

Simple types of anisotropic inflationJohn D. Barrow^{1,*} and Sigbjørn Hervik^{2,†}¹*DAMTP, Centre for Mathematical Sciences, Cambridge University, Wilberforce Road, Cambridge CB3 0WA, United Kingdom*²*Faculty of Science and Technology, University of Stavanger, N-4036 Stavanger, Norway*

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We display some simple cosmological solutions of gravity theories with quadratic Ricci curvature terms added to the Einstein-Hilbert Lagrangian which exhibit anisotropic inflation. The Hubble expansion rates are constant and unequal in three orthogonal directions. We describe the evolution of the simplest of these homogeneous and anisotropic cosmological models from its natural initial state and evaluate the deviations they will create from statistical isotropy in the fluctuations produced during a period of anisotropic inflation. The anisotropic inflation is not a late-time attractor in these models but the rate of approach to a final isotropic de Sitter state is slow and is conducive to the creation of observable anisotropic statistical effects in the microwave background. The statistical anisotropy would not be scale invariant and the level of statistical anisotropy will grow with scale.

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I. INTRODUCTION

The observations of the cosmic microwave background (CMB) made by WMAP and ground-based detectors are in good general agreement with the expectations of a post-inflationary universe containing a residual quintessence field [1,2]. However, the data quality has led to a focus upon the presence of several unexpected features of the sky maps as well as a search for any evidence of non-Gaussianity in their statistics [3]. In particular, there have been studies of the significance and possible explanations for a lower than expected power in the quadrupole signal [4], possible alignments between low multipoles giving rise to a preferred direction, or “axis,” on the sky [5] that can arise in some anisotropic universes [6], and an apparent asymmetry between the northern and southern ecliptic hemispheres [7]. These deviations from statistical isotropy in the data are hard to assess definitively by means of *a posteriori* statistics and may be due to unnoticed biases in the foreground subtraction, but careful studies of this potential problem have yet to find evidence of such an effect at a level which can explain the observations [8]. A detailed study of the evidence for homogeneous statistical anisotropy in the CMB has been conducted by Hanson and Lewis [9], who use quadratic maximum-likelihood estimators to analyse Gaussian models with statistical anisotropy and realistic sources of instrumental noise. They find evidence for anisotropy in the power spectrum with a large angular dependence for the quadrupole, aligned close to the ecliptic plane. Since this is the plane in which the satellite moves there is a suspicion that this observed asymmetry may be associated with a systematic instrumental effect or a beam ellipticity which is not corrected for in the CMB maps analyzed.

There have been several attempts to explore the possibility that small anisotropic features might be imprinted upon the primordial fluctuation spectrum by the process of inflation. In the standard general relativistic model of inflation driven by scalar fields which violate the strong-energy condition during a period of slow rolling, this would only be possible in the very earliest moments of inflation if the prior state was one of extreme expansion- and curvature-anisotropy because the dynamics rapidly approach those of the de Sitter metric in the presence of a positive effective cosmological constant. This situation is studied in Ref. [10], but only for the simplest form of expansion anisotropy ignoring the effects of collisionless particles [11] and curvature anisotropies [12,13], which both make the residual effects larger. It is also possible to induce local expansion anisotropies by the presence of superhorizon scale inhomogeneities, and this scenario is explored by Gao [14]. If we change the underlying inflationary model by adding a vector field [15–17], a Chern-Simons or Kalb-Ramond field [18], or some quadratic curvature corrections to the gravitational Lagrangian [19], then the situation can change. Effective stresses are created which (unlike the scalar inflaton field in general relativity) can violate the dominant or the weak-energy conditions as well as the strong-energy condition. Under these circumstances the de Sitter metric may no longer be an attractor during accelerated expansion and anisotropic inflationary behavior might even be possible. Anisotropic inflation is only possible in general relativity with a positive cosmological constant and matter that obeys the strong energy condition if the 3-curvature is positive. There is a known exact $S^1 \times S^2$ Kantowski-Sachs universe of this type [20] which expands forever, with exponential expansion of its scale factors in two directions and a constant scale factor along the third. This model has been used in a study of inflation [21], however, its anisotropic behavior is unstable within this class of Kantowski-Sachs models and to the addition of matter fields [22].

