp)$ coupled to the Collins function $\Delta^N D_{h/q^i}(z, p_\perp)$ [12,81],

$$\begin{aligned} \Delta \hat{D}_{h/q^i}(z, \mathbf{p}_\perp) &= \hat{D}_{h/q^i}(z, \mathbf{p}_\perp) - \hat{D}_{h/q^i}(z, \mathbf{p}_\perp) \\ &\equiv \Delta^N D_{h/q^i}(z, p_\perp) s_q \cdot (\hat{\mathbf{p}}'_q \times \hat{\mathbf{p}}_\perp) \\ &= \frac{2p_\perp}{z m_h} H_1^{\perp q}(z, p_\perp) s_q \cdot (\hat{\mathbf{p}}'_q \times \hat{\mathbf{p}}_\perp), \end{aligned} \quad (20)$$

Ref. [80], simplified to the case in which particle B is a pointlike lepton and the elementary interaction has only two independent amplitudes. Notice that several TMDs appear in the expression for A_N ; however, numerical evaluations show that the contribution of the Siverson effect is the dominant one. A modest contribution is given by the Collins function (coupled to the transversity distribution), while another contribution involving h_{1T}^\perp (see Appendix C) is totally negligible. Considering only the Siverson and Collins effects, one has

and to the transverse spin transfer elementary interaction ($d\sigma^{\uparrow,\uparrow} - d\sigma^{\uparrow,\downarrow} \propto \hat{M}_1^0 \hat{M}_2^0$). The factor $\cos(\phi' + \phi_q^h)$ arises from phases in the \mathbf{k}_\perp -dependent transversity distribution, the Collins function, and the elementary polarized interaction. ϕ' is the azimuthal angle of the fragmenting quark (with 3-momentum \mathbf{p}'_q) and ϕ_q^h is the azimuthal angle of \mathbf{p}_\perp around the $\hat{\mathbf{p}}'_q$ direction [80]. Their expressions in terms of integration and overall variables can be found in Appendix A.

- The elementary interaction amplitudes are explicitly given in Eqs. (8) and (9). Notice that the elementary Mandelstam variables $\hat{s}, \hat{t}, \hat{u}$ are computed taking into account the full kinematics, and thus depend on the transverse momenta.
- A final issue which needs to be clarified concerns *perturbative QCD corrections*. Our proposed process involves TMDs coupled to lowest order perturbative interactions and is driven by a large angle elementary electromagnetic scattering, $ql \rightarrow ql$. Some QCD effects, like soft gluon emissions, are taken into account in the TMDs, as the emission of soft gluons builds up intrinsic partonic motion. Higher order pQCD corrections due to genuine hard QCD processes, like $ql \rightarrow qlg$ or $gl \rightarrow q\bar{q}l$ are not included in our computation of A_N . These contribute at order α_s to the cross section and can be neglected at large Q^2 values; moreover, one should notice that events induced by these hard pQCD elementary interactions result in final states with two fragmenting partons, i.e. two jets, and could be experimentally excluded. However, these pQCD corrections might be of some relevance and difficult to disentangle at HERMES, COMPASS, or JLab energies.

B. Large P_T jet production

We consider also the most interesting case of SSAs for the inclusive process $p^\dagger l \rightarrow \text{jet} + X$. Although it is a difficult process to detect experimentally and might require future higher energy and luminosity machines, it would certainly give the most direct access to the Sivers effect, as the lack of any fragmentation mechanism forbids other contributions. Even more difficult, the observation of both a jet and a final hadron inside the jet (with a measurement of its transverse momentum \mathbf{p}_\perp), would allow a direct detection of the Collins effect [82].

In the case of the $p^\dagger l \rightarrow \text{jet} + X$ process, with no observation of a single final particle, Eq. (4) simplifies to

$$\begin{aligned} \frac{E_j d\sigma^{(p,S)+l \rightarrow \text{jet}+X}}{d^3\mathbf{P}_j} &= \sum_{q,\{\lambda\}} \int \frac{dx}{16\pi^2 x s} d^2\mathbf{k}_\perp \delta(\hat{s} + \hat{t} + \hat{u}) \\ &\times \rho_{\lambda_q, \lambda'_q}^{q/p, S} \hat{f}_{q/p, S}(x, \mathbf{k}_\perp) \\ &\times \frac{1}{2} \hat{M}_{\lambda_q, \lambda_i; \lambda'_q, \lambda_l} \hat{M}_{\lambda'_q, \lambda_l; \lambda'_q, \lambda_l}^* \end{aligned} \quad (21)$$

while Eq. (16) becomes

$$A_N^{\text{jet}} = \frac{\sum_{q,\{\lambda\}} \int \frac{dx}{16\pi^2 x s} d^2\mathbf{k}_\perp \delta(\hat{s} + \hat{t} + \hat{u}) [\Sigma(\uparrow) - \Sigma(\downarrow)]_{\text{jet}}^{q^l \rightarrow q^l}}{\sum_{q,\{\lambda\}} \int \frac{dx}{16\pi^2 x s} d^2\mathbf{k}_\perp \delta(\hat{s} + \hat{t} + \hat{u}) [\Sigma(\uparrow) + \Sigma(\downarrow)]_{\text{jet}}^{q^l \rightarrow q^l}}. \quad (22)$$

In this case the kinematics is very simple and is shown explicitly in Appendix A 2. For a generic azimuthal direction ϕ_S of the transverse spin \mathbf{S}_T , the Sivers function, Eq. (19), can be written as

$$\begin{aligned} \Delta^N f_{q/p^i}(x, k_\perp) \mathbf{S}_T \cdot (\hat{\mathbf{p}} \times \hat{\mathbf{k}}_\perp) &= \Delta^N f_{q/p^i}(x, k_\perp) \\ &\times \left(\sin\phi_S \frac{k_\perp^x}{k_\perp} - \cos\phi_S \frac{k_\perp^y}{k_\perp} \right) \\ &= \Delta^N f_{q/p^i}(x, k_\perp) \sin(\phi_S - \phi), \end{aligned} \quad (23)$$

and the Σ kernels in Eq. (22) are

$$\begin{aligned} \sum_{\{\lambda\}} [\Sigma(\uparrow) - \Sigma(\downarrow)]_{\text{jet}}^{q^l \rightarrow q^l} \\ = \frac{1}{2} \Delta^N f_{q/p^i}(x, k_\perp) \sin(\phi_S - \phi) [|\hat{M}_1^0|^2 + |\hat{M}_2^0|^2] \end{aligned} \quad (24)$$

$$\sum_{\{\lambda\}} [\Sigma(\uparrow) + \Sigma(\downarrow)]_{\text{jet}}^{q^l \rightarrow q^l} = f_{q/p}(x, k_\perp) [|\hat{M}_1^0|^2 + |\hat{M}_2^0|^2]. \quad (25)$$

The elementary amplitudes are the same as given in Eqs. (8) and (9).

