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Completely regular quantum stress tensor with $w < -1$

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For many quantum field theory computations in cosmology it is not possible to use the flat space trick of obtaining full, interacting states by evolving free states over infinite times. State wave functionals must be specified at finite times and, although the free states suffice to obtain the lowest order effects, higher order corrections necessarily involve changes of the initial state. Failing to correctly change the initial state can result in effective field equations which diverge on the initial value surface, or which contain tedious sums of terms that redshift like inverse powers of the scale factor. In this paper we verify a conjecture from 2004 that the lowest order initial state correction can indeed absorb the initial value divergences and all the redshifting terms of the two-loop expectation value (in free, Bunch-Davies vacuum) of the stress tensor of a massless, minimally coupled scalar with a quartic self-interaction on a nondynamical de Sitter background.

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I. INTRODUCTION

Suppose $\varphi(t, \vec{x})$ is a real scalar field operator whose Lagrangian (by which we mean the spatial integral of the Lagrangian density) at time t is $L[\varphi(t)]$. Then the relation between the in-out functional integral formalism and canonical matrix elements is

$$\langle\Phi|T^*(\mathcal{O}[\varphi])|\Psi\rangle = \int [d\phi] e^{i\int_{t_1}^{t_2} dt L[\phi(t)]} \Phi^*[\phi(t_2)] \times \mathcal{O}[\phi]\Psi[\phi(t_1)]. \tag{1}$$

In this formula $\mathcal{O}[\varphi]$ is some functional of the field for times between t_1 and t_2, and the T^* symbol means that the operator upon which it acts is time-ordered, but with any time derivatives taken *outside* the time-ordering. The Heisenberg states $|\Psi\rangle$ and $|\Phi\rangle$ have $\mathbb{C}$-number wave functionals $\Psi[\phi(t_1)]$ and $\Phi[\phi(t_2)]$ in terms of the eigenkets of $\varphi(t, \vec{x})$ at times t_1 and t_2, respectively.

In flat space physics we typically seek to compute matrix elements between states which are true vacuum in the infinite past and future. This might seem problematic because no one has ever exhibited a normalizable energy eigenstate for an interacting, $D = 4$ dimensional quantum field theory. Of course it would be possible to build up perturbative corrections—which is all that is needed for finite order computations—the same as in quantum mechanics. However, for theories with a mass gap we can avoid this tedious and noncovariant exercise by taking $|\Psi\rangle$ and $|\Phi\rangle$ to be free vacuum, and then considering the limit in which t_1 goes to $-\infty$ and t_2 goes to $+\infty$. Up to a normalization factor, this limit projects out true vacuum in the weak operator sense [1]. Of course the most interesting theories have massless particles, which violate the assumption about a mass gap, but it is believed the procedure still gives correct inclusive rates and cross sections [2].

In cosmology we typically imagine that the Universe began with an initial singularity, and it is often our ignorance about what happens in the far future that is the chief reason for interest in the computation. The canonical operator formalism is of course the same, but its more useful functional integral representation is given by the Schwinger-Keldysh formalism [3–5]. The relation analogous to (1) is [6],

$$\langle\Psi|\bar{T}^*(B[\varphi])T^*(A[\varphi])|\Psi\rangle = \int [d\phi_+][d\phi_-] \times \delta[\phi_-(t_2) - \phi_+(t_2)] \times e^{i\int_{t_1}^{t_2} dt\{L[\phi_+(t)] - L[\phi_-(t)]\}} \times \Psi^*[\phi_-(t_1)]B[\phi_-] \times A[\phi_+]\Psi[\phi_+(t_1)]. \tag{2}$$

Here $|\Psi\rangle$ is the Heisenberg state whose $\mathbb{C}$-number wave functional in terms of the time t_1 eigenkets is $\Psi[\phi(t_1)]$. Like $\mathcal{O}[\varphi]$ in (1), the operators $A[\varphi]$ and $B[\varphi]$ are functionals of the operators $\varphi(t, \vec{x})$ for $t_1 < t < t_2$. As in (1), the T^* symbol stands for time-ordering, with any time derivatives taken outside; the $\bar{T}^*$ symbol stands for anti-time-ordering, again with time derivatives taken outside. The reason for the two $\mathbb{C}$-number integration variables $\phi_\pm(t, \vec{x})$ in (2) is that the functional integration over ϕ_+ evolves the system forward to time t_2, whereas the functional integration over ϕ_- carries it back to the initial time t_1.

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Expression (2) is well adapted to cosmological problems in which the Universe is released in a prepared state $|\Psi\rangle$ at some finite time t_1 and its subsequent evolution is studied through correlators. Unfortunately, we can no longer use infinite time evolution to transform the known free states into fully interacting ones. There have been attempts to achieve the same thing by including an additional evolution in Euclidean time [7–9]. However, the absence of a unique vacuum means that it is not clear what the fully interacting state should be [10]. Of special significance to this work is the fact that this is even true on de Sitter background for the massless, minimally coupled scalar [11].

Of course we are doing perturbation theory so the lowest order results can be obtained using the free vacuum. Certain higher order corrections show secular growth from the coherent superposition of interactions throughout the past light-cone, which is not affected by corrections to the initial state [12–19]. However, in many cases state corrections on the initial value surface are as important as 4-volume effects [20–22]. And even when a higher order correction is dominated by secular growth from a 4-volume effect, failure to include state corrections leads to a number of problems including:

(i) divergences when operators touch the initial value surface [13–16,18,20–23];
(ii) nonvanishing surface terms from partial integrations [17]; and
(iii) complicated collections of terms which redshift like inverse powers of the scale factor [13].