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In this paper we are interested in the situation where a quadratic Ricci term, $R_{\mu\nu}R^{\mu\nu}$, is added to the Einstein-Hilbert term. We have already shown [23] that in the resulting theory with positive cosmological constant it is possible to find simple exact spatially homogeneous cosmological models of Bianchi type I which inflate anisotropically. Here we explore these solutions in more detail and show that the same cosmological models also possess an exact de Sitter attractor solution. However, the evolution towards the de Sitter attractor from an earlier state of anisotropic inflation is unusually slow and there is ample opportunity for anisotropic statistical effects to be imprinted upon the fluctuation spectrum created by the accelerated expansion. These anisotropies will be larger at earlier times in the inflationary phase and therefore will imprint greater anisotropic effects on larger scales than on smaller ones. This model is mathematically simple, with Euclidean space sections, and provides a useful testing ground for computing more detailed effects of anisotropic inflation on the scalar and tensor irregularity spectra. It allows us to determine the sense and relative magnitudes of explicit expansion anisotropies during a period of volume inflation in which the orthogonal scale factors expand by different exponential time factors.

Usually inflation is considered to be driven by an isotropic scalar field; here we will consider a simple cosmological constant as an example of such an effect. The models can easily be extended to include isotropic scalar fields of this kind, however, this may require additional fine-tuning for the scalar field to come to dominate the expansion dynamics at the right time. This is also the case for $f(R)$ effects through the standard conformal transformation [24]. The effects from the quadratic Ricci term, on the other hand, are different as they allow for anisotropic degrees of freedom—indeed, a quadratic Ricci term is the main source of the anisotropic inflation considered in this paper.

II. A BIANCHI TYPE I MODEL

Our starting point is the equations of motion using a dynamical systems approach. For the Bianchi type I (and II) models, these are given in [23]. We will consider the quadratic theory where the Einstein-Hilbert action is modified:

$$S_G = \frac{1}{2\kappa} \int_M d^4x \sqrt{|g|} (R + \alpha R^2 + \beta R_{\mu\nu}R^{\mu\nu} - 2\Lambda).$$

Upon variation, the equations of motion are the modified Einstein equations:

$$G_{\mu\nu} + \Phi_{\mu\nu} + \Lambda g_{\mu\nu} = 0, \quad (1)$$

where $G_{\mu\nu} = R_{\mu\nu} - (1/2)Rg_{\mu\nu}$ is the regular Einstein tensor, and

$$\begin{aligned} \Phi_{\mu\nu} = & 2\alpha R(R_{\mu\nu} - \frac{1}{4}Rg_{\mu\nu}) + (2\alpha + \beta) \\ & \times (g_{\mu\nu}\square - \nabla_\mu\nabla_\nu)R + \beta\square(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}) \\ & + 2\beta(R_{\mu\sigma\nu\rho} - \frac{1}{4}g_{\mu\nu}R_{\sigma\rho})R^{\sigma\rho}, \end{aligned} \quad (2)$$

and $\square \equiv \nabla^\mu\nabla_\mu$. We note that the GR-limit can be obtained by letting $(\alpha, \beta) \rightarrow (0, 0)$. Now consider the spatially homogeneous Bianchi type I metrics. We can always write their metric line elements as

$$ds^2 = -dt^2 + \delta_{ab}\omega^a\omega^b,$$

where ω^a is a triad of one-forms which, for the Bianchi type I model, can be written as $\omega^a = e^a_i(t)dx^i$.