III. ESTIMATES FOR A_N

We have computed the SSA, A_N , as defined in Eq. (12) or (14), for the large P_T production of pions and jets in $p^\dagger l \rightarrow hX$ and $p^\dagger l \rightarrow \text{jet} + X$ processes, according to the expressions given, respectively, in Eqs. (16)–(18) and in Eqs. (22) and (24) (with $\phi_S = \pi/2$), and (25).

Analogous results for the case of leptons moving along the $Z_{\text{c.m.}}$ axis, $lp^\dagger \rightarrow h(\text{jet}) + X$, in the same chosen hadronic frame (that is, keeping fixed the definitions of $x_F = 2P_L/\sqrt{s}$ and of the \uparrow, \downarrow transverse polarization directions) can be easily obtained using rotational invariance:

$$A_N^{lp^\dagger \rightarrow h(\text{jet})+X}(x_F, \mathbf{P}_T) = -A_N^{p^\dagger l \rightarrow h(\text{jet})+X}(-x_F, \mathbf{P}_T). \quad (26)$$

We have used the Sivers distributions as parametrized and extracted—from SIDIS data—in Ref. [14]; even if the Sivers functions, being related to final state interactions [72], are expected to be process dependent [23], they should be the same in SIDIS and the (related) processes considered here, which all originate from the same $ql \rightarrow ql$ elementary interaction and subsequent quark fragmentation. Similarly, we have used the transversity distributions and Collins functions as parametrized and extracted in Ref. [22]. The unpolarized parton distribution functions (PDFs) and fragmentation functions (FFs) are taken, respectively, from Refs. [83,84].

Our results are given for the kinematical configurations of HERMES, COMPASS, JLab at 12 GeV, and a hypothetical ENC future machine operating at an energy $\sqrt{s} = 50$ GeV. For hadron production, the Sivers and Collins contributions are shown separately. We plot A_N as a function of x_F at fixed P_T values; these should be chosen as the hard scale of the process, ensuring a large momentum transfer in the hard scattering, say $Q^2 > 1 \text{ GeV}^2$. In collinear cases, at LO, it might suffice to have $P_T > 1 \text{ GeV}$; however, with TMD factorization, one has to be more careful, as P_T might be partially generated by intrinsic k_\perp . We have checked that a value of $P_T = 2.5 \text{ GeV}$ corresponds to a safe $Q^2 > 1 \text{ GeV}^2$ region in the whole range of x_F , while $P_T = 1.5 \text{ GeV}$ implies a safe Q^2 region only for backward production, $x_F \lesssim 0$. We give predictions for these two values of P_T .

Notice also that for positive x_F the minimum of x is given, roughly, by x_F . This implies that for $x_F > 0.2$ – 0.3 we should employ the parametrizations of the Sivers and transversity functions in a region where they are not constrained by SIDIS data. For this reason we will give our theoretical estimates of A_N only up to $x_F \approx 0.2$. On the other hand, for negative x_F the minimum of x is controlled by the ratio $x_T = 2P_T/\sqrt{s}$, implying that at moderate c.m. energies (i.e. $\sqrt{s} \approx 10$ – 20 GeV) and $P_T \approx 1$ – 2 GeV , we are sensitive to the valence region of the polarized proton, i.e. the region where the Sivers (and the transversity) functions reach their maxima.

Let us comment in detail our results.

- We first stress some aspects peculiar to the $p^\dagger l \rightarrow hX$ process. As in SIDIS processes at leading order accuracy, only one partonic subprocess, $ql \rightarrow ql$, is active, with a simple $1/\hat{t}^2$ dependence (a much simpler dynamics than in the $pp \rightarrow hX$ case). However, since the lepton plane is not identified (we do not require the detection of the outgoing lepton), one cannot access, separately, the Siverts and the Collins effects. Nevertheless, in the backward region [with respect to (w.r.t.) the proton direction] the variable $|\hat{u}|$ becomes smaller and so does the partonic spin transfer cross section $\propto \hat{M}_1^0 \hat{M}_2^0$ [see Eqs. (8) and (9)], entering the Collins contribution to A_N [second term on the rhs of Eq. (17)]. This implies a strong *dynamical* suppression of the Collins effect (reinforced by the integration over the azimuthal phases) at largely and moderately negative values of x_F , leaving active mainly the Siverts contribution. Notice that, contrary to what happens in the $pp \rightarrow hX$ process, no \hat{u} channel in the partonic process is present; moreover, the variable \hat{t} strongly depends on ϕ , the azimuthal phase of the Siverts effect [first term on the rhs of Eq. (17)].
- In Fig. 2 we present our estimates, separately, for the Siverts and Collins contributions to A_N at HERMES kinematics. More precisely, we show the Siverts effect at $P_T = 1.5$ GeV (left panel) and at $P_T = 2.5$ GeV (central panel) and the Collins effect at $P_T = 2.5$ GeV (right panel). The Collins effect at $P_T = 1.5$ GeV (not shown) is almost negligible in the kinematical region considered. For charged pion production at $P_T = 1.5$ GeV (left panel) the statistical uncertainty bands as resulting from our fit [14] are also shown.

The largest A_N values obtained correspond to the x region (of the polarized proton distributions) where the Siverts functions, for u and d quarks, reach their

maxima. It is interesting to note that the sizable value of A_N for π^- production (larger than the corresponding Siverts contribution to A_{UT} in SIDIS) is due to the dominance of the d quark with a small contamination from the u quark. This is related to the fact that the light-cone momentum fraction z is always bigger than the maximum between $|x_F|$ and x_T , implying, at moderate and large $|x_F|$, a dominance of the *leading* fragmentation functions.

