Strong decays of newly observed D_{sJ} states in a constituent quark model with effective Lagrangians

Xian-Hui Zhong[1,3,*] and Qiang Zhao[2,3,†]

[1]*Department of Physics, Hunan Normal University, and Key Laboratory of Low-Dimensional Quantum Structures and Quantum Control of Ministry of Education, Changsha 410081, People's Republic of China*
[2]*Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, People's Republic of China*
[3]*Theoretical Physics Center for Science Facilities, CAS, Beijing 100049, People's Republic of China*
(Received 12 November 2009; published 29 January 2010)

The strong decay properties of the newly observed states $D_{sJ}(3040)$, $D_{sJ}(2860)$, and $D_{sJ}(2710)$ are studied in a constituent quark model with quark-meson effective Lagrangians. We find that the $D_{sJ}(3040)$ could be identified as the low-mass physical state $|2P_1\rangle_L$ ($J^P = 1^+$) from the $D_s(2^1P_1)$-$D_s(2^3P_1)$ mixing. The $D_{sJ}(2710)$ is likely to be the low-mass mixed state $|(SD)\rangle_L$ via the 1^3D_1-2^3S_1 mixing. In our model, the $D_{sJ}(2860)$ cannot be assigned to any single state with a narrow width and compatible partial widths to DK and D^*K. Thus, we investigate a two-state scenario as proposed in the literature. In our model, one resonance is likely to be the 1^3D_3 ($J^P = 3^-$), which mainly decays into DK. The other resonance seems to be the $|1D_2{}'\rangle_H$, i.e. the high-mass state in the 1^1D_2-1^3D_2 mixing with $J^P = 2^-$, of which the D^*K channel is its key decay mode. We also discuss implications arising from these assignments and give predictions for their partner states such as $|(SD)'\rangle_H$, $|2P_1'\rangle_H$, 2^3P_0, and 2^3P_2, which could be helpful for the search for these new states in future experiments.

DOI: 10.1103/PhysRevD.81.014031 PACS numbers: 12.39.Fe, 12.39.Jh, 13.25.Ft, 13.25.Hw

I. INTRODUCTION

Experimental progress on the study of D and D_s states in the past few years provides a great opportunity for theory development. Recently a new broad resonance $D_{sJ}(3040)$ with a mass of $(3044 \pm 8_{\rm stat}(^{+30}_{-5})_{\rm syst})$ MeV and a width of $\Gamma = (239 \pm 35_{\rm stat}(^{+46}_{-42})_{\rm syst})$ MeV is reported in the D^*K channel [1]. Apart from the $D_{sJ}(3040)$ another two states $D_{sJ}(2710)$ and $D_{sJ}(2860)$, which were observed by *BABAR* and Belle two years ago [2,3], are also examined. Their branching ratio fractions between D^*K and DK are measured [1]

$$\frac{D_{sJ}(2710)^+ \to D^*K}{D_{sJ}(2710)^+ \to DK} = 0.91 \pm 0.13_{\rm stat} \pm 0.12_{\rm syst}, \quad (1)$$

$$\frac{D_{sJ}(2860)^+ \to D^*K}{D_{sJ}(2860)^+ \to DK} = 1.10 \pm 0.15_{\rm stat} \pm 0.19_{\rm syst}. \quad (2)$$

These new observations stimulate great interest in the understanding of their nature and strong coupling properties in theory. Different theoretical approaches for the study of the strong coupling properties of heavy-light mesons can be found in the literature, such as the heavy quark effective field theory approach [4–18], QCD sum rules [18–20], $^3P_0$ model [21–25], and chiral quark model [26].

In this work, we present an analysis of these D_s states in a constituent quark model with effective Lagrangians for the quark-meson couplings, and try to clarify the following issues: (i) To gain information about the structure of the newly observed state $D_{sJ}(3040)$ according to its strong decay properties. (ii) With the new data for the $D_{sJ}(2710)$ and $D_{sJ}(2860)$, we reanalyze their strong decays and examine their structures again. The quantum numbers of these two states remain controversial. The $D_{sJ}(2710)$ is identified as a state of $J^P = 1^-$ in B decays [3], while it is explained by various models as the 2^3S_1, 1^3D_1, the admixtures of 2^3S_1-1^3D_1, molecular structure, or tetraquark state [16,21,24,25,27]. There are also a lot of solutions proposed for the $D_{sJ}(2860)$. The assignments of 1^3D_3 or 2^3P_0 have been discussed in Refs. [21,24,25,28–31]. A recent comment by Ref. [32] suggests a two-state structure for the $D_{sJ}(2860)$ in order to understand the controversial aspects arising from its strong decays. (iii) The quark-model assignment of these states will result in implications of their partner multiplets. We discuss some of those relevant states, for which the experimental observations would be able to clarify some of those theoretical and experimental issues.

By treating the light mesons (pseudoscalar and vector mesons) as effective fields, we introduce constituent-quark-meson couplings to describe the charmed meson strong decays into a charmed meson plus a light pseudoscalar or vector meson in the final state. The quark-pseudoscalar-meson coupling is given by the chiral quark model at the leading order as proposed by Manohar and Georgi [33]. Its application to pseudoscalar meson photoproduction in the quark model turns out to be promising and many low-energy phenomena can be highlighted in such a framework [34–38]. In particular, the axial current conservation allows one to extract the axial coupling in

*zhongxh@ihep.ac.cn
†zhaoq@ihep.ac.cn

terms of the meson decay constant and a form factor arising from the microscopic quark model wave functions [39]. With an effective quark-vector-meson coupling, one can also extract the vector couplings in a similar way [39–42].

A natural extension of this picture is to apply this effective Lagrangian approach to heavy-light meson strong decays involving light pseudoscalar or vector mesons, which would be a good place to examine the validity of the light axial and vector fields in such a transition. On the one hand, the quark-meson coupling is the same as that defined in meson photoproduction which is proportional to the meson decay constant. On the other hand, the heavy-light meson in the initial and final state would provide information about the coupling form factor and can be calculated in the quark model framework. Thus, one can study the heavy-light meson strong decays by combining dynamical information from meson photoproduction off nucleons.

The paper is organized as follows. In Sec. II, a brief review of the quark-meson effective Lagrangian approach is given. The numerical results are presented and discussed in Sec. III. Finally, a summary is given in Sec. IV.

II. FRAMEWORK

In Fig. 1, we illustrate the similarity of the quark-meson coupling in meson-baryon and light-meson production in heavy-light meson strong decays. It should be pointed out that since the flavor symmetries beyond the SU(3) are badly broken, the contributions from transitions of treating the final-state heavy-light meson as an effective field are strongly suppressed. We thus can neglect those contributions safely in our approach. An early study of the charmed meson strong decays can be found in Ref. [31].

In the chiral quark model [33], the low-energy quark-pseudoscalar-meson interactions in the SU(3) flavor basis are described by the effective Lagrangian [35–38,43]

$$\mathcal{L}_{Pqq} = \sum_j \frac{1}{f_m} \bar{\psi}_j \gamma_\mu^j \gamma_5^j \psi_j \partial^\mu \phi_m, \tag{3}$$

where ψ_j represents the j-th quark field in the hadron, and ϕ_m is the pseudoscalar meson field.

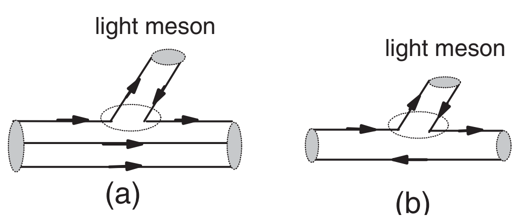

FIG. 1. Diagrams (a) and (b) stand for the quark-meson couplings in meson-baryon interactions and light-meson production in heavy-light meson strong decays, respectively.

