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Non-Gaussianity generated in the inflationary scenario with nonminimally coupled inflaton field

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We investigate a possibility to restrict various inflationary models with nonminimally coupled inflaton field ϕ by future measurement of nonlinear parameter $f_{\rm NL}$ which characterizes the non-Gaussianity in the cosmic microwave background temperature fluctuation. These models are related to the minimally coupled inflationary models by conformal transformation. We show that the curvature perturbation is invariant under the conformal transformation up to the second order. By using this property we show that nonminimal coupling $f(\phi)R$ does not produce large non-Gaussianity, and, in particular, the nonlinear parameter takes a narrow range $-0.022 < f_{\rm NL} < -0.007$ in the case of $f(\phi) = 1 + \xi\phi^2/m_P^2$ with a wide range of parameter ξ.

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I. INTRODUCTION

Although the standard big-bang theory can explain many observed features of the present Universe, it has serious conceptual problems such as the horizon problem and flatness problem. The inflationary scenario is proposed to solve these problems by assuming a sufficient amount of accelerated expansion (inflation) in the very early stage of the Universe [1–4]. It is soon realized that inflationary expansion can produce almost scale independent density perturbation and explain the origin of the structure [5–9]. Recent measurement of cosmic microwave background (CMB) temperature fluctuation by the WMAP satellite [10] strongly suggests the existence of the inflationary scenario. Various models of inflationary scenarios [11,12] have been proposed so far, but we are unable to determine which model is realized in our Universe. It is expected that future detection of non-Gaussian fluctuation as well as polarizations in CMB are used to discriminate various possible models.

Here we are interested in non-Gaussianity. Primordial non-Gaussianity in CMB is described in terms of the 3-point correlation function of Bardeen's curvature perturbations, $\Phi(k)$, in Fourier space:

$$\langle\Phi(\mathbf{k}_1)\Phi(\mathbf{k}_2)\Phi(\mathbf{k}_3)\rangle = (2\pi)^3\delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)F(k_1, k_2, k_3). \quad (1)$$

Different inflationary models predict different functional forms of F [13–15]. In this paper we consider only the local type of non-Gaussianity. The primordial curvature perturbation in position space is known to have the form

$$\Phi(x) = \phi_g(x) + f_{\rm NL}\phi_g^2(x), \quad (2)$$

where $\phi_g(x)$ is a Gaussian field within the context of inflationary scenario [16,17] and $f_{\rm NL}$ characterizes the amplitude of primordial non-Gaussianity. The latest constraint on $f_{\rm NL}$ from the WMAP 5-yr data is $f_{\rm NL} = 38 \pm 21$ (68% C.L.; see [18]).

The purpose of this paper is to investigate the possibility to restrict a certain class of inflationary scenarios by observing non-Gaussianity in CMB. The class we consider is inflationary models with inflaton field which couples nonminimally with background geometry. Many inflationary scenarios with nonminimally coupled inflaton field have been proposed [19–27]. Although a simple chaotic inflationary model with a minimally coupled inflaton field ϕ with a $\lambda\phi^4$ interaction is now rejected by the result of 5-yr WMAP, the same model with a nonminimal coupling $\xi\phi^2R$ does not need a fine-tuning for the self-coupling constant λ and has not been rejected. Since no symmetry except conformal symmetry is known to determine the nonminimal coupling constant ξ, there is no reason not to consider nonminimally coupled inflationary models, and it would be very useful to determine the coupling by observation. We will show that the nonlinear parameter $f_{\rm NL}$ characterizing non-Gaussianity will be restricted in a narrow range if there is a nonminimal coupling in inflaton field with Ricci curvature.

This paper is organized as follow. First we review general inflationary models with nonminimally coupled scalar field and the model is transformed to minimally coupled scalar field by conformal transformation in Sec. II. The frame where the scalar field couples with background curvature nonminimally is called the Jordan frame, and the frame where the field couples with curvature minimally is called the Einstein frame. We can make use of results derived in the Einstein frame in order to study the situation in the Jordan frame. If we consider inflationary scenarios with slowly rolling inflaton in the Jordan frame, the conformally related scenarios have also slowly rolling inflaton and predict very small primordial non-Gaussianity. Thus it is expected that the level of primordial non-Gaussianity will also be very small in our model. We will verify this expectation and use this fact to restrict the magnitude of the

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nonminimal coupling constant by observation. In Sec. III we prove that the curvature perturbations ζ in both frames coincide up to second order under the single field assumption. Using this fact we calculate $f_{\rm NL}$ in the Jordan frame. It is found that the possible range of $f_{\rm NL}$ is tightly constrained, and thus it would be possible to constrain the models with nonminimally coupled inflaton field by observing non-Gaussianity in CMB.

