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Enhancement of quark number susceptibility with an alternative pattern of chiral symmetry breaking in dense matter

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We explore general features of thermodynamic quantities and hadron mass spectra in a possible phase where chiral $SU(2)_L \times SU(2)_R$ symmetry is spontaneously broken while its center Z_2 symmetry remains unbroken. In this phase, chiral symmetry breaking is driven by a quartic quark condensate although a bilinear quark condensate vanishes. A Ginzburg-Landau free energy leads to a new tricritical point between the Z_2 broken and unbroken phases. Furthermore, a critical point can appear even in the chiral limit where explicit breaking is turned off, instead of a tricritical point at which restoration of chiral and its center symmetries takes place simultaneously. The net quark number density exhibits an abrupt change near the restoration of the center symmetry rather than that of the chiral symmetry. Hadron masses in possible phases are also studied in a linear sigma model. We show that, in the Z_2 symmetric phase, the $\bar{q}q$-type scalar meson with zero isospin $I = 0$ splits from the $\bar{q}q$-type pseudoscalar meson with $I = 1$.

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I. INTRODUCTION

Properties of hot and/or dense QCD matter has been extensively studied within chiral approaches [1]. Our knowledge on the phase structure is, however, still limited and the description of the matter around the phase transitions does not reach a consensus, where a typical size of the critical temperature and chemical potential is considered to be of order $\Lambda_{\rm QCD}$. The phases of QCD are characterized by symmetries and their breaking pattern: QCD at asymptotically high density leads to the color-flavor-locked phase as the true ground state under the symmetry breaking pattern, $SU(3)_c \times SU(3)_L \times SU(3)_R$ down to the diagonal subgroup $SU(3)_{c+L+R}$ [2]. The residual discrete symmetries characterize the spectra of excitations.

At zero temperature and density, an alternative pattern of spontaneous chiral symmetry breaking was suggested in the context of QCD [3–5]. This pattern keeps the center of the chiral group unbroken, i.e. $SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V \times (Z_{N_f})_A$, where a discrete symmetry $(Z_{N_f})_A$ is the maximal axial subgroup of $SU(N_f)_L \times SU(N_f)_R$. The Z_{N_f} symmetry protects a theory from the condensate of quark bilinears $\langle \bar{q}q \rangle$. Spontaneous symmetry breaking is driven by quartic condensates which are invariant under both $SU(N_f)_V$ and Z_{N_f} transformation. Although meson phenomenology with this breaking pattern seems to explain the reality reasonably [3], this possibility is strictly ruled out in QCD both at zero and finite temperatures but at zero density since a different way of coupling of Nambu-Goldstone bosons to pseudoscalar density violates QCD inequalities for density-density correlators [6]. However, this does not exclude the unorthodox pattern in the presence of dense baryonic matter. There are several attempts which dynamically generate a similar breaking pattern in an $O(2)$ scalar model [7] and in $\mathcal{N} = 1$ super Yang-Mills theory [8]. It is of particular interest to explore general features of thermodynamic quantities in the phase associated with this breaking pattern, which was not studied.

Within the Skyrme model on crystal, a new intermediate phase where a skyrmion turns into two half skyrmions was numerically found [9]. This phase is characterized by a vanishing quark condensate $\langle \bar{q}q \rangle$ and a nonvanishing pion decay constant. Recently, another novel view of dense matter, quarkyonic phase, has been proposed based on the argument using the large N_c counting where N_c denotes the number of colors [10]: In the large N_c limit there are three phases which are rigorously distinguished using the Polyakov loop expectation value $\langle \Phi \rangle$ and the baryon number density $\langle N_B \rangle$. The quarkyonic phase is characterized by $\langle \Phi \rangle = 0$ indicating the system confined and nonvanishing $\langle N_B \rangle$ above $\mu_B = M_B$ with a baryon mass M_B. The separation of the quarkyonic from the hadronic phase is not clear any more in a system with finite N_c. Nevertheless, an abrupt change in the baryon number density would be interpreted as the quarkyonic transition which separates meson dominant from baryon dominant regions. This might appear near the boundary for chemical equilibrium at which one would expect a rapid change in the number of degrees of freedom [10,11].

A steep increase in the baryon number density and the corresponding maximum in its susceptibility χ_B are driven by a phase transition from a chirally broken to a restored phase in most model-approaches. Interplay between (de) confinement and chiral symmetry breaking has been studied within a Nambu–Jona-Lassinio model with Polyakov loops [12] which describes how the deconfine-

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ment and chiral phase boundaries are changed from $N_c = \infty$ down to $N_c = 3$ [13]. The model study shows that the chiral phase transition at $T = 0$ appears just above the mass threshold $\mu_B = M_B$ and thus a large χ_B is associated with the chiral phase transition. However, a constituent-quark picture does not directly describe the thermodynamics of hadronic matter and there are no *a priori* reasons that the quarkyonic transition should be accompanied by chiral phase transition. Besides, it seems unlikely that the chiral symmetry is (even partially) restored slightly above the freeze-out curve where the baryon density is not high enough to drive a phase transition. From this perspective, further investigations of dense baryonic matter and a possible appearance of the quarkyonic phase in QCD with $N_c = 3$ require a modeling in terms of dynamical hadronic-degrees of freedom in a systematic way.

In this paper we will address this issue under the alternative pattern of chiral symmetry breaking in dense hadronic matter. The aim of this paper is to discuss a possibility of a new phase in dense QCD and to clarify the characteristic behavior of the bulk thermodynamic quantities and the hadronic spectra in this phase. We will show how an intermediate phase with unbroken center symmetry can appear between chiral symmetry broken phases and its restored phases with analyses using a Ginzburg-Landau free energy. This leads to multiple critical points and one of them is associated with the restoration of the center symmetry rather than that of chiral symmetry. The net baryon number susceptibility exhibits a strong enhancement at the restoration point of the center symmetry although the chiral symmetry remains spontaneously broken. This is reminiscent of the quarkyonic transition and our framework provides a theoretical description of the quarkyonic phase on the bases of a chiral Lagrangian with two distinct order parameters. An analysis using a linear sigma model for the hadron mass spectra is also made.

II. A MODEL FOR 2-QUARK AND 4-QUARK STATES

We construct a chiral Lagrangian for 2- and 4-quark states under the following pattern of symmetry breaking:

$$SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V \times (Z_{N_f})_A \to SU(N_f)_V. \tag{2.1}$$

In this paper we will restrict ourselves to a two-flavor case.