The effective Lagrangian for quark-vector-meson interactions in the SU(3) flavor basis is [40–42]

$$\mathcal{L}_{Vqq} = \sum_j \bar{\psi}_j \left(a\gamma_\mu^j + \frac{ib}{2m_j}\sigma_{\mu\nu}q^\nu \right) V^\mu \psi_j, \tag{4}$$

where V^μ represents the vector meson field with four-vector moment q. Parameters a and b denote the vector and tensor coupling strength, respectively.

As follows, we provide the quark-pseudoscalar and quark-vector-meson coupling operators in a nonrelativistic form [35–38,40–43]. Considering light-meson emission in a heavy-light meson strong decay, the effective quark-pseudoscalar-meson coupling operator in the center-of-mass (c.m.) system of the initial meson is

$$H_m = \sum_j \left[-\left(1 + \frac{\omega_m}{E_f + M_f}\right) \boldsymbol{\sigma}_j \cdot \mathbf{q} + \frac{\omega_m}{2\mu_q} \boldsymbol{\sigma}_j \cdot \mathbf{p}_j \right] I_j \varphi_m. \tag{5}$$

In a case that a light vector meson is emitted, the transition operators for producing a transversely or longitudinally polarized vector meson are as follows:

$$H_m^T = \sum_j \left\{ i\frac{b'}{2m_q} \boldsymbol{\sigma}_j \cdot (\mathbf{q} \times \boldsymbol{\epsilon}) + \frac{a}{2\mu_q} \mathbf{p}_j \cdot \boldsymbol{\epsilon} \right\} I_j \varphi_m, \tag{6}$$

and

$$H_m^L = \sum_j \frac{aM_v}{|\mathbf{q}|} I_j \varphi_m. \tag{7}$$

In the above three equations, $\mathbf{q}$ and ω_m are the three-vector momentum and energy of the final-state light meson, respectively. $\mathbf{p}_j$ is the internal momentum operator of the j-th quark in the heavy-light meson rest frame. $\boldsymbol{\sigma}_j$ is the spin operator on the j-th quark of the heavy-light system and μ_q is a reduced mass given by $1/\mu_q = 1/m_j + 1/m'_j$ with m_j and m'_j for the masses of the j-th quark in the initial and final mesons, respectively. Here, the j-th quark is referred to the active quark involved at the quark-meson coupling vertex. M_v is the mass of the emitted vector meson. The plane wave part of the emitted light meson is $\varphi_m = e^{-i\mathbf{q}\cdot\mathbf{r}_j}$, and I_j is the flavor operator defined for the transitions in the SU(3) flavor space [31,35–38,40,43,44]. Parameters a and b are the vector and tensor coupling strengths of the quark-vector-meson couplings, respectively. Studies of vector meson photoproduction [41,42,45,46] suggest that $a = g_{\omega qq} = g_{\rho qq} \simeq -3$ and $b' \equiv b - a \simeq 5$. Because of vector current conservation, one has $a = g_{\rho NN} = g_{\omega NN}/3$ [39,40,42].

The heavy-light meson wave functions have been given in Ref. [31], and some of the decay amplitudes have also been deduced there. In the charmed meson decays, the SU(4) flavor symmetry is broken. Thus, the charm quark is treated as a spectator and the transition amplitude is proportional to the final-state light meson decay constant associated with a form factor arising from the convolution of the initial- and final-state charmed meson wave functions.

In the calculation, the standard quark model parameters are adopted. Namely, we set $m_u = m_d = 330$ MeV, $m_s = 450$ MeV, and $m_c = 1700$ MeV for the constituent quark masses. The harmonic oscillator parameter β is usually adopted in the range of (0.4–0.5) GeV; in this work we take it as $\beta = 0.45$ GeV. The decay constants for K and η mesons are $f_K = f_\eta = 160$ MeV. As shown in Refs. [31,44], the flavor symmetry breaking will lead to corrections to the quark-pseudoscalar-meson coupling vertex, for which an additional global parameter δ is introduced. Here, we fix its value the same as that in Refs. [31,44], i.e. $\delta = 0.557$. For the quark-vector-meson coupling strength which still suffers relatively large uncertainties, we adopt the values extracted from vector meson photoproduction as mentioned earlier, i.e. $a \simeq -3$ and $b' \simeq 5$. The masses of the mesons used in the calculations are adopted from the Particle Data Group [47].

Justification of the nonrelativistic formulation is not obvious for the light quark sector in the heavy-light meson transitions. This is similar to the case of a nonrelativistic quark model for baryons, where the results would rely on the experimental data to tell how far they deviate from reality. Treating the light meson as a chiral field somehow assumes that the light meson is produced at short distance, and the spectators (i.e. the two spectator quarks inside a baryon or the heavy quark in the heavy-light meson transitions) do not respond to the internal structure of the light meson. Instead, the propagation of the light quark pair would feel the hadronic environment from the convolution of initial- and final-state heavy-light mesons. Such an implicated assumption means that only the processes with relatively small momentum transfers between the light quarks inside the light meson would dominantly contribute to the transition matrix element. This empirically supports the validity of the nonrelativistic formulation as a leading order approximation.

III. RESULTS AND DISCUSSIONS

A. $D_{sJ}(3040)$

The $D_{sJ}(3040)$ is observed in the D^*K mode, while there is no sign of $D_{sJ}(3040) \to DK$ in experiment [1]. This allows its quantum number to be $J^P = 0^-$, 1^+, 2^-, etc. The $J^P = 0^-$ state 2^1S_0 seems not a good candidate since its predicted mass, ~2.7 GeV [21,26,48,49], is much less than 3.04 GeV. The predicted masses of $J^P = 1^+$ and 2^- are close to 3.04 GeV. We hence discuss these two possibilities for the $D_{sJ}(3040)$ in this work.

First, we considered it as the $J^P = 1^+$ states 2^1P_1 and 2^3P_1. These two states also can decay into D^*K, DK^*, $D_s^*\eta$, $D_s\phi$, $D_0(2400)K$, $D_1(2430)K$, $D_1(2420)K$, $D_2(2460)K$, $D_s(2317)\eta$, $D_s(2460)\eta$. With a mass of 3.04 GeV, we calculate their decay widths, which are listed in Table I. From the table, it is found that the decay width of 2^1P_1 and 2^3P_1 are ~115 MeV and ~93 MeV, respectively, which are too small to compare with the data, although the decay mode, dominated by the D^*K, is consistent with the observation [1]. Thus, the $D_{sJ}(3040)$ may not be considered as pure 2^1P_1 or 2^3P_1 state.

Since the heavy-light mesons are not charge conjugation eigenstates, state mixing between spin $\mathbf{S} = 0$ and $\mathbf{S} = 1$ states with the same J^P can occur via the spin-orbit interactions [22,50,51]. The physical states with $J^P = 1^-$ can then be described as

$$|2P_1\rangle_L = +\cos(\phi)|2^1P_1\rangle + \sin(\phi)|2^3P_1\rangle, \tag{8}$$

$$|2P_1{}'\rangle_H = -\sin(\phi)|2^1P_1\rangle + \cos(\phi)|2^3P_1\rangle, \tag{9}$$

where the subscripts L and H stand for the low mass and high mass of the physical states after the mixing.

Usually, the low-mass state has a broad width while the high-mass state has a narrow width. We set the mass of $|2P_1\rangle_L$ with 3.04 GeV, and plot its decay width as a function of the mixing angle ϕ, which is shown in Fig. 2. It shows that when the mixing angle is in the range $\phi \simeq -(40 \pm 12)°$, the total decay width, $\Gamma = (162 \sim 170)$ MeV, is in the range of the experimental data (close to the lower limit of the data) [1]. The mixing angle predicted here is consistent with the result $\phi \simeq -55°$ in the heavy quark limit [22,50,51]. The D^*K governs the decays of $|2P_1\rangle_L$, while the DK channel is forbidden. This is also in agreement with the observations. These results suggest that the $D_{sJ}(3040)$ favors the $|2P_1\rangle_L$ classification.