- In Fig. 3 we show the analogous results for COMPASS kinematics. Again at $P_T = 1.5$ GeV only the Siverts effect gives a sizable contribution (left panel), while the Collins effect (not shown) is compatible with zero. At $P_T = 2.5$ GeV the Siverts effect (central panel) dominates only in the backward region, while in the forward region the Collins effect (right panel) becomes sizable. For charged pion production at $P_T = 1.5$ GeV (left panel) the statistical uncertainty bands as resulting from our fit [14] are also shown. The main difference w.r.t. A_N for HERMES kinematics at $P_T = 1.5$ GeV (compare Figs. 2 and 3, left panels) is that at the larger COMPASS energy ($\sqrt{s} \approx 17$ GeV) the valence region for the polarized proton, where the Siverts functions reach their maxima, starts dominating at larger x_F .
- In Fig. 4 we show our results for ENC kinematics at $\sqrt{s} = 50$ GeV. For $P_T = 1.5$ GeV (left panel) only at the upper range of the safe x_F values (i.e. $x_F \lesssim 0$) the Siverts effect gives a sizable contribution, of the order of a few percent (the Collins effect is once again negligible). At $P_T = 2.5$ GeV both the Siverts (central panel) and the Collins (right panel) contributions are comparable and sizable around $x_F \approx 0.2$, therefore hardly distinguishable. This can be understood because at such P_T and energy values the valence region for the polarized proton dominates only for

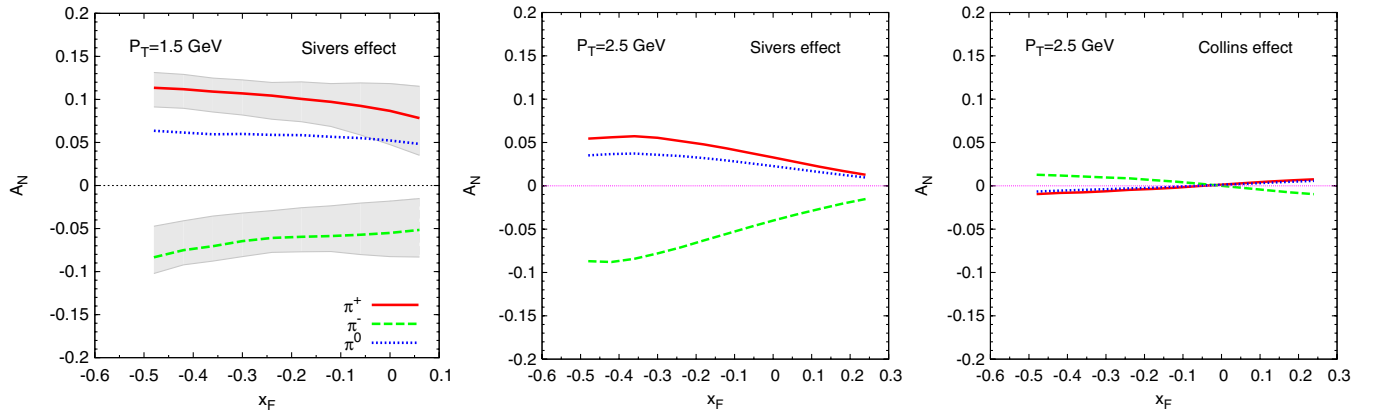
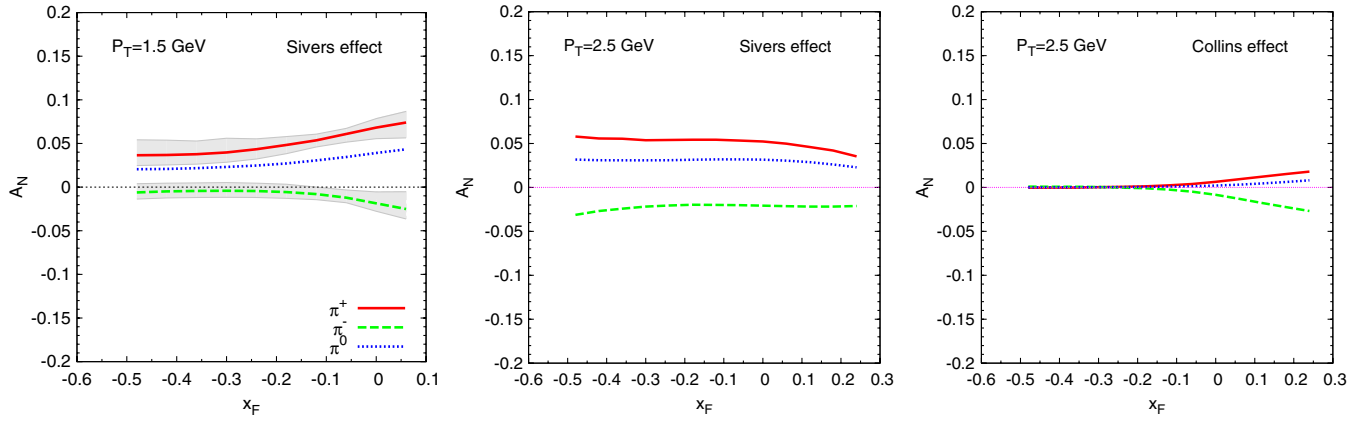
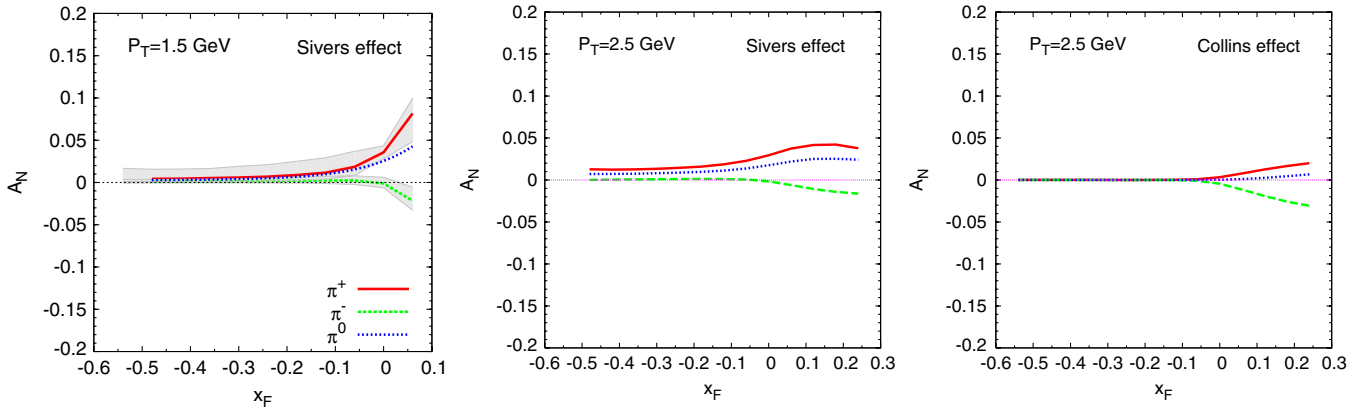


