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Interacting new agegraphic dark energy in nonflat Brans-Dicke cosmology

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We construct a cosmological model of late acceleration based on the new agegraphic dark energy model in the framework of Brans-Dicke cosmology where the new agegraphic energy density $\rho_D = 3n^2 m_p^2/\eta^2$ is replaced with $\rho_D = 3n^2\phi^2/(4\omega\eta^2)$. We show that the combination of the Brans-Dicke field and agegraphic dark energy can accommodate a $w_D = -1$ crossing for the equation of state of *noninteracting* dark energy. When an interaction between dark energy and dark matter is taken into account, the transition of w_D to the phantom regime can be more easily accounted for than when we resort to the Einstein field equations. In the limiting case $\alpha = 0$ ($\omega \to \infty$), all previous results of the new agegraphic dark energy in Einstein gravity are restored.

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I. INTRODUCTION

One of the most dramatic discoveries of modern cosmology in the past decade is that our Universe is currently accelerating [1]. Many scenarios have been proposed to explain this acceleration, but most of them cannot explain all the features of the Universe or they have so many parameters, making them difficult to fit. For a recent review on dark energy proposals, see [2]. Many theoretical studies on dark energy are devoted to understanding and shedding light on the problem in the framework of a fundamental theory such as string theory or quantum gravity. Although a complete theory of quantum gravity has not yet been established, we still can make some attempts to investigate the nature of dark energy according to some principles of quantum gravity. The holographic dark energy model and the agegraphic dark energy (ADE) model are two such examples, which originate from some considerations of the features of the quantum theory of gravity. That is to say, the holographic and ADE models possess some significant features of quantum gravity. The former, which recently created much enthusiasm[3,4], is motivated by the holographic hypothesis [5] and has been tested and constrained by various astronomical observations [6]. The latter (ADE) is based on the uncertainty relation of quantum mechanics together with the gravitational effect in general relativity. The ADE model assumes that the observed dark energy comes from the spacetime and matter field fluctuations in the Universe [7,8]. Following the line of quantum fluctuations of spacetime, Karolyhazy *et al.* [9] discussed that the distance t in Minkowski spacetime cannot be known to a better accuracy than $\delta t = \beta t_p^{2/3} t^{1/3}$, where β is a dimensionless constant of order unity. Based on the Karolyhazy relation, Maziashvili [10] argued that the energy density of spacetime fluctuations is given by

$$\rho_D \sim \frac{1}{t_p^2 t^2} \sim \frac{m_p^2}{t^2}, \tag{1}$$

where t_p and m_p are the reduced Planck time and mass, respectively. On these bases, Cai wrote down the energy density of the original ADE as [7]

$$\rho_D = \frac{3n^2 m_p^2}{T^2}, \tag{2}$$

where T is the age of the Universe and the numerical factor $3n^2$ is introduced to parametrize some uncertainties, such as the species of quantum fields in the Universe. However, the original ADE model has some difficulties [7]. In particular, it has difficult describing the matter-dominated epoch. Therefore, a new model of ADE was proposed [8], in which the time scale was chosen to be the conformal time η instead of the age of the Universe, which is defined by $dt = a d\eta$, where t is the cosmic time. It is worth noting that the Karolyhazy relation $\delta t = \beta t_p^{2/3} t^{1/3}$ was derived for Minkowski spacetime $ds^2 = dt^2 - dx^2$ [9,10]. In the case of the Friedmann-Robertson-Walker (FRW) universe, we have $ds^2 = dt^2 - a^2 dx^2 = a^2(d\eta^2 - dx^2)$. Thus, it might be more reasonable to choose the time scale in Eq. (2) to be the conformal time η since it is the causal time in the Penrose diagram of the FRW universe. The new ADE model contains some new features that are different from the original ADE model and that overcome some unsatisfactory points. The ADE models have been examined and constrained by various astronomical observations [11,12].