Apart from the D^*K mode, the $D_1(2430)K$, $D_2(2460)K$, $D_0(2400)K$, DK^*, and $D_s^*\eta$ are also important in the decays of $|2P_1\rangle_L$ as shown by Fig. 2. In particular, the partial widths of $D_1(2420)K$, $D_s(2317)\eta$, and D^*K^* turn out to be sizable. A search for those channels would be useful for clarifying the property of the $D_{sJ}(3040)$. With the mixing angle $\phi \simeq -55°$, the relative decay ratios

TABLE I. The decay widths (MeV) for the $D_{sJ}(3040)$ as 1^1D_2, 1^3D_2, 2^1P_1, and 2^3P_1 candidates.

	D^*K	DK^*	D^*K^*	$D_s^*\eta$	$D(2430)K$	$D(2420)K$	$D_s(2460)\eta$	$D_s\phi$	$D(2400)K$	$D(2460)K$	$D_s(2317)\eta$	total
1^1D_2	197	27	2	25	3	2	0.4	4	3	345	4	608
1^3D_2	256	21	33	34	1	18	0.01	0.05	3.4	512	1.6	879
2^1P_1	44	9	0.3	5.5	0.02	0.01	7.5×10^{-5}	0.1	33	12	11	115
2^3P_1	41	2	2.5	7.5	24	7	0.5	0.002	0.09	9	0.06	93

among those decay channels are

$$D^*K{:}D_1(2430)K{:}D_2(2460)K{:}D_0(2400)K{:}D_1(2420)K{:}DK^*{:}D_s(2317)\eta{:}D_s^*\eta{:}D^*K^* = 78{:}17{:}19{:}13{:}4{:}8{:}4{:}11{:}2.$$

Since the mass of $D_s(3040)$ still has a large uncertainty, it may bring uncertainties to the theoretical predictions on the decay widths. To investigate this effect, we plot the decay widths as a function of the mass in Fig. 3 with the mixing angle fixed at $\phi = -50°$. It shows that the mass uncertainty gives rise to an uncertainty of about ~70 MeV in the total decay width. The predicted widths are much closer to the central value of the data with the increasing mass. The sensitivity of different decay modes to the mass can also be seen clearly in the plot.

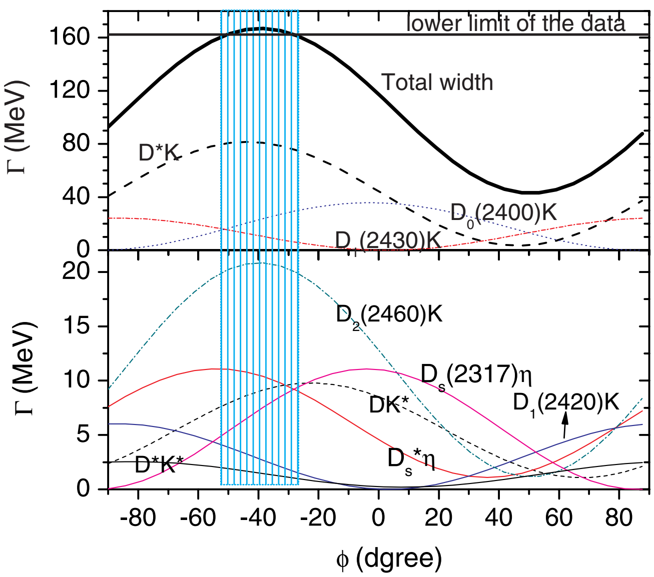

FIG. 2 (color online). The partial decay widths and total width of $|2P_1\rangle_L$ with a mass of 3040 MeV as functions of mixing angle ϕ. The data are from the *BABAR* Collaboration [1].

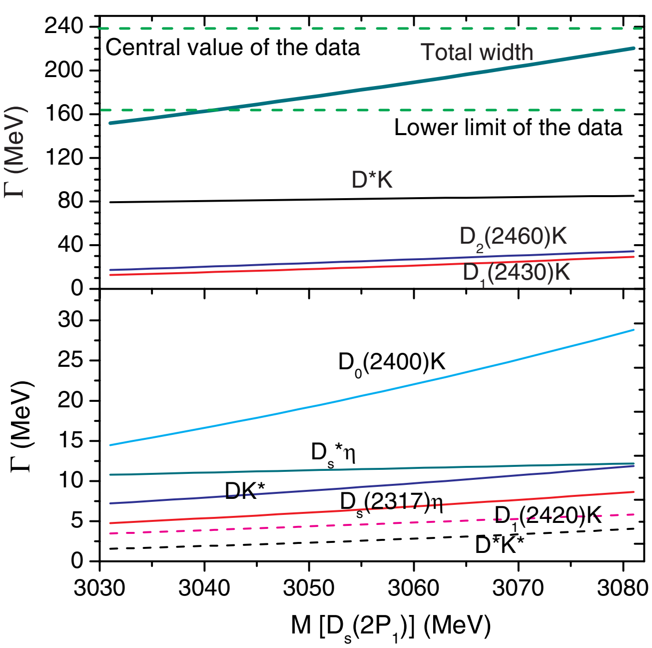

FIG. 3 (color online). The partial decay widths and total width of $|2P_1\rangle_L$ as functions of mass. The data are from *BABAR* [1].

Finally, we discuss the possibilities of $D_{sJ}(3040)$ as a $J^P = 2^-$ candidate. There are two states, 1^1D_2 and 1^3D_2, with $J^P = 2^-$. If 1^1D_2 and 1^3D_2 have a mass of 3.04 GeV, they can decay into the following channels, D^*K, DK^*, D^*K^*, $D_s^*\eta$, $D_s\phi$, $D_0(2400)K$, $D_1(2430)K$, $D_1(2420)K$, $D_2(2460)K$, $D_s(2317)\eta$, and $D_s(2460)\eta$. We calculate these partial decay widths and list the results in Table I. It shows that D^*K and $D_2(2460)K$ are the two main decay channels. The total widths for both 1^1D_2 and 1^3D_2 are very broad, i.e. $\Gamma \sim 608$ MeV and ~ 879 MeV, respectively. They are too large to compare with the data $\Gamma = (239 \pm 35)$ MeV [1]. Nevertheless, it shows that the admixtures between 1^1D_2 and 1^3D_2 are unable to give a reasonable explanation of the decay properties of $D_{sJ}(3040)$ as well. Thus, the $D_{sJ}(3040)$ as a $J^P = 2^-$ candidate is not favored.

In brief, the $D_{sJ}(3040)$ seems to favor a $|2P_1\rangle_L$ state with $J^P = 1^+$, which is an admixture of 2^1P_1 and 2^3P_1 with a mixing angle $\phi \simeq -(40 \pm 12)°$. Our conclusion is in agreement with that of a $^3P_0$ model analysis [23]. The semiclassical flux tube model [52] and relativistic quark model [49] mass calculations also support this picture.

B. $D_{sJ}(2710)$

The $D_{sJ}(2710)$ was first reported by *BABAR* [2], and its quantum number $J^P = 1^-$ was determined by Belle [3]. Recently, the decay ratios of the $D_{sJ}(2710)$ have also been reported [1], which is very useful for understanding its nature. According to the classification of the quark model, only two states 2^3S_1 and 1^3D_1 with the quantum number $J^P = 1^-$ are located around the mass range $(2.7 \sim 2.8)$ GeV. This state is studied by various models, e.g. as a 2^3S_1 state [17,49], 1^3D_1 state [25], or admixture of 2^3S_1-1^3D_1 [21]. It should be mentioned that in our previous work [31] an error occurred in the partial decay amplitude of $1^3D_1 \to DK$, which led to a rather small width for the assignment of the admixture of 2^3S_1-1^3D_1. Here we correct the formulation and reanalyze the mixing scenario for the $D_{sJ}(2710)$.

We first assign the $D_{sJ}(2710)$ as the 2^3S_1 and 1^3D_1 states and calculate its decay widths. The results are listed in Table II, respectively. For the assignment of the 2^3S_1 state, the total decay width and the decay branching ratio fraction between D^*K and DK channels are

TABLE II. The decay widths (MeV) for the $D_{sJ}(2710)$ as 1^3D_1 and 2^3S_1 candidates.