This paper concerns an example of the first and last problems above. Consider a massless, minimally coupled scalar with a $\lambda\varphi^4$ self-interaction on a nondynamical de Sitter background whose scale factor $a = e^{Ht}$ is normalized to be one on the initial value surface. The expectation value of the stress tensor has been computed at one- and two-loop orders in the presence of free Bunch-Davies vacuum [13]. With a slight change in the original renormalization scheme, the energy density and pressure are [24],

$$\rho = \frac{3H^2}{8\pi G} + \frac{\lambda H^4}{(4\pi)^4}\Big\{+2\ln^2(a) + \frac{13}{6}\ln(a) - \frac{43}{18} + \frac{\pi^2}{3} + \frac{8}{9a^3} - 2\sum_{n=2}^{\infty}\frac{(n+1)}{n^2 a^n}\Big\} + O(\lambda^2), \tag{3}$$

$$p = -\frac{3H^2}{8\pi G} + \frac{\lambda H^4}{(4\pi)^4}\Big\{-2\ln^2(a) - \frac{7}{2}\ln(a) + \frac{5}{3} - \frac{\pi^2}{3} - \frac{2}{3}\sum_{n=2}^{\infty}\frac{(n-3)(n+1)}{n^2 a^n}\Big\} + O(\lambda^2). \tag{4}$$

This model violates the classical stability condition $p/\rho \equiv w \geq -1$ through quantum effects, without any intrinsic instability. That holds great interest for cosmologists because the original data revealing the current phase of cosmic acceleration [25] showed a pronounced tendency to favor $w < -1$ [26]. That sparked an explosion of model-building [27], most of which could be immediately ruled out owing to instabilities [28]. Although the phenomenological interest in models which exhibit $w < -1$ has waned in the face of better data [29], understanding how a viable model can exhibit this behavior is still interesting.

In the case of (3) and (4) the secular growth (which is what causes $w < -1$) derives from inflationary particle production. More scalars increases the scalar field strength, driving it up the $\lambda\varphi^4$ potential and thereby increasing the vacuum energy.[1] This part of the result will persist for any initial state which is finitely excited from Bunch-Davies vacuum. That is not true of the exponentially falling terms,

$$\rho_{\text{falling}} = \frac{\lambda H^4}{(4\pi)^4}\Big\{-\frac{3}{2a^2} - 2\sum_{n=4}^{\infty}\frac{(n+1)}{n^2 a^n}\Big\}, \tag{5}$$

$$p_{\text{falling}} = \frac{\lambda H^4}{(4\pi)^4}\Big\{+\frac{1}{2a^2} - \frac{2}{3}\sum_{n=4}^{\infty}\frac{(n-3)(n+1)}{n^2 a^n}\Big\}. \tag{6}$$

Because they are separately conserved, diverge on the initial value surface, and fall off rapidly as one evolves to late times, it was conjectured that ρ_{falling} and p_{falling} could be absorbed into corrections to the initial state wave functional [13]. In this paper we will prove the conjecture by constructing the $\lambda\phi^2$ correction which completely absorbs (5) and (6). We will even explain the curious fact that they contain no $1/a^3$ term.

This paper consists of five sections. In Sec. II we specify the background geometry and the entire apparatus of perturbation theory, even though our own work does not require regularization, renormalization or even the quartic self-interaction. In Sec. III we compute the effect on the expectation value of the stress tensor of a general $\lambda\phi^2$ correction to the initial state wave functionals. The specific correction which absorbs (5) and (6) is worked out in Sec. IV. Our conclusions are given in Sec. V.

II. $\lambda\varphi^4$ THEORY ON DE SITTER

We work on the open conformal coordinate patch of de Sitter space, the invariant element for which is

$$ds^2 \equiv g_{\mu\nu}dx^\mu dx^\nu = a^2[-d\eta^2 + d\vec{x}\cdot d\vec{x}] \quad \text{with} \quad a \equiv -\frac{1}{H\eta} = e^{Ht}. \tag{7}$$

The Hubble constant is H and the conformal time η runs from $-\infty$ to 0. To facilitate dimensional regularization

[1] Because λ is a constant, whereas $\ln(a) = Ht$ grows with time, these secular corrections eventually become nonperturbatively strong. Starobinsky has developed a stochastic formalism for summing the series of leading logarithms [30–32].

(when necessary) we work in D spacetime dimensions, with the indices taking values $\mu,\ \nu = 0, 1, 2, \ldots, (D-1)$. As the name of the coordinate patch suggests, the metric is conformal to the flat space metric $\eta_{\mu\nu}$: $g_{\mu\nu} = a^2\eta_{\mu\nu}$. It is sometimes useful to distinguish the purely spatial parts of tensors with an overline, for example,

$$\langle\Omega|T_{\mu\nu}|\Omega\rangle \equiv a^2\delta^0_\mu\delta^0_\nu \times \rho + a^2\bar{\eta}_{\mu\nu} \times p. \tag{8}$$

The Lagrangian density is

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\varphi_0\partial_\nu\varphi_0 g^{\mu\nu}\sqrt{-g} - \frac{\xi_0}{2}\varphi_0^2 R\sqrt{-g} - \frac{\lambda_0}{4!}\varphi_0^4\sqrt{-g} - \frac{(D-2)\Lambda_0}{16\pi G}\sqrt{-g}. \tag{9}$$