II. INFLATION MODELS WITH NONMINIMAL COUPLED INFLATON FIELD

Here we briefly review the recipe to compute inflationary observable in our model which will be used in the later sections.

The inflaton field model we consider is a real scalar field ϕ nonminimally coupled to gravity via the Ricci scalar $\mathcal{R}$. The action will be the following:

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}m_p^2 f(\phi)\mathcal{R} - \frac{1}{2}g^{ab}\partial_a\phi\partial_b\phi - V(\phi)\right], \tag{3}$$

where we use a coefficient of the Ricci scalar $f(\phi) = 1 + [\xi(\phi^2/m_p^2)]$, and potential $V(\phi) = \frac{\lambda}{4}(\phi^2 - v^2)^2$. Here $m_p \simeq 2.43 \times 10^{18}$ GeV is the reduced Plank mass.

It is well known that the Jordan frame is transformed to the Einstein frame by a conformal transformation $g^E_{ab} = \Omega g_{ab}$. In this case we take $\Omega = f(\phi)$ and we have

$$S = \int d^4x\sqrt{-g_E}\left[\frac{1}{2}m_p^2\mathcal{R}_E - \frac{1}{2}g_E^{ab}\partial_a\sigma\partial_b\sigma - V_E(\sigma)\right], \tag{4}$$

where $V_E(\sigma) = \frac{V(\phi)}{f(\phi)^2}$, and $\mathcal{R}_E$ is the Ricci scalar calculated from g^E_{ab}. A new scalar field σ is defined by

$$\left(\frac{d\sigma}{d\phi}\right)^2 \equiv \frac{1}{f(\phi)} + \frac{3}{2}m_p^2\frac{f(\phi)^2_{,\phi}}{f(\phi)^2}. \tag{5}$$

Note that the transformed scalar field σ in the Einstein frame is also slowly rolling when ϕ is slowly rolling in the Jordan frame.

We use spatially flat Friedmann-Robertson-Walker (FRW) metric as our background:

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j. \tag{6}$$

Then the background equations in the Jordan frame may be written as

$$H^2 = \frac{\dot\phi^2}{6f(\phi)m_p^2} + \frac{V(\phi)}{3f(\phi)m_p^2} - H\dot\phi\frac{f(\phi)_{,\phi}}{f(\phi)} \tag{7}$$

$$\frac{\ddot a}{a} = -\frac{1}{2}\frac{f(\phi)_{,\phi}}{f(\phi)}\ddot\phi - \left(\frac{f(\phi)_{,\phi\phi}}{2f(\phi)} + \frac{1}{3m_p f(\phi)}\right)\dot\phi^2 - H\dot\phi\frac{f(\phi)_{,\phi}}{f(\phi)} + \frac{V(\phi)}{3m_p^2 f(\phi)} \tag{8}$$

$$\ddot\phi\left[1 + \frac{3}{2}m_p^2\frac{f^2_{,\phi}}{f}\right] + 3H\dot\phi\left[1 + \frac{3}{2}m_p^2\frac{f^2_{,\phi}}{f}\right] + V_{,\phi} + \frac{f_{,\phi}}{2f}(\dot\phi^2 - 4V) + \frac{3}{2}m_p^2\frac{f_{,\phi}}{f}\dot\phi^2 f_{,\phi} = 0. \tag{9}$$

We can also calculate the background equations in the Einstein frame

$$H_E = \frac{\hat\sigma}{6m_p^2} + \frac{V_E}{3m_p^2} \tag{10}$$

$$\frac{\hat{\hat a}_E}{a_E} = -\frac{\hat\sigma^2}{3m_p^2} + \frac{V(\sigma)_E}{3m_p^2} \tag{11}$$

$$\hat{\hat\sigma} + 3H_E\hat\sigma + V_{,\sigma} = 0, \tag{12}$$

where

$$a_E = \sqrt{\Omega}a, \qquad dt_E = \sqrt{\Omega}dt \tag{13}$$

and $\hat{} = \frac{d}{dt_E}$, $\hat{\hat{}} = \frac{d^2}{dt_E^2}$.

