Threshold corrections to the radiative breaking of electroweak symmetry and neutralino dark matter in supersymmetric seesaw model

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We study the radiative electroweak symmetry breaking and the relic abundance of neutralino dark matter in the supersymmetric type I seesaw model. In this model, there exist threshold corrections to Higgs bilinear terms coming from heavy singlet sneutrino loops, which make the soft supersymmetry breaking (SSB) mass for up-type Higgs shift at the seesaw scale and thus a minimization condition for the Higgs potential is affected. We show that the required fine-tuning between the Higgsino mass parameter μ and the SSB mass for up-type Higgs may be reduced at the electroweak scale, due to the threshold corrections. We also present how the parameter μ depends on the SSB B-parameter for heavy singlet sneutrinos. Since the property of neutralino dark matter is quite sensitive to the size of μ, we discuss how the relic abundance of neutralino dark matter is affected by the SSB B-parameter. Taking the SSB B-parameter of order of a few hundreds TeV, the required relic abundance of neutralino dark matter can be correctly achieved. In this case, dark matter is a mixture of bino and Higgsino, under the condition that gaugino masses are universal at the grand unification scale.

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I. INTRODUCTION

A supersymmetric (SUSY) seesaw model is a SUSY extension of the seesaw model [1,2] which naturally explains small masses of neutrinos and stabilizes the hierarchy between the electroweak scale and some other high scale without severe fine-tuning, if the mass spectrum of superpartners is less than the TeV scale as well. In the SUSY type I seesaw model, we introduce not only heavy right-handed (RH) Majorana neutrinos but also their superpartner called sneutrinos which are standard model gauge singlet. This leads us to anticipate that some predictions of the minimal supersymmetric standard model (MSSM) can be deviated due to the contributions associated with the heavy RH neutrinos and their superpartners, and new phenomena absent in MSSM may occur in the SUSY type I seesaw model. In this regard, there have been attempts to study lepton flavor violation and neutrino masses in the SUSY type I seesaw model [3–5]. On the other hand, the gauge singlet RH neutrino superfield may affect the Higgs sector as investigated in Ref. [6], where they have shown that there is a sizable negative loop contribution to the mass of the lightest Higgs field in the split-SUSY scenario at the price of giving up the naturalness in supersymmetry.

In this study, we revisit the issue as to how the Higgs sector can be affected by heavy singlet sneutrinos while keeping the naturalness in supersymmetry. It is well known that the lightest CP-even Higgs mass in the MSSM can get large one-loop corrections which increase with the top quark and squark masses [7–10]. The current experimental bound on the lightest CP-even Higgs mass, $m_h \gtrsim 114$ GeV, demands the top squark mass to be larger than 500 GeV [11], which in turn leads to a fairly large correction to the soft supersymmetry breaking (SSB) mass for the up-type Higgs $m^2_{H_2}$. In the MSSM, electroweak symmetry can be broken due to the large logarithmic correction to $m^2_{H_2}$ [12–16]. However, as is known, we need rather large fine-tuning between the Higgsino mass parameter μ and the SSB mass $m^2_{H_2}$ to achieve the Z-boson mass at the electroweak scale through a minimization condition for the Higgs potential of the MSSM. In this study, we show that there exist some new contributions generated from the loops mediated by the heavy singlet sneutrino sector to the SSB mass $m^2_{H_2}$ and the Higgsino mass parameter μ in the SUSY type I seesaw model. The new contributions are given in terms of SSB parameters B_N and SSB mass term for the singlet sneutrino $m^2_{\tilde{N}}$ at the seesaw scale.

Integrating out the singlet neutrino superfield below the seesaw scale, the SUSY type I seesaw becomes equivalent to the MSSM but those new contributions are taken to be threshold corrections to the Higgs bilinear terms. As will be discussed, those threshold corrections can lower the sizes of $m^2_{H_2}$ and μ at the electroweak scale and thus the fine-tuning may be reduced. This means that the fine-tuning required for the radiative electroweak symmetry breaking can be shifted to tuning the size of B_N at the seesaw scale. In this paper, we investigate how the sizes of $m^2_{H_2}$ and μ at the electroweak scale depend on the parameter B_N.

Since the property of neutralino dark matter is quite sensitive to the size of μ, we discuss how the relic abundance of the neutralino dark matter is affected by the parameter B_N. In fact, some literature exists in which the impacts of neutrino Yukawa couplings on neutralino dark matter in the SUSY type I seesaw model have been discussed [17–24]. It was found that some regions of parameter space can significantly affect the neutralino relic density without the threshold corrections associated with the heavy singlet neutrino superfield. In our work, however, we consider the possible existence of the threshold corrections generated from the loops mediated via the heavy singlet neutrino superfield which can also significantly affect the neutralino relic abundance by lowering the sizes of $m^2_{H_2}$ and μ at the electroweak scale. Such a possibility of the impact on the neutralino relic density has not been studied before.

This paper is organized as follows. First, we present the effective potential for Higgs fields in the SUSY type I seesaw model in Sec. II. We show that threshold corrections to Higgs bilinear terms are generated from the loops mediated by heavy singlet neutrino superfields. In Sec. III, we give the alternative derivation for the threshold corrections, using renormalization group equations (RGEs) for a general field theory. In Sec. IV, we study the contributions of the threshold corrections to the radiative electroweak symmetry breaking and investigate how the size of the parameter μ can be affected by them. In Sec. V, we discuss the relic abundance of neutralino dark matter. Finally Sec. VI is devoted to conclusions and discussions. The details of convention for CP phases and the derivation of the effective potential for Higgs fields are given in the Appendixes.

II. THE EFFECTIVE POTENTIAL OF SUSY TYPE I SEESAW MODEL

In this section, we first derive the effective potential of the SUSY type I seesaw model, and then show that there exist threshold corrections to Higgs bilinear terms arisen due to the heavy RH singlet sneutrinos. Those threshold corrections may be modified by wave function renormalization for the Higgs field.

The superpotential of the SUSY seesaw model is given by

$$W = \mu H_1 \cdot H_2 - Y_\nu (\hat{L} \cdot \hat{H}_2)\hat{N}^c - \frac{M_R}{2}\hat{N}^c\hat{N}^c, \quad (1)$$

where $\hat{N}^c$ is a gauge singlet chiral superfield, which contains a RH neutrino and its scalar partner. M_R denotes the mass of the RH neutrino. Here, we do not consider the terms associated with the charged leptons and quarks whose contributions to our study are negligibly small except for the top quark superfield. From now on, we consider only one generation of $\hat{N}^c$ for simplicity, and the extension to three generations is straightforward. The soft breaking terms of the Lagrangian in the SUSY seesaw model are given by

$$\begin{aligned}\mathcal{L}_{\text{soft}} = & -m^2_{\tilde{L}}|\tilde{L}|^2 - m^2_{\tilde{N}}|\tilde{N}|^2 - (\tfrac{1}{2}B^*_N M^*_R \tilde{N}^2 + \text{H.c.}) \\ & + 2\,\text{Re}(B\mu H_1 \cdot H_2) - m^2_{H_1} H_1^\dagger H_1 - m^2_{H_2} H_2^\dagger H_2 \\ & + (A_\nu Y_\nu (H_2 \cdot \tilde{L})N^* + \text{H.c.}), \end{aligned} \quad (2)$$

where we can take M_R, B_N, Y_ν, and μ to be real by superfield rotation and $U(1)_R$ symmetry, whereas A_ν and B are left as complex numbers. We discuss the details of the phase convention in Appendix A. From the superpotential given in Eq. (1), the SUSY part of the Lagrangian is obtained as follows:

$$\begin{aligned}\mathcal{L}_{\text{SUSY}} = & -|Y_\nu \tilde{L} \cdot H_2 + M_R \tilde{N}^*|^2 - |Y_\nu \tilde{N}^* \tilde{L} - \mu H_1|^2 \\ & - |\mu|^2 H_2^\dagger H_2 - Y_\nu^2 |\tilde{N}|^2 H_2^\dagger H_2 \\ & - \tfrac{1}{2} M_R \bar{N}_R N_R{}^c - Y_\nu \bar{N}_R l_L \cdot H_2 + \text{H.c.} \end{aligned} \quad (3)$$