Here φ_0 is the bare field, ξ_0 is the bare conformal coupling constant, λ_0 is the bare quartic coupling constant, and Λ_0 is the bare cosmological constant. The renormalized field φ is defined by field strength renormalization of the bare one as usual,

$$\varphi(x) \equiv \frac{1}{\sqrt{Z}}\varphi_0(x). \tag{10}$$

That brings the Lagrangian density to the form,

$$\mathcal{L} = -\frac{Z}{2}\partial_\mu\varphi\partial_\nu\varphi g^{\mu\nu}\sqrt{-g} - \frac{Z\xi_0}{2}\varphi^2 R\sqrt{-g} - \frac{Z^2\lambda_0}{4!}\varphi^4\sqrt{-g} - \frac{(D-2)\Lambda_0}{16\pi G}\sqrt{-g}. \tag{11}$$

The associated stress tensor is,

$$T_{\mu\nu} = Z\bigg[\delta^\rho_\mu\delta^\sigma_\nu - \frac{1}{2}g_{\mu\nu}g^{\rho\sigma}\bigg]\partial_\rho\varphi\partial_\sigma\varphi - \frac{Z^2\lambda_0}{4!}\varphi^4 g_{\mu\nu} + Z\xi_0\bigg[R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - D_\mu D_\nu + g_{\mu\nu}\Box\bigg]\varphi^2 - \frac{(D-2)\Lambda_0}{16\pi G}g_{\mu\nu}. \tag{12}$$

Conservation is straightforward to verify, as a strong operator equation, using the regulated scalar field equation,

$$T_{\mu\nu;}{}^{\nu} = \bigg[Z\Box\varphi - Z\xi_0 R\varphi - \frac{Z^2\lambda_0}{6}\varphi^3\bigg]\partial_\mu\varphi = 0. \tag{13}$$

Renormalization is accomplished by expressing the bare parameters in terms of the renormalized parameters and counter parameters,

$$Z \equiv 1 + \delta Z, \qquad Z^2\lambda_0 \equiv \lambda + \delta\lambda, \qquad Z\xi_0 \equiv 0 + \delta\xi, \qquad \Lambda_0 \equiv \frac{6H^2}{D-2} + \delta\Lambda. \tag{14}$$

Note that no mass counterterm is necessary because mass is multiplicatively renormalized in dimensional regularization. However, a conformal counterterm is necessary even if the renormalized conformal coupling is zero. The one- and two-loop counterterms were chosen as the following functions of $\epsilon \equiv 4 - D$,

$$\delta Z = -\frac{\lambda^2}{12(4\pi)^4}\bigg(\frac{4\pi}{H^2}\bigg)^{\epsilon}\frac{\Gamma^2(1-\frac{1}{2}\epsilon)}{(1-\frac{3}{2}\epsilon)(1-\epsilon)(1-\frac{3}{4}\epsilon)\epsilon} + O(\lambda^3), \tag{15}$$

$$\delta\lambda = \frac{3\lambda^2}{16\pi^2}\bigg(\frac{4\pi}{H^2}\bigg)^{(1/2)\epsilon}\frac{\Gamma(1-\frac{1}{2}\epsilon)}{(1-\epsilon)\epsilon} + O(\lambda^3), \tag{16}$$

$$\delta\xi = -\frac{\lambda}{192\pi^2}\bigg(\frac{4\pi}{H^2}\bigg)^{(1/2)\epsilon}\frac{\pi\cot(\frac{1}{2}\pi\epsilon)(1-\epsilon)\Gamma(1-\epsilon)}{(1-\frac{1}{3}\epsilon)(1-\frac{1}{4}\epsilon)\Gamma(1-\frac{1}{2}\epsilon)} + O(\lambda^2), \tag{17}$$

$$\delta\Lambda = \frac{8\pi GH^4}{D-2}\bigg\{\frac{3}{16\pi^2}\bigg(\frac{4\pi}{H^2}\bigg)^{(1/2)\epsilon} \times \frac{(1-\epsilon)(1-\frac{1}{2}\epsilon)(1-\frac{1}{3}\epsilon)\Gamma(1-\epsilon)}{(1-\frac{1}{4}\epsilon)\Gamma(1-\frac{1}{2}\epsilon)} - \frac{\lambda}{(4\pi)^4}\bigg(\frac{4\pi}{H^2}\bigg)^{\epsilon} \times \frac{[\pi\cot(\frac{1}{2}\pi\epsilon)\epsilon(1-\epsilon)\Gamma(1-\epsilon)]^2}{4\epsilon(1-\frac{1}{4}\epsilon)\Gamma^2(1-\frac{1}{2}\epsilon)} + O(\lambda^2)\bigg\}. \tag{18}$$

Note that a more complicated renormalization scheme involving a mass counterterm was employed in the original computation [13].