On the other front, it is quite possible that gravity is not given by the Einstein action, at least at sufficiently high energies. In string theory, gravity becomes scalar-tensor in nature. The low energy limit of string theory leads to Einstein gravity, coupled nonminimally to a scalar field [13]. Although the pioneering study on scalar-tensor theories was done several decades ago [14], it has recently obtained a new impetus as it arises naturally as the low energy limit of many theories of quantum gravity such as

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superstring theory or Kaluza-Klein theory. Because the agegraphic energy density belongs to a dynamical cosmological constant, we need a dynamical frame to accommodate it instead of Einstein gravity. Therefore, the investigation of the agegraphic models of dark energy in the framework of Brans-Dicke theory is well motivated. In the framework of Brans-Dicke cosmology, holographic models of dark energy have also been studied [15]. Our aim in this paper is to construct a cosmological model of late acceleration based on the Brans-Dicke theory of gravity and on the assumption that the pressureless dark matter and new ADE models are not conserved separately but interact with each other.

II. NEW ADE IN BRANS-DICKE THEORY

We start from the action of Brans-Dicke theory, which in the canonical form can be written [16]

$$S = \int d^4x\sqrt{g}\left(-\frac{1}{8\omega}\phi^2 R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + L_M\right), \quad (3)$$

where R is the scalar curvature and ϕ is the Brans-Dicke scalar field. The nonminimal coupling term $\phi^2 R$ replaces the Einstein-Hilbert term R/G in such a way that $G_{\rm eff}^{-1} = 2\pi\phi^2/\omega$, where $G_{\rm eff}$ is the effective gravitational constant as long as the dynamical scalar field ϕ varies slowly. The signs of the nonminimal coupling term and the kinetic energy term are properly adopted to the $(+---)$ metric signature. The new ADE model will be accommodated in the nonflat FRW universe, which is described by the line element

$$ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right), \quad (4)$$

where $a(t)$ is the scale factor, and k is the curvature parameter with $k = -1, 0, 1$ corresponding to open, flat, and closed universes, respectively. A closed universe with a small positive curvature ($\Omega_k \simeq 0.01$) is compatible with observations [17]. Varying action (3) with respect to metric (4) for a universe filled with dust and ADE yields the following field equations:

$$\frac{3}{4\omega}\phi^2\left(H^2 + \frac{k}{a^2}\right) - \frac{1}{2}\dot{\phi}^2 + \frac{3}{2\omega}H\dot{\phi}\phi = \rho_m + \rho_D, \quad (5)$$

$$\frac{-1}{4\omega}\phi^2\left(2\frac{\ddot{a}}{a} + H^2 + \frac{k}{a^2}\right) - \frac{1}{\omega}H\dot{\phi}\phi - \frac{1}{2\omega}\ddot{\phi}\phi - \frac{1}{2}\left(1 + \frac{1}{\omega}\right)\dot{\phi}^2 = p_D, \quad (6)$$

$$\ddot{\phi} + 3H\dot{\phi} - \frac{3}{2\omega}\left(\frac{\ddot{a}}{a} + H^2 + \frac{k}{a^2}\right)\phi = 0, \quad (7)$$

where the dot is the derivative with respect to time and $H = \dot{a}/a$ is the Hubble parameter. Here ρ_D, p_D, and ρ_m are, respectively, the dark energy density, dark energy pressure, and energy density of dust (dark matter). We shall assume that the Brans-Dicke field can be described as a power law of the scale factor, $\phi \propto a^\alpha$. A case of particular interest is when α is small and ω is large, so the product $\alpha\omega$ results in order unity [15]. This is interesting because local astronomical experiments set a very high lower bound on ω; in particular, the Cassini experiment implies that $\omega > 10^4$ [18]. Taking the derivative with respect to time of the relation $\phi \propto a^\alpha$, we get

$$\dot{\phi} = \alpha H\phi, \quad (8)$$

$$\ddot{\phi} = \alpha^2 H^2\phi + \alpha\phi\dot{H}. \quad (9)$$

The energy density of the new ADE model can be written [8] as

$$\rho_D = \frac{3n^2 m_p^2}{\eta^2}, \quad (10)$$

where the conformal time is given by

$$\eta = \int_0^a \frac{da}{Ha^2}. \quad (11)$$

In the framework of Brans-Dicke cosmology, we write down the new agegraphic energy density of the quantum fluctuations in the Universe as