Defining the timelike hypersurface-orthogonal vector $\mathbf{u} = \partial/\partial t$, we can define the Hubble scalar H and the shear σ_{ab} as follows:

$$H \equiv \frac{1}{3}u^\mu{}_{;\mu}, \quad \sigma_{ab} = u_{(a;b)} - H\delta_{ab}.$$

We will also restrict attention to cosmological models where the shear is diagonal, so we can write

$$\sigma_{ab} = \text{diag}(-2\sigma_+, \sigma_+ + \sqrt{3}\sigma_-, \sigma_+ - \sqrt{3}\sigma_-).$$

We define the dimensionless expansion-normalized variables by scaling out appropriate powers of H

$$\begin{aligned} B = \frac{1}{(3\alpha + \beta)H^2}, \quad \chi = \frac{\beta}{3\alpha + \beta}, \quad Q = \frac{\dot{H}}{H^2}, \\ Q_2 = \frac{\ddot{H}}{H^3}, \quad \Omega_\Lambda = \frac{\Lambda}{3H^2}, \quad \Sigma_\pm = \frac{\sigma_\pm}{H}, \\ \Sigma_{\pm 1} = \frac{\dot{\sigma}_\pm}{H^2}, \quad \Sigma_{\pm 2} = \frac{\ddot{\sigma}_\pm}{H^3}. \end{aligned} \quad (3)$$

Note the presence of time derivatives of the variables Q_2 and $\Sigma_{\pm 2}$; this reflects the 4th order time derivatives in the field equations of the quadratic theory [25]; χ is a constant. We also introduce the dynamical time variable τ by

$$\frac{d\tau}{dt} = H,$$

and note that since $H = \dot{a}/a$, the dynamical time is related to the length scale as $a = \ell_0 e^\tau$, where $\ell_0 > 0$ is constant. We also assume that the cosmological constant is positive: $\Omega_\Lambda > 0$.

The full set of dynamical equations for the Bianchi type I model (and type II model) are given in [23]. In [23] it was shown that there exists a peculiar set of solutions for this theory, namely, a set of anisotropically inflating solutions. A stability analysis of these solutions was also performed. Here, we intend to study these solutions further; in particular, we will investigate their possible influence on any statistical relic anisotropy at the end of inflation.

For later reference, let us also give the equations of motion. We will, for simplicity, consider the invariant subspace when $(\Sigma_+, \Sigma_-) \propto (\Sigma_{+1}, \Sigma_{-1}) \propto (\Sigma_{+2}, \Sigma_{-2})$. This enables us to write:

$$\begin{aligned}
(\Sigma_+, \Sigma_-) &= \Sigma(\cos\phi, \sin\phi), \\
(\Sigma_{+1}, \Sigma_{-1}) &= \Sigma_1(\cos\phi, \sin\phi), \\
(\Sigma_{+2}, \Sigma_{-2}) &= \Sigma_2(\cos\phi, \sin\phi), \quad \phi' = 0.
\end{aligned} \tag{4}$$

The equations of motion are now:

$$B' = -2QB, \tag{5}$$

$$\Omega'_\Lambda = -2Q\Omega_\Lambda, \tag{6}$$

$$Q' = -2Q^2 + Q_2, \tag{7}$$

$$\begin{aligned}
Q'_2 &= -3(Q+2)Q_2 - \frac{9}{2}(Q+2)Q \\
&\quad - \frac{3}{4}B\left(1 + \Sigma^2 - \Omega_\Lambda + \frac{2}{3}Q\right) - \frac{3}{2}(1+2\chi)\Sigma^4 \\
&\quad - \frac{1}{4}(8+\chi)\Sigma_1^2 - (4-\chi)\Sigma\Sigma_1 \\
&\quad - \frac{1}{4}(4-\chi)(3\Sigma^2 + 2\Sigma\Sigma_2 + 2Q\Sigma^2),
\end{aligned} \tag{8}$$

$$\Sigma' = -Q\Sigma + \Sigma_1, \tag{9}$$

$$\Sigma'_1 = -2Q\Sigma_1 + \Sigma_2, \tag{10}$$

$$\begin{aligned}
\Sigma'_2 &= -3(Q+2)\Sigma_2 + \frac{\Sigma_1}{\chi}[B - (11\chi - 8) + 4Q(1 - \chi) \\
&\quad + 4\Sigma^2(1 + 2\chi)] + \frac{\Sigma}{\chi}[3B + (4 - \chi)(6 + Q_2 + 7Q) \\
&\quad + 4(1 + 2\chi)(3\Sigma^2 + 2\Sigma\Sigma_1)]
\end{aligned} \tag{11}$$