There are no normalizable de Sitter invariant state for the free massless, minimally coupled scalar [11]. We choose to preserve the symmetries of cosmology—homogeneity and isotropy—which is known as the ''E3'' vacuum [33]. It can be realized in terms of plane wave mode sums by making the spatial manifold T^{D-1}, rather than R^{D-1}, with coordinate radius H^{-1} in each direction, and then using the integral approximation with the lower limit cut off at $k = H$ [34–36]. The resulting free field expansion is

$$\varphi(\eta, \vec{x}) = \int\frac{d^{D-1}k}{(2\pi)^{D-1}}\theta(k-H)\{u(\eta,k)e^{i\vec{k}\cdot\vec{x}}\alpha(\vec{k}) + u^*(\eta,k)e^{-i\vec{k}\cdot\vec{x}}\alpha^\dagger(\vec{k})\}. \tag{19}$$

In this expression the creation and annihilation operators are canonically normalized,

$$[\alpha(\vec{k}), \alpha^\dagger(\vec{k}')] = (2\pi)^{D-1}\delta^{D-1}(\vec{k}-\vec{k}'), \tag{20}$$

and the mode functions are

$$u(\eta, k) = \sqrt{\frac{\pi}{4H}}a^{-((D-1)/2)}H^{(1)}_{((D-1)/2)}\bigg(\frac{k}{Ha}\bigg). \tag{21}$$

The mode functions take a particularly simple form in $D = 4$,

$$u(\eta, k)|_{D=4} = \frac{H}{\sqrt{2k^3}}\bigg[1 - \frac{ik}{Ha}\bigg]\exp\bigg[\frac{ik}{Ha}\bigg]. \tag{22}$$

Because time translation is not an invariance of cosmology there is no conserved energy, even at the free level. However, it is still the case that each mode of a free quantum field theory behaves as a harmonic oscillator, in this case with time dependent mass and frequency. Hence there will be a minimum energy Heisenberg state at any instant, although this state will not generally have the minimum energy before or after that instant. The Bunch-Davies vacuum is the state which was minimum energy in the distant past. It corresponds to the condition

$$\alpha(\vec{k})|\Omega\rangle = 0 \quad \forall\ \vec{k} \ni \|\vec{k}\| > H. \tag{23}$$

It is a straightforward exercise to solve for the state wave functional using expressions (19), (21), and (23),

$$\Omega[\phi(\eta_I)] = N\exp\bigg[-\frac{1}{2}\int\frac{d^{D-1}k}{(2\pi)^{D-1}}\theta(k-H)\tilde{\phi}^*(\eta_I,\vec{k}) \times \bigg[\frac{iu'(\eta_I,k)}{u(\eta_I,k)}\bigg]^*\tilde{\phi}(\eta_I,\vec{k})\bigg]. \tag{24}$$

Here $\eta_I \equiv -1/H$ is the initial time (corresponding to $t = 0$), N is a functional normalization factor and $\tilde{\phi}(\eta_I,\vec{k})$ is the spatial Fourier transform of field on the initial value surface,

$$\tilde{\phi}(\eta_I,\vec{k}) \equiv \int d^{D-1}x e^{-i\vec{k}\cdot\vec{x}}\phi(\eta_I,\vec{x}). \tag{25}$$

It remains only to give the Schwinger-Keldysh formalism, which can be read off from the fundamental relation (2). There are some excellent reviews of this subject [37] so we shall just summarize the results:

(i) because the same field operator $\varphi(\eta,\vec{x})$ is represented by two different functional integration variables $\phi_\pm(\eta,\vec{x})$, the endpoints of lines carry a $\pm$ polarity;
(ii) interaction vertices are either all $+$ or all $-$;
(iii) vertices with a $+$ polarity are the same as for the in-out formalism whereas those with a $-$ polarity are conjugated;
(iv) corrections to the initial states take the form of vertices on the initial value surface; and
(v) propagators can be $++$, $+-$, $-+$ or $--$.

The mode sums for the various propagators are

$$i\Delta_{++}(x;x') = \int\frac{d^{D-1}k}{(2\pi)^{D-1}}\theta(k-H)e^{i\vec{k}\cdot(\vec{x}-\vec{x}')} \times\{\theta(\eta-\eta')u(\eta,k)u^*(\eta',k) + \theta(\eta'-\eta)u^*(\eta,k)u(\eta',k)\}, \tag{26}$$

$$i\Delta_{+-}(x;x') = \int\frac{d^{D-1}k}{(2\pi)^{D-1}}\theta(k-H)e^{i\vec{k}\cdot(\vec{x}-\vec{x}')} \times u^*(\eta,k)u(\eta',k), \tag{27}$$

$$i\Delta_{-+}(x;x') = \int\frac{d^{D-1}k}{(2\pi)^{D-1}}\theta(k-H)e^{i\vec{k}\cdot(\vec{x}-\vec{x}')} \times u(\eta,k)u^*(\eta',k), \tag{28}$$

$$i\Delta_{--}(x;x') = \int\frac{d^{D-1}k}{(2\pi)^{D-1}}\theta(k-H)e^{i\vec{k}\cdot(\vec{x}-\vec{x}')} \times\{\theta(\eta-\eta')u^*(\eta,k)u(\eta',k) + \theta(\eta'-\eta)u(\eta,k)u^*(\eta',k)\}. \tag{29}$$

III. ORDER $\lambda\phi^2$ STATE CORRECTION

Consider a change in the initial state,

$$|\Omega\rangle \longrightarrow |\Psi\rangle \equiv |\Omega\rangle + |\Delta\Omega\rangle, \tag{30}$$

where $|\Omega\rangle$ is free, Bunch-Davies vacuum (24). Because the stress tensor is conserved (13) as a strong operator equation, its expectation value must be conserved in any state. Hence we have

$$D^\nu\langle\Omega|T_{\mu\nu}|\Omega\rangle = 0, \tag{31}$$

and also,

$$D^\nu\{\langle\Delta\Omega|T_{\mu\nu}|\Omega\rangle + \langle\Omega|T_{\mu\nu}|\Delta\Omega\rangle + \langle\Delta\Omega|T_{\mu\nu}|\Delta\Omega\rangle\} = 0. \tag{32}$$

Of course this was one reason for suspecting that the separately conserved parts (5) and (6) of the original result (3) and (4) could be absorbed into a change of the initial state.