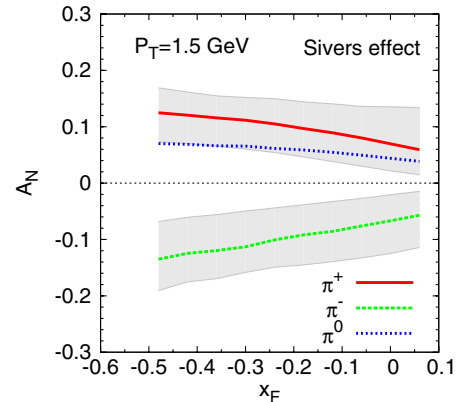
FIG. 2 (color online). Estimates of A_N vs x_F for the $p^\dagger l \rightarrow \pi X$ process at HERMES ($\sqrt{s} \approx 7$ GeV). Left panel: Siverts effect at $P_T = 1.5$ GeV; central panel: Siverts effect at $P_T = 2.5$ GeV; right panel: Collins effect at $P_T = 2.5$ GeV. The computation has been performed according to Eqs. (16)–(18) of the text, adopting the Siverts functions of Ref. [14] and the transversity and Collins functions of Ref. [22], as extracted from SIDIS and e^+e^- data, the unpolarized PDFs of Ref. [83] and the FFs of Ref. [84]. In the left panel we also show, for charged pions, the statistical uncertainty bands coming from the extracted Siverts functions [14].

FIG. 3 (color online). Same as in Fig. 2 but for COMPASS kinematics ($\sqrt{s} \approx 17$ GeV).FIG. 4 (color online). Same as in Fig. 2 but for ENC kinematics at $\sqrt{s} = 50$ GeV.

$x_F > 0$, where both effects are active (see our comment on the suppression of the Collins effect for negative x_F at the beginning of this section).

- In Fig. 5 we show analogous estimates of the Sivers contribution to A_N for JLab kinematics at the upgraded energy $E_{\text{Lab}} = 12$ GeV, corresponding to a c.m. energy $\sqrt{s} \approx 4.9$ GeV. Again, in order to guarantee a sufficiently large momentum transfer we show results at $P_T = 1.5$ GeV vs $x_F \lesssim 0.1$. The results are comparable to the corresponding estimates for HERMES kinematics, see Fig. 3 (left panel), with large asymmetries (in size) for all pions. Given the lower c.m. energy, however, cross sections are in general smaller than those for HERMES and COMPASS kinematics. Larger values of P_T are probably out of reach at Jlab, while the Collins contribution is again negligible in the x_F region considered.
- Finally, in Fig. 6 we show estimates of the Sivers contribution to A_N for the process $p^{\uparrow}l \rightarrow \text{jet} + X$ for ENC kinematics ($\sqrt{s} = 50$ GeV) at $P_T = 1.5$ GeV (left panel) and $P_T = 2.5$ GeV (right panel) vs x_F . The results are similar, both in size and shape, to the corresponding ones for neutral and positive pions, see

Fig. 4, left and central panels (notice the different scale). The asymmetry is almost negligible at negative x_F and becomes sizable only at the upper range of the safe x_F values. We have found that A_N becomes even smaller at larger c.m. energies.

FIG. 5 (color online). Same as in Fig. 2, left panel, but for JLab kinematics at $E_{\text{Lab}} = 12$ GeV ($\sqrt{s} \approx 4.9$ GeV).

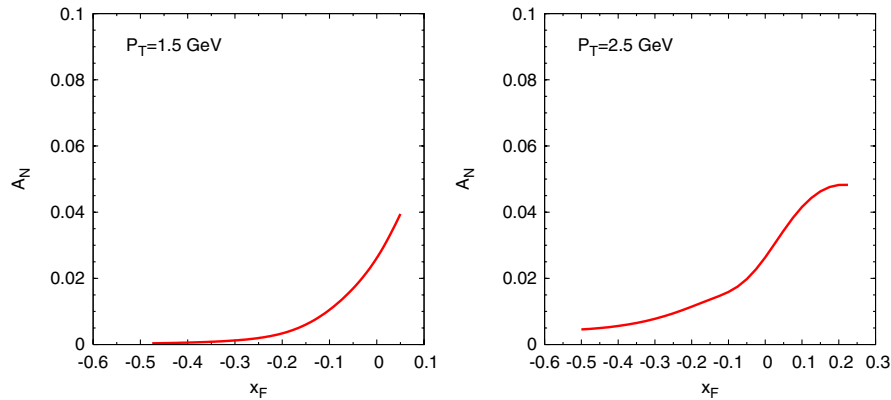


FIG. 6 (color online). Estimates of A_N vs x_F for the $p^l l \rightarrow \text{jet} + X$ process and for ENC kinematics at $\sqrt{s} = 50$ GeV. Left panel: Siverts effect at $P_T = 1.5$ GeV; right panel: Siverts effect at $P_T = 2.5$ GeV. The computation has been performed according to Eqs. (22), (24), and (25) of the text, adopting the Siverts functions of Ref. [14], as extracted from SIDIS data, and the unpolarized PDFs of Ref. [83].

IV. COMMENTS AND CONCLUSIONS

In this paper we have presented a phenomenological study, based on the assumption of TMD factorization, of transverse single spin asymmetries for the inclusive production of large P_T pions and jets in lepton-proton collisions, $p^l l \rightarrow h(\text{jet}) + X$. These asymmetries, measured in the lepton-proton c.m. frame (since the final lepton is not observed, the γ^* -proton c.m. frame cannot be reconstructed) should involve the same TMD distribution and fragmentation functions which contribute to the transverse azimuthal asymmetries measured by the HERMES and COMPASS collaborations in the last years in semi-inclusive deep inelastic scattering.

Using best-fit parametrizations of the TMD functions extracted from HERMES, COMPASS, and Belle data, we have shown that, in the kinematical regions where our perturbative approach should be reliable, the asymmetries dominantly arise from the Siverts effect in the distribution sector and marginally from the Collins effect in the fragmentation sector (not present in the case of jet production). We have presented results for several kinematical configurations corresponding to present experimental setups (HERMES and COMPASS), to the forthcoming 12 GeV upgraded JLab setup and to a class of lepton-proton (ion) colliders (ENC) currently under active study in the QCD and hadron physics community. These results show that for pion production the Siverts A_N can be sizable, at least for HERMES, COMPASS, and JLab at 12 GeV kinematics. For pion and jet production at typical energies of the proposed ENC colliders, the asymmetries are much smaller and become larger only at the boundary of the safe kinematical regions, where, for pions, both the Siverts and the Collins contributions play a role and the two mechanisms cannot be disentangled.

The measurement of these predicted asymmetries allows a test of the validity of the TMD factorization, largely accepted for SIDIS processes with two scales (small P_T

and large Q), but still much debated for processes with only one large scale (P_T), like the one we are considering here. A test of TMD factorization in such processes is of great importance for a consistent understanding of the large SSAs measured in the single inclusive production of large P_T hadrons in proton-proton collisions.

