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Effects of θ on the deuteron binding energy and the triple-alpha process

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We study the effects that a nonzero strong-*CP*-violating parameter θ would have on the deuteron and diproton binding energies and on the triple-alpha process. Both these systems exhibit fine-tuning, so it is plausible that a small change in the nuclear force would produce catastrophic consequences. Such a nuclear force is here understood in the framework of an effective Lagrangian for pions and nucleons, and the strength of the interaction varies with θ. We find that the effects are not too dramatic.

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I. INTRODUCTION

The QCD Lagrangian includes a θ-term which is usually written as

$$\mathcal{L}_\theta = -\frac{g^2\theta}{32\pi^2} F\tilde{F}. \tag{1}$$

For $\theta \neq 0$, this leads to *CP*-violation in the strong interaction. Measurements of the neutron electric dipole moment set the severe bound $|\theta| < 10^{-10}$ [1,2]. The lack of a satisfactory explanation, within the standard model, of why this should be the case is referred to as the strong *CP* problem.

The strong *CP* problem remains an outstanding issue in string theory. In critical string theories, *CP* is a gauge symmetry. Typically, these theories exhibit moduli, and *CP* is spontaneously broken on most of the moduli space. The *CP* odd moduli are candidate axions. If such theories describe nature, the moduli must somehow be fixed, and it is not clear whether axions typically survive this process. For example, in the flux vacua studied in Ref. [3], *all* of the moduli are massive and there are no candidate axions. $\sin(\theta)$ looks like a random variable [4–6] whose typical value is of order one.

In the absence of a principle which ensures a small θ *ab initio*, or a very light axion, one can ask what might account for a small θ. It has been argued [3,7,8] that in theories like string theory, the cosmological constant, Λ, might be a similar random variable, fixed by anthropic considerations. This is plausible because for large Λ, physics would be drastically different than what we observe. For θ, however, it is clear that, say, $\theta = 10^{-6}$ would not significantly change nuclear physics, so anthropic considerations are likely to be ineffective. It is interesting to ask for what values of θ *would* physics be significantly different. This is the topic of this paper. Our goal will be to examine different processes, and ask, in order of magnitude, when θ might make an appreciable difference. This is similar in spirit to the work recently published by Jaffe *et al.* [9]. Varying the quark masses, they investigate which values satisfy the environmental constraint that the quark masses allow for stable nuclei, making organic chemistry possible. In our case, instead, we fix the quark masses to the values[1]

$$m_u = 4\ \text{MeV}, \qquad m_d = 7\ \text{MeV} \tag{2}$$

and let θ vary. We study the effects that this would produce on the binding energies of two among the lightest nuclei, the deuteron and the diproton,[2] and on the abundance of carbon and oxygen. Csoto, Oberhummer and Schlattl [10,11] determined that the abundance of ^{12}C and ^{16}O is extremely sensitive to even small changes in the strength of the nucleon-nucleon force. The models they use to describe the *N*-*N* interaction in their study do not involve explicitly the angle θ. They multiply the strength of the *N*-*N* force by a factor p, which they then vary from 0.996 to 1.004. In our case, the tool for exploring the consequences of $\theta \neq 0$ on the systems just mentioned is provided by a sigma model, intended as an effective Lagrangian that describes the interactions between pions and nucleons. For nuclei like ^{12}C and ^{16}O, the nuclear force is described by contact interactions, the strength of which depends on the pion mass, that in turn depends on θ. The analysis will be somewhat simplistic, since we are only interested in an order-of-magnitude estimate.

The paper is organized as follows. First we write the sigma model Lagrangian and we derive formulas for the pion mass and the proton-neutron mass difference as functions of θ. Then, we compute the correction to the binding energies of the deuteron and the diproton, and we study the consequences of varying θ on the triple-alpha process. We conclude with a few comments on the results.

II. LAGRANGIAN FOR NUCLEON-PION INTERACTIONS

A. θ-dependence in the quark mass matrix

For the purposes of the following discussion, it is convenient to remove the term (1) from the Lagrangian by

[1]These values of the quark masses are typically taken to apply at a scale of 1 GeV.

[2]The diproton does not actually exist as a bound state in nature, but the effect of θ could be such to bind it.

performing a rotation of the quark fields

$$u \to e^{i\phi_u} u \tag{3}$$

$$d \to e^{i\phi_d} d, \tag{4}$$

such that

$$\phi_u + \phi_d = \theta. \tag{5}$$

This introduces an equivalent θ dependence in the quark mass matrix, that we write as MU_0, where

$$M = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix}, \qquad U_0 = \begin{pmatrix} e^{i\phi_u} & 0 \\ 0 & e^{i\phi_d} \end{pmatrix}. \tag{6}$$

B. The sigma model

The sigma model Lagrangian provides a framework for understanding the very low energy limit of QCD. We use the notation of the text by Srednicki [12], and write our effective Lagrangian for pions and nucleons as