The expectation value of the stress tensor must also be conserved order-by-order in perturbations theory. Of course we can expand the initial state correction in powers of λ,

$$|\Delta\Omega\rangle \equiv \sum_{n=1}^{\infty}\lambda^n|\Omega_n\rangle. \tag{33}$$

The purpose of this paper is to find the first-order correction $\lambda|\Omega_1\rangle$ which absorbs the exponentially redshifting terms (5) and (6),

$$\lambda\bigg[\delta^\rho_\mu\delta^\sigma_\nu - \frac{1}{2}g_{\mu\nu}g^{\rho\sigma}\bigg]\{\langle\Omega_1|\partial_\rho\varphi\partial_\sigma\varphi|\Omega\rangle + \langle\Omega|\partial_\rho\varphi\partial_\sigma\varphi|\Omega_1\rangle\} = -a^2\delta^0_\mu\delta^0_\nu\times\rho_{\rm falling} - a^2\bar{\eta}_{\mu\nu}\times p_{\rm falling}. \tag{34}$$

In order for perturbation theory to make sense, all corrections to the initial state must take the form of the free vacuum times powers of the fields. Because the first-order correction of interest to us must link up with the $\partial_\rho\varphi\partial_\sigma\varphi$ part of the stress tensor, we are obviously looking for a correction of the form $\lambda\phi^2$. The two fields in the state correction will each connect with fields in the stress tensor as in Fig. 1, so there will be no ultraviolet divergences and

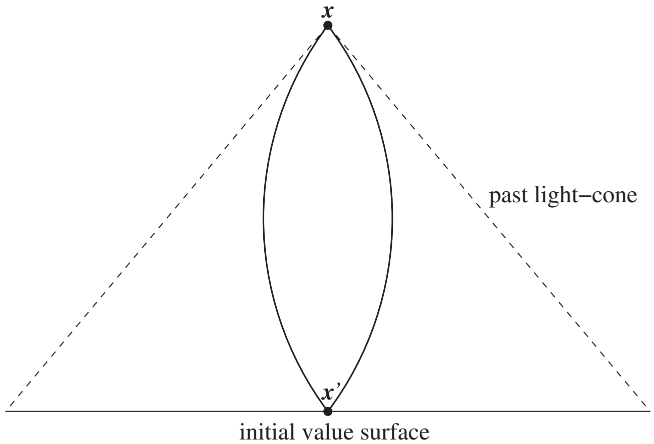

FIG. 1. Feynman diagram for the order λ initial state correction to the expectation value of the stress tensor at x^μ.

we can simplify the discussion by taking $D = 4$. The most general state correction with these properties, which also has the right dimensions and is consistent with homogeneity and isotropy, can be written as

$$\lambda\Omega_1[\phi_+(\eta_I)] = \Omega[\phi_+(\eta_I)] \times \frac{\lambda H}{2} \times \int \frac{d^3k}{(2\pi)^3} F\Big(\frac{k}{H}\Big) \tilde{\phi}^*_+(\eta_I, \vec{k}) \tilde{\phi}_+(\eta_I, \vec{k}), \tag{35}$$

$$\lambda\Omega^*_1[\phi_-(\eta_I)] = \Omega[\phi_-(\eta_I)] \times \frac{\lambda H}{2} \times \int \frac{d^3k}{(2\pi)^3} F^*\Big(\frac{k}{H}\Big) \tilde{\phi}^*_-(\eta_I, \vec{k}) \tilde{\phi}_-(\eta_I, \vec{k}). \tag{36}$$

The function $F(k/H)$ characterizes the state, and is at this stage arbitrary. We will determine it in the next section.

State corrections of the form (35) and (36) are treated just as interaction vertices in the Schwinger-Keldysh formalism, the only differences with the volume terms being that there is no factor of $\pm i$, that the "interactions" are restricted to the initial value surface, and that they are generally not local in position space. We obviously get distinct contributions from the ϕ_+ correction (35) and from the ϕ_- correction (36). The contribution from (35) involves two $++$ propagators between the observation point $(\eta, \vec{x})$ and the initial value surface. Because the observation comes after the initial time this contribution is

$$\Delta T^+_{\mu\nu} = \lambda H\Big[\delta^\rho_\mu \delta^\sigma_\nu - \frac{1}{2} g_{\mu\nu} g^{\rho\sigma}\Big] \int \frac{d^3k}{(2\pi)^3} F\Big(\frac{k}{H}\Big) \times [u^*(\eta_I, k)]^2 \partial_\rho [e^{i\vec{k}\cdot\vec{x}} u(\eta, k)] \partial_\sigma [e^{-i\vec{k}\cdot\vec{x}} u(\eta, k)]. \tag{37}$$

The contribution from (36) involves two $+-$ propagators and is

$$\Delta T^-_{\mu\nu} = \lambda H\Big[\delta^\rho_\mu \delta^\sigma_\nu - \frac{1}{2} g_{\mu\nu} g^{\rho\sigma}\Big] \int \frac{d^3k}{(2\pi)^3} F^*\Big(\frac{k}{H}\Big) \times [u(\eta_I, k)]^2 \partial_\rho [e^{i\vec{k}\cdot\vec{x}} u^*(\eta, k)] \partial_\sigma [e^{-i\vec{k}\cdot\vec{x}} u^*(\eta, k)]. \tag{38}$$