We stress once more that our predictions refer to large P_T production, in the lepton-proton c.m. frame, at leading perturbative order. It implies that, in order to compare experimental data with our results, one has to select large P_T , single-jet events, excluding those events containing a second jet in the opposite hemisphere w.r.t. to the primary observed jet (containing the final observed hadron). This should avoid large P_T jets (or hadrons) coming from next-to-leading order partonic processes (hard pQCD corrections). Although these requirements might correspond to smaller cross sections and difficult selection procedures, we believe that the relevance of testing TMD factorization in this simple process justifies efforts in this direction and motivates our work.

ACKNOWLEDGMENTS

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APPENDIX A: KINEMATICS

1. Hadron production

We work in the proton-lepton center of mass frame, with the incoming proton and lepton moving along the $Z_{\text{c.m.}}$ axis

and the outgoing hadron emitted in the $(XZ)_{\text{c.m.}}$ plane:

$$p = \frac{\sqrt{s}}{2}(1, 0, 0, 1) \quad (\text{A1})$$

$$l = \frac{\sqrt{s}}{2}(1, 0, 0, -1) \quad (\text{A2})$$

$$P_h = (E_h, P_T, 0, P_L) \quad E_h^2 = P_T^2 + P_L^2, \quad (\text{A3})$$

where s is the proton-lepton c.m. square energy and where we have assumed all particles to be massless. The kinematical variables for the elementary underlying process result in $(k_\perp = |\mathbf{k}_\perp|, p_\perp = |\mathbf{p}_\perp|)$

$$p_q = \left(\frac{x\sqrt{s}}{2} + \frac{k_\perp^2}{2x\sqrt{s}}, \mathbf{k}_\perp, \frac{x\sqrt{s}}{2} - \frac{k_\perp^2}{2x\sqrt{s}} \right) \quad (\text{A4})$$

$$l = \frac{\sqrt{s}}{2}(1, 0, 0, -1) \quad (\text{A5})$$

$$p'_q = \frac{E_h + \sqrt{E_h^2 - p_\perp^2}}{2z} \times \left[1, \frac{1}{\sqrt{E_h^2 - p_\perp^2}}(P_T - p_\perp^x, -p_\perp^y, P_L - p_\perp^z) \right] \quad (\text{A6})$$

$$l' = p_q + l - p'_q, \quad (\text{A7})$$

with \mathbf{k}_\perp being the intrinsic transverse momentum of parton q inside the parent proton and \mathbf{p}_\perp being the intrinsic transverse momentum of the detected final hadron h with respect to the fragmenting parton q' . The expression for p'_q has been obtained by requiring z to be the light-cone momentum fraction of the emitted hadron, $z = \tilde{P}_h^+ / \tilde{p}'_q{}^+$, as defined in the helicity frame of the fragmenting quark q' , which we will denote as \tilde{S} . With this kinematics, the partonic Mandelstam invariants are

$$\hat{s} = xs$$

$$\hat{t} = -\frac{x\sqrt{s}}{2z} \left(1 + \frac{E_h}{\sqrt{E_h^2 - p_\perp^2}} \right) \left[\left(1 + \frac{k_\perp^2}{x^2s} \right) \sqrt{E_h^2 - p_\perp^2} - \left(1 - \frac{k_\perp^2}{x^2s} \right) (P_L - p_\perp^z) - \frac{2k_\perp^x (P_T - p_\perp^x) - 2k_\perp^y p_\perp^y}{x\sqrt{s}} \right]$$

$$\hat{u} = -\frac{\sqrt{s}}{2z} \left(1 + \frac{E_h}{\sqrt{E_h^2 - p_\perp^2}} \right) (\sqrt{E_h^2 - p_\perp^2} + P_L - p_\perp^z). \quad (\text{A8})$$

Notice that the orthogonality between \mathbf{p}'_q and \mathbf{p}_\perp , explicitly guaranteed through the delta function $\delta(\mathbf{p}_\perp \cdot \hat{\mathbf{p}}'_q)$ in Eq. (16), allows us to fix one component of the vector \mathbf{p}_\perp in terms of all the others; in particular, it gives

$$|\mathbf{p}_\perp|^2 = P_T p_\perp^x + P_L p_\perp^z$$

$$\Rightarrow p_\perp^y = \pm \sqrt{P_T p_\perp^x + P_L p_\perp^z - (p_\perp^x)^2 - (p_\perp^z)^2}. \quad (\text{A9})$$

Similarly, the other delta function in Eq. (16), $\delta(\hat{s} + \hat{t} + \hat{u})$, can be used to perform the integration over the light-cone fraction z fixing

$$z = \frac{1}{2x\sqrt{s}} \left(1 + \frac{E_h}{\sqrt{E_h^2 - p_\perp^2}} \right) \left[\left(1 + x + \frac{k_\perp^2}{xs} \right) \sqrt{E_h^2 - p_\perp^2} + \left(1 - x + \frac{k_\perp^2}{xs} \right) (P_L - p_\perp^z) - \frac{2k_\perp^x (P_T - p_\perp^x) - 2k_\perp^y p_\perp^y}{\sqrt{s}} \right], \quad (\text{A10})$$

with p_\perp^y given by Eq. (A9).

The angle ϕ_q^h , which identifies the direction of \mathbf{p}_\perp around \mathbf{p}'_q , can be expressed in terms of the \mathbf{p}_\perp components, p_\perp^x , p_\perp^y , and p_\perp^z , simply by noticing that in the helicity frame of parton q' (where the \tilde{Z} axis coincides with the direction of \mathbf{p}'_q) this angle is the azimuth of \mathbf{p}_\perp , that is,

$$\sin\phi_q^h = \hat{\mathbf{p}}_\perp \cdot \tilde{\mathbf{Y}} \quad \cos\phi_q^h = \hat{\mathbf{p}}_\perp \cdot \tilde{\mathbf{X}}. \quad (\text{A11})$$