These equations are subject to the (“Friedmann-like”) constraint:

$$\begin{aligned}
0 &= B(1 - \Omega_\Lambda - \Sigma^2) + 12Q - 2Q^2 + 4Q_2 \\
&\quad - (4 - \chi)(3 + 2Q)\Sigma^2 - 6(1 + 2\chi)\Sigma^4 \\
&\quad - \chi(\Sigma_1^2 - 2\Sigma\Sigma_2) + 4(2 + \chi)\Sigma\Sigma_1.
\end{aligned} \tag{12}$$

The parameter Q is related to the usual deceleration parameter q via

$$q = -(1 + Q),$$

and the variable B measures how greatly the quadratic part of the Lagrangian dominates over the general-relativistic Einstein-Hilbert term $R - 2\Lambda$. In particular, the larger the value of B , the “closer” we are to GR. The $B = 0$ case corresponds to a purely quadratic Lagrangian theory whose equations of motion reduce to $\Phi_{\mu\nu} = 0$.

The inflating solutions

We will now focus on two (sets of) solutions, namely, the de Sitter solution and the anisotropically inflating type I solutions.

The de Sitter solution, dS: The de Sitter solution is characterized by the critical points where

$$\begin{aligned}
Q = Q_2 = \Sigma_\pm = \Sigma_{\pm 1} = \Sigma_{\pm 2} = N = 0, \\
\Omega_\Lambda = 1, \quad B \neq 0.
\end{aligned}$$

Its stability is assured if [23]

$$\begin{aligned}
B > 0 &\Rightarrow (3\alpha + \beta) > 0, \\
\frac{B + 2(4 - \chi)}{\chi} < 0 &\Rightarrow \frac{1 + 2\Lambda(4\alpha + \beta)}{\beta} < 0.
\end{aligned}$$

Anisotropically inflating type I universes, $\mathcal{A}(I)$: For certain values of χ and B , there are also exact solutions that describe anisotropic inflationary solutions of Bianchi type I [23]:

$$\begin{aligned}
(\Sigma_+, \Sigma_-) &= \Sigma(\cos\phi, \sin\phi), \\
\Sigma^2 = \Sigma_0^2 &\equiv -\frac{2(4 - \chi) + B}{4(2\chi + 1)}, \\
Q = \Sigma_{\pm 1} = \Sigma_{\pm 2} = N &= 0.
\end{aligned}$$

There are two classes of such solutions, depending on the values of B and Ω_Λ . Here we will concentrate on the following:

$$B = \text{const}, \quad \Omega_\Lambda = \frac{18\chi - B}{8(2\chi + 1)}.$$

So long as χ and B take values for which $\Sigma_0^2 > 0$, these solutions exist. Moreover, the solution is unstable, but for $B > 0$ contains only *one* unstable mode.

The metrics corresponding to the case where $B \neq 0$ can be written

$$\begin{aligned}
ds^2 &= -dt^2 + e^{2bt}[e^{-4\sigma_+ t} dx^2 + e^{2(\sigma_+ + \sqrt{3}\sigma_-)t} dy^2 \\
&\quad + e^{2(\sigma_+ - \sqrt{3}\sigma_-)t} dz^2], \\
b^2 &= \frac{1 + 8\Lambda(\alpha + \beta)}{9\beta}, \\
(\sigma_+^2 + \sigma_-^2) &= -\frac{1 + 2\Lambda(4\alpha + \beta)}{18\beta}.
\end{aligned} \tag{13}$$

We note that for these solutions to exist we need both these squares to be positive. Furthermore, we require $\sigma_+^2 + \sigma_-^2 + b^2/2 = \Lambda/3 > 0$.

III. ANISOTROPIC INFLATION

Based on the solutions above and their stability, *there are values of the parameters where $\mathcal{A}(I)$ exists, and the de Sitter solution is stable.* In fact, the connection is deeper than this. Assuming $B > 0$, and $\Lambda > 0$, then if $\chi > 4$, $\mathcal{A}(I)$ is connected to dS via a bifurcation at $B = 2(\chi - 4)$. For $B < 2(\chi - 4)$ the de Sitter solution is unstable (1 unstable mode). As the value of B increases, dS bifurcates at $B = 2(\chi - 4)$ creating the equilibrium points $\mathcal{A}(I)$. For