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4} f_\pi^2 \mathrm{Tr}[\partial_\mu U \partial^\mu U^\dagger] + B_0 \mathrm{Tr}[(MU_0)U + (MU_0)^\dagger U^\dagger] + i\bar{N}\gamma_\mu \partial^\mu N - m_N \bar{N}(U^\dagger P_L + UP_R)N \\ &- \frac{1}{2}(g_A - 1)i\bar{N}\gamma^\mu(U\partial_\mu U^\dagger P_L + U^\dagger \partial_\mu U P_R)N - c_1 \bar{N}((MU_0)P_L + (MU_0)^\dagger P_R)N - c_2 \bar{N}(U^\dagger (MU_0)^\dagger U^\dagger P_L \\ &+ U(MU_0)UP_R)N - c_3 \mathrm{Tr}((MU_0)U + (MU_0)^\dagger U^\dagger)\bar{N}(U^\dagger P_L + UP_R)N - c_4 \mathrm{Tr}((MU_0)U \\ &- (MU_0)^\dagger U^\dagger)\bar{N}(U^\dagger P_L - UP_R)N, \end{aligned} \tag{7}$$

where $U = e^{i\pi^a\tau^a/f_\pi}$, π^a is the pion field, τ^a are the isospin matrices, $f_\pi = 92.4$ MeV is the pion decay constant, N is the nucleon field, $P_L = \frac{1}{2}(1 - \gamma_5)$ and $P_R = \frac{1}{2}(1 + \gamma_5)$ are the projection operators, $g_A = 1.27$ is the axial vector coupling, and c_1, c_2, c_3, c_4 are dimensionless constants. B_0 is a constant with dimension of $[\mathrm{mass}]^3$ that can be determined from ratios of meson masses in $SU(3)$. Roughly speaking, $B_0 \sim \Lambda_{\mathrm{QCD}}^3$. In this paper we use $B_0 = 7.6 \times 10^6\ \mathrm{MeV}^3$. In the Lagrangian above we wrote all the possible terms that are invariant under $SU(2)_L \times SU(2)_R$, with the fields obeying the transformation rules

$$\begin{aligned} N_L &\to LN_L, \qquad N_R \to RN_R, \\ U &\to LUR^\dagger, \qquad (MU_0) \to R(MU_0)L^\dagger, \end{aligned} \tag{8}$$

for L, R in $SU(2)$.

The pion mass. We first obtain a formula for the mass of the pion as a function of θ. We can start by writing

$$U = e^{i\pi^a\tau^a/f_\pi} = \cos\frac{|\vec{\pi}|}{f_\pi} + i\frac{\pi^a}{|\vec{\pi}|}\tau^a \sin\frac{|\vec{\pi}|}{f_\pi}. \tag{9}$$

It will prove convenient also to adopt the following parametrization for the quark mass matrix:

$$MU_0 = A\mathbf{1}_2 + iB\mathbf{1}_2 + C\tau^3 + iD\tau^3. \tag{10}$$

Using (9) and (10), the potential V in the Lagrangian (7) reduces to

$$\begin{aligned} V &= -B_0 \mathrm{Tr}[(MU_0)U + (MU_0)^\dagger U^\dagger] \\ &= -B_0\left[4A\cos\frac{|\vec{\pi}|}{f_\pi} - 4D\frac{\pi^3}{|\vec{\pi}|}\sin\frac{|\vec{\pi}|}{f_\pi}\right]. \end{aligned} \tag{11}$$

In order not to have a tadpole in π^3, we impose the condition $D = 0$

$$\begin{aligned} D &= \frac{1}{2}\mathrm{Tr}\left[\tau^3\begin{pmatrix} m_u \sin\phi_u & 0 \\ 0 & m_d \sin\phi_d \end{pmatrix}\right] \\ &= \frac{1}{2}(m_u \sin\phi_u - m_d \sin\phi_d) = 0. \end{aligned} \tag{12}$$

Solving (5) and (12) we find the useful relations

$$\sin\phi_u = \frac{m_d \sin\theta}{[m_u^2 + m_d^2 + 2m_u m_d \cos\theta]^{1/2}} \tag{13}$$

$$\sin\phi_d = \frac{m_u \sin\theta}{[m_u^2 + m_d^2 + 2m_u m_d \cos\theta]^{1/2}} \tag{14}$$

$$\cos\phi_u = \frac{m_u + m_d \cos\theta}{[m_u^2 + m_d^2 + 2m_u m_d \cos\theta]^{1/2}} \tag{15}$$

$$\cos\phi_d = \frac{m_d + m_u \cos\theta}{[m_u^2 + m_d^2 + 2m_u m_d \cos\theta]^{1/2}}. \tag{16}$$

Next we determine A

$$\begin{aligned} A &= \frac{1}{2}\mathrm{Tr}\begin{pmatrix} m_u \cos\phi_u & 0 \\ 0 & m_d \cos\phi_d \end{pmatrix} \\ &= \frac{1}{2}(m_u \cos\phi_u + m_d \cos\phi_d). \end{aligned} \tag{17}$$

We now have all the ingredients to get an expression for the pion mass. From Eq. (11), expanding $\cos\frac{|\vec{\pi}|}{f_\pi}$ to second order we find

$$m_\pi^2 = \frac{2B_0}{f_\pi^2}[m_u^2 + m_d^2 + 2m_u m_d \cos\theta]^{1/2}. \tag{18}$$

Note that this is an even function of θ, therefore CP conserving. This formula generalizes and, for $\theta = 0$, reduces to the well-known $m_\pi^2 = \frac{2B_0}{f_\pi^2}(m_u + m_d)$. Note that, varying θ from 0 to π, the pion mass decreases, and it attains a minimum at $\theta = \pi$.