A. Lagrangian

We introduce a 2-quark state M in the fundamental and a 4-quark state Σ in the adjoint representation as[1]

$$M_{ij} \sim \bar{q}_{R,j} q_{L,i}, \qquad \Sigma_{ab} \sim \bar{q}_L \tau_a \gamma_\mu q_L \bar{q}_R \tau_b \gamma^\mu q_R, \tag{2.2}$$

where the flavor indices run $(i, j) = 1, 2$ and $(a, b, c) = 1, 2, 3$ and Pauli matrices $\tau^a = 2T^a$ with $\mathrm{tr}[T^a T^b] = \delta^{ab}/2$. The M and Σ are expressed as

$$\begin{aligned} M_{ij} &= \frac{1}{\sqrt{2}}(\sigma\delta_{ij} + i\phi^a \tau^a_{ij}), \\ \Sigma_{ab} &= \frac{1}{\sqrt{3}}\chi\delta_{ab} + \frac{1}{\sqrt{2}}\epsilon_{abc}\psi_c, \end{aligned} \tag{2.3}$$

where σ and χ represent scalar fields and ϕ and ψ pseudoscalar fields, and ϵ_{ijk} is the total antisymmetric tensor with $\epsilon_{123} = 1$. In general the field Σ contains an isospin 2 state. One can take appropriate parameters in a Lagrangian in such a way that this exotic particle is very heavy. Thus, we will consider only isospin 0 (χ) and 1 (ψ) states in this paper. The fields transform under $SU(2)_L \times SU(2)_R$ as chiral nonsinglet,

$$M \to g_L^{(2)} M g_R^{(2)\dagger}, \qquad \Sigma \to g_L^{(3)} \Sigma g_R^{(3)\dagger}. \tag{2.4}$$

This transformation property implies that the field M changes its sign under the center Z_2 of $SU(2)_L$ [or $SU(2)_R$], while Σ is invariant:

$$M \to -M, \qquad \Sigma \to \Sigma. \tag{2.5}$$

Up to the fourth order in fields one obtains a potential,

$$\begin{aligned} V(M, \Sigma) = &-\frac{m^2}{2}\mathrm{Tr}[MM^\dagger] + \frac{\lambda^2}{4}(\mathrm{Tr}[MM^\dagger])^2 \\ &- \frac{\bar{m}^2}{2}\Sigma_{ab}\Sigma^T_{ba} + \frac{\bar{\lambda}_1^2}{4}\Sigma_{ab}\Sigma^T_{bc}\Sigma_{cd}\Sigma^T_{da} \\ &+ \frac{\bar{\lambda}_2^2}{4}(\Sigma_{ab}\Sigma^T_{ba})^2 + 2g_1\Sigma_{ab}\mathrm{Tr}[T_a M T_b M^\dagger] \\ &+ g_2\Sigma_{ab}\Sigma^T_{ba}\mathrm{Tr}[MM^\dagger] + g_3\mathrm{Det}\Sigma \\ &+ g_4(\mathrm{Det}M + \mathrm{H.c.}). \end{aligned} \tag{2.6}$$

The last term violates the $U(1)_A$ symmetry. The coefficients of the quartic terms are positive for this potential to be bounded. Other parameters g_i can be both positive and negative and will determine the topology of the phase structure. An explicit chiral symmetry breaking can be introduced through, e.g.,

$$V_{\mathrm{SB}}(M, \Sigma) = -h\sigma - \alpha h^2 \chi, \tag{2.7}$$

with constants h and α. Note that a similar Lagrangian was considered for a system with 2- and 4-quark states under the symmetry breaking pattern without unbroken center symmetry in [14] where their 4-quark states are chiral

[1] We consider Σ as any linear combination of $\bar{q}q$-$\bar{q}q$- and $\bar{q}\,\bar{q}$-qq-type fields allowed by symmetries.

singlet and the potential does not include quartic terms in the fields.

B. Ginzburg-Landau effective potential

We first study possible phases derived from the effective potential (2.6) taking[2]

$$M_{ij} = \frac{1}{\sqrt{2}}\sigma\delta_{ij}, \qquad \Sigma_{ab} = \frac{1}{\sqrt{3}}\chi\delta_{ab}. \tag{2.8}$$

One can reduce Eq. (2.6) as well as an explicit breaking term to

$$V(\sigma,\chi) = A\sigma^2 + B\chi^2 + \sigma^4 + \chi^4 - h\sigma + C\sigma^2\chi + D\chi^3 + F\sigma^2\chi^2. \tag{2.9}$$

We will take $C = -1$ without loss of generality in the following calculations.

We start with the potential for $D = F = 0$ and $h = 0$,

$$V = A\sigma^2 + B\chi^2 + \sigma^4 + \chi^4 - \sigma^2\chi. \tag{2.10}$$

Phases from this potential can be classified by the coefficients A and B. The expression of the phase boundaries is summarized in Appendix A. Here we discuss the obtained phase structure shown in Fig. 1. There are three distinct phases characterized by two order parameters: Phase I represents the system where both chiral symmetry and its center are spontaneously broken due to nonvanishing expectation values χ_0 and σ_0. The center symmetry is restored when σ_0 becomes zero. However, chiral symmetry remains broken as long as one has nonvanishing χ_0, indicated by phase II. The chiral symmetry restoration takes place under $\chi_0 \to 0$ which corresponds to phase III. The phases II and III are separated by a second-order line, while the broken phase I from II or from III is by both first- and second-order lines. Accordingly, there exist two tricritical points (TCPs) and one triple point. One of them, TCP_2 in Fig. 1, is associated with the center Z_2 symmetry restoration rather than the chiral transition.

Two phase transitions are characterized by susceptibilities of the corresponding order parameters. We introduce a 2-by-2 matrix composed of the second derivatives of V as

$$\hat{C} = \begin{pmatrix} C_{\sigma\sigma} & C_{\sigma\chi} \\ C_{\chi\sigma} & C_{\chi\chi} \end{pmatrix}, \tag{2.11}$$

with

[2] The potential (2.6) does not exclude a possibility of $\langle\psi\rangle \neq 0$ leading to pion condensation. This corresponds to a further breaking of the symmetry down to $U(1)$. This is favored in a limited range of the parameters. In this paper we will not consider this case but focus on the specific symmetry breaking pattern Eq. (2.1) and their consequences on the hadronic observables. The pion condensation is in fact unfavored when e.g. $\bar{\lambda}_1 = 0$ is taken.