As mentioned before, if $\dot\phi \sim 0$ then $\hat\sigma \sim 0$ in both frames. In fact, the condition of accelerated expansion of the universe in the Jordan frame and the Einstein frame is

$$\frac{\ddot a}{a} \sim \frac{V(\phi)}{3m_p^2 f(\phi)} > 0 \tag{14}$$

$$\frac{\hat{\hat a}_E}{a_E} \sim \frac{V(\sigma)_E}{3m_p^2} = \frac{V(\phi)}{3m_p^2 f(\phi)^2} > 0. \tag{15}$$

The difference of both frames is only the factor of $\frac{1}{f(\phi)}$.

It is convenient to study our model in the Einstein frame because inflationary models with a single minimally coupled inflaton field are well studied and we can make use of these results.

We assume that the e-folding of inflationary expansion is sufficient to solve puzzles in the standard big-bang theory:

$$N = \int dtH = \frac{1}{8v^2 + 8m_p^2}\left[(2v^2 + 2m_p^2)\log\left(\frac{\phi_i^2\xi + m_p^2}{\phi_f^2\xi + m_p^2}\right) - 2v^2\log\left(\frac{\phi_i}{\phi_f}\right) + (6\xi + 1)(\phi_i^2 - \phi_f^2)\right] > 60 \tag{16}$$

in the Jordan frame. We assume that $\phi_f \sim v$, and we derive the condition of ϕ_i to solve cosmological puzzles.

III. EQUALITY OF CURVATURE PERTURBATION BETWEEN JORDAN AND EINSTEIN FRAMES

Now we prove that the curvature perturbations calculated in both frames coincide up to second order. This property will be used to calculate $f_{\rm NL}$ in the next section.

The metric perturbation of the spatially flat FRW metric and the scalar field perturbation up to second order can be written as [28,29]

$$g_{00} = -(1 + 2A^{(1)} + A^{(2)}) \tag{17}$$

$$g_{0i} = B_i^{(1)} + \frac{1}{2} B_i^{(2)} \tag{18}$$

$$g_{ij} = a^2(1 - 2D^{(1)} - D^{(2)})\delta_{ij} + a^2\left(C_{ij}^{(1)} + \frac{1}{2} C_{ij}^{(2)}\right) \tag{19}$$

$$\phi = \phi_0 + \phi^{(1)} + \frac{1}{2}\phi^{(2)}. \tag{20}$$

In the following we will only consider scalar perturbation.

It is well known that the first and second order gauge-invariant scalar perturbations ζ generated by quantum noise during the inflation may be defined as follows [30,31]:

$$\zeta^{(1)} = D^{(1)} + H\frac{\phi^{(1)}}{\dot{\phi}} \tag{21}$$

$$\zeta^{(2)} = D^{(2)} + H\frac{\phi^{(2)}}{\dot{\phi}^{(0)}} - 2\frac{\phi^{(1)}}{\dot{\phi}^{(0)}}(\dot{D}^{(1)} + 2HD^{(1)}) - 2H\frac{\phi^{(1)}}{\dot{\phi}^{(0)}} + \left(\frac{\phi^{(1)}}{\dot{\phi}^{(0)}}\right)^2\left(H\frac{\ddot{\phi}^{(0)}}{\dot{\phi}^{(0)}} - \dot{H} - 2H^2\right). \tag{22}$$

In inflation theory, we consider first-order perturbation to be Gaussian perturbation, because it behaves as a free-field. The frame independence of the first-order curvature perturbation is known [32,33].

Thus the non-Gaussianity is generated by second order perturbation and higher order. In this paper we only consider the second order perturbation to calculate the nonlinear parameter $f_{\rm NL}$ from the 3-point function of ζ.

We now prove that ζ defined in the Jordan frame coincides with ζ_E defined in the Einstein frame up to second order. Then we can compute the primordial power spectrum P_ζ and nonlinear parameter $f_{\rm NL}$ generated by the primordial 3-point function in the Jordan frame by using corresponding quantities in the Einstein frame.