All this was done in $SU(2)$. One could be more ambitious and try to find a formula for the pion mass in $SU(3)$. In that case, the analysis is carried out in the same way. Requiring the absence of tadpoles translates into two conditions

$$m_u \sin\phi_u = m_d \sin\phi_d = m_s \sin\phi_s, \qquad (19)$$

and Eq. (5) is modified to

$$\phi_u + \phi_d + \phi_s = \theta. \qquad (20)$$

Now (19) and (20) cannot be solved analytically, but if we make the reasonable approximation m_u, $m_d \ll m_s$, they reduce to

$$\phi_u + \phi_d = \theta \qquad (21)$$

$$\phi_s = 0 \qquad (22)$$

$$m_u \sin\phi_u = m_d \sin\phi_d, \qquad (23)$$

which can be solved, leading to the same solution we found previously. The pion mass then turns out be the same as in the $SU(2)$ case.

The nucleons. Let us now examine the part of the Lagrangian involving the nucleons. First we can rewrite it in a more convenient way, using the following field redefinition[3]

$$N = (u_0 u P_L + u_0^\dagger u^\dagger P_R)\mathcal{N}, \qquad (24)$$

where $u_o^2 = U_0$ and $u^2 = U$. The last five lines in (7) become

$$\begin{aligned} &i\bar{\mathcal{N}}\gamma^\mu\partial_\mu\mathcal{N} - m_N\bar{\mathcal{N}}\mathcal{N} + \bar{\mathcal{N}}\gamma^\mu v_\mu\mathcal{N} - g_A\bar{\mathcal{N}}\gamma^\mu\gamma_5 a_\mu\mathcal{N} - \frac{1}{2}c_+\bar{\mathcal{N}}(u(MU_0)u + u^\dagger(MU_0)^\dagger u^\dagger)\mathcal{N} \\ &+ \frac{1}{2}c_-\bar{\mathcal{N}}(u(MU_0)u - u^\dagger(MU_0)^\dagger u^\dagger)\gamma_5\mathcal{N} - c_3\mathrm{Tr}[(MU_0)U + (MU_0)^\dagger U^\dagger]\bar{\mathcal{N}}\mathcal{N} \\ &+ c_4\mathrm{Tr}[(MU_0)U - (MU_0)^\dagger U^\dagger]\bar{\mathcal{N}}\gamma_5\mathcal{N}, \end{aligned} \qquad (25)$$

where $v_\mu = \frac{i}{2}[u^\dagger(\partial_\mu u) + u(\partial_\mu u^\dagger)]$, $a_\mu = \frac{i}{2}[u^\dagger(\partial_\mu u) - u(\partial_\mu u^\dagger)]$, and $c_\pm = c_1 \pm c_2$. This is not yet particularly illuminating. With some more algebra, we can write, to lowest order, the corrections to the nucleon mass

$$\begin{aligned} \mathcal{L}_{\mathrm{mass}} = &-\frac{1}{2}(c_+ + 4c_3)[m_u^2 + m_d^2 + 2m_u m_d\cos\theta]^{1/2}\bar{\mathcal{N}}\mathcal{N} + i(c_- + 4c_4)\frac{m_u m_d\sin\theta}{[m_u^2 + m_d^2 + 2m_u m_d\cos\theta]^{1/2}}\bar{\mathcal{N}}\gamma_5\mathcal{N} \\ &-\frac{1}{2}c_+\frac{m_u^2 - m_d^2}{[m_u^2 + m_d^2 + 2m_u m_d\cos\theta]^{1/2}}\bar{\mathcal{N}}\tau^3\mathcal{N} \end{aligned} \qquad (26)$$

and the nucleon-pion interactions

$$\begin{aligned} \mathcal{L}_{\mathrm{int}} = &-ig_{\pi NN}\pi^a\bar{\mathcal{N}}\tau^a\gamma_5\mathcal{N} + \frac{i}{2}c_-[m_u^2 + m_d^2 + 2m_u m_d\cos\theta]^{1/2}\bar{\mathcal{N}}\frac{\pi^a\tau^a}{f_\pi}\gamma_5\mathcal{N} \\ &+ c_+\frac{m_u m_d\sin\theta}{[m_u^2 + m_d^2 + 2m_u m_d\cos\theta]^{1/2}}\bar{\mathcal{N}}\frac{\pi^a\tau^a}{f_\pi}\mathcal{N} + \frac{i}{2f_\pi}(c_- + 4c_4)\frac{m_u^2 - m_d^2}{[m_u^2 + m_d^2 + 2m_u m_d\cos\theta]^{1/2}}\pi^3\bar{\mathcal{N}}\gamma_5\mathcal{N}. \end{aligned} \qquad (27)$$

From (26) we get the proton-neutron mass difference

$$m_n - m_p = c_+\frac{m_d^2 - m_u^2}{[m_u^2 + m_d^2 + 2m_u m_d\cos\theta]^{1/2}}. \qquad (28)$$

Note that, varying θ from 0 to π, $m_n - m_p$ increases. It reaches the maximum value $c_+(m_d + m_u)$ at $\theta = \pi$.

Estimation of the constants. The constants c_+, c_-, c_3, c_4 can in principle be related to quantities measured in experiments. Since in our world θ is smaller than 10^{-10} (see e.g. [1]), we define these quantities to be measured at $\theta = 0$. Note that, with this definition, we do not learn anything about c_- and c_4 from the nucleon mass, since the second term in (26) vanishes at $\theta = 0$. It would be good, for the sake of completeness if nothing else, if we could determine all the constants, but this task is not so easy and, for the calculation that we will perform in the next section, only c_+ contributes substantially.