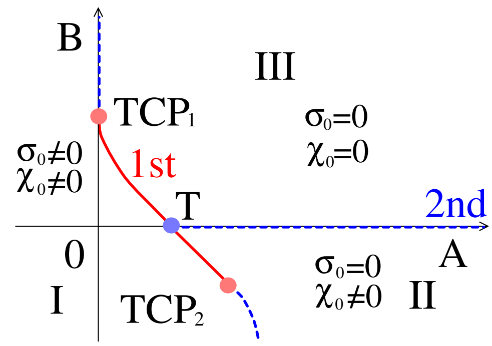

FIG. 1 (color online). Phase diagram with $D = F = 0$ and $h = 0$. The solid and dashed lines indicate first- and second-order phase boundaries, respectively. One tricritical point, TCP_1, is located at $(A, B) = (0, 1/4)$ and another, TCP_2, at $(A, B) = (1/4, -1/8)$. The triple point represented by T is at $(A, B) = (1/8, 0)$.

$$C_{\sigma\sigma} = \frac{\partial^2 V}{\partial\sigma^2}, \qquad C_{\chi\chi} = \frac{\partial^2 V}{\partial\chi^2}, \qquad C_{\sigma\chi} = C_{\chi\sigma} = \frac{\partial^2 V}{\partial\sigma\partial\chi}, \tag{2.12}$$

under the solutions of the gap equations, σ_0 and χ_0. A set of susceptibilities is defined by the inverse of $\hat{C}$ [15];

$$\hat{\chi} = \frac{1}{\det\hat{C}}\begin{pmatrix} C_{\chi\chi} & -C_{\sigma\chi} \\ -C_{\chi\sigma} & C_{\sigma\sigma} \end{pmatrix}. \tag{2.13}$$

We identify the susceptibilities associated with 2-quark and 4-quark states as

$$\chi_{2Q} = \hat{\chi}_{11}, \qquad \chi_{4Q} = \hat{\chi}_{22}. \tag{2.14}$$

The χ_{2Q} is responsible to the Z_2 symmetry and the χ_{4Q} to the chiral symmetry restoration.

We consider χ_{2Q} and χ_{4Q} around the TCP_1 in Fig. 1 where the potential has zero curvature and thus $\det\hat{C} = 0$. When approaching the TCP_1 from the broken phase I by tuning A and B as $A \to A_{\mathrm{critical}} = 0$ and $B = 1/4$, these susceptibilities diverge as

$$\chi_{2Q} \sim t^{-1}, \qquad \chi_{4Q} \sim t^{-2/3}, \tag{2.15}$$

where $A_{\mathrm{critical}} - A \sim t$ with the reduced temperature or chemical potential, e.g. $t = |\mu - \mu_c|/\mu_c$. The gap equations determine the scaling of 2-quark and 4-quark condensates as

$$\sigma_0^2 \sim t^{1/3}, \qquad \chi_0 \sim t^{1/3}. \tag{2.16}$$

Consequently, the quark number susceptibility $\chi_q = -\partial^2 V/\partial\mu^2$ exhibits a singularity as

$$\chi_q \sim \sigma_0^2 \cdot \chi_{2Q} \sim t^{-2/3}. \tag{2.17}$$

This critical exponent is same as the one in the 3-d Ising model. The coincidence can be understood due to the same Z_2 symmetries.[3]

The critical behavior near the TCP_2 involves more: When the A is approached as $1/4 - t$ with $B = -1/8$ fixed, χ_{2Q} and χ_{4Q} diverge as

$$\chi_{2Q} \sim t^{-1}, \qquad \chi_{4Q} \sim t^{-1/2}, \tag{2.18}$$

and only σ_0 vanishes as $\sigma_0^2 \sim t^{1/2}$. As a result, the quark number susceptibility χ_q diverges as

$$\chi_q \sim t^{-1/2}. \tag{2.19}$$

Note that the critical exponent $1/2$ is different from the one near the TCP_1, which may reflect different symmetries possessed by the system at TCP_2, $SU(2)_V$ and the center Z_2, from that at TCP_1, $SU(2)_L \times SU(2)_R$ including its center $(Z_2)_L \times (Z_2)_R$. Those exponents at $\mathrm{TCP}_{1,2}$ are changed when $D \neq 0$ (see below).

When the second-order phase transition separating phase I from II or from III is approached from the broken phase with a fixed B, we have

$$\chi_{2Q} \sim t^{-1}, \qquad \chi_{4Q} \sim \frac{1}{B}, \tag{2.20}$$

where B is a finite number, which thus gives no singularities in χ_{4Q}. The 2-quark condensate scales as $\sigma_0^2 \sim t^1$ and the quark number susceptibility χ_q is finite along the second-order phase transition line:

$$\chi_q \sim \sigma_0^2 \cdot \chi_{2Q} \sim t^0. \tag{2.21}$$

Nevertheless, χ_q is enhanced toward the phase transition induced by χ_{2Q} and becomes small above the phase transition. Such abrupt changes in χ_q indicate the phase transition, especially for a negative B which is driven by the center symmetry restoration rather than the chiral phase transition.

Near the second-order chiral transition between phases II and III, one obtains from $B \sim t$,

$$\chi_0^2 \sim t^1, \qquad \chi_{4Q} \sim t^{-1}. \tag{2.22}$$

Since the chiral symmetry including the center symmetry prohibits the Yukawa-type coupling of χ to a fermion and an antifermion in the fundamental representation, the coupling of χ to the baryon number current would be highly suppressed. Therefore, χ_q shows less sensitivity around the chiral transition.[4]

[3]The Z_2 symmetry in the 3-d Ising system is not the center of the two-flavor chiral group, but emerges in the direction of a linear combination of quark number and scalar densities [16].

[4]As we will show below, the phase transition from phase II to phase III is of first order in a more general parameter choice. Thus, χ_q exhibits a jump at the chiral phase transition point.