With this Lagrangian, we can derive the effective potential by using field dependent masses for the singlet RH neutrinos and sneutrinos. The effective Higgs potential which includes 1-loop contributions mediated by the singlet RH neutrino superfields is written as

$$\begin{aligned} V^{\text{1 loop}}_{\text{eff}} = & (|\mu|^2 + m^2_{H_1}(Q^2))H_1^\dagger H_1 \\ & + (|\mu|^2 + m^2_{H_2}(Q^2))H_2^\dagger H_2 - 2\,\text{Re}(B(Q^2)\mu H_1 \cdot H_2) \\ & + \left(\mu^2 \frac{Y_\nu^2}{16\pi^2} \log\frac{M_R^2}{Q^2}\right) H_1^\dagger H_1 + \frac{Y_\nu^2}{16\pi^2}\left(\log\frac{M_R^2}{Q^2}\right. \\ & \left. \times (m^2_{\tilde{L}} + m^2_{\tilde{N}} + |A_\nu^2|) + 2m^2_{\tilde{N}} + 2\,\text{Re}(A_\nu B_N)\right) \\ & \times H_2^\dagger H_2 - 2\,\text{Re}\left(\frac{Y_\nu^2}{16\pi^2}\left(B_N + A_\nu \log\frac{M_R^2}{Q^2}\right)\right. \\ & \left. \times \mu H_1 \cdot H_2\right) - \mathcal{L}_D, \end{aligned} \quad (4)$$

where Q is a renormalization scale and $\mathcal{L}_D$ is the D-term contributions given by

$$\begin{aligned} \mathcal{L}_D = & -\frac{g'^2}{8}(H_1^\dagger H_1 - H_2^\dagger H_2)^2 \\ & - \frac{g^2}{8}(H_1^\dagger \tau^a H_1 + H_2^\dagger \tau^a H_2)^2. \end{aligned} \quad (5)$$

In Appendix B, we present in detail how the effective potential is derived. Matching this effective potential with that of the MSSM at the seesaw scale, we can obtain some relations between MSSM parameters and corresponding ones in the SUSY seesaw model. Here, we do not include the loop contributions mediated by the top quark and its superpartner because they are identical to each other in both the MSSM and the SUSY seesaw model, and thus canceled in the relations. Therefore those contributions are irrelevant to the threshold corrections for the Higgs bilinear terms. The Higgs potential of the MSSM is given by

$$\begin{aligned} V_{\rm MSSM} = & (|\mu|^2 + \bar{m}^2_{H_1}(Q^2))H_1^{Q\dagger}H_1^Q \\ & + (|\mu|^2 + \bar{m}^2_{H_2}(Q^2))H_2^{Q\dagger}H_2^Q \\ & - (\bar{B}(Q^2)\mu H_1^Q \cdot H_2^Q + \text{H.c.}) - \mathcal{L}_D. \end{aligned} \quad (6)$$

By matching the Higgs potentials Eq. (6) with Eq. (4) at $Q^2 = M_R^2$, we obtain the following relations:

$$\begin{aligned} \bar{m}^2_{H_1}(M_R^2) &= m^2_{H_1}(M_R^2), \\ \bar{m}^2_{H_2}(M_R^2) &= m^2_{H_2}(M_R^2) + \frac{Y_\nu^2}{8\pi^2}(m^2_{\tilde{N}} + \text{Re}(A_\nu B_N)), \\ \bar{B}(M_R^2) &= B(M_R^2) + \frac{Y_\nu^2}{16\pi^2}B_N. \end{aligned} \quad (7)$$

On the other hand, the wave function renormalization for the Higgs field H_2 in the limit of small external momenta is given by

$$\left(1 - \frac{Y_\nu^2}{16\pi^2}\log\frac{M_R^2}{Q^2}\right)\partial_\mu H_2^{Q\dagger}\partial^\mu H_2^Q, \quad (8)$$

where we neglect the terms suppressed by M_R^{-2}. We notice that there exist no contributions from heavy RH neutrino superfields to wave function renormalization for H_1. At $Q^2 = M_R^2$, Eq. (8) becomes $\partial_\mu H_2^\dagger \partial^\mu H_2$, so the relations given in Eq. (7) are not modified by wave function renormalization.

It is worth noting that the soft breaking parameter of the singlet sneutrino, B_N, contributes to the Higgs mass $\bar{m}^2_{H_2}(M_R^2)$ and the parameter B. We use RGEs for the soft breaking parameters of the MSSM to obtain their low energy values below the seesaw scale M_R, whereas the corresponding RGEs given in the SUSY seesaw model are used above the seesaw scale. Thus, the values of the parameters in the RH side of Eq. (7), $m^2_{H_1}(Q^2 = M_R^2)$ and $m^2_{H_2}(Q^2 = M_R^2)$, depend on the boundary condition at further high energy scale, such as $M_{\rm GUT}$ or $M_{\rm Planck}$.

III. THE THRESHOLD CORRECTIONS FROM RENORMALIZATION GROUP EQUATIONS

In this section, we study the alternative derivation of the threshold corrections given in Eq. (7) by using RGEs including threshold effects. The RGEs in the MSSM including threshold effects are discussed in Refs. [25–28]. We derive the one-loop RGEs for Higgs mass-squared parameters in the SUSY seesaw model, by using the formulas for RGEs of dimensional parameters in general gauge field theories [29]. Then we integrate them and obtain the threshold corrections. Here we focus on the effects from the heavy neutrino and sneutrinos.

The key point of the derivation of the threshold corrections is to take into account three different thresholds. One of them corresponds to the mass of the RH neutrino (M_R), and the others correspond to the masses of the heavy sneutrinos, i.e., the superpartners of the RH neutrino. They are two real scalar fields and their masses are deviated from M_R due to soft SUSY breaking terms of the sneutrinos sector, as given by

$$\mathcal{L}_{\rm mass} = -\tfrac{1}{2}M^2_{\tilde{N}_1}N_1^2 - \tfrac{1}{2}M^2_{\tilde{N}_2}N_2^2, \quad (9)$$

where N_1 and N_2 are real and imaginary parts of the complex scalar field $\tilde{N}$, respectively, and are defined as

$$N_1 = (\tilde{N} + \tilde{N}^*)/\sqrt{2}, \qquad N_2 = (\tilde{N} - \tilde{N}^*)/(\sqrt{2}i). \quad (10)$$

The masses of the N_1 and N_2 are then given by

$$\begin{aligned} M^2_{\tilde{N}_1} &= m^2_{\tilde{N}} + M_R^2 + B_N M_R, \\ M^2_{\tilde{N}_2} &= m^2_{\tilde{N}} + M_R^2 - B_N M_R. \end{aligned} \quad (11)$$

Since B_N is real positive, the hierarchy of the three mass scales is given by

$$M^2_{\tilde{N}_1} > M_R^2 > M^2_{\tilde{N}_2}. \quad (12)$$

Then the energy scales at which $\tilde{N}_1$, $\tilde{N}_2$, and N_R are decoupled are different from each other, yielding the threshold corrections to Higgs mass-squared parameters. The Higgs mass terms are given as

$$\mathcal{L}_{\rm Higgs} = -m_{11}^2|H_1|^2 - m_{22}^2|H_2|^2 - m_{12}^2 H_1 \cdot H_2 + \text{H.c.}, \quad (13)$$

where

$$\begin{aligned} m_{11}^2 = |\mu|^2 + m^2_{H_1}, \qquad m_{22}^2 = |\mu|^2 + m^2_{H_2}, \\ m_{12}^2 = -B\mu. \end{aligned} \quad (14)$$

Following [29], we divide all the complex scalar fields into their real and imaginary parts, and derive the beta functions for the Higgs mass-squared parameters by adopting the step functions of the renormalization scale (Q) to take into account the thresholds. Then we obtain the threshold corrections by integrating the beta functions with respect to the energy scale between two mass scales of the singlet sneutrinos.

At the one-loop level, the beta functions for the Higgs mass parameters are given as

$$(4\pi)^2 \frac{dm_{11}^2}{d\ln Q} = Y_\nu^2 \mu^2 [\theta(Q^2 - M_{\tilde{N}_1}^2) + \theta(Q^2 - M_{\tilde{N}_2}^2)],$$
$$(4\pi)^2 \frac{dm_{12}^2}{d\ln Q} = Y_\nu^2 A_\nu \mu [\theta(Q^2 - M_{\tilde{N}_1}^2) + \theta(Q^2 - M_{\tilde{N}_2}^2)] - Y_\nu^2 \mu M_R \theta(M_{\tilde{N}_1}^2 - Q^2)\theta(Q^2 - M_{\tilde{N}_2}^2),$$
$$\begin{aligned}(4\pi)^2 \frac{dm_{22}^2}{d\ln Q} &= Y_\nu^2 (m_{\tilde{N}}^2 + |A_\nu|^2)[\theta(Q^2 - M_{\tilde{N}_1^2}^2) + \theta(Q^2 - M_{\tilde{N}_2}^2)] - Y_\nu^2 [2\,\mathrm{Re}(A_\nu) + B_N] M_R \theta(M_{\tilde{N}_1}^2 - Q^2)\theta(Q^2 - M_{\tilde{N}_2}^2) \\ &\quad + 2Y_\nu^2 M_R^2 [\theta(Q^2 - M_{\tilde{N}_1}^2) + \theta(Q^2 - M_{\tilde{N}_2}^2) - 2\theta(Q^2 - M_R^2)] + 2Y_\nu^2 m_{22}^2 \theta(Q^2 - M_R^2) \\ &\quad + Y_\nu^2 m_{\tilde{L}}^2 [\theta(Q^2 - M_{\tilde{N}_1}^2) + \theta(Q^2 - M_{\tilde{N}_2}^2)]. \end{aligned} \tag{15}$$

Here, we note that only the terms coming from the neutrino-sneutrino sector are presented because the other terms are the same as those in the MSSM. In deriving the RGEs, we take into account the fact that the effective theory changes by passing each threshold corresponding to the heavy degree of freedom. At the energy scale above $M_{\tilde{N}_1}$ where the RH neutrino and sneutrinos are active, our RGEs given in Eq. (15) are consistent with those in the supersymmetric type I seesaw model [30,31]. While the RH neutrino and the lighter sneutrino are active between the two scales $M_{\tilde{N}_1}$ and M_R, only the lighter sneutrino is active between the two scales M_R and $M_{\tilde{N}_2}$. Finally, the effective theory becomes the MSSM below $M_{\tilde{N}_2}$. In each step, we integrate out the heavier degrees of freedom and derive the effective theories which are valid at the lower energy scales.