	D^0K^+	D^+K^0	$D^{*+}K^0$	$D^{*0}K^+$	$D_s\eta$	$D_s^*\eta$	total	$\Gamma(D^*K)/\Gamma(DK)$
1^3D_1	75	73.6	17.8	18.5	14	0.9	200	0.24
2^3S_1	5.4	5.6	9.0	9.1	1.7	0.7	31	1.65

$$\Gamma \simeq 31\ \text{MeV}, \qquad \frac{\Gamma(D^*K)}{\Gamma(DK)} \simeq 1.65. \tag{10}$$

It shows that the predicted width $\Gamma \simeq 31$ MeV is too narrow to compare with the data, and the predicted decay ratio $D^*K/DK \simeq 1.65$ is much larger than the measurement $D^*K/DK \simeq 0.91 \pm 0.13 \pm 0.12$ [1]. The calculations of Ref. [24] also tend to give a small width $\Gamma \simeq 32$ MeV for the 2^3S_1 configuration. The predicted branching ratio fraction is also inconsistent with the observations [1]. In Ref. [21], it is also found that the large branching ratio fraction $D^*K/DK \simeq 3.55$ does not support the $D_{sJ}(2710)$ as a pure 2^3S_1 state.

On the other hand, if the $D_{sJ}(2710)$ is considered as a 1^3D_1 state, the decay width and branching ratio fraction will be

$$\Gamma \simeq 200\ \text{MeV}, \qquad \frac{\Gamma(D^*K)}{\Gamma(DK)} \simeq 0.24. \tag{11}$$

In this case, the branching ratio fraction $\Gamma(D^*K)/\Gamma(DK) \simeq 0.24$ is too small though the decay width $\Gamma \simeq 200$ MeV is roughly consistent with the upper limit of the data [1,3]. These results suggest that either 1^3D_1 or 2^3S_1 is not a good assignment for the $D_{sJ}(2710)$.

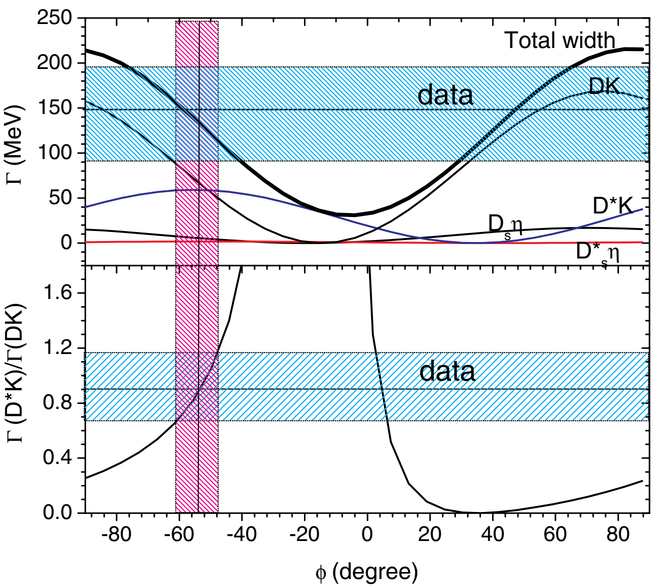

FIG. 4 (color online). The partial decay widths, total width, and the decay branching ratio fraction $\Gamma(D^*K)/\Gamma(D^*K)$ of $|(SD)_1\rangle_L$ as functions of mass, respectively. The data are from *BABAR* [1].

Thus, we consider the possibilities of the $D_{sJ}(2710)$ as a mixed state of 2^3S_1-1^3D_1, for which the physical states can be expressed as [21]

$$|(SD)_1\rangle_L = +\cos(\phi)|2^3S_1\rangle + \sin(\phi)|1^3D_1\rangle, \tag{12}$$

$$|(SD)_1'\rangle_H = -\sin(\phi)|2^3S_1\rangle + \cos(\phi)|1^3D_1\rangle, \tag{13}$$

where the physical partner in the mixing is included. Assuming that the low-mass state $|(SD)_1\rangle_L$ corresponds to the $D_{sJ}(2710)$ [21], we plot the decay properties of $|(SD)_1\rangle_L$ as functions of the mixing angle ϕ in Fig. 4. It shows that with the mixing angle $\phi \simeq (-54 \pm 7)°$, the decay width and branching ratio fraction are

$$\Gamma \simeq (133 \pm 22)\ \text{MeV}, \qquad \frac{\Gamma(D^*K)}{\Gamma(DK)} \simeq 0.91 \mp 0.25, \tag{14}$$

which are in a good agreement with the data [1,3].

Following this scheme, one can examine the high-mass partner $|(SD)_1'\rangle_H$, of which the expected mass is $\sim$2.81 GeV [21]. Taking into account the mass uncertainties of a region $M \simeq (2.71 \sim 2.88)$ GeV, we plot the mass dependence of the partial and total widths in Fig. 5. It shows that the $|(SD)_1'\rangle_H$ also has a broad width $\sim(120 \pm 10)$ MeV, and the DK channel is dominant over others. In contrast, the partial width of $D_s\eta$ is also sizable, while the $D_s^*\eta$ width is negligible. Around $M = 2.81$ GeV, the pre-

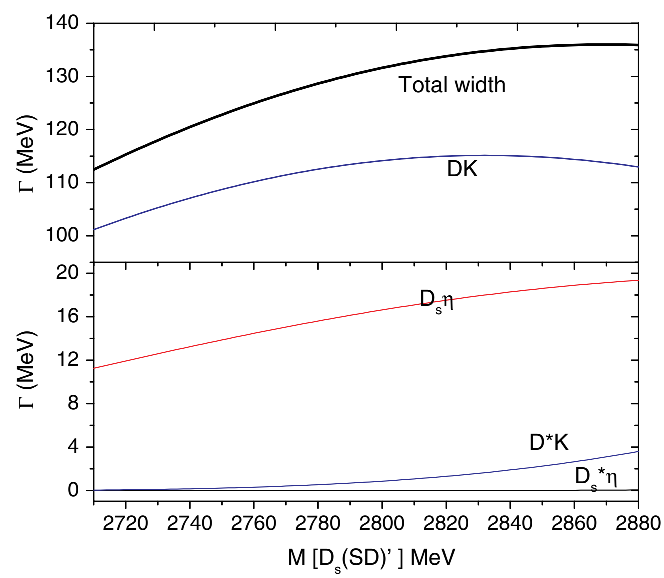

FIG. 5 (color online). The partial decay widths and total width of $|(SD)_1'\rangle_H$ as functions of mass.

dicted branching ratio fractions are

$$\frac{\Gamma(D_s\eta)}{\Gamma(DK)} \simeq 0.15, \qquad \frac{\Gamma(D^*K)}{\Gamma(D_s\eta)} \simeq 0.06. \tag{15}$$

The above mixing scheme is consistent with Ref. [21] for the low-mass state while the predicted suppression of the D^*K decay mode is different from that of Ref. [21]. In Ref. [21] a very broad high-mass state is predicted and would dominantly decay into both DK and D^*K. In our scheme, the predicted decay width for $|(SD)_1'\rangle_H$ is $\sim(120 \pm 10)$ MeV. As a consequence, one would expect that it should appear in the DK spectrum similar to the $D_s(2710)$ signal. Taking into account the still undetermined mass for $|(SD)_1'\rangle_H$, one possible explanation would be that the $|(SD)_1'\rangle_H$ mass may be larger than $M \simeq 2.88$ GeV. If so, its total width would be larger than we estimated above and become much broader; thus, it cannot be easily identified in the present DK spectrum. Interestingly, a recent study of the D_s spectrum suggests a larger mass for the 1^3D_1 state [49].