Once small h is turned on, chiral symmetry and its center are explicitly broken. Second-order phase boundaries are replaced with a crossover and the two TCPs with two critical points. The singularity in χ_q is now governed by the Z_2 universality class of 3-d Ising systems. Thus, the scaling of χ_q at the critical points (CPs) will be given by

$$\chi_q \sim t^{-2/3}. \tag{2.23}$$

A cubic term in χ modifies the previous phase structure shown in Fig. 1. The phase diagram from the potential,

$$V = A\sigma^2 + B\chi^2 + \sigma^4 + \chi^4 - \sigma^2\chi + D\chi^3, \tag{2.24}$$

is classified by the following regions of D: (a) $-1 < D < 0$, (b) $D \leq -1$, (c) $0 < D < 1$, and (d) $1 \leq D$. One observes a deformation of the boundary lines depending on D as in Fig. 2. The phase transition line separating phase II from phase III becomes of first order due to the presence of $D\chi^3$. For any negative D, (a) and (b), a critical point CP_1 appears as a remnant of TCP_1 for $D = 0$. TCP_2 remains on the phase diagram for $-1 < D < 0$, (a), which eventually coincides with the triple point at $D = -1$, (b). For positive D, (c) and (d), the transition line which separates phase I from phase II turns to be of first order everywhere. The triple point approaches the TCP_1 and coincides when a positive D reaches unity. The different order of phase transition between phase I and phase II for $-1 < D < 0$ to that for $0 < D < 1$ can be understood as follows: For $D = 0$ (see Fig. 1) the vacuum expectation value (VEV) χ_0 is positive in phase I near the phase boundary between phases I and II due to the existence of the $-\sigma^2\chi$ term in the potential. In phase II, on the other hand, when the positive χ_0 provides a local minimum of the potential, $-\chi_0$ also does, and both coincide with the global minima. These two vacua are physically equivalent, so that the phase transition from phase I to phase II can be of second order. When we add the $D\chi^3$ term with negative D to the potential, the local minimum corresponding to the positive χ_0 is only the global minimum in phase II. This can be smoothly connected to the vacuum in phase I where the VEV χ_0 is positive. On the other hand, when D is positive, the negative χ_0 gives the global minimum in phase II. Thus, there is a mismatch of χ_0 along the phase boundary separating phase I from phase II, which indicates a first-order transition.

D also affects the quark number susceptibility χ_q. As in the case of $D = 0$, the χ_q exhibits a more relevant increase toward the Z_2 symmetry restoration than at the chiral phase transition. The critical exponents of χ_q are summarized in Table I. One finds that the two regions, $D \leq 0$ and $0 < D$, correspond to different universality. The cubic term plays a similar role to an explicit symmetry breaking term in the potential. This may be an origin for the appearance of a critical point.

For $-1 < D < 0$, TCP_2 for $h = 0$ becomes a critical point, CP_2, for finite h. When the value of h is increased,

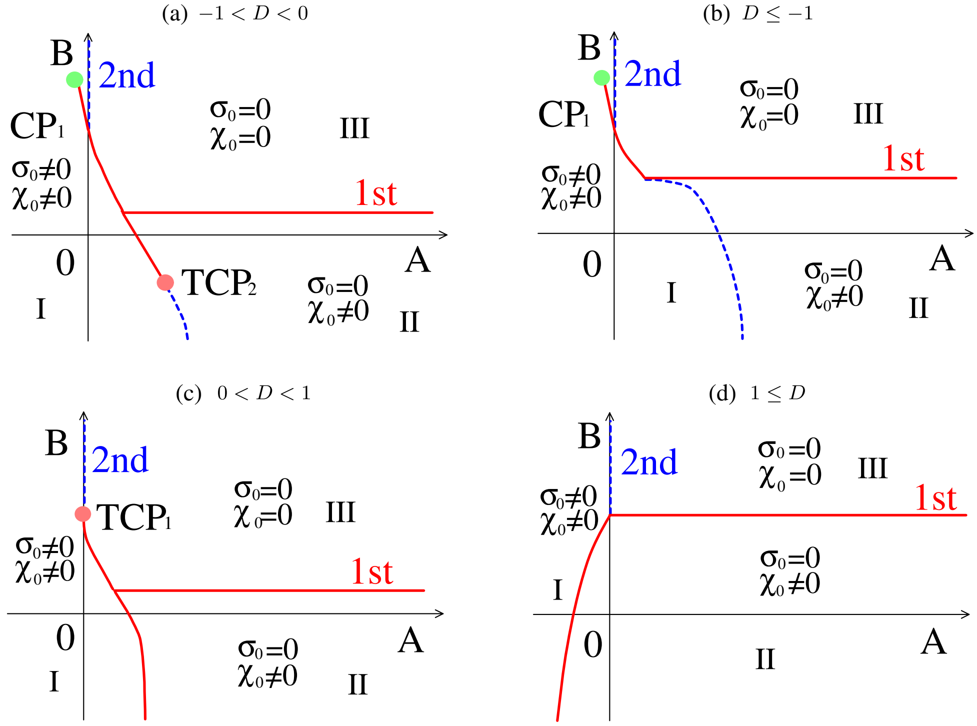

FIG. 2 (color online). Phase diagram for different values of D under $F = 0$ and $h = 0$. The solid and dashed lines indicate first- and second-order phase boundaries, respectively.

TABLE I. The critical exponents of the quark number susceptibility for vanishing and nonvanishing D at two tricritical points (TCP_1 and TCP_2) and at the critical point (CP_1).

	CP_1	TCP_1	TCP_2
$D < 0$	2/3	...	1/2
$D = 0$	...	2/3	1/2
$D > 0$	...	1/2	...

the CP_2 approaches the triple point and coincides with it for a certain value of h, h_0. The topology of the phase diagram for larger $h \geq h_0$ agrees with that for $D \leq -1$. Similarly, the TCP_1 in the $0 < D < 1$ phase diagram becomes a critical point CP_1 and disappears for a sufficiently large h. On the other hand, the CP_1 stays in the phase diagram Fig. 2(a) and 2(b) for any value of h. The scaling of χ_q there will be given by

$$\chi_q \sim t^{-2/3}. \tag{2.25}$$

We note that adding finite F to the potential does not generate any essential differences from the above result with $F = 0$.

III. HYPOTHETICAL PHASE DIAGRAM AND QUARK NUMBER SUSCEPTIBILITY

From the above observations one would expect phase diagrams mapped onto the (T, μ) plane. In the chiral limit a new phase where the center symmetry is unbroken but chiral symmetry remains broken might appear in dense matter since at $\mu = 0$ this phase is strictly forbidden by the no-go theorem. With an explicit breaking of chiral symmetry one would draw a phase diagram as in Fig. 3. The intermediate phase remains characterized by a small condensation $|\sigma_0| \ll |\chi_0|$. One would expect a new critical point associated with the restoration of the center symmetry, CP_2, rather than that of the chiral symmetry if dynamics prefers a negative coefficient of the cubic term in χ. Multiple critical points in principle can be observed as singularities of the quark number susceptibility.