The helicity frame \tilde{S} of parton q' can be reached by performing two rotations, as explained in Appendix C of Ref. [80], in the following way:

$$\tilde{\mathbf{Z}} = \hat{\mathbf{p}}'_q = \frac{1}{\sqrt{E_h^2 - p_\perp^2}} (P_T - p_\perp^x, -p_\perp^y, P_L - p_\perp^z) \quad (\text{A12})$$

$$\tilde{\mathbf{Y}} = \hat{\mathbf{Z}}_{\text{cm}} \times \hat{\mathbf{p}}'_{qT} = \frac{(p_\perp^y, P_T - p_\perp^x, 0)}{\sqrt{E_h^2 - p_\perp^2 - (P_L - p_\perp^z)^2}} \quad (\text{A13})$$

$$\tilde{\mathbf{X}} = \tilde{\mathbf{Y}} \times \tilde{\mathbf{Z}} = \frac{[(P_T - p_\perp^x)(P_L - p_\perp^z), -p_\perp^y(P_L - p_\perp^z), -(p_\perp^y)^2 - (P_T - p_\perp^x)^2]}{\sqrt{E_h^2 - p_\perp^2 - (P_L - p_\perp^z)^2} \sqrt{E_h^2 - p_\perp^2}}, \quad (\text{A14})$$

where $\hat{\mathbf{p}}'_{qT}$ is given by the transverse components of \mathbf{p}'_q in the center of mass reference frame, S:

$$\hat{\mathbf{p}}'_{qT} = \frac{1}{\sqrt{E_h^2 - p_\perp^2 - (P_L - p_\perp^z)^2}} (P_T - p_\perp^x, -p_\perp^y, 0), \quad (\text{A15})$$

and p_\perp^y is fixed by the orthogonality condition of Eq. (A9).

By replacing Eqs. (A12)–(A14) into Eq. (A11) we find

$$\begin{aligned} \sin\phi_q^h &= \frac{p_\perp^y}{p_\perp} \frac{P_T}{\sqrt{E_h^2 - p_\perp^2 - (P_L - p_\perp^z)^2}} \\ \cos\phi_q^h &= -\frac{p_\perp^z}{p_\perp} \frac{\sqrt{E_h^2 - p_\perp^2}}{\sqrt{E_h^2 - p_\perp^2 - (P_L - p_\perp^z)^2}}. \end{aligned} \quad (\text{A16})$$

Alternatively, in terms of angles instead of components we can write

$$\begin{aligned} \sin\phi_q^h &= -\frac{P_T}{p_\perp} \sin\phi' \\ \cos\phi_q^h &= -\frac{p_\perp^z}{p_\perp} \frac{1}{\sin\theta'} = -\frac{\cos\theta_\perp}{\sin\theta'}, \end{aligned} \quad (\text{A17})$$

where ϕ' , θ' are the azimuthal, polar angles of \mathbf{p}'_q and θ_\perp is the polar angle of \mathbf{p}_\perp in our c.m. reference frame.

Finally, the $\cos(\phi' + \phi_q^h)$ azimuthal dependence of the Collins effect, see Eq. (17), can be expressed as

$$\cos(\phi' + \phi_q^h) = \frac{p_\perp^z (p_\perp^x - P_T) \sqrt{E_h^2 - p_\perp^2} + (p_\perp^y)^2 P_T}{p_\perp [E_h^2 - p_\perp^2 - (P_L - p_\perp^z)^2]}, \quad (\text{A18})$$

or, more simply, in terms of angles

$$\cos(\phi' + \phi_q^h) = \frac{P_T}{p_\perp} \sin^2\phi' - \cos\theta_\perp \frac{\cos\phi'}{\sin\theta'}. \quad (\text{A19})$$

2. Jet production

The 4-momenta involved, in our reference frame and neglecting all masses, are

$$p = \frac{\sqrt{s}}{2} (1, 0, 0, 1) \quad l = \frac{\sqrt{s}}{2} (1, 0, 0, -1) \quad (\text{A20})$$

$$p_q = \left(\frac{x\sqrt{s}}{2} + \frac{k_\perp^2}{2x\sqrt{s}}, \mathbf{k}_\perp, \frac{x\sqrt{s}}{2} - \frac{k_\perp^2}{2x\sqrt{s}} \right) \quad (\text{A21})$$

$$p'_q = P_j = (E_j, P_T, 0, P_L) \quad E_j^2 = P_T^2 + P_L^2, \quad (\text{A22})$$

so that the partonic Mandelstam invariants are given by

$$\hat{s} = xs \quad (\text{A23})$$

$$\hat{t} = 2P_T k_\perp^x - x\sqrt{s} \left[E_j - P_L + \frac{k_\perp^2}{x^2 s} (E_j + P_L) \right] \quad (\text{A24})$$

$$\hat{u} = -\sqrt{s}(E_j + P_L). \quad (\text{A25})$$

Notice that there is no linear k_\perp^y dependence in these variables and, as a consequence, in the elementary amplitudes $\hat{M}_{1,2}^0$. The delta function ensuring $\hat{s} + \hat{t} + \hat{u} = 0$ can be used to perform the integration on k_\perp^x in Eq. (22):

$$k_\perp^x = x\sqrt{s} \left[\frac{P_T}{E_j + P_L} \pm \sqrt{\frac{\sqrt{s}}{E_j + P_L} - \frac{1}{x} - \frac{(k_\perp^y)^2}{x^2 s}} \right], \quad (\text{A26})$$

which implies

$$x_{\min} = \frac{E_j + P_L}{2\sqrt{s}} \left[1 + \sqrt{1 + \frac{4(k_\perp^y)^2}{\sqrt{s}(E_j + P_L)}} \right]. \quad (\text{A27})$$

Note that the term proportional to $\cos\phi_s$ in Eq. (23), being odd in k_\perp^y , vanishes when integrating over k_\perp^y , resulting, as it should, in a SSA proportional to $\sin\phi_s$.

APPENDIX B: SPINORS AND HELICITY AMPLITUDES

We compute the helicity amplitudes for the nonplanar $ql \rightarrow ql$ process exploiting the well-known spinor helicity technique (see, for example, Refs. [85,86]). To be precise, we adopt the phase convention and gamma matrix representation of Ref. [86]; that is, our helicity spinors for a massless Dirac particle with 4-momentum $k = (k^0, k^x, k^y, k^z)$ and helicity $\pm 1/2$ are given by

$$\begin{aligned} u_+(k) = v_-(k) &= \begin{pmatrix} \sqrt{k^+} e^{-i\phi/2} \\ \sqrt{k^-} e^{i\phi/2} \\ 0 \\ 0 \end{pmatrix}, \\ u_-(k) = v_+(k) &= \begin{pmatrix} 0 \\ 0 \\ -\sqrt{k^-} e^{-i\phi/2} \\ \sqrt{k^+} e^{i\phi/2} \end{pmatrix}, \end{aligned} \quad (\text{B1})$$

where

$$e^{\pm i\phi} \equiv \frac{k^x \pm ik^y}{\sqrt{(k^x)^2 + (k^y)^2}} = \frac{k^x \pm ik^y}{\sqrt{k^+ k^-}}, \quad k^\pm = k^0 \pm k^z. \quad (\text{B2})$$

