Cosmic string configuration in a five dimensional Brans-Dicke theory

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We consider a scalar field interacting with a cosmic string configuration. The origin of the scalar field is given by a compactification mechanism in the context of a five-dimensional Brans-Dicke theory. We analyze the behavior of a charged cosmic string given by the Maxwell-Chern-Simons term on the 3-brane. The Cosmic Microwave Background Radiation constraint is used to analyze the possibility of optical activity effect in connection with the Brans-Dicke parameter ω. We show that the dilatons produced by a cosmic string can decay into gauge bosons with masses given by the compactification modes. The Brans-Dicke parameter ω imposes stringent constraints on the mass of the dilaton and help us to understand the energy scales. In this scenario the lifetime of the dilaton which decays into light gauge bosons as well as the dependence of this phenomenon with the Brans-Dicke parameter are estimated.

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I. INTRODUCTION

Cosmic strings could have been produced in the early stages of the universe as a consequence of cosmological phase transitions. The conical topology of the associated spacetime [1–5] may produce interesting effects. As examples of these effects we can mention the emission of radiation by a freely moving body [6] and the vacuum fluctuations of quantum fields [7,8], among others. It has been argued that the gravitational interaction may be described by scalar-tensor theories, at least at sufficiently high energy scales [9]. Thus, it seems natural to investigate some physical systems, as, for example, one which contains scalar fields and cosmic strings, at these energy scales. In particular, it is interesting to consider this system in the framework of the Brans-Dicke theory of gravity [10].

At high energy scales, usually, the theories of fundamental physics are formulated in higher dimensional spacetimes in which case is assumed that the extra dimensions are compactified. This compactification of spatial dimensions leads to interesting quantum effects like as topological mass generation [11,12] and symmetry breaking [13].

In despite of the successes of the standard model of elementary particles, the fact that the gravitational interaction is not included in it and that the hierarchy problem is not considered constitutes unsatisfactory aspects of this model. Otherwise, these questions can be explained in the context of a string theory which has been considered as a promise theory. Moreover, based on string theory ideas, the brane-world scenario has been proposed to take into account the hierarchy between the electroweak and Planck scales and becomes the effective form to possibly realize cosmological scenarios.

On the other hand, the consistency of string theory, which is a candidate to describe all fundamental interactions, requires that our world has to have more than four dimensions. Originally, extra dimensions were supposed to be very small (of the order of Planck length). However, it has been proposed, recently, that the solution to the hierarchy problem may arises by considering that some of these extra dimensions are not so small. In the approach used in Ref. [14] the space has one or more flat extra dimensions while, according to the so called Randall-Sundrum (RS) model [15], hierarchy would be explained by one large warped extra dimension. In RS framework, the standard model fields are confined to the 3-brane while gravity propagates in a five-dimensional anti-de Sitter (AdS) bulk with two slices where the background metric is written as

$$ds^2 = e^{2\sigma(\Omega)}\eta_{\mu\nu}dx^\mu dx^\nu + d\Omega^2, \tag{1}$$

where x^μ ($\mu = 0, 1, 2, 3$) are Lorentz coordinates and we are considering the signature $(-, +, +, +, +)$. The coordinate Ω for the extra dimension takes values in the range $-L \leq \Omega \leq L$ (L being the size of the extra dimension), with (x, Ω) and $(x, -\Omega)$ being identified. The two 3-branes are located at $\Omega = 0, L$. In Eq. (1), $\sigma(\Omega) = -k|\Omega|$ with k being a constant with a scale of the order of the Planck scale. The energy scales are related in such a way that the TeV mass scales are produced on the brane from Planck masses through the warp factor $e^{-2k\Omega}$. Then, a field confined to the 3-brane at $\Omega = L$, with mass parameter m_0, corresponds to a physical mass given by

$$m = m_0 e^{-2kL} \tag{2}$$

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As was shown in [15], the four-dimensional Planck scale is given by

$$M_{\rm Pl}^2 = \frac{M^3}{k}(1 - e^{-2kL}) \tag{3}$$

where $M_{\rm Pl}$ is the four-dimensional Planck scale and M is the $(4 + d)$-dimensional Planck scale, where d is the number of extra compact dimensions. In this way, the $(4 + d)$-dimensional spacetime is determined by $M_{\rm pl}$, M and the geometry of spacetime.

This paper is organized as follows. In Sec. II, we discuss the compactification of the scalar field. In Sec. III the charged cosmic string configuration in the brane-world scenario is presented. In Sec. IV we discuss the cosmic optical activity and its connection with the cosmic microwave background radiation. In Sec. V, we consider particle production in the brane-world scenario. Finally, in Sec. VI, we present the conclusions.

II. COMPACTIFICATION OF THE SCALAR FIELD PERTURBATION IN THE BRANE-WORLD SCENARIO

In this section we consider the five-dimensional Brans-Dicke theory with two 3-brane potentials and a scalar field nonminimally coupled to gravity. We study the compactification of the scalar field perturbation in the spacetime generated in the context of Brans-Dicke theory in a five-dimensional brane-world scenario. The complete action that represents the system realization in a five-dimensional spacetime, in Jordan frame, can be written as

$$\mathcal{S} = \int d^5x\sqrt{-\tilde{g}}\left[\tilde{\Psi}\tilde{R} - V(\tilde{\Psi}) + \frac{\omega}{\tilde{\Psi}}\tilde{g}^{\tilde{\mu}\tilde{\nu}}\partial_{\tilde{\mu}}\tilde{\Psi}\partial_{\tilde{\nu}}\tilde{\Psi}\right] - \int_{\Omega=0} d^4x\sqrt{-\tilde{h}}\tilde{\lambda}_1(\tilde{\Psi}) - \int_{\Omega=L} d^4x\sqrt{-\tilde{h}}\tilde{\lambda}_2(\tilde{\Psi}), \tag{4}$$

where $\tilde{R}$ is the curvature scalar in Brans-Dicke theory in five dimensions, $x^{\tilde{\mu}} \equiv \{x^\mu, \Omega\}$. It forms the orbifold space $\frac{S^1}{Z_2}$, which is realized as the circle of the circumference $2L$, with the identification $\Omega = -\Omega$. In this context the metric $\tilde{g}_{\mu\nu}$ and the scalar function $\tilde{\Psi}$ satisfy the following orbifold symmetry conditions

$$\begin{aligned} \tilde{g}_{\mu\nu}(x, -\Omega) &= \tilde{g}_{\mu\nu}(x, \Omega), \\ \tilde{g}_{\mu\Omega}(x, -\Omega) &= -\tilde{g}_{\mu\Omega}(x, \Omega), \\ \tilde{g}_{\Omega\Omega}(x, -\Omega) &= \tilde{g}_{\Omega\Omega}(x, \Omega), \\ \tilde{\Psi}(x, -\Omega) &= \tilde{\Psi}(x, \Omega). \end{aligned} \tag{5}$$

In the action (4), the branes are located at fixed points, namely, $\Omega = 0$ and $\Omega = L$. The quantity $\tilde{h}_{\mu\nu}$ is the induced metric on the two branes, $\tilde{V}(\tilde{\Psi})$ is a bulk potential and $\tilde{\lambda}_1(\tilde{\Psi})$ and $\tilde{\lambda}_2(\tilde{\Psi})$ are scalar potentials. The constant ω is the Brans-Dicke parameter.