The value of c_+ can be estimated in at least two ways:

(i) from the measured proton-neutron mass difference (~ 1.3 MeV at $\theta = 0$). Taking into account also the electromagnetic contribution $\epsilon_{\mathrm{EM}} \sim 0.5$ MeV we have

$$(m_n - m_p)_{\mathrm{measured}} = c_+(m_d - m_u) - \epsilon_{\mathrm{EM}}, \qquad (29)$$

yielding $c_+ = 0.6$. This estimation is crude, because

[3]This is the same field redifinition that the reader finds in [12]

the second contribution on the right-hand side of the above equation is of the same order as the first one;

(ii) more accurately, from the mass splitting $M_\Xi - M_N$ in the baryon octet, as pointed out in [1]. That yields $c_+ = 2.5$. This is the value that we are going to use in the next section.

The constant c_- deserves some comments. If we look at the first line of (27), it appears that $\frac{c_-}{2f_\pi}[m_u^2 + m_d^2 + 2m_u m_d \cos\theta]^{1/2}$ can be considered as some kind of correction to $g_{\pi NN}$. At $\theta = 0$, one could interpret the measured value of $g_{\pi NN}$ as including such a correction, but that would not tell us anything about c_-. In other words, one could trade c_- for a new constant, say $g'_{\pi NN} = g_{\pi NN} + \frac{c_-}{2f_\pi}(m_u + m_d)$. For $\theta \neq 0$, though, one wants to keep the contribution coming from c_- separate from $g_{\pi NN}$ and deal with the fact that there seems to be no obvious physical quantity from which this constant can be estimated. From the construction of the Lagrangian, it makes sense to believe that c_- should be of the same order as c_+, namely, of order unity, because they are both linear combinations of c_1 and c_2 that appear in Eq. (7), but there is no proof of this. On the other hand, a value as big as 10 would be disturbing because it would cancel the suppression $\frac{[m_u^2+m_d^2+2m_u m_d \cos\theta]^{1/2}}{f_\pi} \sim \frac{1}{10}$. As already stated, we will not need c_- for our calculation. We actually need to make this statement more precise: we can forget about the exact value of c_- as long as it is not much greater than one. The reason for this will be discussed in the next section.

Equations (18) and (23) are the main results of this section. They make the θ-dependence of the pion mass and the proton-neutron mass difference explicit and, since these quantities play key roles in determining nuclear properties, they can be used to explore the consequences of a nonzero strong-*CP*-violating parameter in nuclear physics.

III. EFFECTS OF θ IN NUCLEAR PHYSICS

We do not have yet a complete picture to explain nuclear physics in terms of effective field theories, but enough progress has been made to allow us to investigate, at least qualitatively, the effects that $\theta \neq 0$ would have in nuclear physics. In this section we focus our attention on:

(i) *Two-nucleon systems*, namely, the deuteron and the diproton.[4] The former has a binding energy which is relatively small (2.2 MeV); the latter does not exist as a bound state in nature, but we know that it fails to bind by only ~70 keV. In principle, one expects that the θ-dependent nucleon-pion interactions in (25) could give corrections to these energies that might be big enough to unbind the deuteron or to bind the diproton. If either one of these possibilities were realized, the consequences would be dramatic. For instance, if the diproton were bound, all the hydrogen in the Universe would have been burnt to He^2 during the early stages of the Big Bang and no hydrogen compounds or stable stars would exist today. Likewise, an unbound deuteron would significantly change the chain of nucleosynthesis that leads to heavier elements [13]. Another reason for studying these two-nucleon systems is that they are simple, and we have a good control over the calculation. Other authors have studied the dependence of the deuteron binding energy on variations of other parameters, such as the coupling constant [14,15], or the quark masses [16–19];

(ii) *The triple-alpha process*, which is responsible for the production of carbon in stars. The observed abundance of carbon and oxygen results from a peculiar position of various nuclear energy levels, and it is very sensitive to even small shifts of such levels. It is hard to relate the spacing between excited states of a nucleus to first principles, but, with some assumptions, we can qualitatively study how variations of θ affect the triple-alpha process.

A. Two-nucleon systems

The deuteron. The deuteron exists as a bound state only in an isospin singlet and spin triplet configuration, and its binding energy is rather small ($E = -2.22$ MeV).[5] Attempts to derive the nuclear potential starting from a chiral Lagrangian show that the deuteron binding is predominantly a consequence of two-pion and three-pion exchanges. The two-pion can be modeled by $\sigma(600)$ exchange, which gives an attractive medium-range contribution, whereas the three-pion corresponds to an $\omega(783)$ exchange, which is short-range and repulsive. The one-pion exchange is responsible for the long-range contribution.

For the purpose of our study here, however, we can content ourselves with a much simpler form for the potential, a three-dimensional square well

$$V(r) = \begin{cases} -V_0 & r < R \\ 0 & r > R \end{cases}; \qquad \psi(r) = \begin{cases} A\frac{\sin kr}{r} & r < R \\ B\frac{e^{-\rho r}}{r} & r > R \end{cases}$$

with parameters chosen to fit the experimental measurements: $V_0 = 41$ MeV, $R = 8.62 \times 10^{-3}\ \mathrm{MeV}^{-1}$, $k = 212$ MeV, $\rho = 46.4$ MeV, $A = 2.31\ \mathrm{MeV}^{1/2}$, $B = 1.44A$.

We want to compute the first-order corrections to the potential that we get from the theta-dependent terms in the Lagrangian, and see how significant they are. The interaction terms, that we need to look at, are listed in (27). A couple of comments are in due order:

[4]For completeness one could also study the dineutron. The conclusions would be qualitatively the same as for the diproton.