It should be noted that different methods seem to lead to different conclusions on the $D_{sJ}(2710)$ state. In Refs. [16,17], both the decay width and branching ratio fraction of the $D_{sJ}(2710)$ as the 2^3S_1 assignment can be well explained. In Ref. [49], the mass calculation also suggests that the $D_{sJ}(2710)$ is 2^3S_1. However, the recent study of a $^3P_0$ model tends to conclude that the $D_{sJ}(2710)$ is a mixture of 2^3S_1 and 1^3D_1 [53]. Therefore, additional information for $D_{sJ}(2710) \to D_s\eta$ and $D_s^*\eta$, as well as a search for the $|(SD)_1'\rangle_H$ partner in experiment would be useful for understanding the property of the $D_{sJ}(2710)$.

C. $D_{sJ}(2860)$

The situation about the $D_{sJ}(2860)$ is still controversial and different solutions have been proposed in the literature. In Ref. [28], the $D_{sJ}(2860)$ is assigned as a $J^P = 0^+$ state. However, the recent observation of $D_{sJ}(2860) \to D^*K$ does not support this picture. It is also proposed to be a $J^P = 3^-$ state [24,29,31]. However, although the decay width and decay mode are consistent with the observation, the predicted ratio $D^*K/DK \simeq 0.4$ is too small to compare with the data $D^*K/DK \simeq 1.1$ [1].

Since the $D_{sJ}(2860)$ is observed in both D^*K and DK channels, the allowed quantum numbers would be 1^3D_3, 2^3P_2, and 1^3F_2. We calculate the total and partial widths for these configurations and list the results in Table III.

More specifically, as the 1^3D_3 state, the predicted width and branching ratio fraction between the D^*K and DK channel are

$$\Gamma \simeq 36 \text{ MeV}, \qquad \frac{\Gamma(D^*K)}{\Gamma(DK)} \simeq 0.4. \tag{16}$$

The predicted ratio $\Gamma(D^*K)/\Gamma(DK)$ differs from the measurement $D^*K/DK \simeq 1.1$ [1] at the level of 3 standard deviations, although the decay width is in agreement with the data. Our predictions are consistent with those of Refs. [17,29]. It should be mentioned that the QCD-motivated relativistic quark model cannot well explain the mass of $D_{sJ}(2860)$ if it is considered as the 1^3D_3 state [49]. This could be a signal indicating the chiral symmetry in association with the heavy quark symmetry in the heavy-light meson transitions.

As a candidate of the 2^3P_2 state, the decay width and branching ratio fraction of $D_{sJ}(2860)$ are

$$\Gamma \simeq 8 \text{ MeV}, \qquad \frac{\Gamma(D^*K)}{\Gamma(DK)} \simeq 1.53, \tag{17}$$

where both the predicted width and ratio are inconsistent with the data. It is interesting to mention that our predicted ratio agrees with the estimation of Ref. [32].

If the $D_{sJ}(2860)$ is a 1^3F_2 state, the predicted width and branching ratio fraction are

$$\Gamma \simeq 49 \text{ MeV}, \qquad \frac{\Gamma(D^*K)}{\Gamma(DK)} \simeq 0.005, \tag{18}$$

where the decay mode of D^*K turns out to be negligible in comparison with the DK mode, and disagrees with the experimental observation.

It can be seen from the above analysis that a simple assignment of the $D_{sJ}(2860)$ to be a pure 2^3P_0, 1^3D_3, 2^3P_2, or 1^3F_2 cannot well explain the data. We also point out that the 2^3P_2 and 1^3F_2 mixing is unable to overcome the problem either because of the narrow width of the 2^3P_2 state or small branching ratio fraction $\Gamma(D^*K)/\Gamma(DK) \simeq 0.005$ of 1^3F_2.

In Ref. [32], van Beveren and Rupp recently proposed an alternative solution that there might exist two largely overlapping resonances at about 2.86 GeV, i.e. a radially excited tensor (2^+) and a scalar (0^+) $c\bar{s}$ state. Following this two-state assumption, one would expect that one state $D_{sJ_1}(2860)$ dominantly decays into DK, while the other one $D_{sJ_2}(2860)$ dominantly decays into D^*K. Both states have a mass around 2.86 GeV, and comparable width $\Gamma \sim 50$ MeV. This idea may shed some light on the controver-

TABLE III. The decay widths (MeV) for the $D_{sJ}(2860)$ as 1^3D_3, 2^3P_2, and 1^3F_2 candidates.

	D^0K^+	D^+K^0	$D^{*+}K^0$	$D^{*0}K^+$	$D_s\eta$	$D_s^*\eta$	DK^*	total	$\Gamma(D^*K)/\Gamma(DK)$
1^3D_3	12.3	11.8	5	4.7	1.7	0.3	0.2	36	0.40
2^3P_2	1.3	1.3	2.1	1.9	0.01	1.7	0.02	8	1.53
1^3F_2	21.9	21.4	0.1	0.1	5.5	0.02	0.005	49	0.005

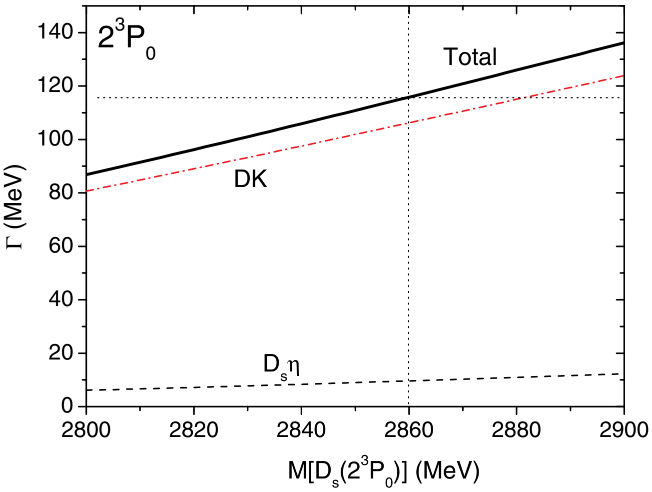

FIG. 6 (color online). The partial decay widths and total width of 2^3P_0 as functions of mass.

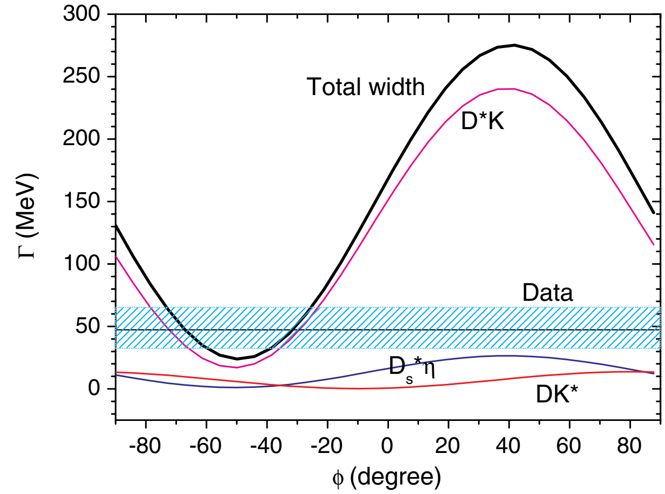

FIG. 7 (color online). The partial decay widths and total width of $|1D_2\rangle_H$ with a mass of 2860 MeV as functions of mixing angle ϕ. The data are from *BABAR* [1,2].

sial issues. As follows, we shall investigate such a possibility in our approach.

It shows that the decays of 2^3P_0, 1^3D_3, and 1^3F_2 is dominated by the DK channel, while the decay of 1^3D_2, 1^1D_2, 2^3P_2 is dominated by the D^*K channel. We shall identify which states are more appropriate candidates in the two-state scenario.

First, we analyze the states dominated by DK decays, i.e. 2^3P_0, 1^3D_3, and 1^3F_2. In Fig. 6 the total and partial decay widths for the 2^3P_0 state are revealed. It shows that the 2^3P_0 possesses a broad decay width $\Gamma \simeq 115$ MeV at about 2.86 GeV, which is inconsistent with the data. The 1^3F_2 is not considered as a good candidate of $D_{sJ_1}(2860)$ as well since its mass is expected to be larger than 3.1 GeV [26,49]. Furthermore, our earlier analysis suggests that the $D_{sJ}(3040)$ may favor a configuration of $|2P_1\rangle_L$ such that the mass of the 1^3F_2 should be larger than the P wave state $|2P_1\rangle_L$ as a consequence. In contrast, we find that the 1^3D_3 could be a good candidate for $D_{sJ_1}(2860)$ since it is dominated by the DK decay mode and has a narrow width $\Gamma \simeq 36$ MeV. The calculation results for the total and partial decay widths have been listed in Table III.