In order to preserve the Poincaré invariance in the four-dimensional subspace $\Omega =$ constant, we will assume that the metric and the scalar field can be written as

$$\begin{aligned} &\tilde{\Psi} = \tilde{\Psi}(\Omega) \\ &ds^2 = \tilde{g}_{\tilde{\mu}\tilde{\nu}}dx^{\tilde{\mu}}dx^{\tilde{\nu}} = e^{2\sigma}\eta_{\mu\nu}dx^\mu dx^\nu + d\Omega^2, \end{aligned} \tag{6}$$

where $\eta_{\mu\nu}$ is the flat Minkowski spacetime metric. Substituting Eq. (6) in the Lagrangian density given by Eq. (4), we find

$$S = \int d^4x\sqrt{-g}\int d\Omega\left\{e^{4\sigma}\left[-\tilde{\Psi}(8\sigma'' + 10\sigma'^2) - \omega\frac{\tilde{\Psi}'^2}{\Psi} - \tilde{V}(\tilde{\Psi}) - \tilde{\lambda}_1(\tilde{\Psi})\delta(\Omega) - \tilde{\lambda}_2(\tilde{\Psi})\delta(\Omega - L))\right]\right\} \tag{7}$$

In order to use the BPS mechanism [16], we rewrite Eq. (7) in the following form

$$\begin{aligned} S = \int d^4x\sqrt{-g}\int d\Omega\Bigg\{e^{4\sigma}\Bigg[&-\tilde{\Psi}\left(\omega + \frac{4}{3}\right)\left(\frac{\tilde{\Psi}'}{\tilde{\Psi}} - \hat{W} + 3\tilde{\Psi}\frac{\partial\hat{W}}{\partial\tilde{\Psi}}\right)^2 + 12\tilde{\Psi}\left[\sigma' + \frac{1}{3}\frac{\tilde{\Psi}'}{\tilde{\Psi}} - \left(\omega + \frac{4}{3}\right)\hat{W}\right]^2 \\ &- \left[\tilde{V} + 2\tilde{\Psi}^2\hat{W}\frac{\partial\hat{W}}{\partial\tilde{\Psi}} - 3\tilde{\Psi}^3\left(\frac{\partial\hat{W}}{\partial\tilde{\Psi}}\right)^2 + (3\omega + 4)(4\omega + 5)\tilde{\Psi}\hat{W}^2\right] \\ &- \left[2(3\omega + 4)\frac{\partial\hat{W}}{\partial\tilde{\Omega}}\tilde{\Psi} - \lambda_1(\tilde{\Psi})\delta(\Omega) - \lambda_2(\tilde{\Psi})\delta(\Omega - L)\right] - 8(\sigma'\tilde{\Psi}e^{4\sigma})' + 2(3\omega + 4)(\tilde{\Psi}\hat{W}e^{4\sigma})'\Bigg\}. \end{aligned} \tag{8}$$

where we have introduced an arbitrary odd function of $\tilde{\Psi}$

$$\hat{W}(\tilde{\Psi}) = \begin{cases} W(\tilde{\Psi}) & \text{if } 0 < \Omega < L; \\ -W(\tilde{\Psi}) & \text{if } -L < \Omega < 0. \end{cases} \tag{9}$$

The BPS mechanism implies that the first and second terms can vanish, and therefore, we have

$$\tilde{\Psi}' = \left[\hat{W}\tilde{\Psi} - 3\tilde{\Psi}^2\frac{\partial\hat{W}}{\partial\tilde{\Psi}}\right] \tag{10}$$

$$\sigma' = -\frac{1}{3}\frac{\tilde{\Psi}'}{\tilde{\Psi}} - \left(\omega + \frac{4}{3}\right)\hat{W}. \tag{11}$$

In this way we find a class of bulk potentials in five dimensions which can be written as

$$\tilde{V} = -(3\omega + 4)\left[(4\omega + 5)\tilde{\Psi}\hat{W}^2 + 2\tilde{\Psi}^2\hat{W}\frac{\partial\hat{W}}{\partial\tilde{\Psi}} - 3\tilde{\Psi}^2\left(\frac{\partial\hat{W}}{\partial\tilde{\Psi}}\right)^2\right], \tag{12}$$

with the brane conditions

$$2(3\omega + 4)\frac{\partial\hat{W}}{\partial\Omega} = -\frac{\tilde{\lambda}_1(\tilde{\Psi})}{\tilde{\Psi}}\delta(\Omega) - \frac{\tilde{\lambda}_2(\tilde{\Psi})}{\tilde{\Psi}}\delta(\Omega - L). \tag{13}$$

This structure of the potentials (12) is similar to the one introduced in [17]. Then, the scalar field, $\tilde{\Psi}$, satisfies the equation of motion in the interval $[0, L]$, provided that the following boundary conditions on the branes are satisfied

$$(3\omega + 4)W(\tilde{\Psi})|_{\Omega=0^+} = -\frac{\tilde{\lambda}_1(\tilde{\Psi})}{4\tilde{\Psi}}\bigg|_{\Omega=0^+} \tag{14}$$

and

$$(3\omega + 4)W(\tilde{\Psi})|_{\Omega=L^-} = \frac{\tilde{\lambda}_2(\tilde{\Psi})}{4\tilde{\Psi}}\bigg|_{\Omega=L^-}. \tag{15}$$

These relations can be obtained by using the symmetry conditions (5).

Considering the solution of the first order Eqs. (10) and (11) similar to Randall- Sundrum model we find that $W =$ constant. In this case, we find that the potential, arising from (12) is

$$\tilde{V}(\tilde{\Psi}) = \Lambda\tilde{\Psi} \tag{16}$$

with

$$\Lambda = -(3\omega + 4)(4\omega + 5)W^2 \tag{17}$$

and the background solution is given by

$$\tilde{\Psi} = Ce^{(\sigma/(\omega+1))} \tag{18}$$

$$\sigma = -k|\Omega|, \tag{19}$$

where C is a constant and

$$W = -\frac{k}{(\omega + 1)}. \tag{20}$$

Using the brane boundary conditions (14) and (15) we have

$$\tilde{\lambda}_{1,2} = \pm\lambda\tilde{\Psi}, \tag{21}$$

which is compatible with (14) and (15). The constant λ can be written as

$$\lambda = 4\left(\frac{3\omega + 4}{4\omega + 5}\right)^{1/2}\sqrt{-\Lambda}. \tag{22}$$