$$\rho_D = \frac{3n^2\phi^2}{4\omega\eta^2}, \quad (12)$$

where $\phi^2 = \omega/2\pi G_{\rm eff}$. In the limit of Einstein gravity, $G_{\rm eff} \to G$, expression (12) recovers the standard new agegraphic energy density in Einstein gravity. We define the critical energy density $\rho_{\rm cr}$ and the energy density of the curvature, ρ_k, as

$$\rho_{\rm cr} = \frac{3\phi^2 H^2}{4\omega}, \qquad \rho_k = \frac{3k\phi^2}{4\omega a^2}. \quad (13)$$

We also introduce, as usual, the fractional energy densities such as

$$\Omega_m = \frac{\rho_m}{\rho_{\rm cr}} = \frac{4\omega\rho_m}{3\phi^2 H^2}, \quad (14)$$

$$\Omega_k = \frac{\rho_k}{\rho_{\rm cr}} = \frac{k}{H^2 a^2}, \quad (15)$$

$$\Omega_D = \frac{\rho_D}{\rho_{\rm cr}} = \frac{n^2}{H^2\eta^2}. \quad (16)$$

A. Noninteracting case

Let us begin with the noninteracting case, in which the dark energy and dark matter evolve according to their conservation laws,

$$\dot{\rho}_D + 3H\rho_D(1 + w_D) = 0, \quad (17)$$

$$\dot{\rho}_m + 3H\rho_m = 0, \tag{18}$$

where $w_D = p_D/\rho_D$ is the equation of state parameter of the new ADE model. Differentiating Eq. (12) and using Eqs. (8) and (16), we have

$$\dot{\rho}_D = 2H\rho_D\left(\alpha - \frac{\sqrt{\Omega_D}}{na}\right). \tag{19}$$

Inserting this equation in the conservation law (17), we obtain the equation of state parameter of the new ADE model,

$$w_D = -1 - \frac{2\alpha}{3} + \frac{2}{3na}\sqrt{\Omega_D}. \tag{20}$$

It is important to note that when $\alpha = 0$, the Brans-Dicke scalar field becomes trivial and Eq. (20) reduces to its respective expression in the new ADE model in general relativity [8],

$$w_D = -1 + \frac{2}{3na}\sqrt{\Omega_D}. \tag{21}$$

In this case ($\alpha = 0$), the present accelerated expansion of our Universe can be derived only if $n > 1$ [8]. Note that we take $a = 1$ for the present time. In addition, w_D is always larger than -1 and cannot cross the phantom divide $w_D = -1$. However, in the presence of the Brans-Dicke field ($\alpha > 0$) the condition $n > 1$ is no longer necessary to derive the present accelerated expansion. Besides, from Eq. (20) one can easily see that w_D can cross the phantom divide, provided that $na\alpha > \sqrt{\Omega_D}$. If we take $\Omega_D = 0.73$ and $a = 1$ for the present time, the phantomlike equation of state can be accounted for if $n\alpha > 0.85$. For instance, for $n = 1$ and $\alpha = 0.9$, we get $w_D = -1.03$. Therefore, with the combination of new agegraphic energy density and the Brans-Dicke field w_D of the *noninteracting* new ADE model, the phantom divide can be crossed.

Let us examine the behavior of w_D in two different stages. In the late time where $\Omega_D \to 1$ and $a \to \infty$, we have $w_D = -1 - \frac{2\alpha}{3}$. Thus $w_D < -1$ for $\alpha > 0$. This implies that in the late time, w_D necessarily crosses the phantom divide in the framework of Brans-Dicke theory. In the early time, where $\Omega_D \to 0$ and $a \to 0$, we cannot find w_D from Eq. (20) directly. Let us consider the matter-dominated epoch, $H^2 \propto \rho_m \propto a^{-3}$. Therefore $\sqrt{a}da \propto dt = ad\eta$. Thus $\eta \propto \sqrt{a}$. From Eq. (12) we have $\rho_D \propto a^{2\alpha-1}$. Putting this in conservation law, $\dot{\rho}_D + 3H\rho_D(1 + w_D) = 0$, we obtain $w_D = -2/3 - 2\alpha/3$. Substituting this w_D in Eq. (20), we find that $\Omega_D = n^2a^2/4$ in the matter-dominated epoch, as expected. We will see below that this is exactly the result one obtains for Ω_D from its equation of motion in the matter-dominated epoch.