By integrating the beta functions with respect to Q from $M_{\tilde{N}_1}$ down to $M_{\tilde{N}_2}$, we obtain the threshold corrections. The integrals can be approximated as follows:

$$\begin{aligned}\int_{M_{\tilde{N}_2}}^{M_{\tilde{N}_1}} d\ln Q &= \ln\frac{M_{\tilde{N}_1}}{M_{\tilde{N}_2}} = \frac{B_N}{M_R} + \mathcal{O}(M_R^{-3}), \\ \int_{M_R}^{M_{\tilde{N}_1}} d\ln Q &= \ln\frac{M_{\tilde{N}_1}}{M_R} \\ &= \frac{1}{2}\left[\frac{B_N}{M_R} + \frac{m_{\tilde{N}^2}}{M_R^2} - \frac{B_N^2}{2M_R^2} + \mathcal{O}(M_R^{-3})\right].\end{aligned} \tag{16}$$

Only the terms proportional to M_R or M_R^2 in Eq. (15) contribute to the threshold corrections. The results of integrating the beta functions give

$$\begin{aligned}\delta m_{H_1}^2 &= \mathcal{O}(M_R^{-1}), \\ \delta m_{H_2}^2 &= \frac{Y_\nu^2}{8\pi^2}[m_{\tilde{N}}^2 + \mathrm{Re}(A_\nu)B_N] + \mathcal{O}(M_R^{-1}), \\ \delta B &= \frac{Y_\nu^2}{16\pi^2} B_N + \mathcal{O}(M_R^{-1}),\end{aligned} \tag{17}$$

which are the same as Eq. (7).

Next, we discuss how the numerical value of the parameter μ can be affected by threshold corrections for the Higgs bilinear terms in the radiative electroweak symmetry breaking scenario [12–16]. In the calculation, we assume that gaugino masses, scalar masses, and A terms are universal at the grand unified theory (GUT) scale.

IV. MU TERM AND RADIATIVE ELECTROWEAK SYMMETRY BREAKING

As we have shown, the soft breaking parameter for the Higgs mass $m_{H_2}^2$ in the MSSM at the seesaw scale M_R is determined by not only $\bar{m}_{H_2}^2(M_R^2)$ calculated via RGEs in the SUSY seesaw model but also additional contribution due to the loops mediated by light and heavy sneutrinos in the seesaw model at the scale M_R. From Eq. (7), the shift of $m_{H_2}^2$ from $\bar{m}_{H_2}^2$ at the scale M_R is approximately given as

$$\begin{aligned}\delta m_{H_2}^2 &\approx \frac{Y_\nu^2}{8\pi^2}\,\mathrm{Re}(A_\nu B_N) \\ &\approx 1.6\times 10^5\ (\mathrm{GeV})^2\left(\frac{Y_\nu}{0.5}\right)^2\left(\frac{\mathrm{Re}A_\nu}{100\ \mathrm{GeV}}\right)\left(\frac{B_N}{500\ \mathrm{TeV}}\right).\end{aligned} \tag{18}$$

Therefore the soft breaking parameter B_N of the order of 500 TeV may significantly affect $m_{H_2}^2$ at the scale M_R. This observation in turn indicates that the shift of $m_{H_2}^2$ at the scale M_R affects electroweak symmetry breaking in the MSSM when we take the MSSM as an effective theory of the SUSY type I seesaw model at the low energy scale.

Let us discuss how electroweak symmetry breaking can be affected by the parameter B_N. In the MSSM, radiative breaking of electroweak symmetry can occur when SSB parameters for Higgs sectors satisfy the following relation:

$$\frac{1}{2}m_Z^2 = -|\mu|^2 + \frac{m_{H_1}^2(m_Z^2) - m_{H_2}^2(m_Z^2)\tan^2\beta}{\tan^2\beta - 1}. \tag{19}$$

In the limit of large $\tan\beta$, this relation becomes

$$\tfrac{1}{2}m_Z^2 \approx -|\mu|^2 - m_{H_2}^2(m_Z^2). \tag{20}$$

Therefore we see that the values of μ and $m_{H_2}^2$ are directly related. In order to satisfy this condition, $m_{H_2}^2$ has to be negative at the scale m_Z. In the radiative electroweak symmetry breaking scenario, $m_{H_2}^2$ is generally taken to be positive at high energy scale, but it receives quite large radiative corrections due to a heavy stop mass and large top quark Yukawa couplings between high and low energy scales, which drive $m_{H_2}^2$ negative so that electroweak symmetry can break at the low energy scale. At the scale above M_R, soft breaking masses and couplings are subject to the RGEs of the SUSY seesaw model. The RGE for $m_{H_2}^2$ in the SUSY seesaw model is given by [30,31]

$$\frac{dm_{H_2}^2}{dt} = \frac{2}{16\pi^2}\left[-\frac{3}{5}g_1^2M_1^2 - 3g_2^2M_2^2 + 3Y_t^2X_t + Y_\nu^2X_n\right], \tag{21}$$

where $t = \ln\frac{Q}{Q_0}$, $X_t = m_{\tilde{Q}_3}^2 + m_{\tilde{t}_R}^2 + m_{H_2}^2 + |A_t|^2$, and $X_n = m_{\tilde{L}}^2 + m_{\tilde{N}}^2 + m_{H_2}^2 + |A_\nu|^2$. Here, M_1 and M_2 denote the bino mass and the wino mass, respectively. The last term comes from the presence of RH neutrino superfields and other terms are the same as those in the MSSM.

It is expected that the RGE for $m_{H_2}^2$ can be significantly affected by the Yukawa coupling of the neutrino sector Y_ν when it is quite large. We can estimate the deviation of $m_{H_2}^2$ from that without a neutrino sector by integrating out Eq. (21) explicitly. The deviation at the scale M_R is approximately given as

$$\delta_{\log}m_{H_2}^2 \approx \frac{Y_\nu^2}{8\pi^2}(3m_0^2 + A_0^2)\ln\frac{M_R}{M_X}, \tag{22}$$

where m_0 and A_0 are the universal values for scalar masses and A terms, respectively. For $M_R = 6\times 10^{13}$ GeV and $M_X \approx 2\times 10^{16}$ GeV, this contribution can be written approximately as

$$\delta_{\log}m_{H_2}^2 \approx -5.5\times 10^4\ (\text{GeV})^2\left(\frac{Y_\nu}{0.5}\right)^2\left(\frac{m_0}{1\ \text{TeV}}\right)^2. \tag{23}$$

As we can see from Eq. (18), $\delta_{\log}m_{H_2}^2$ is easily dominated by the threshold correction when B_N is large.

Without threshold corrections, the weak scale value of $m_{H_2}^2$ becomes more negative than that of the minimal supergravity (mSUGRA) case. This affects the condition for electroweak symmetry breaking and the allowed regions for the observed relic density of dark matter [17,18]. Especially, the allowed region where $|\mu|$ is small is changed significantly. Universal scalar mass at the GUT scale, m_0 is larger than that of mSUGRA. However, with the inclusion of the threshold corrections, m_0 can be smaller than that of mSUGRA when B_N is large.

Figure 1 shows the RG evolution of $m_{H_2}^2$ and $m_{\tilde{t}}^2$ with the energy scale. Here, $m_{\tilde{t}}$ is defined as $m_{\tilde{t}}^2 = m_{\tilde{Q}}m_{\tilde{t}_R}$. We assume that soft breaking masses, gaugino masses, and A terms are universal at the GUT scale ($\approx 2\times 10^{16}$ GeV). The calculations are performed with the ISASUGRA code which is included in the ISAJET package [32]. The input values used in the calculations are given in the caption and neutrino masses m_ν and M_R are taken to be 0.1 eV and 6×10^{13} GeV, respectively, in both panels so that Y_ν and Y_t become the same order of magnitude. The pink, blue, and red curves correspond to the predictions of $\text{sign}(m_{H_2}^2)|m_{H_2}|$ including threshold corrections for $B_N =$ 500, 50, and 5 TeV, respectively. The green curves show how the predictions of $m_{\tilde{t}}^2$ evolve from the GUT scale to the electroweak scale. When $B_N = 50$ TeV, $A_\nu \sim 300$ GeV, and $m_0 \sim 1$ TeV, the threshold correction and the running effects from the neutrino Yukawa sector are almost canceled, i.e. $\delta m_{H_2}^2 + \delta_{\log}m_{H_2}^2 \sim 0$. Therefore the blue lines below the scale M_R behave as if there are no effects from the neutrino Yukawa sector. As we can see from Fig. 1, the value of $m_{H_2}^2$ at the scale m_Z obtained in the SUSY seesaw model is significantly deviated from that obtained in the MSSM for given input values of m_0, $m_{1/2}$, A_0, $\tan\beta$, and $B_N = 500$ TeV, whereas such a deviation disappears for $B_N \lesssim 5$ TeV.