Let us consider that the cosmic string field is a fluctuation on the brane and that this fact does not modify the solutions for $\tilde{\Psi}$ and σ. Thus, we can write that the gauge metric fluctuation is given by

$$\tilde{g}_{\mu\nu}(x, \Omega) = e^{2\sigma}g_{\mu\nu}(x), \qquad \tilde{\Psi}(x, \Omega) = \tilde{\Psi}(\Omega)$$
$$g_{\Omega\Omega} = 1 \qquad g_{\mu\Omega}(x, \Omega) = 0, \tag{23}$$

where $g_{\mu\nu}$ is the cosmic string fluctuation metric. Considering these relations in the action (4) and the solutions (18) and (19), we get, after performing the integration in Ω, the following result

$$S_{\rm eff}^{\rm Grav} = \frac{1}{16\pi G_{\rm eff}}\int d^4x\sqrt{-g}R_4, \tag{24}$$

where R_4 is the Ricci scalar in four dimensions and

$$\frac{1}{16\pi G_{\rm eff}} = \frac{2C(\omega + 1)}{4\omega + 5}\frac{1}{k}(1 - e^{-((4\omega+5)/(\omega+1))kL}) = 2M_{\rm pl}^2. \tag{25}$$

Now, let us study the compactification of the scalar field fluctuation in the Jordan frame. To understand the form of the scalar action in Jordan frame let us consider the Einstein frame where the Ricci scalar and scalar field $\tilde{\Psi}$ in the action (4) decouples as

$$\mathcal{S} = \int d^5x\sqrt{-\tilde{g}}\left[\frac{1}{16\pi G_5}R - \frac{1}{2}g^{MN}\partial_M\Psi\partial_N\Psi - V(\Psi)\right] - \int_{\Omega=0} d^4x\sqrt{-h}\lambda_1(\Psi) - \int_{\Omega=L} d^4x\sqrt{-h}\lambda_2(\Psi), \tag{26}$$

In the Brans-Dicke theory there are relations between the Einstein and Jordan frames, which in five dimensions are given as follows

$$\tilde{g}_{\hat{\mu}\hat{\nu}} = e^{2\alpha\Psi}g_{\mu\nu}, \tag{27}$$

$$\tilde{\Psi} = \frac{1}{16\pi G_5}e^{-3\alpha\Psi}, \tag{28}$$

$$\tilde{V}(\tilde{\Psi}) = e^{-5\alpha\Psi}V(\Psi), \tag{29}$$

$$\tilde{\lambda}_i(\tilde{\Psi}) = e^{-4\alpha\Psi}\lambda_i(\Psi), \tag{30}$$

with

$$\alpha^2 = \frac{1}{288\pi G_5}e^{-3\alpha\Psi}. \tag{31}$$

Using solution (18) we find that the metric in Einstein frame is given by

$$g_{\hat{\mu}\hat{\nu}} = e^{(2\sigma/3(\omega+1))}\tilde{g}_{\hat{\mu}\hat{\nu}} \tag{32}$$

$$\Psi = \frac{\sigma}{3\alpha(\omega + 1)} \tag{33}$$

$$C = \frac{1}{16\pi G_5}. \tag{34}$$

In fact we know that in the Einstein frame the invariante action is given by

$$S_{\text{Einstein}} = D \int d^5x \sqrt{-g} \partial_{\hat{\mu}} \Phi \partial^{\hat{\mu}} \Phi, \tag{35}$$

where D is a constant. In this case we have

$$ds^2 = e^{(2\sigma)/3(\omega+1)} [e^{2\sigma} \eta_{\mu\nu} dx^\mu dx^\nu + d\Omega^2] \tag{36}$$

To find the corresponding Einstein action (35) in Jordan frame, we can use the conformal transformation and choice $D = -\frac{2(\omega+1)C}{(4\omega+5)k} = -\frac{1}{8\pi G_{\text{eff}}}$, which is compatible with the Brans-Dicke theory in four dimensions [18]. Using this procedure we can assume that the action which represents this fluctuation in Jordan frame is given by

$$\mathcal{S}^5_{\text{scalar}} = -\frac{1}{8\pi G_{\text{eff}} C} \int d^5x \sqrt{-g_5} \tilde{\Psi} \partial_{\tilde{\mu}} \Phi \partial^{\tilde{\mu}} \Phi, \tag{37}$$

where Φ is the scalar field in the bulk. This expression is compatible with the scalar action in the Jordan frame. Substituting Eqs. (18) and (19) into (37) and taking into account Eq. (23), the action S^5_{scalar} turns into

$$S^5_{\text{scalar}} = -\frac{1}{8\pi G_{\text{eff}}} \int d^5x \sqrt{-g} [e^{((2\omega+3)/(\omega+1))\sigma} g^{\mu\nu} \times \partial_\mu \Phi \partial_\nu \Phi + \Phi \partial_\Omega (e^{((4\omega+5)/(\omega+1))\sigma} \partial_\Omega \Phi)], \tag{38}$$

Now, let decompose the scalar field $\Phi(x, \Omega)$ as a sum over modes as follows:

$$\Phi(x, \Omega) = \frac{1}{\sqrt{L}} \sum_{n=0}^{\infty} X_n(x) \xi_n(\Omega), \tag{39}$$

where the modes $\xi_n(\Omega)$ satisfy

$$\frac{1}{L} \int_{-L}^{L} d\Omega e^{((2\omega+3)/(\omega+1))\sigma} \xi_n(\Omega) \xi_m(\Omega) = \delta_{nm} \tag{40}$$

and

$$-\frac{e^{2\sigma}}{v(\Omega)} \frac{d}{d\Omega} \left(v(\Omega) \frac{d\xi_n}{d\Omega} \right) = m_n^2 \xi_n, \tag{41}$$

with $v(\Omega) = e^{2\nu\sigma}$ and $\nu = \frac{4\omega+5}{2(\omega+1)}$. After the compactification we have

$$S_{X_n} = \frac{1}{8\pi G_{\text{eff}}} \sum_{n=0}^{\infty} \int d^4x \sqrt{-g} (\partial_\mu X_n \partial^\mu X_n - U(X_n)), \tag{42}$$

where $U(X_n)$ is the scalar potential given by

$$U(X_n) = m_n^2 X_n^2. \tag{43}$$

As in the usual Kaluza-Klein compactification, the bulk field $\Phi(x, \Omega)$ manifests itself to a four-dimensional observer as an infinite set of scalars $X_n(x)$ with masses m_n, which can be determined by solving Eq. (41).

The equation for the zero mode came from Eq. (41) when $n = 0$. In this case we have zero mass ($m_0 = 0$) and the solution

$$\xi_0 = \frac{1}{N_0} e^{-2\nu\sigma}, \tag{44}$$

where N_0 is the normalization constant that can be calculated using Eq. (40). It is given by

$$N_0 = \frac{1}{\sqrt{(\nu+1)kL}} e^{(\nu+1)kL}. \tag{45}$$

In order to obtain the modes for $n \neq 0$, we change the variables to $z = e^{k\Omega}/k$. Then, Eq. (41) turns into

$$z^u \frac{d}{dz} \left(z^{-u} \frac{d\xi_n}{dz} \right) = -m_n^2 \xi_n, \tag{46}$$

where $u = \frac{3\omega+4}{\omega+1}$. The solutions of this equation are combinations of Bessel functions of order $\nu = \frac{4\omega+5}{2(\omega+1)}$ and are given by

$$\xi_n(\Omega) = \frac{e^{-\nu\sigma(\Omega)}}{N_n} \left[J_\nu\left(\frac{m_n}{k} e^{-\sigma} \right) + b_{n\nu} Y_\nu\left(\frac{m_n}{k} e^{-\sigma} \right) \right]. \tag{47}$$

Putting Eq. (47) in the normalization condition (40) we find that the normalization constant is

$$N_n = \frac{2m_n e^{kL}}{x_{n\nu} \sqrt{2kL}} \left(\int dz z [J_\nu(m_n z) + b_{n\nu} Y_\nu(m_n z)]^2 \right)^{1/2}, \tag{48}$$

with $x_{n\nu} = \frac{m_n}{k} e^{kL}$.