Candidates for the $D_{sJ_2}(2860)$ could be 1^3D_2, 1^1D_2, or 2^3P_2 which dominantly decay into D^*K. As discussed earlier in this section and shown in Table III, the 2^3P_2 is not a good candidate since its total width is too small to compare with the data. Nevertheless, its expected mass should be larger than 2.86 GeV [26,49].

If the $D_{sJ_2}(2860)$ is considered as pure 1^3D_2 or 1^1D_2 state, their decay widths would be $\Gamma \simeq 170$ MeV and $\Gamma \simeq 130$ MeV, respectively, which are inconsistent with the data as well. In fact, the physical states should be the admixtures between 1^3D_2 and 1^1D_2 due to the presence of the spin-orbit interactions [22,50,51]. Thus, the mixed states can be expressed as

$$|1D_2\rangle_L = +\cos(\phi)|1^1D_2\rangle + \sin(\phi)|1^3D_2\rangle, \quad (19)$$

$$|1D_2{}'\rangle_H = -\sin(\phi)|1^1D_2\rangle + \cos(\phi)|1^3D_2\rangle, \quad (20)$$

where the subscripts L and H denote the low-mass and high-mass state due to the mixing. Usually, the $|1D_2{}'\rangle_H$ have a narrow width [22,50,51]. We thus consider the $|1D_2{}'\rangle_H$ as the $D_{sJ_2}(2860)$ in the calculation. In Fig. 7 the decay properties as a function of the mixing angle are plotted. We see that around $\phi = -65°$ or $\phi = -35°$ the decay width is $\Gamma \simeq 40$ MeV, which is compatible with the observation, and the decay mode is dominated by the D^*K. With $\phi = -35°$, the corresponding decay branching ratio fractions are

$$\frac{\Gamma(D^*K)}{\Gamma(D_s^*\eta)} \simeq 1.2, \qquad \frac{\Gamma(D^*K)}{\Gamma(DK^*)} \simeq 13, \quad (21)$$

which fit in the experimental data quite well. This result turns out to support the $|1D_2{}'\rangle_H$ to be a candidates of $D_{sJ_2}(2860)$ in the two-state scenario. In the range of $\phi = -65° \sim -35°$ the partial widths do not change drastically with the mixing angle. In contrast, the suggested value is consistent with that ($\phi = -50.7°$) obtained in the heavy quark effective theory [22,49–51].

In brief, it seems likely that the abnormal property with the $D_{sJ}(2860)$ arises from two overlapping resonances with the same mass but different decay modes. One is 1^3D_3 and the other is $|1D_2{}'\rangle_H$ from the 1^3D_2 and 1^1D_2 mixing. The 1^3D_3 state mainly decays into DK and the $|1D_2{}'\rangle_H$ into D^*K. With these two largely overlapping resonances at about 2.86 GeV, we can understand both the observed decay widths and branching ratio fractions of the $D_{sJ}(2860)$. It shows that the 1^3D_3 has a sizable partial width in the $D_s\eta$ channel, while the $|1D_2{}'\rangle_H \to D_s^*\eta$ also turns out to be measurable. Further measurements of

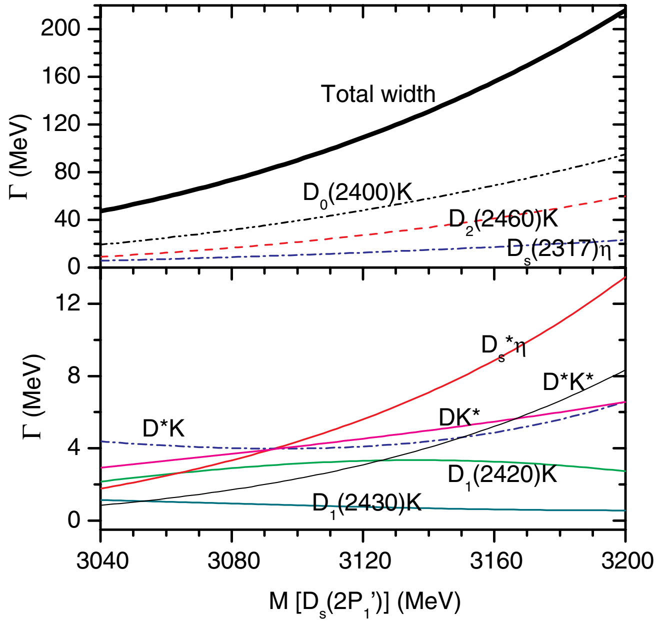

FIG. 8 (color online). The partial decay widths and total width of the $|2P_1'\rangle_H$ state as functions of mass.

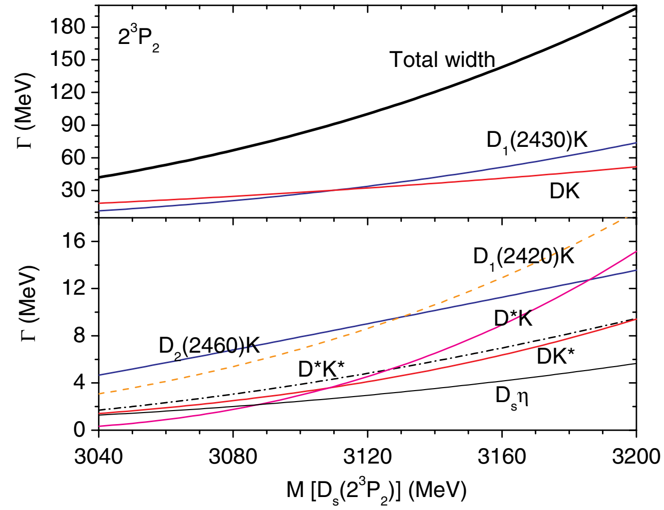

FIG. 9 (color online). The partial decay widths and total width of 2^3P_2 as functions of mass.

$\Gamma(D_s^*\eta)/\Gamma(D^*K)$ and $\Gamma(DK)/\Gamma(D_s\eta)$ may be able to distinguish the 1^3D_3 and $|1D_2{}'\rangle_H$ and test the two-state scenario in experiment.

D. $D_{sJ}(2|P_1'\rangle_H)$, $D_{sJ}(2^3P_0)$, and $D_{sJ}(2^3P_2)$

In this subsection we discuss the implications of other states following the consequence of the assignments for the $D_{sJ}(3040)$, $D_{sJ}(2710)$, and $D_{sJ}(2860)$.

Since the $D_{sJ}(3040)$ seems to favor a P wave with $J^P = 1^+$ ($|2P_1\rangle_L$), experimental evidences for the other P waves, $D_{sJ}(2|P_1'\rangle_H)$, $D_{sJ}(2^3P_0)$ and $D_{sJ}(2^3P_2)$, would be important to establish the spectrum. In particular, its high-mass partner $|2P_1'\rangle_H$ should be searched in experiments. Supposing that the $|2P_1'\rangle_H$ has a mass in the range of $(3.04 \sim 3.2)$ GeV, we plot in Fig. 8 the decay widths as functions of the mass with the mixing angle $\phi = -50°$ fixed by $D_{sJ}(3040)$. It shows that the $|2P_1'\rangle_H$ width is indeed relatively narrower around $M = 3.04$ GeV, although we should note that the decay width increases fast with the increasing mass. The decay channels, $D_0(2400)K$, $D_2(2460)K$, and $D_s(2317)\eta$, are predicted to be the dominant ones, which can be investigated in experiments. In contrast, the D^*K channel plays a less important role in the decays.