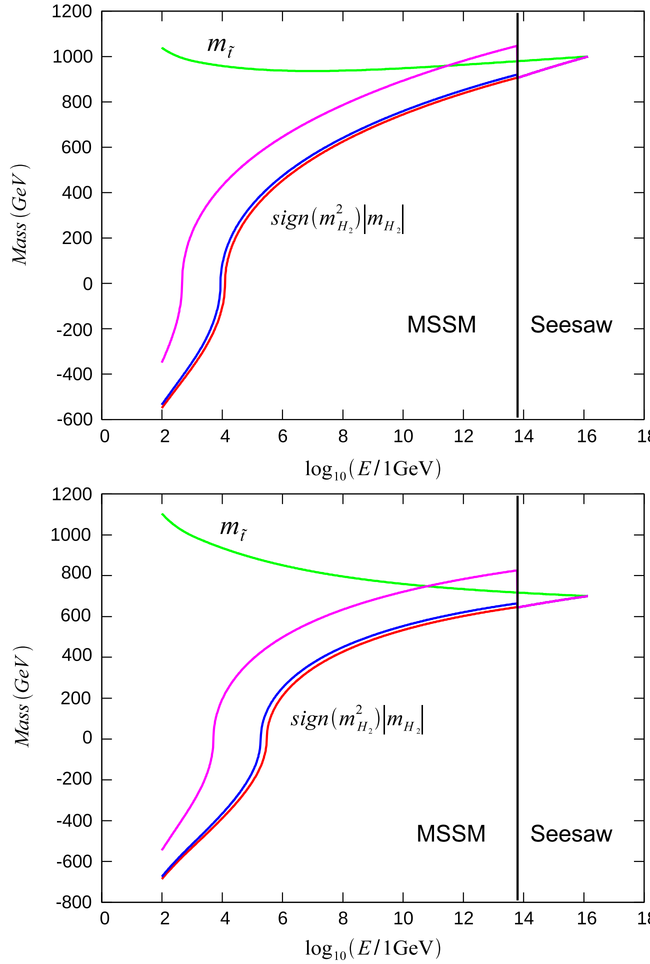

FIG. 1 (color online). The renormalization group evolutions of soft scalar masses for up-type Higgs and stops are shown. The calculation is performed by taking m_0, $m_{1/2}$, A_0, and $\tan\beta$ to be 1 TeV, 400 GeV, 300 GeV, and 10, respectively, in the upper panel and 700 GeV, 500 GeV, 300 GeV, and 20, respectively, in the lower panel. We take neutrino masses m_ν and M_R to be 0.1 eV and 6×10^{13} GeV, respectively, in both figures so that Y_ν and Y_t become the same order of magnitude. The pink, blue, and red curves correspond to the predictions of $\text{sign}(m_{H_2}^2)|m_{H_2}|$ for $B_N = 500$, 50, and 5 TeV, respectively. The green curves correspond to the MSSM prediction of $m_{\tilde{t}}$.

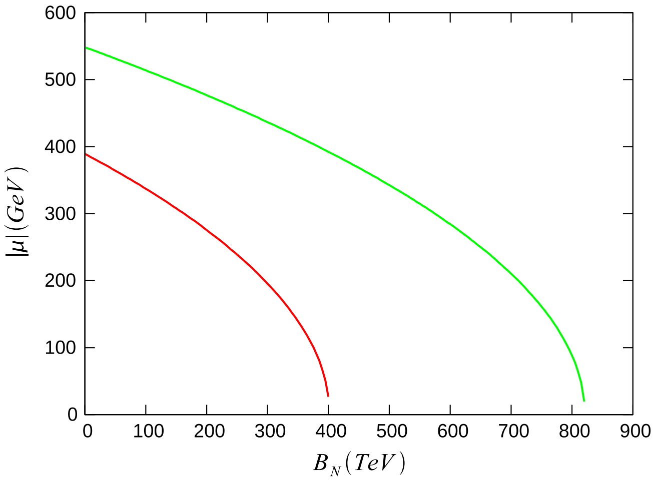

FIG. 2 (color online). The values of $|\mu|$ are plotted as a function of B_N. The lower red line is obtained for $m_0 = 1$ TeV, $m_{1/2} = 400$ GeV, $A_0 = 300$ GeV, and $\tan\beta = 10$, and the upper green line for $m_0 = 700$ GeV, $m_{1/2} = 500$ GeV, $A_0 = 300$ GeV, and $\tan\beta = 20$. We take the same values of m_ν and M_R as in Fig. 1.

In the case without threshold corrections, the running of the $m^2_{H_2}$ in mSUGRA with a RH neutrino superfield (mSUGRA + RHN) is discussed in Refs. [18,23]. The weak scale values of $\sqrt{|m^2_{H_u}|}$ tend to be larger than those in the mSUGRA scenario. The difference between mSUGRA and mSUGRA + RHN is up to a few hundred GeV, when $m_0 \gtrsim 1.5$ TeV and $Y_\nu \gtrsim Y_t$. On the other hand, our results show that the threshold correction increases $m_{H_u}(Q^2 = M_R^2)$ by several hundred GeV and therefore the weak scale values of $\sqrt{|m^2_{H_u}|}$ can be smaller than those in the mSUGRA scenario when B_N is large.

The significant deviation of $m^2_{H_2}$ at the scale m_Z in turn leads to a significant change in $|\mu|$ through the stationary condition, Eq. (19). In Fig. 2, we present how $|\mu(M_Z)|$ depends on the value of B_N. As the value of B_N increases, $|\mu|$ becomes smaller, due to the threshold corrections to $m^2_{H_2}(M_R)$.

It is worthwhile to notice that the size of the mass parameter μ characterizes the property of neutralino dark matter. Since μ is the Higgsino mass term, changing μ may affect the composition of the neutralino dark matter. This indicates that relic abundance of the dark matter is affected by B_N, especially on the condition that gaugino masses are universal at the GUT scale.

V. BINO-HIGGSINO DARK MATTER

In this section, we show that the lightest SUSY particle is a bino-Higgsino mixture state when the size of parameter B_N is of the order of several hundred TeV, and the result of the WMAP observation can be well accounted for. Here, we assume that soft scalar masses, gaugino masses, and A terms are universal at the GUT scale. We consider the lightest neutralino as a dark matter candidate.

The neutralinos are the physical states that are composed of the bino, wino, and two Higgsinos. The neutralino mass matrix in the $\tilde{B} - \tilde{W} - \tilde{H}_1 - \tilde{H}_2$ basis is given by

$$\mathcal{M}_\chi = \begin{pmatrix} M_1 & 0 & -m_Z\cos\beta\sin\theta_W & m_Z\sin\beta\sin\theta_W \\ 0 & M_2 & m_Z\cos\beta\cos\theta_W & -m_Z\sin\beta\cos\theta_W \\ -m_Z\cos\beta\sin\theta_W & m_Z\cos\beta\cos\theta_W & 0 & -\mu \\ m_Z\sin\beta\sin\theta_W & -m_Z\sin\beta\cos\theta_W & -\mu & 0 \end{pmatrix}, \tag{24}$$

where M_1 and M_2 are the bino and wino masses, respectively, and θ_W is the Weinberg angle. This matrix is diagonalized by the unitary matrix N,

$$\mathcal{M}_\chi^{\text{diag}} = N^* \mathcal{M}_\chi N^{-1}. \tag{25}$$

In terms of N, the lightest neutralino χ^0 is expressed as a mixture of the gauginos and the Higgsinos:

$$\chi^0 = N_{11}\tilde{B} + N_{12}\tilde{W} + N_{13}\tilde{H}_1 + N_{14}\tilde{H}_2. \tag{26}$$

Since we assumed a universal value for the gaugino masses at the GUT scale, gaugino masses M_i are related to gauge couplings g_i as follows:

$$\frac{M_i(Q)}{M(\Lambda_{\text{GUT}})} = \frac{g_i^2(Q)}{g^2(\Lambda_{\text{GUT}})}, \tag{27}$$

and this relation is easily derived from the renormalization group equations for gauginos,

$$\frac{dM_i}{dt} = \frac{2}{16\pi^2} b_i g_i^2 M_i, \tag{28}$$

where b_i are coefficients of beta functions for g_i. From Eq. (27), the bino mass M_1 is written in terms of the wino mass M_2:

$$M_1 = \tfrac{5}{3}\tan^2\theta_W M_2 \approx 0.5 M_2, \tag{29}$$

at the scale m_Z.