Let us consider the solution continuous on the branes, which means that the Neumann condition is valid. Therefore, at $\Omega = 0$, we have

$$b_{n\nu} = -\frac{J_{\nu-1}(x_{n\nu} e^{-kL})}{Y_{\nu-1}(x_{n\nu} e^{-kL})}, \tag{49}$$

Applying the same condition at $\Omega = L$, we find

$$x_{n\nu}^2 e^{-kL} [J_{\nu-1}(x_{n\nu}) Y_{\nu-1}(x_{n\nu} e^{-kL}) - Y_{\nu-1}(x_{n\nu}) J_{\nu-1}(x_{n\nu} e^{-kL}] = 0. \tag{50}$$

The action (42) differ from the dilaton action in Einstein frame in four dimensions only by the presence of n massive modes. For the zero mode the mass vanishes and we use it to calculate the zero mode function ξ_0 given by Eq. (44). In this case the action is similar to the four-dimensional dilaton action. The masses given by the zeros arise from the Neumann conditions in the brane localized at $\Omega = L$. It is important to call attention to the fact that the conditions on the brane and the normalization condition determine the form of the solution given by Eq. (47). In the Fig. 1 is showed the dependence of the modes $x_{n\nu}$ as a function of the Brans-Dicke parameter which in the next sections will be interpreted as the fine tune parameter. Note that this

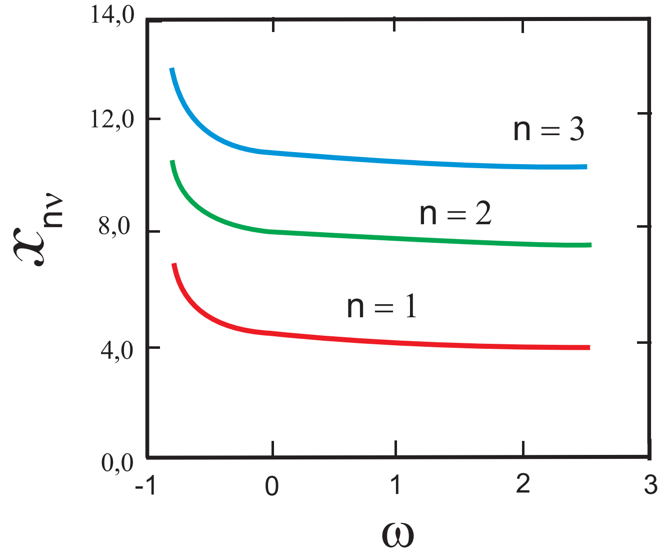

FIG. 1 (color online). A plot of the first mode $x_{n\nu}$ as a function of ω.

dependence presents an asymptotic behavior around $\omega = -1$.

III. CHARGED COSMIC STRING CONFIGURATION IN THE BRANE-WORLD SCENARIO

In this section, we study how this framework, with the scalar field Φ in the bulk, interact with the cosmic string configuration on the brane $\Omega = L$. The form of this interaction is arbitrary, then we consider that the coupling with the cosmic string and others couplings stay only on the brane. Thus, the components of the energy-momentum tensor of the cosmic string are given by

$$T_{\Omega\Omega} = 0 \qquad T_{\Omega\mu} = 0 \qquad T_{\mu\nu} \neq 0. \tag{51}$$

The action that represents the cosmic string in this framework is

$$\mathcal{S}_{\rm CS} = \int d^4x\Big[-\frac{1}{2}e^{2\alpha\Phi}D_\mu\phi(D^\mu\phi)^* - \frac{1}{4}H_{\mu\alpha}H^{\mu\alpha} - e^{4\alpha\Phi}V(\phi)\Big], \tag{52}$$

with α being a coupling constant analogous to the Brans-Dicke parameter [18].

The action given by (52) has a $U(1)$ symmetry associated with the ϕ-field which is broken. Thus, assuming, for simplicity, that the spacetime on the brane is approximately flat, we obtain the vortices solutions of the Nielsen-Olesen type [19]

$$\phi = \varphi(r)e^{i\theta - kL}, \qquad H_\mu = \frac{1}{q}[P(r) - 1]\delta^\theta_\mu, \tag{53}$$

with (t, r, θ, z) being the usual cylindrical coordinates. The boundary conditions for the fields $\varphi(r)$ and $P(r)$ are the same as those of ordinary cosmic strings, namely,

$$\begin{aligned}\phi(r) &= \eta e^{i\theta - kL} \quad r \to \infty, \qquad P(r) = 0 \quad r \to \infty,\\ \phi(r) &= 0 \quad r = 0, \qquad P(r) = 1 \quad r = 0,\end{aligned} \tag{54}$$

where $D_\mu\phi = (\partial_\mu + iqH_\mu)\phi$. The potential $V(\phi)$ triggering the spontaneous symmetry breaking can be fixed by

$$V(\phi) = \frac{\lambda_\phi}{4}(|\phi|^2 - \tilde{\eta}^2)^2, \tag{55}$$

where $\tilde{\eta} = \eta e^{-kL}$ and λ_ϕ is the coupling constant. It is worth calling attention to the fact that this potential can induces the formation of an ordinary cosmic string.

In order to study the effect of the charge, let us consider that the dilaton field, Φ, propagating in the bulk, interact on the brane $\Omega = L$ through the Maxwell-Chern-Simons three-tensor term. In this situation, the gauge fields stay on the brane and couple with the dilaton on the brane.

The Maxwell-Chern-Simons action can be written as

$$\mathcal{S}_{\rm int} = \beta_+^{(1)}\int d^4x\partial_\mu\Phi(x)H_\nu\tilde{H}^{\mu\nu}, \tag{56}$$

where we used the notation $\Phi(x) \equiv \Phi(x, \Omega = L)$, $H_{\mu\nu}$ is the field strength associated with a gauge field and $\tilde{H}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}H^{\alpha\beta}$ is the dual of the field strength. This term is responsible for the topological charge. If we consider the equation of motion, we have

$$\partial_\mu H^{\mu\nu} + \beta_+^{(1)}\epsilon^{\nu\alpha\beta\mu}\sum_{n=0}^{\infty}\xi_n H_{\alpha\beta}\partial_\mu X_n = J^\nu, \tag{57}$$

which contains the sum of the modes. The current in (57) is chosen as $J_\mu = (J_0, 0, 0, 0)$ and is given by

$$J_\mu = \frac{-iq}{2}[\phi^* D_\mu\phi - \phi(D_\mu\phi)^*]. \tag{58}$$

Note that if the charge J_0 is different from zero, then $H_t \neq 0$. In order to obtain a solution compatible with the current given by (58), H_t must behaves as

$$H_t(r) = 0, \quad r \to \infty \qquad H_t(r) = \text{constant}, \quad r = 0. \tag{59}$$