It has been suggested that a similar critical point in lower temperature could appear in the QCD phase diagram based on the two-flavored Nambu–Jona-Lasinio model with vector interaction [17] and a Ginzburg-Landau potential with the effect of an axial anomaly [18]. There the interplay between the chiral (2-quark) condensate and BCS pairings plays an important role. In our framework without diquarks, the critical point discussed in Fig. 3 (left) is driven by the interplay between the 2-quark and 4-quark condensates, and is associated with the restoration of the center symmetry where anomalies have nothing to do with its appearance. Nevertheless, the crossover in low temperatures may have a close connection to the quark-hadron continuity [19] and it is an interesting issue to explore a possibility of dynamical center symmetry breaking in microscopic calculations. The present potential (2.9) leads to a first-order transition of chiral symmetry even with an explicit breaking. This may be replaced with a crossover

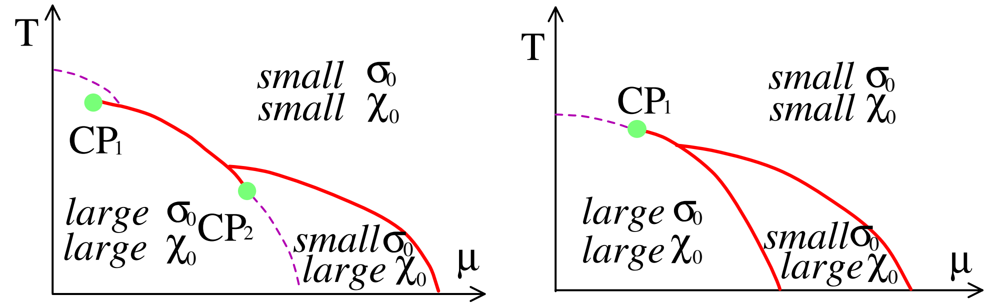

FIG. 3 (color online). Schematic phase diagram mapped onto the (T, μ) plane with a negative D (left) and with a positive D (right). The solid lines indicate first-order phase boundaries, and dashed lines correspond to the crossover.

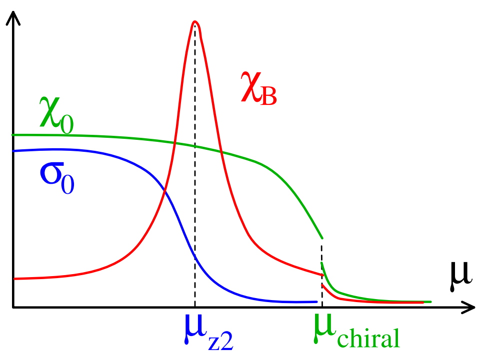

FIG. 4 (color online). The behavior of the baryon number susceptibility as a function of the chemical potential assuming the phase diagram of Fig. 3 (left). The condensates and the susceptibility show a jump also at μ_{z2} when the phase structure of Fig. 3 (right) is preferred.

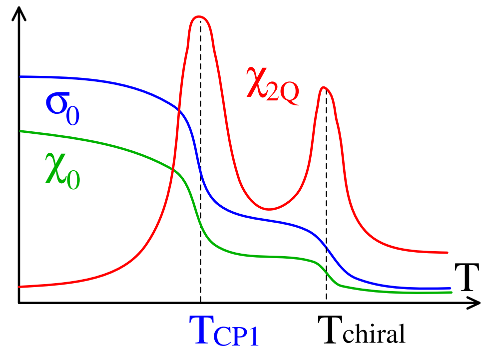

FIG. 5 (color online). A schematic behavior of the susceptibility χ_{2Q} near the CP_1 as a function of the temperature assuming the phase diagram of Fig. 3 (left).

when one considers higher order terms in fields and other symmetry breaking terms as well as in-medium correlations to baryonic excitations, which is beyond the scope of this paper.

The appearance of the above intermediate phase seems to have a similarity to the notion of quarkyonic phase [10,13], which is originally proposed as a phase of dense matter in the large N_c limit. The transition from the hadronic to quarkyonic world can be characterized by a rapid change in the net baryon number density. This feature is driven by the restoration of center symmetry and is due to the fact that the Yukawa coupling of χ to baryons is not allowed by the Z_2 invariance. Figure 4 shows an expected behavior of the quark (baryon) number susceptibility which exhibits a maximum when across the Z_2 crossover. This can be interpreted as the realization of the quarkyonic transition in the $N_c = 3$ world. How far μ_{z2} from μ_{chiral} is depends crucially on its dynamical-model description.[5]

[5]Thus, the present analysis does not exclude the possibility that both transitions take place simultaneously and in such a case enhancement of χ_B is driven by chiral phase transition. The phase with $\chi_0 \neq 0$ and $\sigma_0 = 0$ does not seem to appear in the large N_c limit [5–7]. It would be expected that including $1/N_c$ corrections induce a phase with unbroken center symmetry.

It should be noticed that the critical point in the low density region, indicated by CP_1 in Fig. 3 (left), is different from a usually considered CP [20] in the sense that the CP_1 is not on the crossover line attached to the $T = 0$ axis. When we take a path from the broken phase (both σ_0 and χ_0 are large) to the symmetric phase (both σ_0 and χ_0 are small) passing near the CP_1, the χ_{2Q} may exhibit two peaks; one is located near CP_1 and another is on the crossover line. We show a schematic behavior of χ_{2Q} as a function of temperature, together with σ_0 and χ_0 in Fig. 5. The appearance of two peaks in χ_{2Q} reflects the fact that σ_0 becomes small across the CP_1 and the crossover. The first decrease in σ_0 near CP_1 is caused by a dropping of χ_0, while the second is by the chiral symmetry restoration.

IV. HADRON MASS SPECTRA AND PION DECAY CONSTANT

In this section we derive meson mass spectra in a linear sigma model. The Lagrangian with the potential (2.6) is expressed in terms of the mesonic fields as

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \vec{\phi} \cdot \partial^\mu \vec{\phi}) + \tfrac{1}{2}(\partial_\mu \chi \partial^\mu \chi + \partial_\mu \vec{\psi} \cdot \partial^\mu \vec{\psi}) - \mathcal{U}(\sigma, \phi, \chi, \psi), \quad (4.1)$$

with

$$\mathcal{U} = -\frac{m^2}{2}(\sigma^2 + \vec{\phi}^2) + \frac{\lambda^2}{4}(\sigma^2 + \vec{\phi}^2)^2 - \frac{\bar{m}^2}{2}(\chi^2 + \vec{\psi}^2) + \frac{\bar{\lambda}_1^2}{4}\left[\frac{1}{3}\chi^4 + \frac{2}{3}\chi^2\vec{\psi}^2 + \frac{1}{2}(\vec{\psi}^2)^2\right] + \frac{\bar{\lambda}_2^2}{4}(\chi^2 + \vec{\psi}^2)^2 - g\left[\frac{1}{2\sqrt{3}}\chi(3\sigma^2 - \vec{\phi}^2) + \sqrt{2}\sigma\vec{\phi}\cdot\vec{\psi}\right] + \frac{g_3}{\sqrt{3}}\left(\frac{1}{3}\chi^3 + \frac{1}{2}\chi\vec{\psi}^2\right), \quad (4.2)$$

where $g_1 \equiv -g(g > 0)$ and $g_2 = 0$ were taken. In addition, we set $g_4 = 0$ since the g_4 term generates only a shift in m^2 for $N_f = 2$. We also set the explicit breaking being zero.