For this purpose we first note the transformation of the metric perturbations and scalar field between the Jordan frame and the Einstein frame:

$$\Omega = \Omega^{(0)} + \Omega^{(1)} + \Omega^{(2)} \tag{23}$$

$$\Omega^{(1)} = \frac{d\Omega}{d\phi}\phi^{(1)} \tag{24}$$

$$\Omega^{(2)} = \frac{1}{2}\left(\frac{d^2\Omega}{d\phi^2}\phi^{(1)^2} + \frac{d\Omega}{d\phi}\phi^{(2)}\right) \tag{25}$$

$$H_E = \frac{1}{\sqrt{\Omega}}\left(H + \frac{\dot{\Omega}}{2\Omega}\right) \tag{26}$$

$$D_E^{(1)} = D^{(1)} - \frac{1}{2}\frac{\Omega^{(1)}}{\Omega^{(0)}} \tag{27}$$

$$D_E^{(2)} = D^{(2)} + \frac{2\Omega^{(1)}D^{(1)}}{\Omega^{(0)}} - \frac{\Omega^{(2)}}{\Omega^{(0)}} \tag{28}$$

$$\sigma^{(1)} = \frac{d\sigma}{d\phi}\phi^{(1)} \tag{29}$$

$$\sigma^{(2)} = \frac{d\sigma}{d\phi}\phi^{(2)} + \frac{d^2\sigma}{d\phi^2}\phi^{(1)^2} \tag{30}$$

$$\dot{\sigma}^{(1)} = \dot{\phi}\frac{d^2\sigma}{d\phi^2}\phi^{(1)} + \frac{d\sigma}{d\phi}\dot{\phi}^{(1)}. \tag{31}$$

Using these transformations, it is straightforward to show the equality of $\zeta^{(2)}$ in both frames as follows:

$$\begin{aligned}
\zeta_E^{(2)} &= D_E^{(2)} + H_E\frac{\sigma^{(2)}}{\hat{\sigma}^{(0)}} - 2\frac{\sigma^{(1)}}{\hat{\sigma}^{(0)}}(\hat{D}_E^{(1)} + 2H_E D_E^{(1)}) - 2H_E\frac{\sigma^{(1)}}{\hat{\sigma}^{(0)}} + \left(\frac{\sigma^{(1)}}{\hat{\sigma}^{(0)}}\right)^2\left(H_E\frac{\hat{\hat{\sigma}}^{(0)}}{\hat{\sigma}^{(0)}} - \hat{H}_E - 2H_E^2\right)\\
&= D_E^{(2)} + H_E\frac{\sigma^{(2)}}{\hat{\sigma}} - 2\zeta^{(1)^2}{}_E - \frac{\sigma^{(1)}}{\hat{\sigma}}\hat{\zeta}_E^{(1)} + 2D_E^{(1)^2} - \frac{\sigma^{(1)}}{\hat{\sigma}}\hat{D}_E^{(1)} - H_E\frac{\hat{\sigma}^{(1)}}{\hat{\sigma}}\frac{\sigma^{(1)}}{\hat{\sigma}}\\
&= D_E^{(2)} + H_E\sqrt{\Omega}\frac{\phi^{(2)}}{\dot{\phi}} - 2\zeta^{(1)^2} - \frac{\phi^{(1)}}{\dot{\phi}}\dot{\zeta}^{(1)} + 2D_E^{(1)^2} - \frac{\phi^{(1)}}{\dot{\phi}}\dot{D}_E^{(1)} - H_E\sqrt{\Omega}\frac{\dot{\phi}^{(1)}}{\dot{\phi}}\frac{\phi^{(1)}}{\dot{\phi}}\\
&= D^{(2)} + H\frac{\phi^{(2)}}{\dot{\phi}^{(0)}} - 2\frac{\phi^{(1)}}{\dot{\phi}^{(0)}}(\dot{D}^{(1)} + 2HD^{(1)}) - 2H\frac{\phi^{(1)}}{\dot{\phi}^{(0)}} + \left(\frac{\phi^{(1)}}{\dot{\phi}^{(0)}}\right)^2\left(H\frac{\ddot{\phi}^{(0)}}{\dot{\phi}^{(0)}} - \dot{H} - 2H^2\right) = \zeta^{(2)}.
\end{aligned} \tag{32}$$