We obviously wish to solve for $F(k/H)$ to enforce the condition,

$$\Delta T^+_{\mu\nu} + \Delta T^-_{\mu\nu} = -a^2 \delta^0_\mu \delta^0_\nu \times \rho_{\text{falling}} - a^2 \bar{\eta}_{\mu\nu} \times p_{\text{falling}}. \tag{39}$$

We can eliminate the tensor algebra by distinguishing the temporal and spatial derivatives,

$$A \equiv \lambda H \int \frac{d^3k}{(2\pi)^3} F\Big(\frac{k}{H}\Big) [u^*(\eta_I, k)]^2 [\partial_0 u(\eta, k)]^2, \tag{40}$$

$$= \frac{\lambda H a^2}{2\pi^2} \int_0^\infty dk k^2 F\Big(\frac{k}{H}\Big) [u^*(\eta_I, k)]^2 \Big[\frac{1}{a} \partial_0 u(\eta, k)\Big]^2, \tag{41}$$

$$B \equiv \lambda H \int \frac{d^3k}{(2\pi)^3} F\Big(\frac{k}{H}\Big) [u^*(\eta_I, k)]^2 [k u(\eta, k)]^2, \tag{42}$$

$$= \frac{\lambda H a^2}{2\pi^2} \int_0^\infty dk k^2 F\Big(\frac{k}{H}\Big) [u^*(\eta_I, k)]^2 \Big[\frac{k}{a} u(\eta, k)\Big]^2. \tag{43}$$

Decomposing $\Delta T^+_{\mu\nu}$ into its induced energy density and pressure gives

$$\Delta\rho^+ = \frac{1}{a^2}[A + B], \tag{44}$$

$$= \frac{\lambda H}{4\pi^2} \int_0^\infty dk k^2 F\Big(\frac{k}{H}\Big) [u^*(\eta_I, k)]^2 \Big\{\Big[\frac{1}{a} \partial_0 u(\eta, k)\Big]^2 + \Big[\frac{k}{a} u(\eta, k)\Big]^2\Big\}, \tag{45}$$

$$\Delta p^+ = \frac{1}{2a^2}\Big[A - \frac{1}{3} B\Big], \tag{46}$$

$$= \frac{\lambda H}{4\pi^2} \int_0^\infty dk k^2 F\Big(\frac{k}{H}\Big) [u^*(\eta_I, k)]^2 \Big\{\Big[\frac{1}{a} \partial_0 u(\eta, k)\Big]^2 - \frac{1}{3}\Big[\frac{k}{a} u(\eta, k)\Big]^2\Big\}. \tag{47}$$

The $-$ contributions follow from complex conjugation, and we will be able to completely absorb the exponentially falling terms (5) and (6) if the function $F(k/H)$ can be chosen such that

$$\Delta\rho^+ + (\Delta\rho^+)^* = -\rho_{\text{falling}} \quad \text{and} \quad \Delta p^+ + (\Delta p^+)^* = -p_{\text{falling}}. \tag{48}$$

IV. RECONSTRUCTING $F(k/H)$

Because the stress tensor is conserved it suffices to enforce just the first condition of (48). The key to doing this is expanding out the exponentially falling terms in the curly brackets of expression (45). As we saw in expression (22) the mode function and its time derivative are simple in $D = 4$ spacetime dimensions,

$$u(\eta, k) = \frac{H}{\sqrt{2k^3}}\left[1 - \frac{ik}{Ha}\right]\exp\left[\frac{ik}{Ha}\right] \Rightarrow \partial_0 u(\eta, k) = \frac{H}{\sqrt{2k^3}}\left[-\frac{k^2}{Ha}\right]\exp\left[\frac{ik}{Ha}\right]. \tag{49}$$

It follows that the curly bracketed term of (45) is

$$\left[\frac{1}{a}\partial_0 u(\eta, k)\right]^2 + \left[\frac{k}{a}u(\eta, k)\right]^2 = \frac{H^4}{2k^3}\left(\frac{k}{Ha}\right)^2\left[1 - \frac{2ik}{Ha}\right]\exp\left[\frac{2ik}{Ha}\right], \tag{50}$$

$$= \frac{H^4}{2k^3}\left(\frac{k}{Ha}\right)^2\left\{1 - \sum_{n=2}^{\infty}\frac{(n-1)}{n!}\left(\frac{2ik}{Ha}\right)^n\right\}. \tag{51}$$

It is immediately apparent why there are no $1/a^3$ terms in $\rho_{\rm falling}$!

Substituting (51) into expression (45) and making the change of variable $k = Hx$ gives

$$\Delta\rho^+ = \frac{\lambda H^4}{16\pi^2}\int_0^\infty dx \frac{F(x)}{x^2}(1+ix)^2 e^{-2ix} \times \left\{\frac{1}{a^2} - \sum_{n=4}^{\infty}\frac{(n-3)}{(n-2)!}\frac{(2ix)^{n-2}}{a^n}\right\}. \tag{52}$$

Employing this relation in (48) and comparing with expression (5) for $\rho_{\rm falling}$ implies that we need the function $F(x)$ to obey the relations,

$$\int_0^\infty dx \frac{F(x)}{x^2}(1+ix)^2 e^{-2ix} + {\rm c.c.} = \frac{3}{32\pi^2}, \tag{53}$$

$$\int_0^\infty dx (ix)^{n-4}F(x)(1+ix)^2 e^{-2ix} + {\rm c.c.} = \frac{(n+1)(n-2)}{2^{n+1}n^2\pi^2}(n-4)! \quad \forall\ n \geq 4. \tag{54}$$