The two independent helicity amplitudes $\hat{M}_{\lambda_3, \lambda_4; \lambda_1, \lambda_2}$ for the $q(k_1, \lambda_1) + l(k_2, \lambda_2) \rightarrow q(k_3, \lambda_3) + l(k_4, \lambda_4)$ elementary lowest order QED interaction are given by

$$\hat{M}_{++++} = \hat{M}_{----}^* = -2 \frac{e_q e^2}{\hat{t}} [34] \langle 12 \rangle \quad (\text{B3})$$

$$\hat{M}_{+-;+-} = \hat{M}_{-+;-+}^* = +2 \frac{e_q e^2}{\hat{t}} [23] \langle 14 \rangle, \quad (\text{B4})$$

with

$$\begin{aligned} \bar{u}_-(k_i) u_+(k_j) &\equiv \langle ij \rangle = -[ij]^* \\ &= \sqrt{k_i^+ k_j^-} e^{-i(\phi_i - \phi_j)/2} - \sqrt{k_i^- k_j^+} e^{i(\phi_i - \phi_j)/2}. \end{aligned} \quad (\text{B5})$$

Equation (B5) can be rewritten as [85]

$$\begin{aligned} \langle ij \rangle &= -e^{-i(\phi_i + \phi_j)/2} [\sqrt{k_i^- k_j^+} e^{i\phi_i} - \sqrt{k_i^+ k_j^-} e^{i\phi_j}] \\ &= -e^{-i(\phi_i + \phi_j)/2} \sqrt{|s_{ij}|} e^{i\phi_{ij}}, \end{aligned} \quad (\text{B6})$$

where $s_{ij} = (k_i + k_j)^2 = 2k_i \cdot k_j$, and

$$\begin{aligned} \cos \phi_{ij} &= \frac{k_i^x k_j^+ - k_j^x k_i^+}{\sqrt{|s_{ij}| k_i^+ k_j^+}}, \quad \sin \phi_{ij} = \frac{k_i^y k_j^+ - k_j^y k_i^+}{\sqrt{|s_{ij}| k_i^+ k_j^+}}, \\ \phi_{ij} &= \phi_{ji} + \pi. \end{aligned} \quad (\text{B7})$$

With our kinematical configuration ($\phi_2 = \pi$) we obtain

$$\begin{aligned} \hat{M}_{++++} &= \hat{M}_{----}^* \\ &= -8\pi e_q \alpha \frac{\hat{s}}{\hat{t}} e^{-i\phi_{34}} e^{i(\phi_3 + \phi_4 - \phi_1 + \pi)/2} \end{aligned} \quad (\text{B8})$$

$$\hat{M}_{+-;+-} = \hat{M}_{-+;-+}^* = 8\pi e_q \alpha \frac{\hat{u}}{\hat{t}} e^{i\phi_{14}} e^{-i(\phi_1 + \phi_4 - \phi_3 + \pi)/2}. \quad (\text{B9})$$

In addition, one can show that $\phi_{34} = \phi_{14}$.

Notice that the combinations of helicity amplitudes contributing to the SSA, Eq. (17), are simply given by

$$|\hat{M}_{++++}|^2 \equiv |\hat{M}_1^0|^2 = 64\pi^2 \alpha^2 e_q^2 \frac{\hat{s}^2}{\hat{t}^2} \quad (\text{B10})$$

$$|\hat{M}_{+-;+-}|^2 \equiv |\hat{M}_2^0|^2 = 64\pi^2 \alpha^2 e_q^2 \frac{\hat{u}^2}{\hat{t}^2} \quad (\text{B11})$$

$$\hat{M}_{++++} \hat{M}_{-+;-+}^* = 64\pi^2 \alpha^2 e_q^2 \frac{\hat{s}(-\hat{u})}{\hat{t}^2} e^{-i(\phi_1 - \phi_3)}. \quad (\text{B12})$$

In the (transversity) \otimes (Collins) contribution to the SSA, the phase dependence of the last term above ($\phi_1 - \phi_3 = \phi - \phi'$) combines with the \mathbf{k}_\perp phase in the transversity distribution (ϕ) and the Collins function phase (ϕ_q^h), resulting in the simple expression given in Eq. (17).

APPENDIX C: DETAILS FOR THE COMPUTATION OF $A_{TU}^{\sin \phi_S}$

In this section we show some details of the explicit calculation of the transverse single spin asymmetry $A_{TU}^{\sin \phi_S}$, Eq. (14), for the process $p^1 l \rightarrow hX$, starting from the general expression for the polarized cross section given in Eq. (4). By performing the sum over all the helicity indices and taking into account that the helicity density matrix of a quark q can be written in terms of the quark polarization vector components, $\mathbf{P}^q = (P_x^q, P_y^q, P_z^q)$, as

$$\begin{aligned} \rho_{\lambda_q, \lambda'_q}^{q/p,S} &= \begin{pmatrix} \rho_{++}^q & \rho_{+-}^q \\ \rho_{-+}^q & \rho_{--}^q \end{pmatrix}_{p,S} \\ &= \frac{1}{2} \begin{pmatrix} 1 + P_z^q & P_x^q - iP_y^q \\ P_x^q + iP_y^q & 1 - P_z^q \end{pmatrix}_{p,S}, \end{aligned} \quad (\text{C1})$$

one obtains, for a spinless hadron h ,

$$\begin{aligned} \frac{E_h d\sigma^{(p,S)+l \rightarrow hX}}{d^3 \mathbf{P}_h} &= \sum_q \int \frac{dx dz}{16\pi^2 x z^2 s} d^2 \mathbf{k}_\perp d^3 \mathbf{p}_\perp \delta(\mathbf{p}_\perp \cdot \hat{\mathbf{p}}'_q) J(p_\perp) \delta(\hat{s} + \hat{t} + \hat{u}) \\ &\times \frac{1}{2} \{ \hat{f}_{q/p,S}(x, \mathbf{k}_\perp) (|\hat{M}_{++++}|^2 + |\hat{M}_{-+;-+}|^2) D_{h/q}(z, p_\perp) \\ &+ [P_y^q \hat{f}_{q/p,S}(x, \mathbf{k}_\perp) [\text{Re}(\hat{M}_{++++} \hat{M}_{-+;-+}^*) \cos \phi_q^h - \text{Im}(\hat{M}_{++++} \hat{M}_{-+;-+}^*) \sin \phi_q^h] \\ &- P_x^q \hat{f}_{q/p,S}(x, \mathbf{k}_\perp) [\text{Im}(\hat{M}_{++++} \hat{M}_{-+;-+}^*) \cos \phi_q^h + \text{Re}(\hat{M}_{++++} \hat{M}_{-+;-+}^*) \sin \phi_q^h] \} \Delta^N D_{h/q^1}(z, p_\perp). \end{aligned} \quad (\text{C2})$$