IV. CONSTRAINT ON $f_{\rm NL}$

Since we have shown equality of ζ between both frames, we can make use of the results in the Einstein frame to calculate $f_{\rm NL}$. The non-Gaussianity in the Jordan frame has been studied [34] where it has been shown that the non-Gaussianity becomes very small with the slowly rolling inflaton, but where no attempt has been made to restrict the nonminimal coupling parameter. Although a part of our conclusion that the non-Gaussianity is very small is the same, we will show it using the result of the Einstein frame and will obtain a very severe constraint on the nonminimal parameter.

The following results for spectral index n_s and nonlinear parameter $f_{\rm NL}$ are known in the inflationary model with a single minimally coupled inflaton field [35,36]

$$n_s = 1 - 6\epsilon + 2\eta \tag{33}$$

$$f_{\rm NL} = \frac{5}{6}(\eta - \epsilon(3 - g(k))), \tag{34}$$

where ϵ, η are slow-roll parameters, and $g(k)$ satisfies $0 \le g(k) \le \frac{5}{6}$ which is a function of the shape of the triangle made by 3 momentums k_i which goes to zero when two sides become much larger than the third and becomes $\frac{5}{6}$ when the k_i's form an equilateral triangle. The slow-roll parameter is defined in the Einstein frame:

$$\epsilon = \frac{1}{2}m_p^2\left(\frac{V_{E,\phi}}{V_E}\right)^2\left(\frac{d\sigma}{d\phi}\right)^{-2} \tag{35}$$

$$\eta = m_p^2\left[\frac{V_{E,\phi\phi}}{V_E}\left(\frac{d\sigma}{d\phi}\right)^{-2} - \frac{V_{E,\phi}}{V_E}\left(\frac{d\sigma}{d\phi}\right)^{-3}\left(\frac{d^2\sigma}{d\phi^2}\right)\right]. \tag{36}$$

These slow-roll parameters are constant and small in the Einstein frame if we assume so in the Jordan frame. Under these conditions, $\epsilon(\phi) \sim \epsilon(\phi_i)$ and $\eta(\phi) \sim \eta(\phi_i)$. We write $\phi_i \rightarrow \phi$ to simplify the notation bellow.

We can divide the situation into two cases.

(i) *$\phi < v$ and $\phi \rightarrow 0$.*—This case contains a new inflationary scenario, and the slow-roll parameters take the following limits:

$$\epsilon \rightarrow 0, \qquad \eta \rightarrow -\frac{4(v^2\xi + m_p^2)}{v^2}. \tag{37}$$

Note that the condition $\phi \rightarrow 0$ is necessary to have sufficient e-folding N. Thus we have

$$n_s = 1 - 8\left(\xi + \frac{m_p^2}{v^2}\right). \tag{38}$$

Since the value of n_s is observed to be $n_s = 0.960 \pm 0.14$ by 5-yr WMAP and $m_p/v \sim 10^3$ if we take the grand unified theory (GUT) scale as our energy scale, the value of ξ is $\xi \sim -10^6$. If we adopt a lower energy scale for v, we would get a larger negative value for ξ.

(ii) *$v < \phi$.*—In this case it is convenient to define $\psi = (\phi/m_P)^2\xi$ and $\chi = (v/m_P)^2\xi$. The slow-roll parameters are expressed in terms of ψ and χ from Eqs. (35) and (36) and simplified for $\xi \ge 10$ as follows:

$$\epsilon \sim \frac{4}{3\psi^2}(2\chi + 1) \tag{39}$$

$$\eta \sim -\frac{4}{3\psi^2}(\psi - 1)(\chi + 1). \tag{40}$$

If we assume that the scalar field ϕ is an electroweak Higgs field such as [25,26], then v is about 262.2 GeV and we can totally neglect χ as far as $\xi \ll (m_p/v)^2 \sim 10^{34}$. Then we have

$$n_s \sim 1 - \frac{8}{3\psi^2}(\psi + 2). \tag{41}$$

This can be solved for ψ for positive ξ as

$$\psi = -\frac{4\sqrt{4 - 3n_s} + 4}{3n_s - 3}. \tag{42}$$

This shows that $\psi = (\frac{\phi}{m_P})^2\xi \sim$ const for $\xi \ge 10$.