[5]We take the convention that the binding energy is a negative number for a bound state.

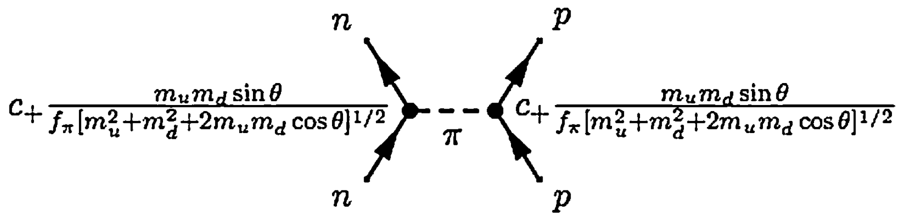

FIG. 1. Feynman diagram.

(i) all the terms in (27), except for the first one, are suppressed by $\frac{m_q}{f_\pi}$, where m_q stands for the quark mass and is, roughly speaking, a few MeV;

(ii) the terms containing a γ_5 get an extra spin-suppression that goes as $\frac{m_\pi}{2m_N}$ at each nucleon-nucleon-pion vertex. Note that $\frac{m_\pi}{2m_N}$ is of the same order as $\frac{m_q}{f_\pi}$.

Thus, a one-pion exchange diagram with $g_{\pi NN}$ at one vertex and $c_- \frac{m_q}{f_\pi}$ at the other vertex is suppressed with respect to a diagram with $c_+ \frac{m_q}{f_\pi}$ at both vertices, as long as c_- *is at the most of order 1*. To lowest order, then, we only need to evaluate the diagram shown in Fig. 1. In the nonrelativistic limit it gives

$$+ ic_+^2 \frac{m_u^2 m_d^2 \sin^2\theta}{f_\pi^2 [m_u^2 + m_d^2 + 2m_u m_d \cos\theta]} \frac{\vec{\tau}_n \cdot \vec{\tau}_p}{\mathbf{q}^2 + m_\pi^2} \tag{30}$$

where $\mathbf{q}$ is the three-momentum of the exchanged pion. Using $\vec{\tau}_n \cdot \vec{\tau}_p = -3$ for the isosinglet, and Fourier transforming to position space, we find the following correction to the potential:

$$V_1(r, \theta) = \frac{3}{4\pi} \frac{c_+^2}{f_\pi^2} \frac{m_u^2 m_d^2 \sin^2\theta}{[m_u^2 + m_d^2 + 2m_u m_d \cos\theta]} \frac{e^{-m_\pi r}}{r}. \tag{31}$$

This is repulsive for all values of θ. We can now use this result to compute the shift in the deuteron binding energy using first-order perturbation theory:

$$\Delta(\theta) = \langle \psi(r) | V_1(r, \theta) | \psi(r) \rangle. \tag{32}$$

The energy shift is plotted as a function of θ in Fig. 2. At $\cos\theta \simeq -0.7$, we read from the plot that $\Delta \simeq 0.15$ MeV. As this shift is positive, the deuteron becomes more weakly bound. 0.15 MeV is a small number compared to 2.22 MeV, but might still have an appreciable effect on the early stages of Big Bang nucleosynthesis, since the reaction rates depend exponentially on the deuteron binding energy.

Before studying the diproton, let us see what would happen if c_- was 10 instead of order 1. In this case, we would have $c_- \frac{m_q}{f_\pi} \sim 1$, and the diagram with $g_{\pi NN}$ at one vertex and $c_- \frac{m_q}{f_\pi}$ at the other vertex would not be suppressed anymore with respect to the one in Fig. 1. Including its contribution we would come to a qualitatively different conclusion as shown in Fig. 3: the maximum value of the energy shift would be at $\theta = 0$.

The diproton. The diproton almost exists as a bound state, so it is conceivable that the correction to the potential, that we get by calculating the diagram analogous to the one in Fig. 1 (just replacing the neutron with a proton), might be significant enough to bind this system. We will proceed along the same line as for the deuteron. Here we adopt again a three-dimensional square well potential with the following parameters: $V_0 = 14$ MeV, $R = 13.1 \times 10^{-3}$ MeV^{-1}, $k = 114$ MeV, $\rho = 8.2$ MeV, $A = 1.09$ MeV$^{1/2}$, $B = 1.11A$. With this choice of parameters,

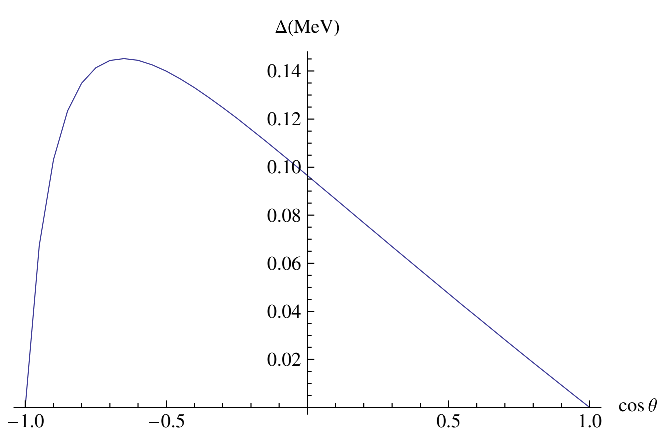

FIG. 2 (color online). Shift in the deuteron binding energy as a function of $\cos\theta$.