In the above expression we have already extracted from the fragmentation functions $\hat{D}_{\lambda_q, \lambda'_q}(z, \mathbf{p}_\perp)$ their azimuthal dependence and exploited their parity properties (see Ref. [80] for details):

$$\hat{D}_{++}(z, \mathbf{p}_\perp) = \hat{D}_{--}(z, \mathbf{p}_\perp) = D_{h/q}(z, p_\perp) \quad (\text{C3})$$

$$\hat{D}_{+-}(z, \mathbf{p}_\perp) = D_{+-}(z, p_\perp) e^{i\phi_q^h} = \frac{i}{2} \Delta^N D_{h/q^1}(z, p_\perp) e^{i\phi_q^h} \quad (\text{C4})$$

$$\hat{D}_{-+}(z, \mathbf{p}_\perp) = [\hat{D}_{+-}(z, \mathbf{p}_\perp)]^*. \quad (\text{C5})$$

When computing the azimuthal asymmetry, one has the difference of cross sections with opposite transverse spin, $d\sigma(\phi_S) - d\sigma(\phi_S + \pi)$; using Eq. (19) and the definitions [80]

$$\begin{aligned}
P_y^q \hat{f}_{q/p, S_T}(x, \mathbf{k}_\perp) - P_y^q \hat{f}_{q/p, -S_T}(x, \mathbf{k}_\perp) &= \Delta \hat{f}_{s_y/S_T}(x, \mathbf{k}_\perp) - \Delta \hat{f}_{s_y/-S_T}(x, \mathbf{k}_\perp) = 2\Delta^- \hat{f}_{s_y/S_T}(x, \mathbf{k}_\perp) \\
P_x^q \hat{f}_{q/p, S_T}(x, \mathbf{k}_\perp) - P_x^q \hat{f}_{q/p, -S_T}(x, \mathbf{k}_\perp) &= \Delta \hat{f}_{s_x/S_T}(x, \mathbf{k}_\perp) - \Delta \hat{f}_{s_x/-S_T}(x, \mathbf{k}_\perp) = 2\Delta \hat{f}_{s_x/S_T}(x, \mathbf{k}_\perp),
\end{aligned} \tag{C6}$$

one obtains

$$\begin{aligned}
d\sigma(\phi_S) - d\sigma(\phi_S + \pi) &= \sum_q \int \frac{dx dz}{16\pi^2 x z^2 s} d^2 \mathbf{k}_\perp d^3 \mathbf{p}_\perp \delta(\mathbf{p}_\perp \cdot \hat{\mathbf{p}}'_q) J(p_\perp) \delta(\hat{s} + \hat{t} + \hat{u}) \\
&\times \left\{ \frac{1}{2} \Delta \hat{f}_{q/p, S_T}(x, \mathbf{k}_\perp) (|\hat{M}_{++++}|^2 + |\hat{M}_{-+;-+}|^2) D_{h/q}(z, p_\perp) \right. \\
&+ [\Delta^- \hat{f}_{s_y/S_T}(x, \mathbf{k}_\perp) [\text{Re}(\hat{M}_{++++} \hat{M}_{-+;-+}^*) \cos \phi_q^h - \text{Im}(\hat{M}_{++++} \hat{M}_{-+;-+}^*) \sin \phi_q^h] \\
&+ \Delta \hat{f}_{s_x/S_T}(x, \mathbf{k}_\perp) [\text{Im}(\hat{M}_{++++} \hat{M}_{-+;-+}^*) \cos \phi_q^h + \text{Re}(\hat{M}_{++++} \hat{M}_{-+;-+}^*) \sin \phi_q^h] \left. \Delta^N D_{h/q^1}(z, p_\perp) \right\}.
\end{aligned} \tag{C7}$$

Finally, using Eqs. (8), (9), (19), (23), and (B10)–(B12) and the relations [80]

$$\Delta^- \hat{f}_{s_y/S_T}(x, \mathbf{k}_\perp) = \left[h_1(x, k_\perp) - \frac{k_\perp^2}{2M^2} h_{1T}^\perp(x, k_\perp) \right] \sin(\phi_S - \phi) \tag{C8}$$

$$\Delta \hat{f}_{s_x/S_T}(x, \mathbf{k}_\perp) = \left[h_1(x, k_\perp) + \frac{k_\perp^2}{2M^2} h_{1T}^\perp(x, k_\perp) \right] \cos(\phi_S - \phi), \tag{C9}$$

yields

$$\begin{aligned}
d\sigma(\phi_S) - d\sigma(\phi_S + \pi) &= \sum_q \int \frac{dx dz}{16\pi^2 x z^2 s} d^2 \mathbf{k}_\perp d^3 \mathbf{p}_\perp \delta(\mathbf{p}_\perp \cdot \hat{\mathbf{p}}'_q) J(p_\perp) \delta(\hat{s} + \hat{t} + \hat{u}) \\
&\times \left\{ \frac{1}{2} \Delta^N f_{q/p^1}(x, k_\perp) \sin(\phi_S - \phi) (|\hat{M}_1^0|^2 + |\hat{M}_2^0|^2) D_{h/q}(z, p_\perp) \right. \\
&+ h_{1q}(x, k_\perp) \sin(\phi_S - \phi' - \phi_q^h) \hat{M}_1^0 \hat{M}_2^0 \Delta^N D_{h/q^1}(z, p_\perp) \\
&- \left. \frac{k_\perp^2}{2M^2} h_{1T}^{\perp q}(x, k_\perp) \sin(\phi_S - 2\phi + \phi' + \phi_q^h) \hat{M}_1^0 \hat{M}_2^0 \Delta^N D_{h/q^1}(z, p_\perp) \right\}.
\end{aligned} \tag{C10}$$

The first term on the rhs of Eq. (C10) gives the Sivers contribution while the second term gives the transversity \otimes Collins effect. We have numerically checked that the third term gives negligible contributions. Notice that the various terms of the type $\sin(\phi_S - \Phi)$ appearing in Eq. (C10) can be decomposed as $\sin \phi_S \cos \Phi - \cos \phi_S \sin \Phi$: similarly to what has been explicitly shown in Appendix A 2 [see the comment after Eq. (A27)], the $\cos \phi_S$ terms integrate to zero. Thus, one obtains the simple expression of Eq. (17), given for $\phi_S = \pi/2$.

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