The relic density of cold dark matter, $\Omega_{\text{CDM}}h^2$, is determined by the WMAP observation [33] and its value is given by

$$\Omega_{\text{CDM}}h^2 = 0.1131 \pm 0.0034. \tag{30}$$

For $|\mu| \gg M_2$, the dark matter is binolike, whereas for $|\mu| \ll M_2$ the dark matter is Higgsino-like. In general, a binolike dark matter leads to a large relic abundance of a dark matter, which cannot accommodate the result from the WMAP observation. This is because couplings for the bino are smaller than those for the Higgsino and the wino. When the value of $|\mu|$ decreases, the Higgsino fraction defined by $|N_{13}|^2 + |N_{14}|^2$ increases, which leads to larger annihilation cross sections for Higgsino-like dark matter. Therefore we can fit the right amount of relic abundance derived from the result of the WMAP observation with a dark matter candidate composed of a bino-Higgsino mixture.

As we can see from Eq. (19), the value of $|\mu(m_Z)|^2$ becomes smaller as $B_N(M_R)$ increases. A larger value of $B_N(M_R)$ leads to a larger Higgsino fraction, which makes the relic abundance of dark matter decreased. Figure 3 presents the predictions of relic abundance of the lightest neutralino and corresponding contributions of Higgsino components as a function of B_N. Our numerical calculation is performed by using the MICROMEGAS 2.2 code [34,35]. The straight blue line represents the value of the relic abundance obtained from the WMAP observation. In this figure, we can see that as $B_N(M_R)$ increases, Higgsino fractions get larger, which makes relic abundances smaller. From our numerical analysis, it turned out that the right amount of the relic abundance of the dark matter could be explained by taking the parameter B_N to be of the order of several hundred TeV which makes Higgsino fractions large. The allowed regions of parameter space for the observed relic density of the dark matter are most conveniently shown in the $(m_{1/2}, m_0)$ plane. In the mSUGRA + RHN scenario without threshold correction, the allowed regions are given in Refs. [18,23]. One of the regions corresponding to the small μ is located along the region where electroweak symmetry breaking cannot take place. This region corresponds to $m_0 \gtrsim 1.3$ TeV. The values of m_0 depend on the renormalization group running effect from the neutrino Yukawa sector and it decreases the low energy value of $m^2_{H_2}$. When this effect becomes larger, we need to choose larger m_0 as the GUT boundary condition. With the inclusion of the threshold correction to $m^2_{H_2}$, however, the consequences change. In our scenario, as shown in Fig. 3, we can take m_0 as small as 700 GeV, since the threshold correction is added to $m^2_{H_2}$ at the scale M_R. Therefore, we conclude that the allowed regions where the observed relic density is explained by the bino-Higgsino dark matter are very different from those of mSUGRA and the mSUGRA + RHN scenario.

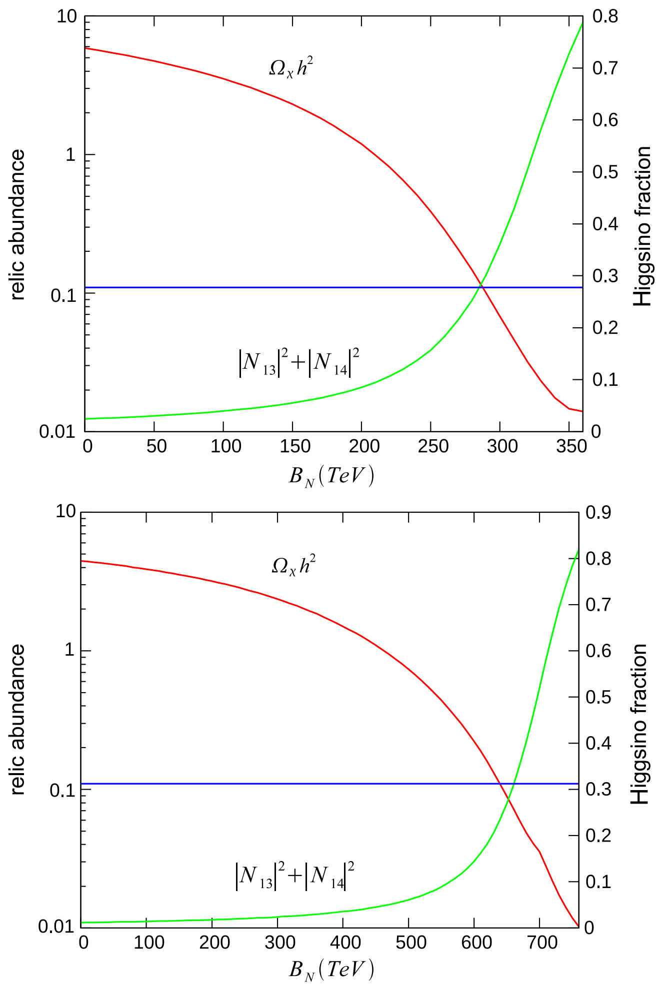

FIG. 3 (color online). The relic abundances of the lightest neutralino (red curves) and corresponding Higgsino contributions (green curves) are drawn as a function of B_N. The relic abundances are shown as decreasing functions with respect to B_N and the corresponding Higgsino contributions are shown as increasing functions with respect to B_N. We take m_0, $m_{1/2}$, A_0, and $\tan\beta$ to be 1 TeV, 400 GeV, 300 GeV, and 10, respectively, in the upper panel and to be 700 GeV, 500 GeV, 300 GeV, and 20, respectively, in the lower panel. μ is positive, and the values of m_ν and M_R are taken to be the same as in Fig. 1. The straight blue lines correspond to the value of the relic abundance obtained from the WMAP observation.

VI. CONCLUSION AND DISCUSSION

We have investigated the effective low energy Higgs potential of the SUSY type I seesaw model. We found that Higgs bilinear terms got threshold corrections at the scale below M_R, due to heavy singlet sneutrino loops. These threshold corrections are proportional to the B term of heavy singlet sneutrino B_N. Therefore, if B_N is large enough, the mass parameters of Higgs bilinear terms are significantly shifted at the scale M_R, which in turn leads to a shift of the parameter $|\mu|$ and reduction of the fine-tuning between the Higgsino mass parameter μ and SSB mass for up-type Higgs at the electroweak scale. We presented how the parameter μ depends on B_N. We have shown that dark matter becomes a mixture of bino and Higgsino for B_N of the order of several hundred TeV and the observed relic abundance can be consistently explained by the bino-Higgsino dark matter. It turned out that the allowed region of parameter space constrained by the relic abundance of dark matter in this model is very different from the MSSM without seesaw under the assumption that

SSB terms are universal at the GUT scale, mainly because of the threshold corrections to $m_{H_2}^2$. Our results are also different from those of conventional mSUGRA with a type I seesaw which does not include the threshold corrections to $m_{H_u}^2$.

The naturalness problem for such a large value of B_N is beyond the scope of this work. Since the size of B_N of the order of several hundred TeV is much larger than the scale of soft breaking parameters, the origin of B_N must be different from those of other SUSY breaking parameters. $U(1)_{B-L}$ extension of the MSSM might provide the origin of large B_N. It would be interesting to see if such a large value of B_N can be naturally possible.

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APPENDIX A: *CP* VIOLATION AND PHASE CONVENTION

Here, we discuss the *CP* violation of the SUSY type I seesaw model and identify the independent phases by choosing a phase convention. One can assign the R charge 0 to the Higgs superfields $\hat{H}_1$ and $\hat{H}_2$, and 1 to the lepton superfields $\hat{L}$ and $\hat{N}^c$. Under the R transformation and the phase redefinition of the superfields $\hat{L}$, $\hat{H}_1$, $\hat{H}_2$, and $\hat{N}^c$, the superpotential is transformed as

$$W \to e^{2i\theta_R}\bigg(-Y_\nu \exp(i(\theta_{N^c} + \theta_L + \theta_2))\hat{N}^c\hat{L}\cdot\hat{H}_2 - \frac{M_R}{2}\exp(2i\theta_{N^c})\hat{N}^c\hat{N}^c + \mu\exp(i(\theta_1 + \theta_2 - 2\theta_R))\hat{H}_1\cdot\hat{H}_2\bigg). \tag{A1}$$

Therefore one can remove the phases of the parameters Y_ν, μ, and M_R in W by choosing the phases of the superfields as follows:

$$\theta_{N^c} = -\tfrac{1}{2}\arg M_R, \qquad \theta_1 + \theta_2 - 2\theta_R = -\arg\mu, \qquad \theta_L + \theta_2 + \theta_{N^c} = -\arg Y_\nu. \tag{A2}$$

The trilinear couplings of the soft breaking terms transform in the same way as the superpotential, so one cannot remove those phases. For the soft breaking parameters of the bilinear form, one can take one of them to be real. We then rotate the phase of B_N away by choosing the phase parameter of R transformation as follows:

$$\theta_R = -\tfrac{1}{2}\arg(B_N). \tag{A3}$$

In Eq. (A2), we still have the freedom of choosing the phase of θ_2. Here, we choose the phase θ_2 so that the vacuum expectation value of H_2 becomes real

$$\theta_2 = -\arg(v_2). \tag{A4}$$

To summarize, we choose the phases as

$$\theta_{N^c} = -\tfrac{1}{2}\arg M_R, \qquad \theta_1 = -\arg\mu + \arg(v_2) - \arg B_N, \qquad \theta_L = \arg(v_2) + \tfrac{1}{2}\arg M_R - \arg Y_\nu. \tag{A5}$$