$B > 2(\chi - 4)$, $\mathcal{A}(I)$ has acquired the unstable mode and made dS stable. Hence, the only unstable mode of $\mathcal{A}(I)$ is actually connected to the de Sitter solution and these inflationary solutions should therefore be connected via a heteroclinic orbit (at least close to the bifurcation point). It is this fact we shall exploit here because this opens up the possibility that the universe approaches the point $\mathcal{A}(I)$ and so starts to inflate anisotropically. This state is almost stable but it does have an unstable mode. This unstable mode will therefore eventually drive the evolution away from anisotropic inflation, towards isotropic inflation (represented by the stable point dS).

In Fig. 1 the evolution of universes with three different initial values are plotted. The universes start to inflate around $\tau = 0$, at which $Q \approx 0$, however, as can be seen, the shear is nonzero; hence, the universe starts inflating anisotropically. During a transient period the universe can stay arbitrary close to the anisotropically inflating point $\mathcal{A}(I)$; however, as we can see, eventually the universe will move towards isotropic inflation. For the universes displayed, we see that during the time from $\tau = 0$ to $\tau = 60$ (which corresponds to 60 e-folds), we have $Q \approx 0$ and therefore they are inflating during the entire period, even if

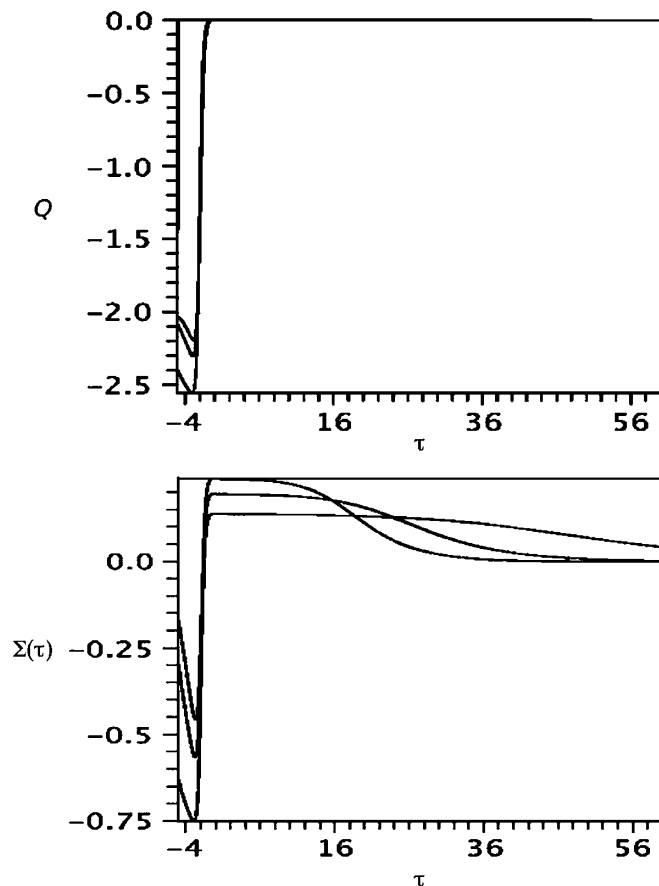


FIG. 1. Inflation: The evolution of Q and Σ with time, τ , for universes with three different initial values. Here, we set $\chi = 6$.

the universes are undergoing a transition between anisotropic inflation and isotropic inflation.

Interestingly, the smaller the shear is at $\tau = 0$, the longer it takes to enter the state of isotropic inflation. The reason for this can be seen from the unstable mode of $\mathcal{A}(I)$. From the eigenvalues derived in [23], we see that the unstable mode goes as $\propto \exp(\lambda_1 \tau)$, where $\lambda_1 = (3/2)(\sqrt{1 + 8\Sigma_0^2} - 1)$. Consequently, the smaller the shear, the more time it would take for the universe to enter a state of isotropic inflation. Furthermore, since the universe inflates during the transition, $Q \approx 0$ and the value of $B \approx \text{const.}$ Therefore, close to the de Sitter isotropic point the shear will decay as $\Sigma \propto \exp(\lambda_2 \tau)$ where λ_2 can be estimated to be

$$\lambda_2 \approx -\frac{3}{2} \left(1 - \sqrt{1 - \frac{16(2\chi + 1)}{9\chi} \Sigma_0^2} \right).$$

For small Σ_0 , this reduces to $\lambda_2 \approx -4(2\chi + 1)\Sigma_0^2/(3\chi)$. This implies that the shear during the anisotropic inflation, Σ_0 , is imprinted in the decay rate of the shear on the approach to isotropic de Sitter state and the smaller the shear Σ_0 , the slower the approach towards the isotropic de Sitter inflation.