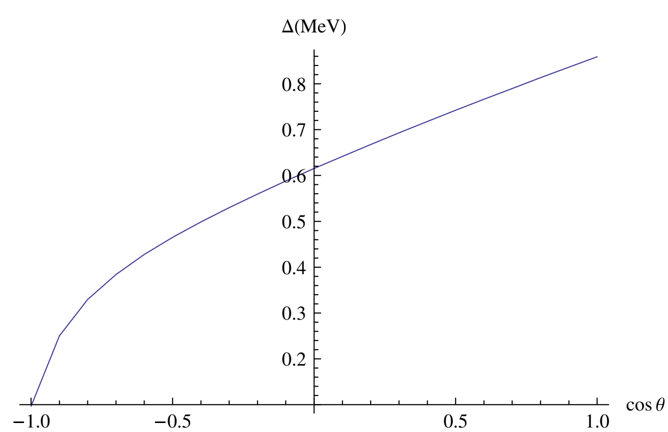

FIG. 3 (color online). Shift in the deuteron binding energy with $c_- = 10$.

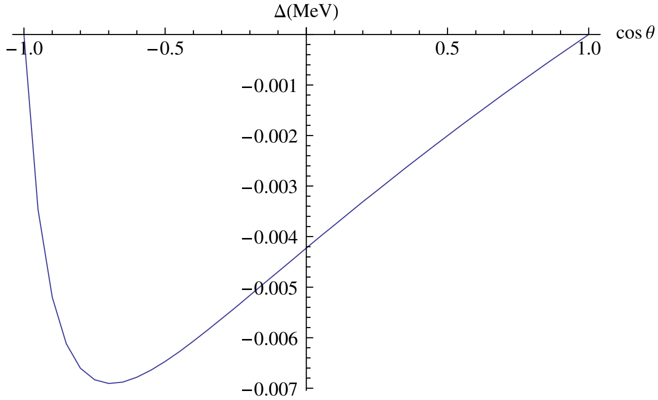

FIG. 4 (color online). Shift in the diproton binding energy as a function of cosθ.

the diproton fails to be bound by an energy $E = 72$ keV. The evaluation of the Feynman diagram in the nonrelativistic limit gives (30), with $\vec{\tau}_n \cdot \vec{\tau}_p$ replaced by $\vec{\tau}_p \cdot \vec{\tau}_p$. If the diproton were bound, it would be in an isosinglet state, in which case $\vec{\tau}_p \cdot \vec{\tau}_p = +1$. The correction to the potential is then the following:

$$V_1(r, \theta) = -\frac{1}{4\pi}\frac{c_+^2}{f_\pi^2}\frac{m_u^2 m_d^2 \sin^2\theta}{[m_u^2 + m_d^2 + 2m_u m_d \cos\theta]}\frac{e^{-m_\pi r}}{r}, \tag{33}$$

and is attractive. The energy shift $\Delta(\theta) = \langle\psi(r)|V_1(r,\theta)|\psi(r)\rangle$ is plotted in Fig. 4. At cos$\theta \simeq -0.7$, Δ attains the minimum value of ~ -7 keV. This represents a 10% correction to the energy, which is not enough to bind the diproton, but might still have consequences on the early stages of the nucleosynthesis chain.

To summarize, the effects of the angle θ on the binding energies of the deuteron and the diproton are at most 10% corrections (6–7% for the deuteron), which could be significant enough to affect the early stages of the Big Bang nucleosynthesis. So large values of θ might have appreciable consequences.

B. The triple-alpha process

The production of carbon in stars results from the reaction $3\alpha \leftrightarrow \alpha + {}^8\text{Be} \leftrightarrow {}^{12}\text{C}^{**}$. The ^{8}Be nucleus, in the second step, is unbound, but it lives long enough to allow for the possibility of capturing another alpha particle to form ^{12}C. However, to produce the observed abundance of carbon, this second reaction must be resonant. The 0_2^+ state of ^{12}C, lying at 380 keV, relative to the 3α threshold (and 7654 keV above the ^{12}C ground state), provides such a resonance. The reaction rate for the triple-alpha process goes as [20]

$$r \sim \frac{\Gamma_\alpha \Gamma_{\text{rad}}}{\Gamma} \exp\left(\frac{-Q_{3\alpha}}{T}\right) \tag{34}$$

where

$$Q_{3\alpha} = M_{^{12}\text{C}^{**}} - 3M_\alpha, \tag{35}$$

Γ_α is the alpha particle width, $\Gamma_{\text{rad}} = \Gamma_\gamma + \Gamma_{\text{pair}}$ is the sum of electromagnetic decay widths to the ^{12}C ground state via gamma-ray emission or via electron-positron pair emission and $\Gamma = \Gamma_\alpha + \Gamma_{\text{rad}}$. The following approximations hold: (i) $\Gamma_\alpha \gg \Gamma_{\text{rad}}$ and (ii) $\Gamma_{\text{rad}} \simeq \Gamma_\gamma$, so that $\frac{\Gamma_\alpha \Gamma_{\text{rad}}}{\Gamma} \simeq \Gamma_\gamma$ and we can write