In fact we evaluate (35) and (36) numerically and plugged the results into the expression (33) for n_s to have χ as a function of ξ. The result is shown in Fig. 1.

The figure shows that $\psi \sim$ const is a very good approximation for $\xi \ge 10$. Thus we adopt this approximation to evaluate e-folding N. When $\xi > 10$, e-folding N is

$$N \sim \frac{1}{4}\log(\psi + 1) + \psi > 60. \tag{43}$$

It turns out that this condition is satisfied for the observed $n_s = 0.960 \pm 0.14$ as shown in Fig. 2 and gives no extra constraint on ξ.

Next we calculate $f_{\rm NL}$ using (34) under the same condition $\psi \sim$ const. We find

$$f_{\rm NL} \sim -\frac{10}{9\psi^2}\{\psi + (2 - g(k))\} \qquad (\xi > 10). \tag{44}$$

Thus the value of $f_{\rm NL}$ depends only on n_s and $g(k)$ for $\xi > 10$. In the Jordan frame $f_{\rm NL}$ takes a minimum value for $n_s = 0.946$ and $g(k) = 0$ and a maximum value for $n_s = 0.974$, $g(k) = \frac{5}{6}f_{\rm NL}$. Thus the possible range for $f_{\rm NL}$ becomes as follows:

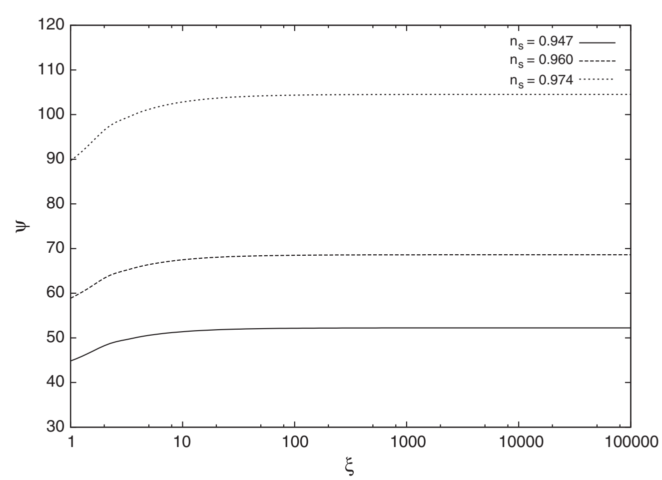

FIG. 1. ψ become constant when $\xi > 10$.

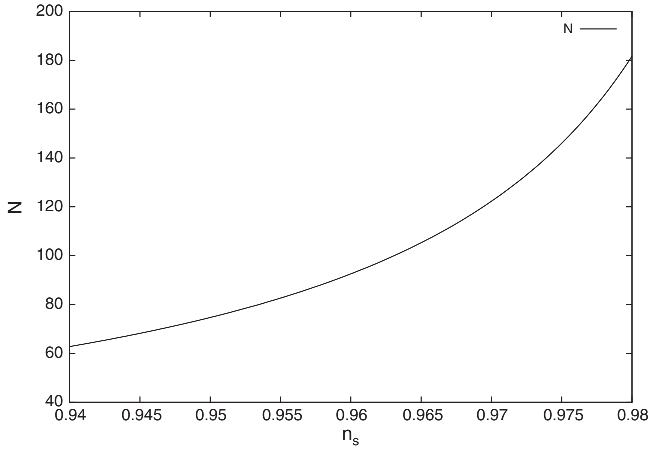

FIG. 2. The e-folding N is bigger than 60 when $n_s = 0.96 \pm 0.14$, $\xi > 10$.