Using the conditions (59) and the Eq. (57), we find that the charge is given by

$$Q = -\beta_+^{(1)}\epsilon^{0\alpha\beta\mu}\sum_{n=0}^{\infty}\xi_n\int d^4x H_{\alpha\beta}\partial_\mu X_n. \tag{60}$$

This result already appeared in [18], but has a problem related with this prescription, which implies that it does not have long range electric field associated to this charge. In order to have a long range electric field, it is necessary an extra $U(1)$ group, which can be introduced if we consider a $U(1)$ invariant action corresponding to the electromagnetic field where the free part is given by

$$\mathcal{S}_{\rm free} = -\frac{1}{4}\int d^4xF_{\mu\nu}F^{\mu\nu}. \tag{61}$$

Let us consider that the interaction part is given by the Maxwell-Chern-Simons term, which can be written as

$$\mathcal{S}_{\rm int} = \beta_+^{(2)}\int d^4x\partial_\mu\Phi(x)A_\nu\tilde{F}^{\mu\nu} \tag{62}$$

The constant coefficients $\beta_+^{(1)}$ and $\beta_+^{(2)}$ couple to Maxwell-Chern-Simons terms and determine the intensity of the external electric field. Now, decomposing the action given by (56) and (63) in terms of the zero modes given by (44) and (47), with $\Phi(x, \Omega = L)$, we obtain from (39) putting $\Omega = L$, the following result

$$\mathcal{S}_{\rm int} = \int d^4x\bigg[\sqrt{(\nu+1)kL}e^{-(\nu-1)kL}\partial_\mu X_0 + e^{\nu kL}\sum_{n=1}\frac{J_\nu(x_{n\nu})}{N_n}\partial_\mu X_n\bigg] \times[\beta_+^{(1)}H_\nu\tilde{H}^{\mu\nu} + \beta_+^{(2)}A_\nu\tilde{F}^{\mu\nu}]. \tag{63}$$

The action (63) is the general form of the interaction terms and is dictated by the effects that we want to analyze. The consequences of the action (56) to the cosmic string configuration can be analyzed by taking into account the equation of motion for the gauge fields. Let us consider that $A_\mu = A_\mu(r)$. Thus, the equation of the motion for the gauge field H_μ, in Minkowski spacetime, is

$$\partial_\mu H^{\mu\nu} + \epsilon^{\nu\alpha\beta\mu}\sum_{n=0}^{\infty}\xi_n[\beta_+^{(1)}H_{\alpha\beta} + \beta_+^{(2)}F_{\alpha\beta}]\partial_\mu X_n = J^\nu, \tag{64}$$

The equation obeyed by the gauge field A_μ is given by

$$\partial_\lambda F^{\lambda\nu} - \beta_+^{(2)}\epsilon^{\mu\nu\alpha\beta}\sum_n\xi_n\partial_\mu X_nH_{\alpha\beta} = 0. \tag{65}$$

Before analyzing the implications of Eqs. (64) and (65), let us study the equation of motion for the scalar fields X_n. To do this, let us consider the weak field approximation and use the following expansions

$$X_n = X_{(0)n} + \epsilon X_{(1)n}, \tag{66}$$

$$A(\Phi) = A(\Phi_{(0)}) + A'(\Phi_{(0)})\Phi_{(1)}, \tag{67}$$

where $X_{(0)n}$ is the constant dilaton value in the background without the cosmic string and $A(\Phi) = e^{\alpha\Phi(x,\Omega=L)}$, where $\Phi(x, \Omega = L)$ is given by (39) with $\Omega = L$.

In our framework the equation for the field X_n reads as

$$\Box X_n = -\frac{1}{2}\frac{dU(X_n)}{dX_n} - 4\pi G_{\rm eff}\xi_n[\alpha T + 2(\beta_+^{(1)}H_{\mu\nu} + \beta_+^{(2)}F_{\mu\nu})\tilde{H}^{\mu\nu}], \tag{68}$$

where the coefficients $\beta_+^{(1)}$ and $\beta_+^{(2)}$ are associated with the topological contributions. The terms whose coefficients are $\beta_+^{(1)}$ and $\beta_+^{(2)}$ in the equation of the motion (68) vanish, if we consider solutions (53) and (59). In this case, we obtain

$$\Box X_n = -\frac{1}{2}\frac{dU(X_n)}{dX_n} - 4\pi G_{\rm eff}\xi_n\alpha T \tag{69}$$

with $U(X_n)$ given by (43). Now, let us assume that Eq. (69) can be written as

$$X_n(t, r, z) = \chi_n(r) + f(r)\Xi_n(t, z), \tag{70}$$

where $f(r)$ vanishes outside the string core. The ansatz (70) is also assumed in [18].

Consider the electric and magnetic fields, E^i and B^i, defined as $E^i = F^{0i}$ and $B^i = -\epsilon^{ijk}F_{jk}$. Using Eqs. (64) and (70), we find that the charge induced by the dilaton can be written as

$$Q = 2\pi\beta_+^{(1)}\bigg[\sqrt{(\nu+1)kL}e^{-(\nu-1)kL}\partial_z\Xi_0e^{\nu kL} \times\sum_{n=1}\frac{J_\nu(x_{n\nu})}{N_n}\partial_z\Xi_n\bigg]\int_0^{r_0}f(r)B(r)rdr. \tag{71}$$

Now, using Eqs. (65) and (71), we find that the induced electric field is given by

$$E_{\rm ext} = \frac{Q}{2\pi\epsilon r}, \tag{72}$$

where $\varepsilon = \beta_+^{(1)}/\beta_+^{(2)}$. This relation tell us that the constants $\beta_+^{(1)}$ and $\beta_+^{(2)}$ are related with the electric conductivity and therefore, determine the intensity of the electric field. This result show up that the external field generated by the charged cosmic string depends on both interaction appearing in Eq. (63), evaluated on the brane.

Using solution (70), we obtain up to first order in $G_{\rm eff}$, the equation

$$\chi_n'' + \frac{1}{r}\chi_n' + m_n^2\chi_n = -4\pi G_{\rm eff}\xi_n\alpha T \tag{73}$$

where T is the trace of the energy-momentum tensor in flat spacetime in the presence of the dilaton. This energy-momentum tensor $T_{\mu\nu}$ is given by

$$T_t^t = -\frac{1}{2}A^2\bigg[\varphi'^2 + \frac{1}{r^2}\varphi^2P^2 + A^{-2}\bigg(\frac{A_t'^2}{4\pi e^2}\bigg) + \varphi^2H_t^2 + A^{-2}\frac{1}{r^2}\bigg(\frac{P'^2}{4\pi q^2}\bigg) + 2A^2V(\varphi) + A^{-2}\bigg(\frac{H_t'^2}{4\pi e^2}\bigg)\bigg] \tag{74}$$