The condensate of the mesonic fields in the phase where both the chiral symmetry and its center Z_2 are broken are determined from the coupled gap equations given by

$$\sigma_0^2 = \frac{2}{\sqrt{3}g}\left(\frac{\bar{\lambda}^2}{3}\chi_0^2 - \bar{m}^2 + \frac{g_3}{\sqrt{3}}\chi_0\right)\chi_0, \qquad \chi_0 = \frac{1}{\sqrt{3}g}(\lambda^2\sigma_0^2 - m^2), \quad (4.3)$$

with $\bar{\lambda}^2 \equiv \bar{\lambda}_1^2 + 3\bar{\lambda}_2^2$. Shifting the fields as

$$\sigma \to \sigma + \sigma_0, \qquad \chi \to \chi + \chi_0, \quad (4.4)$$

the potential reads

$$\mathcal{U} = \tfrac{1}{2}m_\sigma^2\sigma^2 + \tfrac{1}{2}m_\phi^2\vec{\phi}^2 + \tfrac{1}{2}m_\chi^2\chi^2 + \tfrac{1}{2}m_\psi^2\vec{\psi}^2 - \sqrt{3}g\sigma_0\sigma\chi - \sqrt{2}g\sigma_0\vec{\phi}\cdot\vec{\psi} + \ldots, \quad (4.5)$$

where ellipses stand for the terms including the fields more than three, and

$$m_\sigma^2 = 2\lambda^2\sigma_0^2, \qquad m_\chi^2 = \frac{\sqrt{3}}{2}\frac{g}{\chi_0}\sigma_0^2 + \frac{2}{3}\bar{\lambda}^2\chi_0^2 + \frac{1}{\sqrt{3}}g_3\chi_0, \qquad m_\phi^2 = \frac{4}{\sqrt{3}}g\chi_0, \qquad m_\psi^2 = \frac{\sqrt{3}}{2}\frac{g}{\chi_0}\sigma_0^2. \quad (4.6)$$

The mass terms thus become

$$\mathcal{U}^{(2)} = \frac{1}{2}(\sigma, \chi)\begin{pmatrix} m_\sigma^2 & -\sqrt{3}g\sigma_0 \\ -\sqrt{3}g\sigma_0 & m_\chi^2 \end{pmatrix}\begin{pmatrix} \sigma \\ \chi \end{pmatrix} + \frac{1}{2}(\vec{\phi}, \vec{\psi})\begin{pmatrix} m_\phi^2 & -\sqrt{2}g\sigma_0 \\ -\sqrt{2}g\sigma_0 & m_\psi^2 \end{pmatrix}\begin{pmatrix} \vec{\phi} \\ \vec{\psi} \end{pmatrix}. \quad (4.7)$$

Obviously, the determinant of the above mass matrix for ϕ and ψ is zero and thus massless pseudoscalar fields are a mixture of 2-quark and 4-quark states.

The mass eigenstates are introduced with a rotation matrix as

$$\begin{pmatrix} S \\ S' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \sigma \\ \chi \end{pmatrix}, \qquad \begin{pmatrix} \vec{P} \\ \vec{P}' \end{pmatrix} = \begin{pmatrix} \cos\bar{\theta} & \sin\bar{\theta} \\ -\sin\bar{\theta} & \cos\bar{\theta} \end{pmatrix}\begin{pmatrix} \vec{\phi} \\ \vec{\psi} \end{pmatrix}, \quad (4.8)$$

with the angles

$$\tan(2\theta) = \frac{2\sqrt{3}g\sigma_0}{m_\chi^2 - m_\sigma^2}, \qquad \tan(2\bar{\theta}) = \frac{4\sqrt{6}\sigma_0\chi_0}{3\sigma_0^2 - 8\chi_0^2}. \quad (4.9)$$

The masses of scalar mesons are given by

$$m_S^2 = m_\sigma^2\cos^2\theta + m_\chi^2\sin^2\theta - \sqrt{3}g\sigma_0\sin(2\theta), \qquad m_{S'}^2 = m_\chi^2\cos^2\theta + m_\sigma^2\sin^2\theta + \sqrt{3}g\sigma_0\sin(2\theta), \quad (4.10)$$

and those of pseudoscalar mesons by

$$m_P = 0, \qquad m_{P'}^2 = \frac{g(3\sigma_0^2 + 8\chi_0^2)}{2\sqrt{3}\chi_0}, \quad (4.11)$$

with

$$\cos\bar{\theta} = \frac{\sqrt{3}\sigma_0}{\sqrt{3\sigma_0^2 + 8\chi_0^2}}, \qquad \sin\bar{\theta} = \frac{2\sqrt{2}\chi_0}{\sqrt{3\sigma_0^2 + 8\chi_0^2}}. \quad (4.12)$$

The pion decay constant is read from the Noether current, $J_A^\mu \sim \sigma_0\partial^\mu\phi + 4/\sqrt{6}\chi_0\partial^\mu\psi$, as

$$F_\pi = \sqrt{\sigma_0^2 + \frac{8}{3}\chi_0^2}. \quad (4.13)$$

Since we consider a system in the chiral limit, the massive P' state is decoupled from the current and $F_{\pi'} = 0$, as it should be. It should be noted that, when $|\sigma_0| \gg |\chi_0|$, the Nambu-Goldstone (NG) boson is dominantly the 2-quark state. The 4-quark component becomes more relevant for $\sqrt{3}|\sigma_0| < \sqrt{8}|\chi_0|$, i.e. $\bar{\theta} > \pi/4$.