It is useful to eliminate the factors of i by defining real functions $\alpha(x)$ and $\beta(x)$ as

$$F(x)(1+ix)^2 e^{-2ix} \equiv \alpha(x) + i\beta(x). \tag{55}$$

Then conditions (53) and (54) can be rewritten as

$$\int_0^\infty dx x^{-2}\alpha(x) = \frac{3}{64\pi^2}, \tag{56}$$

$$\int_0^\infty dx x^{2m}\alpha(x) = \frac{(-1)^m(2m+5)(m+1)}{2^{2m+7}\pi^2(m+2)^2} \times (2m)! \quad \forall\ m \geq 0, \tag{57}$$

$$\int_0^\infty dx x^{2m+1}\beta(x) = \frac{(-1)^{m+1}(m+3)(2m+3)}{2^{2m+6}\pi^2(2m+5)^2} \times (2m+1)! \quad \forall\ m \geq 0. \tag{58}$$

Let us begin with (58). We can eliminate the factors of 2 and π by defining,

$$\beta(x) \equiv \frac{b(2x)}{32\pi^2}, \tag{59}$$

and making the change of variable $y = 2x$. This implies,

$$\int_0^\infty dy y^{2m+1}b(y) = (-1)^{m+1}(2m+1)! \times \frac{(2m+6)(2m+3)}{(2m+5)^2}, \tag{60}$$

$$= (-1)^{m+1}(2m+1)!\left\{1 - \frac{1}{2m+5} - \frac{2}{(2m+5)^2}\right\}. \tag{61}$$

Now suppose we have found a function $b_1(y)$ which obeys,

$$\int_0^\infty dy y^{2m+1}b_1(y) = (-1)^{m+1}(2m+1)!. \tag{62}$$

We can employ it to construct functions $b_2(y)$ and $b_3(y)$ which will add one and two factors of $1/(2m+5)$, respectively,

$$b_2(y) \equiv y^3\int_y^\infty dz \frac{b_1(z)}{z^4}, \tag{63}$$

$$b_3(y) \equiv y^3\int_y^\infty dz \frac{b_2(z)}{z^4} = y^3\int_y^\infty dz \frac{b_1(z)}{z^4}\ln\left(\frac{z}{y}\right). \tag{64}$$

Changing the order of integration shows that $b_2(y)$ has the desired property,

$$\int_0^\infty dy y^{2m+1}b_2(y) = \int_0^\infty dy y^{2m+4}\int_y^\infty dz \frac{b_1(z)}{z^4}, \tag{65}$$

$$= \int_0^\infty dz \frac{b_1(z)}{z^4}\int_0^z dy y^{2m+4}, \tag{66}$$

$$= \frac{(-1)^{m+1}(2m+1)!}{2m+5}. \tag{67}$$

Of course the same manipulations show that $b_3(y)$ has two powers of $1/(2m+5)$. So if we can find $b_1(y)$ to enforce (62) then we can construct $b_2(y)$ according to (63) and $b_3(y)$ according to (64) to give the function $\beta(x)$,

$$\beta(x) = \frac{1}{32\pi^2}[b_1(2x) - b_2(2x) - 2b_3(2x)]. \tag{68}$$

A solution for $b_1(y)$ seems to be just $\cos(y)$, provided we use a convergence factor to make sense of the integral,

$$\int_0^\infty dy e^{-\epsilon y} y^{2m+1}\cos(y) = \left(-\frac{\partial}{\partial\epsilon}\right)^{2m+1} \times \int_0^\infty dy e^{-\epsilon y}\cos(y), \quad (69)$$

$$= \left(-\frac{\partial}{\partial\epsilon}\right)^{2m+1}\frac{1}{2}\left\{\frac{1}{\epsilon - i} + \frac{1}{\epsilon + i}\right\}, \quad (70)$$

$$= (2m+1)!\frac{1}{2}\left\{\left(\frac{1}{\epsilon - i}\right)^{2m+2} + \left(\frac{1}{\epsilon + i}\right)^{2m+2}\right\}. \quad (71)$$

Taking the limit $\epsilon \to 0^+$ gives the desired relation,

$$\lim_{\epsilon\to 0^+}\int_0^\infty dy e^{-\epsilon y} y^{2m+1}\cos(y) = (-1)^{m+1}(2m+1)!. \quad (72)$$

With a few partial integrations we can even express the function $b_2(y)$ as a sine integral,

$$b_2(y) = y^3\left[\frac{\cos(y)}{3y^3} - \frac{\sin(y)}{6y^2} - \frac{\cos(y)}{6y} - \frac{1}{6}\mathrm{Si}(y)\right]. \quad (73)$$

No similar expression can be obtained for $b_3(y)$.

The same pattern is followed in finding a function $\alpha(x)$ which obeys (57). We first extract the factors of 2 and π,

$$\alpha(x) = \frac{a(2x)}{32\pi^2}, \quad (74)$$

which implies,

$$\int_0^\infty dy y^{2m} a(y) = (-1)^m(2m)!\frac{(2m+5)(m+1)}{2(m+2)^2}, \quad (75)$$

$$= (-1)^m(2m)!\left\{1 - \frac{1}{2(m+2)} - \frac{1}{2(m+2)^2}\right\}. \quad (76)$$

Hence we seek a function $a_1(y)$ with the property,

$$\int_0^\infty dy y^{2m} a_1(y) = (-1)^m(2m)!. \quad (77)$$

From $a_1(y)$ we can construct $a_2(y)$ and $a_3(y)$ to insert factors of $1/(2m+4)$ and $1/(2m+4)^2$, respectively,

$$a_2(y) = y^3\int_y^\infty dz\frac{a_1(z)}{z^4}, \quad (78)$$

$$a_3(y) = y^3\int_y^\infty dz\frac{a_2(z)}{z^4} = y^3\int_y^\infty dz\frac{a_1(z)}{z^4}\ln\left(\frac{z}{y}\right). \quad (79)$$