$$T_z^z = -\frac{1}{2}A^2\left[\varphi'^2 + \frac{1}{r^2}\varphi^2 P^2 - A^{-2}\left(\frac{A_t'^2}{4\pi e^2}\right) - \varphi^2 H_t^2 + A^{-2}\frac{1}{r^2}\left(\frac{P'^2}{4\pi q^2}\right) + 2A^2 V(\varphi) - A^{-2}\left(\frac{H_t'^2}{4\pi e^2}\right)\right] \quad (75)$$

$$T_r^r = \frac{1}{2}A^2\left[\varphi'^2 - \frac{1}{r^2}\varphi^2 P^2 - A^{-2}\left(\frac{A_t'^2}{4\pi e^2}\right) + \varphi^2 H_t^2 + A^{-2}\frac{1}{r^2}\left(\frac{P'^2}{4\pi q^2}\right) - 2A^2 V(\varphi) - A^{-2}\left(\frac{H_t'^2}{4\pi e^2}\right)\right] \quad (76)$$

$$T_\theta^\theta = -\frac{1}{2}A^2\left[\varphi'^2 - \frac{1}{r^2}\varphi^2 P^2 - A^{-2}\left(\frac{A_t'^2}{4\pi e^2}\right) + \varphi^2 H_t^2 - A^{-2}\frac{1}{r^2}\left(\frac{P'^2}{4\pi q^2}\right) + 2A^2 V(\varphi) - A^{-2}\left(\frac{H_t'^2}{4\pi e^2}\right)\right]. \quad (77)$$

The function $f(r)$ and the field $\Xi(t, z)$ obey the following equations

$$\frac{\partial^2 f}{\partial r^2} + \frac{1}{r}\frac{\partial f}{\partial r} = \omega_n^2 f, \quad (78)$$

$$\frac{\partial^2 \Xi_n}{\partial t^2} - \frac{\partial^2 \Xi_n}{\partial z^2} = \omega_n^2 \Xi_n, \quad (79)$$

which satisfies the boundary condition, namely, $f(r) = 0$ when $r \to \infty$. The arbitrary constant w_n can assume both positive and negative values for each n and is given by the relation $\omega_n^2 = k_n^2 - \omega_{0n}^2$. Now, let us consider the solutions of Eqs. (78) and (79) for the case $w_n = k_n$. Thus, the solution for $f(r)$ is

$$f(r) = f_I I_0(kr) + f_k K_0(kr), \quad (80)$$

where I_0 and K_0 are modified Bessel functions of zero order. The function I_0 is exponentially divergent for large values of the argument, then we choose $f_I = 0$ in such a way that $f(r) = 0$ obeys the stated boundary condition. The solution $\Xi(t, z)$ has an oscillatory behavior and can be written as

$$\Xi_n(z) = \Xi_{(0)n}\cos k_n z. \quad (81)$$

Now, let us turn our attention to the energy-momentum tensor that is relevant in the weak field approximation, which in this charged model is given by

$$T_{tt} = U\delta(x)\delta(y) + \frac{Q}{4\pi}\nabla^2\left(\ln\frac{r}{r_0}\right) \quad (82)$$

$$T_{zz} = -\tau\delta(x)\delta(y) + \frac{Q}{4\pi}\nabla^2\left(\ln\frac{r}{r_0}\right) \quad (83)$$

$$T_{(ij} = -Q^2\delta_{ij}\delta(x)\delta(y) + \frac{Q}{2\pi}\partial_i\partial_j\left(\ln\frac{r}{r_0}\right) \quad (84)$$

where, U and τ are the energy per unit length and the tension per unit length, respectively.

As the mass m_n is small compared with the oscillation frequency of the loop, then, the relevant part of the dilaton contribution is the χ_n solution which in our model is given by

$$\chi_0 = 2G_{\rm eff}\alpha\sqrt{(\nu+1)kL}e^{-(\nu-1)kL}\left(U + \tau + \frac{Q^2}{\varepsilon^2}\right)\ln\left(\frac{r}{r_0}\right)$$
$$\chi_n = 2G_{\rm eff}\alpha e^{\nu kL}\frac{J_\nu(x_{n\nu})}{N_n}\left(U + \tau + \frac{Q^2}{\varepsilon^2}\right)\ln\left(\frac{r}{r_0}\right). \quad (85)$$

This result is different from the usual case [18] because in this framework there are n scalar fields corresponding to the oscillation-modes and each frequency is related with one energy scale.

IV. COSMIC MICROWAVE BACKGROUND RADIATION AND COSMIC OPTICAL ACTIVITY

In this section, we consider the framework studied previously to construct a model to understand the cosmic optical activity. Some authors analyzed the possibility that optical polarization of light from quasars and galaxies can provide evidence for cosmic anisotropy [20]. The idea is that under some conditions the spacetime can exhibit different properties such as optical activity or birefringence [21,22].

It is well known that, as a consequence of vacuum polarization effects in QED [23–26], the electromagnetic vacuum presents a birefringent behavior [27–30]. For example, in the presence of an intense static background magnetic field, there are two different refraction indices depending on the polarization of the incident electromagnetic wave. This is in fact a very tiny effect which could be measured in a near future [31]. The present bounds for this anisotropy, followed from several experiments [32–35], are still far from the expected value from QED. For this study let us consider the cosmic string background in the context of the perturbed scalar compactification from five dimensions. In this case, taking into account a scalar-tensor theory brane-world scenario, the action can be written as

$$\mathcal{S} = -\int d^4x\left[\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \beta_+^{(3)}\sum_n \xi_n\partial_\mu X_n A_\nu \tilde{F}^{\mu\nu}\right], \quad (86)$$

with $\beta_+^{(3)}$ being the parameter which couples the electromagnetic field strength, $F_{\mu\nu}$, and the vector potential, A_ν, with the n modes of the dilaton. In this context the equation of motion for the electromagnetic field becomes

$$\partial_\mu F^{\mu\nu} = 2\beta_+^{(3)}\sum_n \xi_n\partial_\mu X_n\tilde{F}^{\mu\nu}. \quad (87)$$

The solutions of the Eq. (87) give us the corresponding dispersion relation

$$(k^{\alpha n}k^n_\alpha)^2 + (k^{\alpha n})k^n_\alpha(v^{\beta m}v^m_\beta) = (k^{\alpha n}v^n_\alpha)^2, \qquad (88)$$

where the indices (n, m) indicate the modes; (α, β) are the spacetime labels and the vector $v_{\mu n} = \partial_\mu X_n$ is responsible for the appearance of the preferred cosmic direction, as suggested by observations [20]. We can expand the dispersion relation given by Eq. (88) in powers of $v_{\alpha n}$. In this case, we find the dispersion relation in power of ϕ_α, considering the linearized solution in ϕ to the first order. The result is the following

$$k_{n\pm} = \omega_n \pm 2\beta^{(3)}_+\alpha_n\xi^2_n G_{\rm eff}\mu\hat{s}\cos(\gamma) \qquad (89)$$

with $\mu = U + \tau + (\frac{Q}{\epsilon})^2$, ω_n and k_n being the wave frequency and wave vector, respectively. The 4-vector k^α_n has components $k^\alpha_n = (\omega_n, k^i_n)$; $k_n = |k_n|$ and γ is the angle between the propagation wave vector k of the radiation and the unit vector $\hat{s}$.