A consequence of this is that at the end of 60 e-folds ($\tau = 60$), the universe may still have considerable amount of shear in spite of the fact that the universe has inflated. However, as we can see from Fig. 1, this shear can also be arbitrary small.

In Fig. 2 the numerics have been started away from the anisotropic inflationary solutions. With a bit of fine-tuning we can see that the solutions experience a transient period

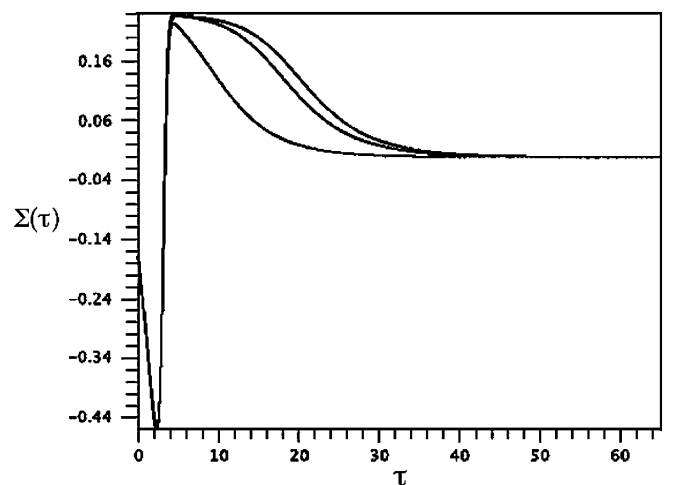


FIG. 2. The evolution of the dimensionless expansion-normalized shear, Σ , with time, τ , for three universes starting away from the inflating solutions. They experience a transient period during which the universes inflate anisotropically. The evolution displayed requires some fine-tuning but the plot illustrates that there is a set of nonzero measure that gives the desired behavior.

during which the universe inflates anisotropically. This shows that there is a set of nonzero measure of initial values that experience anisotropic inflation.

A. Consequences for the CMB

In Ref. [16], the imprints of a preferred direction on the CMB was studied for an anisotropic model in which rotational invariance is broken by the presence of a vector picking out a preferred direction with unit vector \hat{n} . If parity is preserved ($\vec{k} \rightarrow -\vec{k}$) then the leading order of the anisotropic power spectrum has a quadrupole form, with

$$P(\vec{k}) = P(k)[1 + g(k)(\hat{n} \cdot \hat{k})^2 + \text{higher order}],$$

where $P(k)$ is the isotropic part of the power spectrum and $g(k)$ measures the power of the statistical anisotropy. If the anisotropic contributions are scale independent then $g(k)$ will be a constant. However, this need not always be the case, as in the study of anisotropies induced by super-horizon inhomogeneities [14].

The model considered in Ref. [16] was an axisymmetric anisotropically inflationary model. Our model allows for axisymmetry in the special case $\Sigma_- = 0$ (the general case has no rotational symmetry whatsoever). Their model actually corresponds to the exact equilibrium point, but in our model this is unstable and evolves toward the isotropic de Sitter state. In their eq. (39) they define the anisotropy parameter ϵ_H , which in our notation is just the dimensionless shear distortion (in the special case $\Sigma_- = 0$):

$$g(k) \propto \epsilon_H \equiv \frac{2\sigma_+}{H} = 2\Sigma,$$

(recall that the dynamical time τ is related to the scale factor a through $a = a_0 e^\tau$). In our model $g(k)$ will evolve and so cause the perturbation spectrum to depend on the time when each perturbation scale left the horizon. Hence, the statistical anisotropy will not be scale invariant. In our models the anisotropy is larger at the beginning of the inflation which means that long wavelengths will be more anisotropic than the short wavelengths.