With this phase convention, the soft breaking terms are written as

$$\mathcal{L}_{\rm soft} = (|A_\nu||Y_\nu|\tilde{N}^* e^{i[\arg(A_\nu/B_N)]} + \text{H.c.}) + 2|\mu||B|\text{Re}(e^{i[\arg(B/B_N)]}H_1\cdot H_2) - \frac{|M_R|}{2}|B_N|\tilde{N}^*\tilde{N}^* - m_{\tilde{L}}^2|\tilde{L}|^2 - m_{\tilde{N}}^2|\tilde{N}|^2, \tag{A6}$$

and two independent irremovable *CP* violating phases are presented as

$$B = |B|e^{i\arg(B/B_N)}, \qquad A_\nu = |A_\nu|e^{i\arg(A_\nu/B_N)}. \tag{A7}$$

APPENDIX B: DERIVATION OF THE EFFECTIVE POTENTIAL

In this Appendix, we derive the effective potential of Higgs fields in the SUSY type I seesaw model. The contribution to the effective potential for Higgs fields from the loops mediated by neutrino superfields is written as

$$V_{\rm eff}(v_1, v_2) = \int \frac{Q^{4-d}d^dk}{(2\pi)^d i}\frac{1}{2}(\ln\det(M_s{}^2 - k^2) - \ln\det(M_F - \not{k})), \tag{B1}$$

where M_F is the mass matrix of one of the neutrino sectors and M_s^2 is the 4×4 mass-squared matrix of the sneutrino sector given by

$$M_s^2 = \begin{pmatrix} (m_{\tilde{L}}^2 + m_D^2) & 0 & \hat{A}_\nu^* m_D & |M_R m_D| \\ 0 & (m_{\tilde{L}}^2 + m_D^2) & |M_R m_D| & \hat{A}_\nu m_D \\ \hat{A}_\nu m_D & |m_D M_R| & |M_R|^2 + m_{\tilde{N}}^2 & |B_N M_R| \\ |m_D M_R| & \hat{A}_\nu^* m_D & |B_N^* M_R| & |M_R|^2 + m_{\tilde{N}}^2 \end{pmatrix}, \tag{B2}$$

where

$$m_D = \frac{Y_\nu v_2}{\sqrt{2}}, \qquad \hat{A}_\nu = A_\nu - \frac{v_1^*}{v_2}\mu, \qquad A_\nu = |A_\nu| e^{i\arg(A_\nu/B_N)}. \tag{B3}$$

The effects of CP violation appear through the parameter $\hat{A}_\nu$. We compute the following quantity:

$$\ln\det({M_s}^2 - k^2) = \mathrm{Tr}\ln({M_s}^2 - k^2). \tag{B4}$$

To compute the scalar contribution, we diagonalize M_s^2 approximately and treat the A term as perturbation. We first split M_s^2 as

$$M_s^2 = M_0^2 + \Delta_A, \tag{B5}$$

where

$$M_0^2 = \begin{pmatrix} (m_{\tilde{L}}^2 + m_D^2) & 0 & 0 & |M_R m_D| \\ \mathbf{0} & (m_{\tilde{L}}^2 + m_D^2) & |M_R m_D| & 0 \\ 0 & |m_D M_R| & |M_R|^2 + m_{\tilde{N}}^2 + m_D^2 & |B_N M_R| \\ |m_D M_R| & 0 & |B_N^* M_R| & |M_R|^2 + m_{\tilde{N}}^2 + m_D^2 \end{pmatrix}, \tag{B6}$$

and

$$\Delta_A = \begin{pmatrix} 0 & \mathbf{0} & \hat{A}_\nu^* m_D & 0 \\ \mathbf{0} & 0 & 0 & \hat{A}_\nu m_D \\ \hat{A}_\nu m_D & 0 & 0 & 0 \\ 0 & \hat{A}_\nu^* m_D & 0 & 0 \end{pmatrix}. \tag{B7}$$

One can find the orthogonal matrix O which diagonalizes M_0^2. Using this matrix, ${M_s}^2$ is transformed as

$$OMs^2O^T = \mathrm{diag}(m_1^2, m_2^2, m_3^2, m_4^2) + O\Delta_A O^T. \tag{B8}$$

Here, m_1, m_2 are the mass of lighter sneutrinos and m_3, m_4 are those of heavier sneutrinos given by

$$\begin{aligned} m_1^2 &= \frac{M_R^2 + m_{\tilde{N}}^2 + 2m_D^2 + B_N M_R + m_{\tilde{L}}^2}{2} - \frac{1}{2}\sqrt{(M_R^2 + m_{\tilde{N}}^2 + B_N M_R - m_{\tilde{L}}^2)^2 + 4m_D^2 M_R^2}, \\ m_2^2 &= \frac{M_R^2 + m_{\tilde{N}}^2 + 2m_D^2 - B_N M_R + m_{\tilde{L}}^2}{2} - \frac{1}{2}\sqrt{(M_R^2 + m_{\tilde{N}}^2 - B_N M_R - m_{\tilde{L}}^2)^2 + 4m_D^2 M_R^2}, \\ m_3^2 &= \frac{M_R^2 + m_{\tilde{N}}^2 + 2m_D^2 - B_N M_R + m_{\tilde{L}}^2}{2} + \frac{1}{2}\sqrt{(M_R^2 + m_{\tilde{N}}^2 - B_N M_R - m_{\tilde{L}}^2)^2 + 4m_D^2 M_R^2}, \\ m_4^2 &= \frac{M_R^2 + m_{\tilde{N}}^2 + 2m_D^2 + B_N M_R + m_{\tilde{L}}^2}{2} + \frac{1}{2}\sqrt{(M_R^2 + m_{\tilde{N}}^2 + B_N M_R - m_{\tilde{L}}^2)^2 + 4m_D^2 M_R^2}. \end{aligned} \tag{B9}$$

These sneutrino masses should be compared with the neutrino masses written as

$$\begin{aligned} m_H^2 &= \frac{M_R^2}{2} + m_D^2 + \frac{\sqrt{M_R^4 + 4m_D^2 M_R^2}}{2}, \\ m_L^2 &= \frac{M_R^2}{2} + m_D^2 - \frac{\sqrt{M_R^4 + 4m_D^2 M_R^2}}{2}. \end{aligned} \tag{B10}$$

Using Eq. (B1) and the mass eigenvalues, one can find the effective potential as follows:

$$\begin{aligned} V_{\mathrm{eff}} = V_{\mathrm{eff}}^{(0)} &+ \int \frac{d^d k}{(2\pi)^d i}\frac{1}{2} \\ &\times \left(+ \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} \mathrm{Tr}\left(\frac{1}{m^2 - k^2} O\Delta_A O^T\right)^n \right), \end{aligned} \tag{B11}$$

where

$$\begin{aligned} V_{\mathrm{eff}}^{(0)} &= \frac{1}{2}\int \frac{d^d k Q^{4-d}}{(2\pi)^d i}\left(\sum_{i=1}^4 \log(m_i^2 - k^2) - 2\log(m_H^2 - k^2) \right. \\ &\quad \left. - 2\log(m_L^2 - k^2)\right) \\ &= \frac{1}{64\pi^2} C_{UV}\left(2(m_H^4 + m_L^4) - \sum_{i=1}^4 m_i^4\right) \\ &\quad + \frac{1}{64\pi^2}\left(\sum_{i=1}^4 m_i^4\left(\log\frac{m_i^2}{Q^2} - \frac{3}{2}\right) - 2m_H^4\left(\log\frac{m_H^2}{Q^2} - \frac{3}{2}\right)\right. \\ &\quad \left. - 2m_L^4\left(\log\frac{m_L^2}{Q^2} - \frac{3}{2}\right)\right), \end{aligned} \tag{B12}$$

where $C_{UV} = \frac{1}{\epsilon} - \gamma + \log 4\pi$ and Q is the renormalization scale. The renormalization point dependent finite part of the effective potential $V_{\mathrm{eff}}^{(0)}$ is given as

$$V^{(0)}(Q^2) = \frac{1}{64\pi^2}\left(\sum_{i=1}^{4} m_i^4\left(\log\frac{m_i^2}{Q^2} - \frac{3}{2}\right) - 2m_H^4\left(\log\frac{m_H^2}{Q^2} - \frac{3}{2}\right) - 2m_L^4\left(\log\frac{m_L^2}{Q^2} - \frac{3}{2}\right)\right). \tag{B13}$$