$$r \sim \Gamma_\gamma \exp\left(\frac{-Q_{3\alpha}}{T}\right). \tag{36}$$

The measured values that enter the above equation are $Q = 380$ keV, $T \simeq 10$ keV and $\Gamma_\gamma \simeq 3.6$ meV. Let us take these to be our values at $\theta = 0$ and let us now see what would happen if θ were not zero. For simplicity, we make the assumption that the energy of the excited state ^{12}C** with respect to the ground state ^{12}C does not vary with θ. This assumption is probably unrealistic, but we use it to get a feeling for the various possibilities. It follows that Γ_γ is nearly constant as well. But a small variation of $Q_{3\alpha}$ can have significant effects, because it appears in the exponential. We have

$$\begin{aligned} M_{^{12}\text{C}^{**}} &= 6m_p + 6m_n + \text{BE}_\text{C} + 7.65, \\ M_\alpha &= 2m_p + 2m_n + \text{BE}_\alpha, \end{aligned} \tag{37}$$

where BE is the binding energy (negative) and everything is measured in MeV. Thus

$$Q_{3\alpha} = \text{BE}_\text{C} - 3\text{BE}_\alpha + 7.65. \tag{38}$$

Following the work done by Furnstahl and Serot [21], and by Donoghue and Damour [22], we can parametrize the binding energy per nucleon BE/A as [22]

$$\frac{\text{BE}}{A} = -\left(120 - \frac{97}{A^{1/3}}\right)\eta_S + \left(67 - \frac{57}{A^{1/3}}\right)\eta_V + \text{residual terms.} \tag{39}$$

This formula comes from considering the nuclear force as due to contact interactions. For all but the lightest nuclei, the key aspect of binding comes from a spin singlet and isospin singlet central potential, for which one can write a scalar and a vector contribution

$$H_{\text{contact}} = G_S(\bar{N}N)(\bar{N}N) + G_V(\bar{N}\gamma_\mu N)(\bar{N}\gamma^\mu N), \tag{40}$$

where G_S is negative (i.e. attractive) and G_V is positive (i.e. repulsive). In the traditional meson exchange models, the scalar component corresponds to the exchange of the $\sigma(600)$ meson and the vector component to the exchange of the $\omega(783)$ meson. We define η_S and η_V, that appear in Eq. (39), as[6]

$$\eta_S \equiv \frac{G_S(\theta)}{G_S(\theta = 0)} \tag{41}$$

[6]Our $G_{S,V}(\theta = 0)$ is the same as what Damour and Donoghue [22] call $G_{S,V}|_{\text{physical}}$.

$$\eta_V \equiv \frac{G_V(\theta)}{G_V(\theta = 0)}. \tag{42}$$

The scalar channel is the only portion of the central force that receives large effects from low energy. The sensitivity of the vector channel to m_π^2 leads to subleading corrections compared to the effects linked to the m_π^2 sensitivity of the scalar channel (the reader should refer to [23] for the details). For this reason, we will take $\eta_V = 1$ for our discussion and focus on the dominant scalar-channel effects. We parametrize η_S from Fig. 2 in [22]

$$\eta_S = -0.4\frac{m_\pi^2(\theta)}{m_{\rm phys}^2} + 1.4, \tag{43}$$

where $m_{\rm phys}^2 = m_\pi^2(\theta = 0)$ is the physical mass of the pion. The residual terms in Eq. (39), which we assume not to depend on θ, take care of all the other contributions that are not encoded by η_S or η_V, such as the Coulomb repulsion, for example, and can be adjusted to get the measured BE/A for each element at $\eta_S = \eta_V = 1$. For ^{12}C we have

$$\left(\frac{\rm BE}{A}\right)_{\rm C} = -7.67 - 78(\eta_S - 1), \tag{44}$$

for ^{4}He

$$\left(\frac{\rm BE}{A}\right)_{\alpha} = -7.06 - 59(\eta_S - 1). \tag{45}$$

Thus, we can write $Q_{3\alpha}$ as a function of θ

$$Q_{3\alpha}(\theta) = 12\left(\frac{\rm BE}{A}\right)_C - 12\left(\frac{\rm BE}{A}\right)_\alpha + 7.65 \tag{46}$$

$$= 0.38 + 91\left(\frac{[m_u^2 + m_d^2 + 2m_u m_d \cos\theta]^{1/2}}{m_u + m_d} - 1\right). \tag{47}$$

For the resonant reaction to occur, $Q_{3\alpha}$ must be a positive quantity, which is equivalent to require that the excited state ^{12}C** be above threshold. The condition $Q_{3\alpha} > 0$ translates into the constraint

$$\cos\theta > 0.98 \qquad (\theta < 11^\circ). \tag{48}$$

We can plot $r(\theta)/r$ vs $\cos\theta$, where $r(\theta)$ is the reaction rate (36) as a function of θ and $r \equiv r(\theta = 0)$. The result is shown in Fig. 5.

We see that for $\theta \neq 0$ the reaction rate increases dramatically. To understand how the abundance of carbon varies with such a change in the rate one needs more astrophysical input about the stellar processes that produce carbon.[7] This is beyond the scope of the current paper. Here we just note that for $\theta \sim 2^\circ$ or 3° the reaction rate would already be 10 times larger, which would lead to a greater abundance of carbon in the Universe.

There is a catch in the discussion above. If the ^{8}Be that appears in the second step of the reaction were bound, the whole situation would change drastically and the amount of carbon produced would be greatly increased. It is unclear what kind of change in θ would bind the ^{8}Be; in this case it does not make much sense to use the semiempirical Eq. (39) because we are not dealing with a bound nucleus. We can have an indication looking at η_S, which parametrizes the attractive contribution to the potential. From Eq. (43) we see that this attractive contribution becomes stronger for negative or small positive values of $\cos\theta$. Therefore, for $\cos\theta$ close to 1, as we found in Eq. (48), it seems safe to assume that the ^{8}Be will stay unbound.