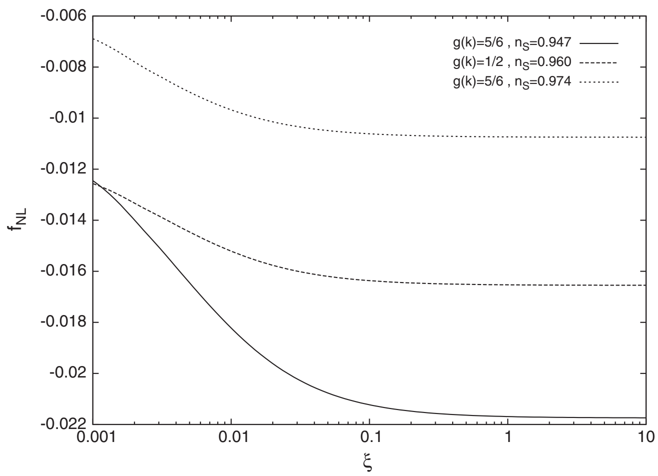

FIG. 3. Numerical result of $f_{\rm NL}$ for $v \sim 0$. $f_{\rm NL}$ becomes constant when $\xi > 0.1$.

$$-0.022 < f_{\rm NL} < -0.011.$$

We evaluate (34) numerically to have $f_{\rm NL}$ as a function of ξ without making the approximation. The result is shown in Fig. 3.

This figure shows that $f_{\rm NL}$ becomes constant, and the range of order of $f_{\rm NL}$ becomes

$$-0.022 < f_{\rm NL} < -0.007$$

for arbitrary ξ. Thus if we measure the 3-point function and obtain the nonlinear parameter $f_{\rm NL}$ outside of the above range for a particular combination of 3 momentums, then

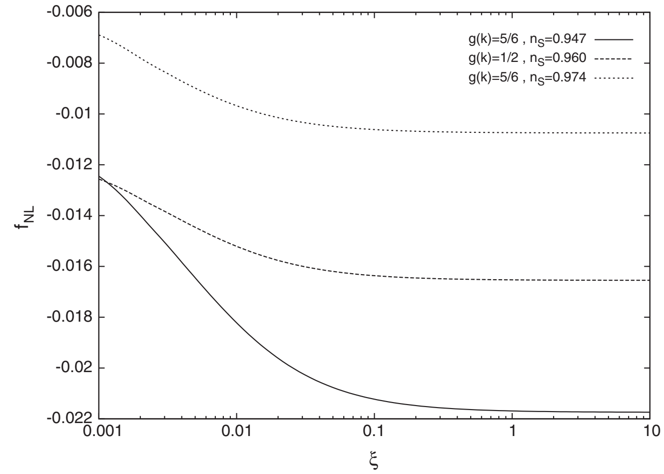

FIG. 4. Numerical result of $f_{\rm NL}$ for $v \sim 5.0 \times 10^{15}$ GeV. This result is rarely different from Fig. 3.

we can reject inflationary models with a nonminimally coupled inflaton field. Finally we note that we will get almost the same result even if the order of v is about GUT scale $v \sim 5.0 \times 10^{15}$ GeV. This can be seen in Fig. 4.

V. CONCLUSION

We have investigated a possibility to constrain inflationary scenarios with a nonminimally coupled inflaton field by measuring the nonlinear parameter $f_{\rm NL}$ which characterizes the non-Gaussianity of CMB fluctuation.

By using the equality of $f_{\rm NL}$ between the Jordan and the Einstein frame and the fact that $f_{\rm NL}$ is very small of the order of 10^{-2} in the Einstein frame, we can conclude that any theory with nonminimal coupling of the form $f(\phi)R$ in action predicts small nonlinear parameter $f_{\rm NL}$.

Furthermore, we show that inflationary scenarios with a nonminimal coupling such as $\xi R \phi^2$ predict a narrow range of nonlinear parameter $-0.022 < f_{\rm NL} < -0.007$ for arbitrary values of ξ. Thus if the value of $f_{\rm NL}$ is observed outside this range, any theory with this type of coupling is rejected.

The measurement of $f_{\rm NL}$ formed from the primordial 3-point function is very difficult, because the second order perturbation is generated not only by inflationary scenarios but by a secondary source [37–42]. We must extract only the primordial perturbation from the measured second order perturbation.

[1] K. Sato, Phys. Lett. B **99**, 66 (1981).
[2] A. H. Guth, Phys. Rev. D **23**, 347 (1981).
[3] A. D. Linde, Phys. Lett. B **108**, 389 (1982).
[4] A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. **48**, 1220 (1982).
[5] V. F. Mukhanov and G. V. Chibisov, Pis'ma Zh. Eksp.