Since in our model the dynamics of the scale factor is governed not only by the dark matter and new ADE model, but also by the Brans-Dicke field, the signature of the deceleration parameter, $q = -\ddot{a}/(aH^2)$, has to be examined carefully. When the deceleration parameter is combined with the Hubble parameter and the dimensionless density parameters, they form a set of useful parameters for the description of the astrophysical observations. Dividing Eq. (6) by H^2, and using Eqs. (8), (9), and (12)–(16), we obtain

$$q = \frac{1}{2\alpha + 2}[(2\alpha + 1)^2 + 2\alpha(\alpha\omega - 1) + \Omega_k + 3\Omega_D w_D]. \tag{22}$$

Substituting w_D from Eq. (20), we reach

$$q = \frac{1}{2\alpha + 2}\left[(2\alpha + 1)^2 + 2\alpha(\alpha\omega - 1) + \Omega_k - (2\alpha + 3)\Omega_D + \frac{2}{na}\Omega_D^{3/2}\right]. \tag{23}$$

When $\alpha = 0$, Eq. (23) restores the deceleration parameter of the new ADE model in general relativity [12],

$$q = \frac{1}{2}(1 + \Omega_k) - \frac{3}{2}\Omega_D + \frac{\Omega_D^{3/2}}{na}. \tag{24}$$

Finally, we obtain the equation of motion for Ω_D. Taking the derivative of Eq. (16) and using relation $\dot{\Omega}_D = H\Omega'_D$, we get

$$\Omega'_D = \Omega_D\left(-2\frac{\dot{H}}{H^2} - \frac{2}{na}\sqrt{\Omega_D}\right), \tag{25}$$

where the prime denotes the derivative with respect to $x = \ln a$. Using relation $q = -1 - \frac{\dot{H}}{H^2}$, we have

$$\Omega'_D = 2\Omega_D\left(1 + q - \frac{\sqrt{\Omega_D}}{na}\right), \tag{26}$$

where q is given by Eq. (23). Let us examine the above equation for the matter-dominated epoch, where $a \ll 1$ and $\Omega_D \ll 1$. Substituting q from Eq. (23) in (26) with $\Omega_k \ll 1$, $\alpha \ll 1$, and $\alpha\omega \approx 1$, this equation reads as

$$\frac{d\Omega_D}{da} \simeq \frac{\Omega_D}{a}\left(3 - \frac{2}{na}\sqrt{\Omega_D}\right). \tag{27}$$

Solving this equation we find $\Omega_D = n^2a^2/4$, which is consistent with our previous result. Therefore, all things are consistent. The confusion in the original ADE model is removed in this new model.

B. Interacting case

Next we generalize our study to the case where the pressureless dark matter and the new ADE model are not conserved separately but interact with each other. Given the unknown nature of both dark matter and dark energy, there is nothing, in principle, against their mutual interaction, and it seems very special that these two major components in the Universe are entirely independent. Indeed, this possibility is receiving growing attention in the literature [19] and appears to be compatible with SNIa and CMB data [20]. The total energy density satisfies a conservation law,

$$\dot{\rho} + 3H(\rho + p) = 0. \tag{28}$$

However, since we consider the interaction between dark matter and dark energy, ρ_m and ρ_D are not conserved separately; rather, they must enter the energy balances

$$\dot{\rho}_m + 3H\rho_m = Q, \tag{29}$$

$$\dot{\rho}_D + 3H\rho_D(1 + w_D) = -Q, \tag{30}$$

where $Q = \Gamma\rho_D$ stands for the interaction term with $\Gamma > 0$. Using Eqs. (8) and (13), we can rewrite the first Friedmann equation (5) as

$$\rho_{\rm cr} + \rho_k = \rho_m + \rho_D + \rho_\phi, \tag{31}$$

where we have defined

$$\rho_\phi \equiv \frac{1}{2}\alpha H^2\phi^2\left(\alpha - \frac{3}{\omega}\right). \tag{32}$$