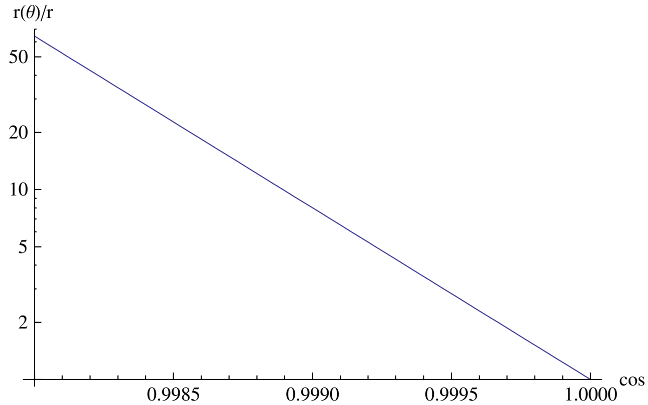

FIG. 5 (color online). Reaction rate for the triple-alpha process as a function of $\cos\theta$.

Carbon is then involved in the reaction ^{12}C + $\alpha \to$ ^{16}O to produce oxygen. In a world where $\theta = 0$, there is no energy level in ^{16}O to allow for this last reaction to be resonant, and that is why a substantial amount of ^{12}C survives. The closest level that could give a resonance is 2.42 MeV above the ^{12}C + α threshold, too high to be resonant. There are two levels that are just subthreshold, though, one at -45 keV, the other at -245 keV. It is conceivable that in our framework, when we vary θ, we shift these levels enough to allow for a resonant reaction that would burn most of the carbon to form oxygen. Let us check if this happens.

We assume again that the energy of the excited states is fixed with respect to the ground state of ^{16}O, and we consider the Q-value for the reaction ^{12}C + $\alpha \to$ ^{16}O

$$Q(\theta) = M_{\rm O} - M_{\rm C} - M_\alpha$$
$$= 16\left(\frac{\rm BE}{A}\right)_{\rm O} - 12\left(\frac{\rm BE}{A}\right)_C - 4\left(\frac{\rm BE}{A}\right)_\alpha \tag{49}$$

where

$$\left(\frac{\rm BE}{A}\right)_{\rm O} = -7.96 - 82(\eta_S - 1). \tag{50}$$

At $\theta = 0$ we get the measured result $Q = -7.16$ MeV. In Fig. 6 we plot $Q(\theta)$ in the region of interest that we found in our study of the triple-alpha reaction.

It is evident from the plot that increasing θ shifts the ground state of ^{16}O down, therefore the subthreshold levels remain such. We see also that the level that could potentially give a resonance moves down by $\sim$120 keV at the

[7]The interested reader can find more details in Ref. [11]

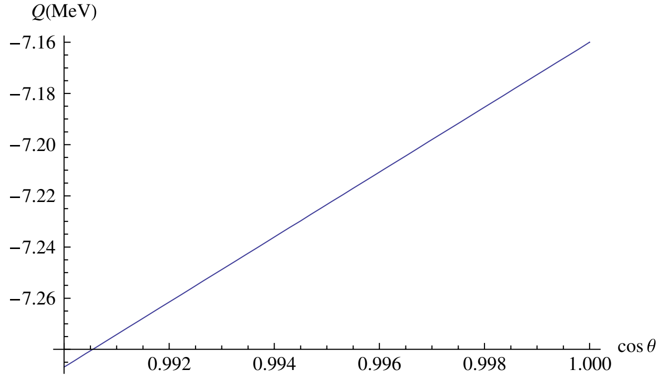

FIG. 6 (color online). Q-value as a function of θ for the reaction $^{12}C + \alpha \rightarrow {}^{16}O$.

most, which is still 2.30 MeV above the threshold, still too far. We conclude that there are not any dramatic effects in the reaction $^{12}C + \alpha \rightarrow {}^{16}O$, so that the ratio carbon/oxygen does not change appreciably, but even small values of the angle θ would result in a way greater abundance of both these elements.

IV. CONCLUSIONS

The question raised in this paper can be phrased in the following way: Would a nonzero angle θ change dramatically some aspects of nuclear physics? In order to find an answer, we singled out two examples, (i) the two-nucleon systems and (ii) the triple-alpha process, and studied the effects of θ on them.

For (i) we found that the nuclear binding energies of deuteron and diproton would change by 10% at $\theta \sim 130°$–$133°$. Even if this effect does not look so dramatic, we believe that it would still affect the outcome of Big Bang nucleosynthesis. For (ii) we found that, even for values of θ as small as 2° or 3°, the reaction rate for the triple-alpha process would be 10 times larger (see Fig. 5), leading to a greater abundance of carbon and oxygen than what measured in our Universe. Would such a greater abundance still be consistent with the evolution of intelligent observer? We do not know with certainty the answer to this question. If negative, it would pose the anthropic bound that θ be less than $\sim 2°$; if a factor of 1000 for the reaction rate, instead of 10, were not compatible with life, then the constraint on θ would be weaker: $\theta < 4.5°$.

We must stress that the numerical values given in (ii) are rough estimates. The main source of error is in the assumption that the energies of the excited states, with respect to the ground state, are not a function of θ, which they most likely are, but it is very difficult to relate the spacing between these levels to first principles.

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