We further study the $D_{sJ}(2^3P_0)$ in detail here. The decay widths as a function of the possible mass range $M = (2.8 \sim 2.9)$ GeV are plotted in Fig. 6. In this range the total decay width is $\Gamma \simeq (90 \sim 140)$ MeV, and increases with the increasing mass. It shows that the DK channel dominates its decays. Taking the mass of the $D_{sJ}(2^3P_0)$ as $(2.82 \sim 2.84)$ GeV [21,54], the total width and branching ratio fractions between $D_s\eta$ and DK are

$$\Gamma \simeq (101 \pm 5)\ \text{MeV}, \qquad \frac{\Gamma(D_s\eta)}{\Gamma(DK)} \simeq 0.08. \tag{22}$$

It should be pointed out that the decay properties of 2^3P_0 are similar to those of $|(SD)_1'\rangle_H$ in the mass range $M < 2.9$ GeV (see Fig. 5 and 6). Both of them have comparable decay widths $\Gamma \sim 100$ MeV, and mainly decay into DK. To distinguish them from each other, the measurements of their decay ratio $\Gamma(D_s\eta)/\Gamma(DK)$ are important. We also note that a recent calculation suggests a larger mass of $M \simeq 3.054$ GeV for 2^3P_0 [49]. As a consequence of this scenario, its total decay width would become much broader than we estimated above. Thus, it may not be easily isolated in experiment.

As discussed earlier the $D_{sJ}(2860)$ does not favor the assignment of 2^3P_2. Thus, we investigate its decay properties and implications of experimental measurement. We also plot its total and partial decay widths as functions of the mass in the possible range $M = (3.04 \sim 3.2)$ GeV in Fig. 9. If 2^3P_2 has a mass larger than 3.04 GeV, decay channels, DK, D^*K, DK^*, D^*K^*, $D_s^*\eta$, $D_s\phi$, $D_s\eta$, $D_1(2430)K$, $D_1(2420)K$, $D_2(2460)K$, $D_s(2460)\eta$, will open in which $D_1(2430)K$ and DK channels are dominant. In Fig. 9, we do not show the results for the $D_s^*\eta$, $D_s\phi$, and $D_s(2460)\eta$ channels since they are negligibly small (<1 MeV). If we adopt the mass $\sim$3.15 GeV as predicted by Refs. [26,49,54], the predicted width is $\Gamma \simeq 140$ MeV, and the relative decay strengths are

$$DK{:}D^*K{:}D_1(2430)K{:}D_1(2420)K{:}D_2(2460)K{:}DK^*{:}D^*K^*{:}D_s\eta \simeq 41{:}9{:}50{:}13{:}11{:}6{:}7{:}4.$$

It suggests that the DK, $D_1(2430)K$, $D_1(2420)K$ channels may be the optimal ones for searching for the $D_{sJ}(2^3P_2)$ state in experiment.

E. Sensitivity to the harmonic oscillator parameter

It should be mentioned that a model-dependent feature of our model arises from the simple treatment of harmonic oscillator potential for the heavy-light quark system. Therefore, uncertainties with the theoretical results are present in the choice of the quark model parameter values. The most important parameter in our model should be the harmonic oscillator strength β, which controls the size effect or coupling form factor from the convolution of the heavy-light meson wave functions. The commonly adopted range of this quantity is $\beta = (0.4 \sim 0.5)$ GeV, and we apply $\beta = 0.45$ GeV in the above calculations.

In order to examine the sensitivity of the calculation results to β, we plot the decay widths and ratios of 2^3S_1, 1^3D_3, mixed state $|(SD)\rangle_L$ of 2^3S_1-1^3D_1, and mixed state $|2P_1\rangle_L$ of 2^1P_1-2^3P_1 as a function of β in Fig. 10. It shows that the decay widths of these excited D_s states exhibit some sensitivities to the parameter β. Within the range of $\beta = (0.45 \pm 0.05)$ GeV, about 30% uncertainties of the decay widths would be expected. This is a typical order of accuracy for the constituent quark model, and can be regarded as reasonable.

The ratio $\Gamma(D^*K)/\Gamma(DK)$ appears to behave differently. For the 2^3S_1, the sensitivity of the ratio to β is apparent. In contrast, the ratios of $|(SD)\rangle_L$ and 1^3D_3 are quite insensitive to β. The ratio $\Gamma(D^*K)/\Gamma(DK)$ is not shown for $|2P_1\rangle_L$ since its decay into DK is forbidden.

In brief, although the harmonic oscillator parameter β can bring some uncertainties to the final results, within the range of $\beta = (0.4 \sim 0.5)$ GeV, our major conclusions will still hold.

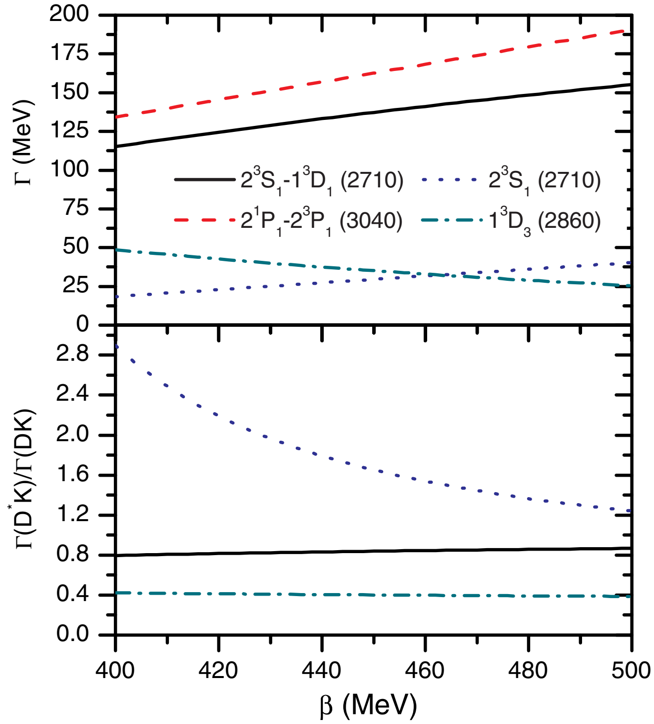

FIG. 10 (color online). The decay widths (upper panel) and ratios (lower panel) of different configuration assignments as a function of β: the solid lines are for the mixed state $|(SD)\rangle_L$ of 2^3S_1-1^3D_1 with a mass of 2710 MeV and mixing angle $-55°$; the dotted lines are for 2^3S_1 with a mass of 2710 MeV; the dot-dashed lines for 1^3D_3 with a mass of 2860 MeV; and the dashed line in the upper panel is for the mixed state $|2P_1\rangle_L$ of 2^1P_1-2^3P_1 with a mass of 3040 MeV and mixing angle $-50°$.

IV. SUMMARY

In this work we investigate the strong decays of several newly observed charmed mesons in a constituent quark model with effective Lagrangians for the quark-meson interactions. The decay amplitudes are extracted for light pseudoscalar meson or vector meson productions via axial or vector current conservation between the quark level and hadronic level couplings. The quark-meson couplings can then be determined by independent measurements such as meson photoproduction and meson-baryon scatterings.

We find that the new state $D_{sJ}(3040)$ can be identified as the low-mass physical state $|2P_1\rangle_L$ from the $D_s(2^1P_1)$-$D_s(2^3P_1)$ mixing with a mixing angle $\phi \simeq -(40 \pm 12)°$. Further experimental search for decay modes of $D_1(2430)K$, $D_2(2460)K$, $D_0(2400)K$, DK^*, and $D_s^*\eta$ should be able to disentangle its property and test our model predictions.

The $D_{sJ}(2710)$ seems to favor a low-mass physical state $|(SD)\rangle_L$ from the 2^3S_1-1^3D_1 mixing with a mixing angle $\phi \simeq (-54 \pm 7)°$. Both the ratio and width are in a good agreement with the data. The decay properties of its heavy partner $|(SD)_1'\rangle_H$ are also discussed. It has a broad width $\Gamma \simeq (110 \sim 140)$ MeV at the 2.8 GeV mass region, and dominated by the DK mode. We also point out that the $|(SD)_1'\rangle_H$ state may be searched in the DK spectrum as the $D_{sJ}(2710)$ if its mass is $\sim$2.8 GeV. Whether the present data have contained its signal could be a crucial criteria for various model predictions.