Teor. Fiz. **33**, 544 (1981) [JETP Lett. **33**, 532 (1981)].
[6] A. H. Guth and S. Y. Pi, Phys. Rev. Lett. **49**, 1110 (1982).
[7] S. Hawking, Phys. Lett. B **117**, 175 (1982).
[8] A. D. Linde, Phys. Lett. B **116**, 335 (1982).
[9] A. Starobinsky, Phys. Lett. B **115**, 295 (1982).
[10] E. K. *et al.* (WMAP Collaboration), Astrophys. J. Suppl. Ser. **180**, 330 (2009).
[11] D. H. Lyth and A. R. Liddle, *The Primordial Density Perturbation* (Cambridge University Press, Cambridge, England, 2009).
[12] S. Weinberg, *Cosmology* (Oxford University Press, New York, 2008).
[13] D. Babich, P. Creminelli, and M. Zaldarriaga, J. Cosmol. Astropart. Phys. 08 (2004) 009.
[14] E. Komatsu, D. N. Spergel, and B. D. Wandelt, Astrophys. J. **634**, 14 (2005).
[15] D. Babich, Phys. Rev. D **72**, 043003 (2005).
[16] E. Komatsu and D. N. Spergel, Phys. Rev. D **63**, 063002 (2001).
[17] E. Komatsu, arXiv:astro-ph/0206039v1.
[18] K. M. Smith, L. Senatore, and M. Zaldarriaga, J. Cosmol. Astropart. Phys. 09 (2009) 006.
[19] F. S. Accetta, D. J. Zoller, and M. S. Turner, Phys. Rev. D **31**, 3046 (1985).
[20] T. Futamase and K. I. Maeda, Phys. Rev. D **39**, 399 (1989).
[21] R. Fakir and W. G. Unruh, Phys. Rev. D **41**, 1783 (1990).
[22] E. Komatsu and T. Futamase, Phys. Rev. D **59**, 064029 (1999).
[23] T. Futamase and M. Tanaka, Phys. Rev. D **60**, 063511 (1999).
[24] M. V. Libanov, V. A. Rubakov, and P. G. Tinyakov, Phys. Lett. B **442**, 63 (1998).
[25] F. L. Bezrukov and M. Shaposhnikov, Phys. Lett. B **659**, 703 (2008).
[26] A. D. Simone, M. P. Hertzberg, and F. Wilczek, Phys. Lett. B **678**, 1 (2009).
[27] A. O. Barvinsky *et al.*, arXiv:0904.1698.
[28] N. Bartolo, E. Komatsu, S. Matarrese, and A. Riotto, Phys. Rep. **402**, 103 (2004).
[29] S. Matarrese, S. Mollerach, and M. Bruni, Phys. Rev. D **58**, 043504 (1998).
[30] D. H. Lyth and D. Wands, Phys. Rev. D **68**, 103515 (2003).
[31] K. A. Malik and D. Wands, Classical Quantum Gravity **21**, L65 (2004).
[32] M. Makino and M. Sasaki, Prog. Theor. Phys. **86**, 103 (1991).
[33] V. F. Mukhanov, H. A. Feldman, and R. H. Brandenberger, Phys. Rep. **215**, 203 (1992).
[34] S. Koh, S. P. Kim, and D. J. Song, Phys. Rev. D **71**, 123511 (2005).
[35] J. Maldacena, J. High Energy Phys. 05 (2003) 013.
[36] P. Creminelli and M. Zaldarriaga, J. Cosmol. Astropart. Phys. 10 (2004) 006.
[37] D. Nitta, E. Komatsu, N. Bartolo, S. Matarrese, and A. Riotto, J. Cosmol. Astropart. Phys. 05 (2009) 014.
[38] N. Bartolo and A. Riotto, arXiv:0811.4584v3.
[39] N. Bartolo, S. Matarrese, and A. Riotto, Phys. Rev. Lett. **93**, 231301 (2004).
[40] D. Hanson, K. M. Smith, A. Challinor, and M. Liguori, Phys. Rev. D **80**, 083004 (2009).
[41] A. Mangilli and L. Verde, Phys. Rev. D **80**, 123007 (2009).
[42] R. Khatri and B. D. Wandelt, Phys. Rev. D **79**, 023501 (2009).