The function $\alpha(x)$ is

$$\alpha(x) = \frac{1}{32\pi^2}[a_1(2x) - a_2(2x) - 2a_3(2x)]. \quad (80)$$

It is straightforward to see that the desired solution for $a_1(y)$ is $\sin(y)$

$$\lim_{\epsilon\to 0^+}\int_0^\infty dy e^{-\epsilon y} y^{2m}\sin(y) = \lim_{\epsilon\to 0^+}\left(\frac{\partial}{\partial\epsilon}\right)^{2m}\frac{1}{2i}\left\{\frac{1}{\epsilon - i} - \frac{1}{\epsilon + i}\right\}, \quad (81)$$

$$= (-1)^m(2m)!. \quad (82)$$

The function $a_2(y)$ can be expressed as a cosine integral,

$$a_2(y) = y^3\left\{\frac{\sin(y)}{3y^3} + \frac{\cos(y)}{6y^2} - \frac{\sin(y)}{6y} + \frac{1}{6}\mathrm{Ci}(y)\right\}. \quad (83)$$

It remains to note that relation (56) follows from analytic continuation of (57) that we have just solved. First write (57) in a form that makes sense for arbitrary m,

$$\frac{(-1)^m(2m+5)(m+1)}{2^{2m+7}\pi^2(m+2)^2}\times(2m)! = \frac{e^{im\pi}(2m+5)(m+1)}{2^{2m+7}\pi^2(m+2)^2}\times\Gamma(2m+1). \quad (84)$$

Then set $m = -1 + \epsilon$ and take the limit as ϵ approaches zero,

$$\lim_{\epsilon\to 0}\frac{e^{i(-1+\epsilon)\pi}(3+2\epsilon)\epsilon}{2^{5+2\epsilon}\pi^2(1+\epsilon)^2}\times\Gamma(-1+2\epsilon) = \frac{3}{64\pi^2}. \quad (85)$$

Assembling the various results of this section gives the following final expression for the kernel function $F(k/H)$ of the state corrections (35) and (36),

$$F(x) = \frac{ie^{2ix}}{32\pi^2(1+ix)^2}\left\{e^{-2ix} - x^3\int_x^\infty\frac{dz}{z^4}e^{-2iz} - 2x^3\int_x^\infty\frac{dz}{z^4}\ln\left(\frac{z}{x}\right)e^{-2iz}\right\}. \quad (86)$$

V. CONCLUSIONS

We have verified the conjecture [13] that the exponentially redshifting parts (5) and (6) of the two-loop energy density and pressure of $\lambda\varphi^4$ theory on de Sitter background can be completely absorbed into a redefinition of the initial state. Our technique was to explicitly construct the corrections (35) and (36), with the kernel function $F(k/H)$ given in expression (86). It might be worried that we have only established the possibility of making this modification of the initial state, not the necessity. However, note that the parts of the free vacuum stress tensor we have absorbed are not only exponentially falling, they also diverge on the initial value surface. There is no alternative to absorbing these terms initially, and making all time derivatives of the stress tensor regular at least requires that the asymptotically large powers of $1/a$ should be canceled.

It seems at least possible to give our state correction an elegant interpretation. That would be to regard it as the finite remainder of the $\lambda\phi^2$ correction that must come from the conformal counterterm (17). The idea is that a nonzero conformal coupling $\delta\xi$ will change the mode functions

$u(\eta, k)$ from (21) to

$$u(\eta, k) \to \sqrt{\frac{\pi}{4H}} a^{-((D-1)/2)} H_\nu^{(1)}\left(\frac{k}{Ha}\right) \quad \text{with} \quad \nu^2 = \left(\frac{D-1}{2}\right)^2 - D(D-1)\delta\xi. \tag{87}$$

Because the conformal counterterm changes only the quadratic part of the Lagrangian density, the wave functional must still have the form (24) but with the new mode functions. Because $\delta\xi$ is of order λ one would expand the mode functions, keeping only the first-order correction for our current purposes.

The obvious problem with the interpretation we have just offered is, *what becomes of the divergent part of* $\delta\xi$? We think a possible answer is that there is also a correction of the form $\lambda\phi^4$ which can contribute if two of the fields are taken up by a coincident propagator and the other two connect to the stress tensor at x^μ. It then seems possible that the divergence in the coincident propagator cancels against the divergent part of $\delta\xi$, leaving the finite state correction we have found. More work needs to be done to check this possibility.

We are obviously just at the beginning of systematically studying and exploiting initial state corrections. Previous work has been done for free scalar fields by regarding the mass and the conformal coupling as interactions [38] but, as far as we know, this is the first result to be obtained for a genuinely interacting theory. One obvious application for initial state corrections is to cancel the surface terms that have been encountered when two loop diagrams are simplified by a partial integration [17]. Far from simply being a complication, these surface terms would actually lead, at higher orders, to new ultraviolet divergences which could not be canceled by the usual volume counterterms.[2] Another important application will be to make the evolution equations for quantum corrections to the mode function reliable at finite times so that momentum dependent but temporally constant changes in the normalization of mode functions can be reliably determined [22]. The possibility for observable tilts in the power spectrum of primordial perturbations has already been noted [22].

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