When the coupling g_3 is negative, which corresponds to $D < 0$ in the Ginzburg-Landau potential given in Sec. II, the phase transition from phase I ($\sigma_0 \neq 0$ and $\chi_0 \neq 0$) to phase II ($\sigma_0 = 0$ and $\chi_0 \neq 0$) can be of second order. In such a case, the restoration of the center Z_2 symmetry is characterized by vanishing σ_0. Approaching the restoration from the broken phase, one finds the lowest scalar meson mass degenerate with the P state, while the pion decay constant remains finite due to $\chi_0 \neq 0$;

$$m_S \to m_P = 0, \qquad F_\pi \to \sqrt{\frac{8}{3}}\chi_0, \quad (4.14)$$

with

$$\chi_0 = \sqrt{\frac{3\bar{m}^2}{\bar{\lambda}^2} + \left(\frac{\sqrt{3}g_3}{2\bar{\lambda}^2}\right)^2} - \frac{\sqrt{3}g_3}{2\bar{\lambda}^2}. \quad (4.15)$$

The vanishing S-state mass corresponds to a divergence of the susceptibility χ_{2Q}, which is responsible to the restoration of the center symmetry. The scalar S and pseudoscalar P states thus become the chiral partners on the phase boundary. In the Z_2 symmetric phase the meson masses are found from the potential (4.2) as

$$m_\sigma^2 = -m^2 - \sqrt{3}g\chi_0, \qquad m_\phi^2 = -m^2 + \frac{g}{\sqrt{3}}\chi_0,$$
$$m_\chi^2 = \frac{2}{3}\bar{\lambda}^2\chi_0^2 + \frac{g_3}{\sqrt{3}}\chi_0, \qquad m_\psi^2 = 0. \tag{4.16}$$

There is no mixing in this phase, $\tan\theta = \tan\bar{\theta} = 0$, so that σ, ϕ, χ, ψ are the mass eigenstates.[6] This implies that the pure 4-quark state ψ is the massless NG boson in the Z_2 symmetric phase. Because of the broken chiral symmetry, σ and ϕ states are not degenerate in mass.[7] The vector and axial-vector states are neither degenerate in mass [6], since both vector and axial-vector currents are invariant under the Z_2 transformation but broken chiral symmetry does not dictate the same masses.

When $|g_3/g| \ll 1$, the chiral phase transition from phase II ($\sigma_0 = 0$ and $\chi_0 \neq 0$) to phase III ($\sigma_0 = 0$ and $\chi_0 = 0$) will be of weak first order. In this case, χ_0 then F_π approach zero near the phase transition point. This is controlled by $\bar{m}$ approaching zero, which corresponds to B approaching zero in the Ginzburg-Landau potential discussed in Sec. II B. The isospin 2 state will become very light near the phase transition. This may suggest that, when the $g_3\mathrm{Det}\Sigma$ term is small and the chiral phase transition is of weak first order, a light exotic state with $I = 2$ might exist in dense baryonic matter. When there exists the non-negligible $g_3\mathrm{Det}\Sigma$ term, on the other hand, such a state never becomes light since the chiral phase transition is of strong first order.

In two flavors, the system would prefer the parity doubling for baryons in the Z_2 symmetric phase where the VEV χ_0 does not generate the baryon masses [6]. In the parity doubling scenario [21], all the baryons have their parity partners and then each pair of parity partners has a degenerate mass. On the other hand, in the naive scenario the lightest baryon does not have a parity partner, so that it becomes massless in the Z_2 symmetric phase. We list hadron mass spectra expected in phase I and phase II in Table II.

[6]When we approach the phase boundary from the Z_2 symmetric phase to the Z_2 broken phase, m_σ^2 in Eq. (4.16) approaches zero, since $-m^2 = \sqrt{3}g\chi_0$ is satisfied at the phase boundary. The pseudoscalar mass m_ϕ^2 approaches $\frac{4}{\sqrt{3}}g\chi_0$ which coincides with the mass of P' in the Z_2 broken phase [see Eq. (4.11)].

[7]In Ref. [6] the degeneracy of the massive scalar and pseudoscalar mesons made of 4-quarks carrying the same isospin for a general number of flavors was shown. In the case of $N_f = 2$ the $U(1)_A$ anomaly generates a mass difference between the σ state and the pseudoscalar meson with $I = 0$ (η). In the present analysis, we did not include the $I = 0$ pseudoscalar and the $I = 1$ scalar mesons from the beginning by assuming that they are very heavy.

TABLE II. The mass spectra of mesons and baryons in different phases for $N_f = 2$. Baryons transform with the naive chirality assignment as $\psi_{R,L} \to g_{R,L}\psi_{R,L}$, while with the mirror assignment as $\psi_{1R,L} \to g_{R,L}\psi_{1R,L}$ and $\psi_{2R,L} \to g_{L,R}\psi_{2R,L}$ with $g_{R,L} \in SU(2)_{R,L}$ where two nucleons ψ_1 and ψ_2 belong to the same chiral multiplets.

Phase I: $\sigma_0 \neq 0$, $\chi_0 \neq 0$	Phase II: $\sigma_0 = 0$, $\chi_0 \neq 0$
$SU(2)_V$	$SU(2)_V \times (Z_2)_A$
$m_S \neq 0$, $m_P = 0$	$m_S \neq m_P \neq 0$, $m_{P'} = 0$
$m_V \neq m_A$	$m_V \neq m_A$
$F_\pi = \sqrt{\sigma_0^2 + (8/3)\chi_0^2}$	$F_\pi = \sqrt{8/3}\chi_0$
$m_{N^+} \neq 0$	(i) naive: $m_{N^+} = 0$ (ground state); $m_{N'^+} = m_{N'^-} \neq 0$ (excited states)
	(ii) mirror: $m_{N^+} = m_{N^-} \neq 0$ (all states)

V. CONCLUSIONS

We have discussed a new phase where chiral symmetry is spontaneously broken while its center symmetry is restored. This might appear as an intermediate state between chirally broken and restored phases in the (T, μ) plane. The appearance of the intermediate phase with unbroken Z_2 also suggests a new critical point associated with the center symmetry in low temperatures. A tendency of the center symmetry restoration is carried by the net baryon number density which shows a rapid increase and this is reminiscent of the quarkyonic transition. The $U(1)_A$ symmetry remains broken and the heavy η mass can be controlled with a certain anomaly coefficient.

There are subtleties in baryon masses since the existence of the center symmetry does not immediately dictate the parity doubling for a general number of flavors: Here we consider the case in massless three flavors. The g_3 term in (2.6) now generates a χ^8 contribution, while the g_4 term does a σ^3 one. It follows that $D\chi^3$ is removed from (2.9) and another cubic term σ^3 is added. Omitting the cubic term σ^3 results in the same phase diagram as Fig. 1 with two TCPs. When the cubic term σ^3 is included, it is conceivable that phase II and phase III in Fig. 2 are separated by a second-order phase boundary, which will become a first-order one when we take quantum fluctuations into account [22]. The topologies are expected to be quite similar to those shown in Fig. 2, so that we expect a strong enhancement of the quark number susceptibility at the Z_3 restoration point. Unlike the case for $N_f = 2$, the Σ_{ab} field is allowed to couple to the octet baryon states as, e.g. $\bar{B}_a\Sigma_{ab}B_b$, and the baryon number current couples to the χ state which becomes massless at the chiral restoration point. As a result, the quark number susceptibility might show another peak at the chiral restoration. Hadron masses in the Z_3 symmetric phase are slightly different from those under Z_2 invariance: In the mesonic sector the

TABLE III. Same as in Table II but for $N_f = 3$.