B. Other Bianchi models

The model considered here is a very special Bianchi model. However, the existence of anisotropic inflation seems to be a widespread feature of quadratic theories of gravity. In [26] we showed the existence of anisotropic inflation for Bianchi type II models. Among the most general Bianchi models are the Bianchi type VI_h and VII_h models and anisotropic inflation is also present there. The following metrics can be shown to be solutions to $\Phi_{,\mu\nu} = 0$ in Eq. (1):

$$ds^2 = -dt^2 + dx^2 + e^{2r(t+x)}[e^{2a}(Ady + Bdz)^2 + e^{-2a}(Cdy + Adz)^2].$$

Here, r and a are constants. For the various Bianchi types, the functions A , B and C are as follows:

- (1) Type VII_h : $A = \cos[\omega(x+t)]$, $B = -C = \sin[\omega(x+t)]$.
- (2) Type VI_h : $A = \cosh[\omega(x+t)]$, $B = C = \sinh[\omega(x+t)]$.
- (3) Type IV: $A = 1$, $B = \omega(x+t)$, $C = 0$.

There are no analogues of these solutions in general relativity because they would violate the weak-energy condition. Interestingly, these are also so-called plane-wave spacetimes. We note that the 3-space volume expands exponentially, $V \propto \exp(2rt)$ in terms of the comoving proper time, t ; hence, these are indeed inflationary solutions. However, as we see that one orthogonal direction is actually fixed and does not expand at all.

The existence of such solutions for these Bianchi types indicates that anisotropic inflation is a more general feature of quadratic theories than previous thought and is present in even the most general class of anisotropic universes, which includes types VI_h and VII_h .

IV. CONCLUSIONS

Simple Bianchi type I universes in gravity theories in which the Einstein-Hilbert lagrangian is augmented by the addition of terms quadratic in the scalar curvature (R^2) and Ricci invariant ($R_{\mu\nu}R^{\mu\nu}$) display evolution that commences from a near-isotropic singularity. It mimics the behavior of a radiation-dominated Friedmann universe [27] and asymptotes towards a de Sitter late-time attractor. However, the evolution spends a long time evolving slowly through a period of anisotropic inflation during which the 3-volume expands exponentially in comoving proper time and the three directional scale factors increase at different rates. We display exact solutions which display this transitional anisotropic inflationary behavior and show that it does not arise in the general relativity limit when the higher-order curvature terms vanish from the Lagrangian. With a small amount of fine-tuning of the preinflationary evolution this behavior can leave a distinctive imprint on the spectrum of inhomogeneities created by any period of inflation defined by the exponential increase in the 3-volume. Using the characterization of the leading statistically anisotropic contribution to the power spectrum of inhomogeneities that was introduced by Ackerman, Carroll, and Wise [16], we determine the amplitude of the statistical anisotropy in the spectrum and show that it will not be scale invariant because the anisotropic effects increase with scale.

There have been a number of studies of the possible sources of statistical anisotropy in the power spectrum of the microwave background. The most detailed study [9] finds significant evidence for anisotropy but no proof that it is primordial in origin. Our analysis identifies a broad class of higher-order gravity theories in which there are solutions that differ significantly from those in general relativ-

ity. In particular, ever-expanding vacuum solutions with a positive cosmological constant do not approach the de Sitter solution. The higher-order curvature terms contribute effective stress terms which violate the energy conditions that are needed for the cosmic no hair theorems of general relativistic cosmology to hold. Hence, they allow new types of exact solution to exist in which different directions accelerate at different rates. We have shown that solutions of this anisotropic inflationary type also exist in some of the most general Bianchi type universe and are not

confined to the type I case we have used for simplicity. In the general Bianchi I universe we have studied, there is attraction to an asymptotic de Sitter solution, as in general relativity, but the evolution spends a large number of e-folds of expansion in the neighborhood of an anisotropic inflationary solution. The existence of such behavior near the Planck scale and the persistence of anomalies in the sky maps derived so far from observations of the microwave sky suggest that there is a possibility the two are related.

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