Let us contextualize our theoretical results in the framework of the conclusions drawn from the analysis of observational data from quasar emission performed by [21,22,36]. The angle γ between the polarization vector and the galaxies major axis is defined as $\langle\Theta_n\rangle = \frac{r}{2}\Lambda^{-1}_n\cos(\vec{k}, \vec{s})$, where $\langle\Theta_n\rangle$ represents the mean rotation angle after Faraday rotation is removed, r is the distance to the galaxy, $\vec{k}$ the wave vector of the radiation and $\vec{s}$ a unit vector defined by the direction on the sky. The Lorentz breaking imposes that the preferred vector is in the radial direction. The rotation of the polarization plane is a consequence of the difference between the propagation velocity of the two modes k_+, k_-, which are the main dynamical quantities. This difference, defined as the angular gradient with respect to the radial (coordinate) distance, is expressed as $\frac{1}{2}(k_+ - k_-) = \frac{d\Theta_n}{dr}$, where Θ_n measures the specific entire rotation of the polarization plane per unit length r, and is given by $\Theta_n = \frac{1}{2}\Lambda^{-1}_n r\cos\gamma$. In the case of the cosmic string solution in a compactified scalar braneworld scenarios, we have

$$\Lambda^{-1}_n = 2\beta^{(3)}_+\alpha_n\xi^2_n G_{\rm eff}\mu. \qquad (90)$$

If we consider that this result is compatible with the cosmic microwave background radiation, up to mode n, we obtain

$$2\alpha_n\xi_n G_{\rm eff}\mu \sim 10^{-6}. \qquad (91)$$

Other important aspect that we must analyze is the hierarchy problem related with the constant α_0. In order to have compatibility with CMB radiation, constraint (91) must be satisfied. Then, for $n = 0$, we have

$$\alpha_0 \sim 0, \qquad 5[12(\nu + 1)]^{-1/2}e^{-12(\nu-1)} \qquad (92)$$

where $G_{\rm eff}\mu \sim 10^{-6}$ and the energy density per unit of length μ of the cosmic string is equal to the symmetry breaking scale η^2, where η is of the order of 10^{16}.

Let us analyze the numerical constraints imposed by CMB radiation and the Brans-Dicke parameter ω, taking into account the compactification modes. In this scenario we use the value of Λ^{-1} as of the order of 10^{-32} eV [22].

In order to analyze the contribution of the Lorentz breaking term in the Universe we must understand how the parameters of our model are related with the physical effects. For each mode, one important aspect to consider is the dependence of the coupling constant Λ^{-1} with the Brans-Dicke parameter ω. This dependence must impose a constraint in the range of validity of the coupling constant Λ^{-1}. In this framework, the zero mode is responsible for important phenomenological implications [37–40]. The dependence of the zero mode coupling constant Λ^{-1}_0 with the Brans-Dicke parameter ω is given by

$$\Lambda^{-1}_0 = 10^{-40}[(\nu + 1)kL]^{1/2}e^{(\nu-1)kL}, \qquad (93)$$

where we have chosen the parameter $\beta^{(3)}_+ = 10^{-34}$. We can see from Fig. 2 that the parameter ν as a function of ω is dimension dependent and they are related by $\nu = \frac{(D-1)w+D}{2(w+1)}$ [41]. In our case $D = 5$ and the expression reduces to $\nu = \frac{4w+5}{2(w+1)}$.

Note in Fig. 2 that when $\omega \to \infty$, then ν goes to 2 and gives us the asymptotic limit $\Lambda^{-1}_0 = 0, 98 \times 10^{-32}$, if we choose $kL = 12$ [42]. This limit corresponds to a lower bound. In this model when ω goes to -1 we have that $\Lambda^{-1}_0 \to \infty$. This upper asymptotic value does not have physical meaning. Note that the values of ω taken into account for the fine tune in our model must be related with the experimental data. The range of values that we are considering here is showed in Fig. 3 which presents also comparison with the massive mode for $n = 1$.

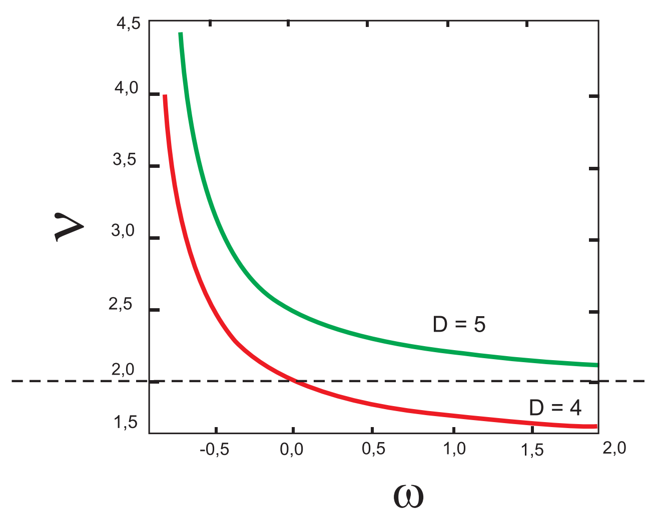

FIG. 2 (color online). A plot of the parameter ν as a function of ω corresponding to $D = 4$ and $D = 5$.

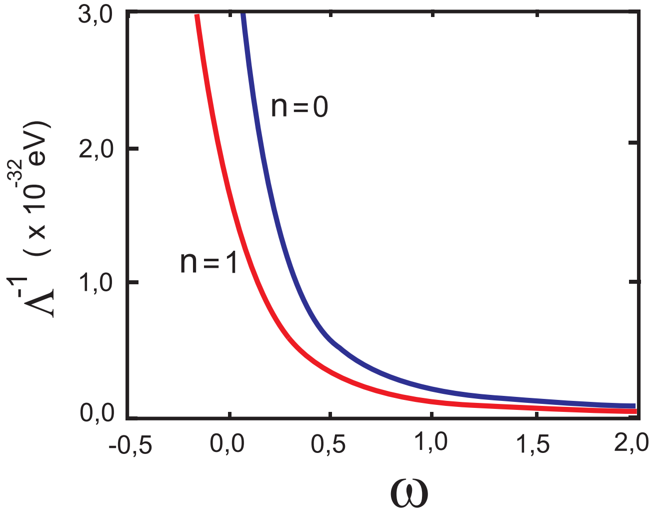

FIG. 3 (color online). A plot of the parameter Λ^{-1} for $n \neq 0$ in terms of ω.

For the massive modes, we obtain the following result

$$\Lambda_1^{-1} = \frac{10^{-40}}{N_1} J_\nu(x_{1\nu}). \tag{94}$$

We can analyze the $n = 1$ massive mode in Fig. 3. We note, by comparing with the $n = 0$ case that the value of Λ^{-1} decreases. This fact can be understood if we consider that in order to activate this mode we need more energy due to the fact that this mode is massive.

V. PARTICLE PRODUCTION IN THE BRANE-WORLD SCENARIO

In this section we consider the dilaton production given by the oscillating loops of the cosmic string which can decay into gauge bosons.