Phase I: $\sigma_0 \neq 0$, $\chi_0 \neq 0$	Phase II: $\sigma_0 = 0$, $\chi_0 \neq 0$
$SU(3)_V$	$SU(3)_V \times (Z_3)_A$
$m_S \neq 0$, $m_P = 0$	$m_S = m_P \neq 0$, $m_{P'} = 0$
$m_V \neq m_A$	$m_V \neq m_A$
$m_{N^+} \neq 0$	(i) naive: $m_{N^+} \neq 0$
	(ii) mirror: $m_{N^+} = m_{N^-} \neq 0$

party partners are degenerate and the degeneracy does not generally occur in the baryonic sector [6]. Following Ref. [6], possible operators for the baryons are expressed as

$$\begin{aligned} B_L^1 &= (q_L q_L q_R)_L, & B_L^2 &= (q_L q_R q_R)_L, \\ B_L^3 &= (q_L q_L q_L)_L, & B_R^1 &= (q_R q_R q_L)_R, \\ B_R^2 &= (q_R q_L q_L)_R, & B_R^3 &= (q_R q_R q_R)_R, \end{aligned} \tag{5.1}$$

where the color and flavor indices are omitted. For the octet baryons, the representations under the chiral $SU(3)_L \times SU(3)_R$ of these baryonic fields are assigned as

$$\begin{aligned} B_L^1 &\sim (\bar{3}, 3), & B_L^2 &\sim (3, \bar{3}), & B_L^3 &\sim (8, 1), \\ B_R^1 &\sim (3, \bar{3}), & B_R^2 &\sim (\bar{3}, 3), & B_R^3 &\sim (1, 8). \end{aligned} \tag{5.2}$$

When the B^3 is the lightest octet baryon, which we call the naive assignment, it is still massive in the Z_3 symmetric phase, since the Yukawa coupling of the 4-quark state Σ_{ab} is possible as, e.g. $\bar{B}_a \Sigma_{ab} B_b$. When the lightest baryons are described by a combination of B^1 and B^2, which we call the mirror assignment, they are degenerate with each other in the Z_3 symmetric phase. We summarize these features in Table III. The baryon masses crucially depend on a way of chirality assignment. It would be an interesting issue to clarify this within a more elaborated model.

The main assumption in this paper is a dynamical breaking of chiral symmetry $SU(N_f)_L \times SU(N_f)_R$ down to a nonstandard $SU(N_f)_V \times (Z_{N_f})_A$ although this seems to be theoretically self-consistent. Calculations using the Swinger-Dyson equations or Nambu–Jona-Lasinio–type models with careful treatment of the quartic operators may directly evaluate this reliability. Besides, anomalously light NG bosons, $m_\pi^2 \sim \mathcal{O}(m_q^2)$, could lead to an s-wave pion condensation as discussed in [23]. It is interesting to explore how this phase is embedded in the current analysis. This will be reported elsewhere. A calculation using the Skyrme model shows a similar intermediate phase [9]. Although the above nonstandard pattern of symmetry breaking was not imposed in the Skyrme Lagrangian, the result could suggest an emergent symmetry in dense medium. This intermediate phase would be an intriguing candidate of the quarkyonic phase if it could sustain in actual QCD at finite density and would lead to a new landscape of dense baryonic matter.

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APPENDIX A: PHASE BOUNDARIES FROM GINZBURG-LANDAU POTENTIAL

The relevant expressions for the phase boundaries obtained from the potential (2.9) are given below. We will take $D = F = 0$ and the chiral limit $h = 0$.

(i) Second-order phase transition when $B \geq 1/4$:

$$A = 0. \tag{A1}$$

The solutions for σ and χ on this boundary are given by

$$(\sigma_0, \chi_0) = (0, 0). \tag{A2}$$

(ii) First-order phase transition when $0 \leq B < 1/4$ and $0 < A \leq 1/8$:

$$A = \left(\frac{3}{8} - \sqrt{\left(\frac{3}{8}\right)^2 + \frac{1}{2}\left(B - \frac{1}{4}\right)}\right) \times \sqrt{-\frac{3}{8} - 2\left(B - \frac{1}{4}\right) + \sqrt{\left(\frac{3}{8}\right)^2 + \frac{1}{2}\left(B - \frac{1}{4}\right)}}. \tag{A3}$$

The solutions for σ and χ on this boundary are given by

$$(\sigma_0, \chi_0) = (0, 0), \tag{A4}$$

$$\left(\pm\left[\frac{1}{2}\left(\frac{1}{8} + \sqrt{\left(\frac{3}{8}\right)^2 + \frac{1}{2}\left(B - \frac{1}{4}\right)}\right) \times \sqrt{-\frac{3}{8} - 2\left(B - \frac{1}{4}\right) + \sqrt{\left(\frac{3}{8}\right)^2 + \frac{1}{2}\left(B - \frac{1}{4}\right)}}\right]^{1/2}, \frac{1}{2}\sqrt{-\frac{3}{8} - 2\left(B - \frac{1}{4}\right) + \sqrt{\left(\frac{3}{8}\right)^2 + \frac{1}{2}\left(B - \frac{1}{4}\right)}}\right). \tag{A5}$$

(iii) First-order phase transition when $-1/8 < B < 0$ and $1/8 < A < 1/4$:

$$A = \frac{1}{8} - B. \tag{A6}$$

The solutions for σ and χ on this boundary are given by

$$(\sigma_0, \chi_0) = \left(0, \pm\sqrt{\frac{-B}{2}}\right), \qquad \left(\pm\sqrt{\frac{1}{2}\left(B + \frac{1}{8}\right)}, \frac{1}{4}\right). \tag{A7}$$

(iv) Second-order phase transition when $B \leq -1/8$ and $A \geq 1/4$:

$$A = \sqrt{-\frac{B}{2}}. \tag{A8}$$

The solutions for σ and χ on this boundary are given by

$$(\sigma_0, \chi_0) = \left(0, \pm\sqrt{\frac{-B}{2}}\right). \tag{A9}$$

(v) Second-order phase transition when $A > 1/8$:

$$B = 0. \tag{A10}$$

The solutions for σ and χ on this boundary are given by

$$(\sigma_0, \chi_0) = (0, 0). \tag{A11}$$

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