We note that $V^{(0)}$ depends on the Higgs vacuum expectation value through m_D^2 where $m_D = \frac{Y_\nu v_2}{\sqrt{2}}$. To obtain the contribution to the Higgs mass term $m_{H_2}^2 H_2^\dagger H_2$, one can differentiate the effective potential with respect to m_D^2, while keeping the terms which remain nonzero in the large limit of M_R,

$$\begin{aligned}\frac{\partial V^{(0)}}{\partial m_D^2} &\simeq \frac{1}{64\pi^2}\left(\left(\log\frac{M_R^2}{Q^2} - C_{UV} - 1\right)\left(2m_3^2\frac{\partial m_3^2}{\partial m_D^2} + 2m_4^2\frac{\partial m_4^2}{\partial m_D^2} - 4m_H^2\frac{\partial m_H^2}{\partial m_D^2}\right) + 2\left(m_3^2\log\frac{m_3^2}{M_R^2}\frac{\partial m_3^2}{\partial m_D^2} + m_4^2\log\frac{m_4^2}{M_R^2}\frac{\partial m_4^2}{\partial m_D^2} - 2m_H^2\log\frac{m_H^2}{M_R^2}\frac{\partial m_H^2}{\partial m_D^2}\right)\right)\\ &\simeq \frac{1}{16\pi^2}\left(\left(\log\frac{M_R^2}{Q^2}\right)(m_{\tilde{L}}^2 + m_{\tilde{N}}^2) + 2m_{\tilde{N}}^2\right) - \frac{1}{16\pi^2}(C_{UV} + 1)(m_{\tilde{L}}^2 + m_{\tilde{N}}^2).\end{aligned} \tag{B14}$$

The terms which are proportional to the derivative of the lighter mass also vanish in the large limit of M_R, because $m_1^2 \sim m_2^2 \simeq m_{\tilde{L}}^2 \simeq \frac{m_D^4}{M_R^2}$ and the derivatives with respect to m_D^2 are suppressed as $\frac{B_N}{M_R}$ and $\frac{m_D^2}{M_R^2}$, respectively. From Eq. (B14), one can read off the coefficient of the Higgs mass term $H_2^\dagger H_2$. The contribution to the Higgs mass term including the counterterm is given as

$$V_{\rm eff}^{(0)}(Q^2) = V_{\rm eff}^{(0)} + V_c^{(0)} = \frac{Y_\nu^2}{16\pi^2}(H_2^\dagger H_2)\left(\log\frac{M_R^2}{Q^2}(m_{\tilde{L}}^2 + m_{\tilde{N}}^2) + 2m_{\tilde{N}}^2\right), \tag{B15}$$

where the counterterm is given as

$$V_c^{(0)} = \frac{Y_\nu^2}{16\pi^2}(C_{UV} + 1)(m_{\tilde{L}}^2 + m_{\tilde{N}}^2)H_2^\dagger H_2. \tag{B16}$$

Next we compute the corrections to $V^{(0)}$ due to the A_ν terms up to the second order of Δ_A, because they give the nonvanishing contribution to the effective potential in the large limit of M_R. To compute the corrections, one needs to derive the orthogonal matrix O in Eq. (B6). To diagonalize M_0^2, we follow two steps. First, we diagonalize M_0^2 with the help of orthogonal matrices O_L and O_H as follows:

$$M_0'^2 = \begin{pmatrix} O_L & 0 \\ 0 & O_H \end{pmatrix} M_0^2 \begin{pmatrix} O_L^T & 0 \\ 0 & O_H^T \end{pmatrix} = \begin{pmatrix} m_{\tilde{L}}^2 + m_D^2 & 0 & 0 & m_D M_R \\ 0 & m_{\tilde{L}}^2 + m_D^2 & m_D M_R & 0 \\ 0 & m_D M_R & M_R^2 + m_{\tilde{N}}^2 + m_D^2 - B_N M_R & 0 \\ m_D M_R & 0 & 0 & M_R^2 + m_{\tilde{N}}^2 + m_D^2 + B_N M_R \end{pmatrix}, \tag{B17}$$

where O_L and O_H are given as

$$O_L = O_H^T = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}. \tag{B18}$$

We note the degenerate diagonal masses of the heavy sneutrinos are split after the rotation. The mass-squared matrix $M_0'^2$ has the separated 2×2 parts as submatrices. Each of them has the form of the seesaw type. Thus, the mass matrix M_0' can be diagonalized as

$$\begin{pmatrix} m_1^2 & 0 & 0 & 0 \\ 0 & m_2^2 & 0 & 0 \\ 0 & 0 & m_3^2 & 0 \\ 0 & 0 & 0 & m_4^2 \end{pmatrix} = \begin{pmatrix} \cos\theta_+ & 0 & 0 & -\sin\theta_+ \\ 0 & \cos\theta_- & -\sin\theta_- & 0 \\ 0 & \sin\theta_- & \cos\theta_- & 0 \\ \sin\theta_+ & 0 & 0 & \cos\theta_+ \end{pmatrix} M_0'^2 \begin{pmatrix} \cos\theta_+ & 0 & 0 & \sin\theta_+ \\ 0 & \cos\theta_- & \sin\theta_- & 0 \\ 0 & -\sin\theta_- & \cos\theta_- & 0 \\ -\sin\theta_+ & 0 & 0 & \cos\theta_+ \end{pmatrix}. \tag{B19}$$

Then the orthogonal matrix O is given as

$$O = \begin{pmatrix} \cos\theta_+ & 0 & 0 & -\sin\theta_+ \\ 0 & \cos\theta_- & -\sin\theta_- & 0 \\ 0 & \sin\theta_- & \cos\theta_- & 0 \\ \sin\theta_+ & 0 & 0 & \cos\theta_+ \end{pmatrix} \times \begin{pmatrix} O_L & 0 \\ 0 & O_H \end{pmatrix}. \tag{B20}$$

Using the above form of orthogonal matrix O, $O\Delta_A O^T$ is given as

$$O\Delta_A O^T = m_D \,\mathrm{Re}(\hat{A}_\nu)\begin{pmatrix} -\sin2\theta_+ & 0 & 0 & \cos2\theta_+ \\ 0 & \sin2\theta_- & -\cos2\theta_- & 0 \\ 0 & -\cos2\theta_- & -\sin2\theta_- & 0 \\ \cos2\theta_+ & 0 & 0 & \sin2\theta_+ \end{pmatrix} + im_D\,\mathrm{Im}(\hat{A}_\nu)$$
$$\times \begin{pmatrix} 0 & \sin(\theta_- + \theta_+) & -\cos(\theta_- + \theta_+) & 0 \\ -\sin\theta_- + \theta_+ & 0 & 0 & \cos(\theta_- + \theta_+) \\ \cos(\theta_- + \theta_+) & 0 & 0 & \sin(\theta_- + \theta_+) \\ 0 & -\cos(\theta_- + \theta_+) & -\sin(\theta_- + \theta_+) & 0 \end{pmatrix}. \tag{B21}$$

We then obtain the corrections to the effective potential at the first order of Δ_A given as

$$\begin{aligned}\delta V_{\rm eff}^{(1)} &= \frac{1}{2}\left(\sum_{i=1}^{4}\int\frac{d^dk}{(2\pi)^d i}\frac{(O\hat{A}_\nu O^T)_{ii}}{m_i^2 - k^2}\right) \\ &= -\frac{\mathrm{Re}\hat{A}_\nu m_D}{32\pi^2}\left(m_4^2\sin2\theta_+\left(C_{UV} + 1 - \ln\frac{m_4^2}{Q^2}\right)\right. \\ &\quad - m_1^2\sin2\theta_+\left(C_{UV} + 1 - \ln\frac{m_1^2}{Q^2}\right) \\ &\quad - m_3^2\sin2\theta_-\left(C_{UV} + 1 - \ln\frac{m_3^2}{Q^2}\right) \\ &\quad \left.+ m_2^2\sin2\theta_-\left(C_{UV} + 1 - \ln\frac{m_2^2}{Q^2}\right)\right).\end{aligned} \tag{B22}$$

Now, let us show how the divergences are canceled so that the correction is finite. To do this, we use the relation

$$(m_4^2 - m_1^2)\sin2\theta_+ = (m_3^2 - m_2^2)\sin2\theta_-. \tag{B23}$$

Then, the corrections to the effective potential become

$$\begin{aligned}\delta V_{\rm eff}^{(1)} &= \frac{m_D\mathrm{Re}\hat{A}_\nu}{32\pi^2}\left(m_1^2\sin2\theta_+\ln\frac{m_3m_4}{m_1^2} - m_2^2\sin2\theta_-\ln\frac{m_3m_4}{m_2^2}\right. \\ &\quad \left.+\frac{m_3^2\sin2\theta_- + m_4^2\sin2\theta_+}{2}\ln\frac{m_4^2}{m_3^2}\right) \\ &\simeq \frac{m_D\mathrm{Re}(\hat{A}_\nu)}{32\pi^2}(m_4^2 - m_3^2)(\theta_+ + \theta_-) \\ &\simeq \frac{m_D^2}{8\pi^2}\mathrm{Re}(\hat{A}_\nu B_N) \simeq \frac{Y_\nu^2}{8\pi^2}\mathrm{Re}\left(A_\nu B_N\frac{v_2^2}{2} - \mu B_N\frac{v_1^* v_2}{2}\right) \\ &\simeq \frac{Y_\nu^2}{8\pi^2}\left(\mathrm{Re}(A_\nu B_N)H_2^\dagger H_2 - \mu B_N\mathrm{Re}(H_1\cdot H_2)\right),\end{aligned} \tag{B24}$$

where we have used the relation which is valid in the large limit of M_R, $\theta_\pm \sim \frac{m_D}{M_R}$, and $m_4^2 - m_3^2 = 2M_R B_N$. The correction at the second order of the Δ_{A_ν} term is given as

$$\delta V_{\rm eff}^{(2)} = -\frac{1}{4}\int\frac{d^dk}{(2\pi)^d i}\frac{1}{m_i^2 - k^2}(O\Delta_A O^T)_{ij}\frac{1}{m_j^2 - k^2} \times (O\Delta_A O^T)_{ji}. \tag{B25}$$