Dividing Eq. (31) by $\rho_{\rm cr}$, this equation can be written as

$$\Omega_m + \Omega_D + \Omega_\phi = 1 + \Omega_k, \tag{33}$$

where

$$\Omega_\phi = \frac{\rho_\phi}{\rho_{\rm cr}} = -2\alpha\left(1 - \frac{\alpha\omega}{3}\right). \tag{34}$$

We also assume $\Gamma = 3b^2(1 + r)H$, where $r = \rho_m/\rho_D$ and b^2 is a coupling constant. Therefore, the interaction term Q can be expressed as

$$Q = 3b^2H\rho_D(1 + r), \tag{35}$$

where

$$r = \frac{\Omega_m}{\Omega_D} = -1 + \Omega_D^{-1}\left[1 + \Omega_k + 2\alpha\left(1 - \frac{\alpha\omega}{3}\right)\right]. \tag{36}$$

Combining Eqs. (19), (35), and (36) with Eq. (30), we can obtain the equation of state parameter

$$w_D = -1 - \frac{2\alpha}{3} + \frac{2}{3na}\sqrt{\Omega_D} - b^2\Omega_D^{-1}\left[1 + \Omega_k + 2\alpha\left(1 - \frac{\alpha\omega}{3}\right)\right]. \tag{37}$$

When $\alpha = 0$, Eq. (37) recovers its respective expression of the interacting new ADE model in general relativity [12]. From Eq. (37) we see that with the combination of the new ADE model and the Brans-Dicke field, the transition of w_D from the phantom divide can be more easily accounted for than in Einstein gravity. For completeness we also present the deceleration parameter for the interacting case,

$$q = \frac{1}{2\alpha + 2}\left[(2\alpha + 1)^2 + 2\alpha(\alpha\omega - 1) + \Omega_k - (2\alpha + 3)\Omega_D + \frac{2}{na}\Omega_D^{3/2} - 3b^2\left(1 + \Omega_k + 2\alpha\left(1 - \frac{\alpha\omega}{3}\right)\right)\right]. \tag{38}$$

In the limiting case $\alpha = 0$, Eq. (38) restores the deceleration parameter for the standard interacting new ADE model in a nonflat universe [12],

$$q = \frac{1}{2}(1 + \Omega_k) - \frac{3}{2}\Omega_D + \frac{\Omega_D^{3/2}}{na} - \frac{3b^2}{2}(1 + \Omega_k). \tag{39}$$

For a flat universe, $\Omega_k = 0$, and we recover exactly the result of [8]. The equation of motion for Ω_D takes the form (26), where q is now given by Eq. (38).

III. CONCLUSIONS

An interesting attempt to probe the nature of dark energy within the framework of quantum gravity is the so-called ADE proposal. Since ADE models belong to a dynamical cosmological constant, it is more natural to study them in the framework of Brans-Dicke theory than in Einstein gravity. In this paper, we studied a cosmological model of late acceleration based on the new ADE model in the framework of nonflat Brans-Dicke cosmology, where the new agegraphic energy density $\rho_D = 3n^2m_p^2/\eta^2$ is replaced with $\rho_D = 3n^2\phi^2/(4\omega\eta^2)$. With this replacement in Brans-Dicke theory, we found that the acceleration of the Universe expansion will be more easily achieved than when the standard new ADE model in general relativity is employed. Interestingly enough, we found that with the combination of the Brans-Dicke field and the ADE model, the equation of state of the *noninteracting* new ADE model can cross the phantom divide. This is in contrast to Einstein gravity, where the equation of state of the *noninteracting* new ADE model cannot cross the phantom divide [8]. When an interaction between dark energy and dark matter is taken into account, the transition to the phantom regime for the equation of state of the new ADE model can be more easily accounted for than when we resort to the Einstein field equations.

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