The loop cosmic string studied in Sec. III couples with the compactified dilaton modes at different energies that coincides with the loop oscillator frequencies. During this process, massive dilatons are emitted with frequency of oscillation greater than the dilaton mass.

To take into account the dilaton decay into gauge bosons we need the interaction with the electromagnetic field. Therefore, we must consider another type of term in which the dilaton with spin-0 couples with the electromagnetic field through the mass term.

In this section, the zero mode is not considered because this mode corresponds to zero mass. The interaction action responsible for decays into gauge bosons on the brane is given by

$$\mathcal{S}_{\rm int} = \beta_- \sum_{n=1} e^{\nu kL} \frac{J_\nu(x_{n\nu})}{N_n} \int d^4x \sqrt{-g} X_n F_{\mu\nu} F^{\mu\nu}, \tag{95}$$

where β_- is a coupling constant. The mass of the dilaton in our model is given by the compactification scales and the coupling strength is related with the mode function ξ_n.

In our model the energy spectrum and angular distribution of the dilaton radiation can be determined by generalization of the results obtained in [43] to include the n modes and considering a charged cosmic string.

If we have a loop with length L, thus $\omega_n = 4\pi n/L$. The sums are taken over $n > L/L_c$, where $L_c = 4\pi/m_n$. The important point of our work is that for each n we have a different mass for the dilaton given by $m_n = x_{n\nu} k e^{-kL}$, in accordance with we have already obtained in Sec. II. These masses also have dependence on ν which is constrained by the Brans-Dicke parameter as showed in Fig. 4.

The parameters $x_{n\nu}$ can be determined as a function of the Brans-Dicke parameter ω. The energy range that we consider as a function of kL are specified in Fig. 5, taking into account the constant k in Planck scale. In the context of the dilaton, the energy scales are compatible with ones in Ref. [44]. In our model, in order to find masses with energy of the order of TeV for $n = 1$, we must have $kL \sim 40$. In this case, it is possible to generate light gauge bosons. However, if we consider $kL = 12$, as in last section, the particles masses are of the order of $\sim 10^{14}$ GeV. These values for the masses can be mensurable in the Pierre Auger experiment, for example. The decays of these massive particles can be detected, in principle, with lower energies.

The energy and particle radiation rates when the loop is such that $L \gg L_c$ have a mode dependence which can be represented as

$$\dot{E}_{X_n} = \Gamma_{X_n} \alpha_n^2 G_{\rm eff} U^2 (L/L_c)^{-1/3} \tag{96}$$

$$\dot{N}_{X_n} = \tilde{\Gamma}_{X_n} \alpha_n^2 G_{\rm eff} U^2 m_n^{-1} (L/L_c)^{-1/3} \tag{97}$$

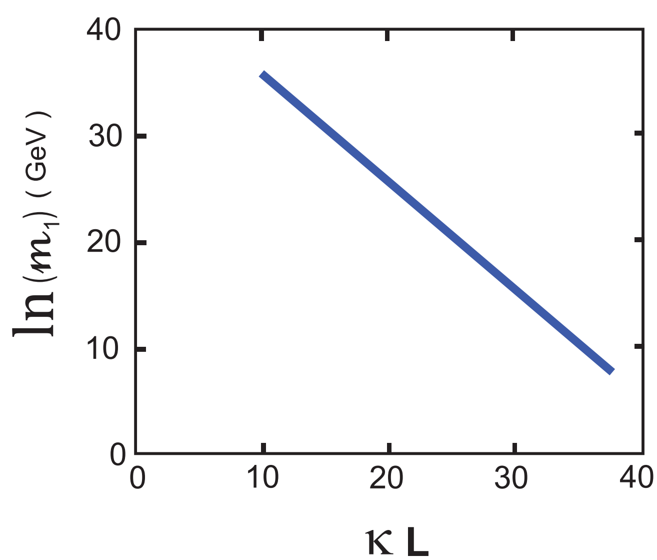

FIG. 4 (color online). A plot of the mass, for $n = 1$, as a function of kL.

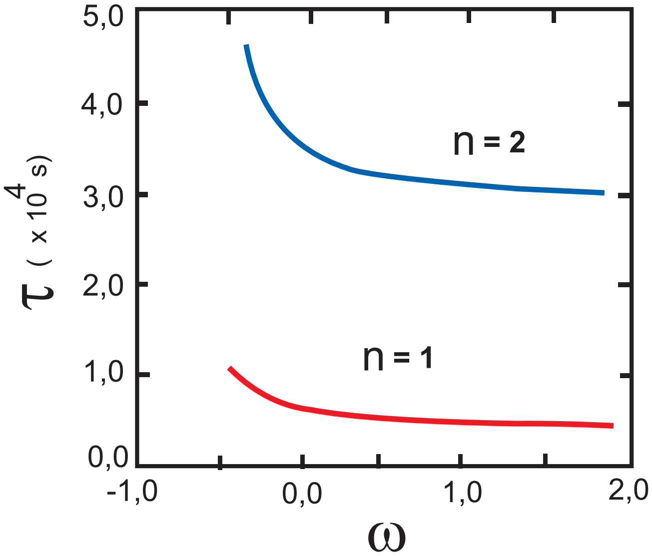

FIG. 5 (color online). A plot of the lifetime of the particles as a function of the Brans-Dicke parameter.

The resulting constraints are relevant if we consider the lifetime of the dilaton, that are determined by the mass and couplings. The corresponding lifetime is

$$\tau_n = \frac{4M_{\rm Pl}^2}{N_F \tilde{\alpha}_{F_n}^2 m_n^3}, \tag{98}$$

where $\tilde{\alpha}_{F_n} = \alpha_n \beta_- \xi_n$ and N_F is the number of gauge bosons with masses much less than m_n. In Fig. 5 we consider $m_n \sim 1$ TeV. In this case all standard-model gauge bosons should be included ($N_F = 12$). We note that the lifetime of the dilatons has a fine structure constant given by the Brans-Dicke parameter. It is important to adjust with the experimental data. Here we also have the constraints given by the Brans-Dicke parameter ω. For $\omega = -1$ the lifetime agrees at infinity, giving us the possibility that massive particles decay in the primordial Universe and probably could be measured in the detectors nowadays.

VI. CONCLUSIONS

It is possible to construct a brane-world scenario including gravity in which we can analyze the effects of the Lorentz breaking in the framework of a cosmic string configuration. With this aim we can assume that the scalar-tensor theories is realized in five dimensions. In this context, it is showed that the cosmological birefringence is connected with the Brans-Dicke parameter which is constrained by the date of the cosmic microwave background radiation and by the compactification modes. The limits of the rotation angle depends on the Brans-Dicke parameter and decreases with the increasing of masses of the corresponding massive modes.

In the context of scalar-tensor theory of gravity in five dimensions, the interaction terms can be considered with and without parity violation. These terms are connected with the cosmic string configuration we are taking into account. The Lorentz breaking term is responsible for generation of charge and the part that does not present the parity violation put limits on the mass scale of the particles which decay into light dilatons. The interaction Lagrangian (95) is responsible for the decay into light gauge bosons, which are the sources of cosmic strings. These depend on the modes given by the five-dimensional brane-world scenario.

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