The term which is not suppressed by $\frac{1}{M_R^n}$ is given as

$$\begin{aligned}\delta V_{\rm eff}^{(2)} &= -\frac{1}{16\pi^2}\left(C_{UV} + 1 - \ln\frac{M_R^2}{Q^2}\right)m_D^2|\hat{A}_\nu|^2 \\ &= -\frac{Y_\nu^2}{16\pi^2}\left(C_{UV} + 1 - \ln\frac{M_R^2}{Q^2}\right)(|A_\nu|^2H_2^\dagger\cdot H_2 \\ &\quad - 2\,\mathrm{Re}(A_\nu\mu H_1\cdot H_2) + \mu^2H_1^\dagger\cdot H_1).\end{aligned} \tag{B26}$$

The divergences are canceled by adding the counterterm,

$$\begin{aligned}V_c^{(2)} &= \frac{Y_\nu^2}{16\pi^2}(C_{UV} + 1)(|A_\nu|^2H_2^\dagger\cdot H_2 \\ &\quad - 2\,\mathrm{Re}(A_\nu\mu H_1\cdot H_2) + \mu^2H_1^\dagger\cdot H_1).\end{aligned} \tag{B27}$$

The effective potential at the one-loop level is finally written as

$$\begin{aligned}V_{\rm eff}^{1\,\rm loop} &= (|\mu|^2 + m_{H_1}^2(Q^2))H_1^\dagger H_1 + (|\mu|^2 + m_{H_2}^2(Q^2))H_2^\dagger H_2 \\ &\quad - 2\mathrm{Re}(B(Q^2)\mu H_1\cdot H_2) + \left(\mu^2\frac{Y_\nu^2}{16\pi^2}\log\frac{M_R^2}{Q^2}\right)H_1^\dagger H_1 \\ &\quad + \frac{Y_\nu^2}{16\pi^2}\left(\log\frac{M_R^2}{Q^2}(m_{\tilde{L}}^2 + m_{\tilde{N}}^2 + |A_\nu^2|) + 2m_{\tilde{N}}^2\right. \\ &\quad \left.+ 2\mathrm{Re}(A_\nu B_N)\right)H_2^\dagger H_2 \\ &\quad - 2\mathrm{Re}\left(\frac{Y_\nu^2}{16\pi^2}\left(B_N + A_\nu\log\frac{M_R^2}{Q^2}\right)\mu H_1\cdot H_2\right) - \mathcal{L}_D,\end{aligned}$$

where $\mathcal{L}_D$ is the D-term contribution. To complete the renormalization of the effective potential, we consider the relation between the renormalized mass parameters and the bare ones. We first note that the bilinear part of the Higgs sector including the tree and the counterterms in the present model can be derived from the following

Lagrangian:

$$\mathcal{L} = Z_1\hat{H}_1^\dagger\hat{H}_1|_D + Z_2\hat{H}_2^\dagger\hat{H}_2|_D + \mu\hat{H}_1\cdot\hat{H}_2|_F + \text{H.c.} - (m_{H_1}^2(Q^2) + \delta m_{H_1}^2)H_1^\dagger H_1 - (m_{H_2}^2(Q^2) + \delta m_{H_2}^2)H_2^\dagger H_2 + 2\,\text{Re}((B(Q^2) + \delta B)\mu H_1\cdot H_2). \quad \text{(B28)}$$

After integrating out the F terms of the superfields, one obtains

$$\mathcal{L} = Z_1\partial_\mu H_1^\dagger\partial^\mu H_1 + Z_2\partial_\mu H_2^\dagger\partial^\mu H_2 - \frac{|\mu|^2}{Z_2}H_1^\dagger H_1 - \frac{|\mu|^2}{Z_1}H_2^\dagger H_2 + 2\,\text{Re}((B(Q^2) + \delta B)\mu H_1\cdot H_2) - (m_{H_1}^2(Q^2) + \delta m_{H_1}^2)H_1^\dagger H_1 - (m_{H_2}^2(Q^2) + \delta m_{H_2}^2)H_2^\dagger H_2. \quad \text{(B29)}$$

We define bare superfields and the bare parameter μ as $\hat{H}_i^0 = \sqrt{Z_i}\hat{H}_i$ $(i = 1, 2)$ and $\mu_0\sqrt{Z_1}\sqrt{Z_2} = \mu$, respectively. One can write the Lagrangian in terms of the bare fields as

$$\mathcal{L} = \partial_\mu H_1^{0\dagger}\partial^\mu H_1^0 + \partial_\mu H_2^{0\dagger}\partial^\mu H_2^0 - |\mu_0|^2 H_1^{0\dagger}H_1^0 - |\mu_0|H_2^{0\dagger}H_2^0 + 2\,\text{Re}(B_0\mu_0 H_1^0\cdot H_2^0) - \frac{(m_{H_1}^2(Q^2) + \delta m_{H_1}^2)}{Z_1}H_1^{0\dagger}H_1^0 - \frac{m_{H_2}^2(Q^2) + \delta m_{H_2}^2}{Z_2}H_2^{0\dagger}H_2^0. \quad \text{(B30)}$$

Then one can define the bare mass parameters as

$$m_{0H_1}^2 Z_1 = m_{H_1}^2(Q^2) + \delta m_{H_1}^2, \quad m_{0H_2}^2 Z_2 = m_{H_2}^2(Q^2) + \delta m_{H_2}^2, \quad B_0 = B(Q^2) + \delta B. \quad \text{(B31)}$$

Equation (B29) leads to the following counterterms for the bilinear parts of the Higgs potential:

$$V_c = (\delta m_{H_1}^2 + (Z_2^{-1} - 1)|\mu|^2)H_1^\dagger H_1 + (\delta m_{H_2}^2 + (Z_1^{-1} - 1)|\mu|^2)H_2^\dagger H_2 - 2\,\text{Re}(\delta B\mu H_1\cdot H_2). \quad \text{(B32)}$$

Comparing V_c with the sum of the counterterms $V_c^{(0)} + V_c^{(2)}$ given by

$$V_c^{(0)} + V_c^{(2)} = \frac{Y_\nu^2}{16\pi^2}(C_{UV} + 1)(|A_\nu|^2 + m_{\tilde{N}}^2 + m_{\tilde{L}}^2)H_2^\dagger H_2 + \frac{Y_\nu^2}{16\pi^2}(C_{UV} + 1)\mu^2 H_1^\dagger H_1 - 2\frac{Y_\mu^2}{16\pi^2}(C_{UV} + 1)\text{Re}(A_\nu\mu H_1\cdot H_2), \quad \text{(B33)}$$

we obtain the following relations:

$$\delta m_{H_1}^2 + (Z_2^{-1} - 1)\mu^2 = \frac{Y_\nu^2}{16\pi^2}(C_{UV} + 1)\mu^2, \quad \delta m_{H_2}^2 + (Z_1^{-1} - 1)\mu^2 = \frac{Y_\nu^2}{16\pi^2}(C_{UV} + 1)(|A_\nu|^2 + m_{\tilde{N}}^2 + m_{\tilde{L}}^2), \quad \delta B = \frac{Y_\mu^2}{16\pi^2}(C_{UV} + 1)A_\nu. \quad \text{(B34)}$$

Using the results of the wave function renormalization,

$$Z_1 = 1, \qquad Z_2 = 1 - \frac{Y_\nu^2}{16\pi^2}C_{UV}, \quad \text{(B35)}$$

we obtain

$$\delta m_{H_1}^2 = \frac{Y_\nu^2}{16\pi^2}\mu^2, \quad \text{(B36)}$$

$$\delta m_{H_2}^2 = \frac{Y_\nu^2}{16\pi^2}(|A_\nu|^2 + m_{\tilde{N}}^2 + m_{\tilde{L}}^2)(C_{UV} + 1). \quad \text{(B37)}$$

Finally, we find the following relations between the renormalized parameters and the bare ones:

$$m_{H_1}^2(Q^2) = m_{0H_1}^2 - \frac{Y_\nu^2}{16\pi^2}\mu^2, \quad m_{H_2}^2(Q^2) = m_{0H_2}^2 Z_2 - \frac{Y_\nu^2}{16\pi^2}(|A_\nu^2| + m_{\tilde{N}}^2 + m_{\tilde{L}}^2)(C_{UV} + 1), \quad B(Q^2) = B_0 - \frac{Y_\nu^2}{16\pi^2}A_\nu(C_{UV} + 1), \qquad \mu(Q^2) = \mu_0\sqrt{Z_2}. \quad \text{(B38)}$$

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