The $D_{sJ}(2860)$ cannot be easily explained by a single configuration of 2^3P_0, 2^3P_2, 1^3F_2, or 1^3D_3. To overcome this problem we follow the proposal of a two-state picture by Ref. [32] and assume that two narrow resonances may have been observed around 2.86 GeV with a width $\Gamma \simeq (40 \sim 50)$ MeV. It shows that one resonance seems to be the 1^3D_3, which mainly decays into DK. The other resonance could be the $|1D_2{}'\rangle_H$, which is the high-mass state from the 1^1D_2-1^3D_2 mixing, and dominantly decays into D^*K. Further theoretical and experimental efforts are needed to disentangle the mysterious properties about this state.

We also study the implications arising from the assignments for those observed resonances, e.g. their partner states in the mixing. In particular, if the $D_{sJ}(3040)$ is indeed a P-wave state $|2P_1\rangle_L$, the other three P-wave states $D_{sJ}(|2P_1'\rangle_H)$, $D_{sJ}(2^3P_0)$, and $D_{sJ}(2^3P_2)$ may also have measurable effects in experiment. Their strong decay properties are predicted, which could be useful for future experimental studies.

ACKNOWLEDGMENTS

This work is supported, in part, by the National Natural Science Foundation of China (Grant Nos. 10675131 and 10775145), Chinese Academy of Sciences (KJCX3-SYW-N2), and Ministry of Science and Technology of China (2009CB825200).

[1] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. D **80**, 092003 (2009).
[2] B. Aubert (*BABAR* Collaboration), Phys. Rev. Lett. **97**, 222001 (2006).
[3] J. Brodzicka *et al.* (Belle Collaboration), Phys. Rev. Lett. **100**, 092001 (2008).
[4] E. J. Eichten, C. T. Hill, and C. Quigg, Phys. Rev. Lett. **71**, 4116 (1993).
[5] Y. B. Dai and S. L. Zhu, Phys. Rev. D **58**, 074009 (1998).
[6] A. Hiorth and J. O. Eeg, Phys. Rev. D **66**, 074001 (2002).
[7] S. Fajfer and J. F. Kamenik, Phys. Rev. D **74**, 074023 (2006).
[8] S. Fajfer and J. F. Kamenik, Int. J. Mod. Phys. A **23**, 3196 (2008).
[9] J. Kamenik and S. Fajfer, J. Phys. Conf. Ser. **110**, 122015 (2008).
[10] W. A. Bardeen and C. T. Hill, Phys. Rev. D **49**, 409 (1994).
[11] W. A. Bardeen, E. J. Eichten, and C. T. Hill, Phys. Rev. D **68**, 054024 (2003).
[12] P. Colangelo, F. De Fazio, and R. Ferrandes, Mod. Phys. Lett. A **19**, 2083 (2004).
[13] M. A. Nowak, M. Rho, and I. Zahed, Phys. Rev. D **48**, 4370 (1993).
[14] A. Deandrea, R. Gatto, G. Nardulli, and A. D. Polosa, J. High Energy Phys. 02 (1999) 021.
[15] A. Deandrea, N. Di Bartolomeo, R. Gatto, G. Nardulli, and A. D. Polosa, Phys. Rev. D **58**, 034004 (1998).
[16] P. Colangelo, F. De Fazio, S. Nicotri, and M. Rizzi, Phys. Rev. D **77**, 014012 (2008).
[17] F. De Fazio, arXiv:0910.0412.
[18] Y. B. Dai, C. S. Huang, M. Q. Huang, H. Y. Jin, and C. Liu, Phys. Rev. D **58**, 094032 (1998); **59**, 059901(E) (1999).
[19] P. Colangelo, F. De Fazio, G. Nardulli, N. Di Bartolomeo, and R. Gatto, Phys. Rev. D **52**, 6422 (1995).
[20] Z. G. Wang, Phys. Rev. D **77**, 054024 (2008).
[21] F. E. Close, C. E. Thomas, O. Lakhina, and E. S. Swanson, Phys. Lett. B **647**, 159 (2007).
[22] F. E. Close and E. S. Swanson, Phys. Rev. D **72**, 094004 (2005).
[23] Zhi-Feng Sun and Xiang Liu, Phys. Rev. D **80**, 074037 (2009).
[24] B. Zhang, X. Liu, W. Z. Deng, and S. L. Zhu, Eur. Phys. J. C **50**, 617 (2007).
[25] W. Wei, X. Liu, and S. L. Zhu, Phys. Rev. D **75**, 014013 (2007).
[26] M. Di Pierro and E. Eichten, Phys. Rev. D **64**, 114004 (2001).
[27] Z. G. Wang, Chin. Phys. C **32**, 797 (2008).
[28] E. van Beveren and G. Rupp, Phys. Rev. Lett. **97**, 202001 (2006).
[29] P. Colangelo, F. De Fazio, and S. Nicotri, Phys. Lett. B **642**, 48 (2006).
[30] J. Koponen, Phys. Rev. D **78**, 074509 (2008).
[31] X. H. Zhong and Q. Zhao, Phys. Rev. D **78**, 014029 (2008).
[32] E. van Beveren and G. Rupp, arXiv:0908.1142.
[33] A. Manohar and H. Georgi, Nucl. Phys. **B234**, 189 (1984).
[34] Z. P. Li, Phys. Rev. D **50**, 5639 (1994).
[35] Z. P. Li, H. X. Ye, and M. H. Lu, Phys. Rev. C **56**, 1099 (1997).
[36] Q. Zhao, J. S. Al-Khalili, Z. P. Li, and R. L. Workman, Phys. Rev. C **65**, 065204 (2002).
[37] X. H. Zhong, Q. Zhao, J. He, and B. Saghai, Phys. Rev. C **76**, 065205 (2007).
[38] X. H. Zhong and Q. Zhao, Phys. Rev. C **79**, 045202 (2009).
[39] D. O. Riska and G. E. Brown, Nucl. Phys. **A679**, 577 (2001).
[40] Q. Zhao, Z. P. Li, and C. Bennhold, Phys. Rev. C **58**, 2393 (1998); Phys. Lett. B **436**, 42 (1998).
[41] Q. Zhao, Phys. Rev. C **63**, 025203 (2001).
[42] Q. Zhao, J. S. Al-Khalili, and C. Bennhold, Phys. Rev. C **64**, 052201(R) (2001).
[43] Q. Zhao, B. Saghai, and Z. P. Li, J. Phys. G **28**, 1293 (2002).
[44] X. H. Zhong and Q. Zhao, Phys. Rev. D **77**, 074008 (2008).
[45] I. Hleiqawi *et al.* (CLAS Collaboration), Phys. Rev. C **75**, 042201 (2007); **76**, 039905(E) (2007).
[46] M. Nanova *et al.* (CBELSA/TAPS Collaboration), Eur. Phys. J. A **35**, 333 (2008).
[47] C. Amsler *et al.* (Partical Data Group), Phys. Lett. B **667**, 1 (2008).
[48] S. Godfrey and N. Isgur, Phys. Rev. D **32**, 189 (1985).
[49] D. Ebert, R. N. Faustov, and V. O. Galkin, arXiv: 0910.5612.
[50] S. Godfrey and R. Kokoski, Phys. Rev. D **43**, 1679 (1991).
[51] E. S. Swanson, Phys. Rep. **429**, 243 (2006).
[52] B. Chen, D. X. Wang, and A. Zhang, Phys. Rev. D **80**, 071502(R) (2009).
[53] D. M. Li and B. Ma, arXiv:0911.2906 [Phys. Rev. D (to be published)].
[54] T. Matsuki, T. Morii, and K. Sudoh, Eur. Phys. J. A **31**, 701 (2007).