Charmed meson decays to two pseudoscalars

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A recent update of data on the decays of D^0, D^+, and D_s^+ to pairs of light pseudoscalars calls for a renewed analysis of key decay amplitudes and tests of flavor symmetry. The present data change our previous understanding of relative phases between amplitudes that describe Cabibbo-favored decays of the charmed mesons. The new data also seem to favor a smaller octet-singlet mixing angle for the η and η' mesons when singly Cabibbo-suppressed processes are taken into account. We also discuss the effects of the new data on interference between Cabibbo-favored and doubly Cabibbo-suppressed decays.

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I. INTRODUCTION

The CLEO Collaboration has recently reported new results on charmed meson decay rates and branching ratios [1], many with experimental errors less than present world averages [2]. This calls for an update of a previous work [3] on extraction of flavor-topology amplitudes and relative phases between them. SU(3) flavor symmetry, applied here, has been shown useful in finding relative strong phases of amplitudes in $D \to PP$ decays, where P represents a light pseudoscalar meson [4–6].

The diagrammatic approach is once again reviewed in Sec. II. In Sec. III we talk about Cabibbo-favored (CF) decays and a new and previously unexpected relative phase between two of the amplitudes in light of the new data. We also perform an analysis of the Cabibbo-favored decay amplitudes using a variable angle for the octet-singlet mixing in η and η'. Section IV updates the singly Cabibbo-suppressed (SCS) amplitudes and their role in determining amplitudes associated with disconnected diagrams. In Sec. V we discuss doubly Cabibbo-suppressed decays. We conclude in Sec. VI.

II. DIAGRAMMATIC AMPLITUDE EXPANSION

The flavor-topology technique for analyzing charmed meson decays makes use of SU(3) invariant amplitudes. The key amplitudes that describe the physics of Cabibbo-favored decays have been defined in Ref. [3], and include a color-favored tree (T), a color-suppressed tree (C), an exchange (E), and an annihilation (A) amplitude. The CF amplitudes are proportional to the product $V_{ud}V_{cs}$ of the Cabibbo-Kobayashi-Maskawa matrix elements, the singly Cabibbo-suppressed amplitudes are proportional to $V_{us}V_{cs}$ or $V_{ud}V_{cd}$, and the doubly Cabibbo-suppressed amplitudes are proportional to $V_{us}V_{cd}$. We denote the Cabibbo-favored, singly Cabibbo-suppressed, and doubly Cabibbo-suppressed amplitudes by unprimed, primed, and quantities with a tilde, respectively. The relative hierarchy of these amplitudes in terms of the Wolfenstein parameter $\lambda = \tan\theta_C = 0.2317$ [2] is $1{:}\lambda{:} -\lambda{:} -\lambda^2$, where θ_C is the Cabibbo angle.

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III. CABIBBO-FAVORED DECAYS

Cabibbo-favored D decays have been discussed at length in Refs. [3,6], where the C, E, and A amplitudes were found to have large phases relative to the dominant T amplitude. In particular, we found [3] that the amplitudes E and A had a relative phase of approximately 180°. This conclusion changes when we make use of new branching ratios for Cabibbo-favored D decays [1]. The phases of C and E relative to T are unchanged, but in the favored solution E and A now are no longer separated by a 180° phase difference, but are much closer to one another in phase. The magnitude of A is smaller than was found in Ref. [3] and hence the ratio between $|A|$ and $|E|$ is also smaller: $|A| = (0.21 \pm 0.09)|E|$.

In Table I we list the branching ratios $\mathcal{B}$ [1] corresponding to the Cabibbo-favored decay modes and the amplitudes extracted using $\mathcal{A} = M_D[8\pi\mathcal{B}\hbar/(p^*\tau)]^{1/2}$, where M_D is the mass of the decaying meson, τ is its lifetime, and p^* is the center-of-mass 3-momentum of a final state pseudoscalar meson. We have described octet-singlet mixing in the η and η' mesons in terms of an angle θ_η:

$$\begin{aligned} \eta &= -\eta_8 \cos\theta_\eta - \eta_1 \sin\theta_\eta, \\ \eta' &= -\eta_8 \sin\theta_\eta + \eta_1 \cos\theta_\eta, \end{aligned} \tag{1}$$

where

$$\begin{aligned} \eta_8 &\equiv (u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}, \\ \eta_1 &\equiv (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}, \end{aligned} \tag{2}$$

and have taken $\theta_\eta = \arcsin(1/3) = 19.5°$ so that [7]

$$\begin{aligned} \eta &= (s\bar{s} - u\bar{u} - d\bar{d})/\sqrt{3}, \\ \eta' &= (2s\bar{s} + u\bar{u} + d\bar{d})/\sqrt{6}. \end{aligned} \tag{3}$$

We can extract T, C, E, and A uniquely (up to a complex conjugation) by defining T to be purely real. The extracted

TABLE I. Branching ratios and invariant amplitudes for Cabibbo-favored decays of charmed mesons to a pair of pseudoscalars. Here an octet-singlet mixing angle of $\theta_\eta = \arcsin(1/3) = 19.5°$ has been assumed.

Meson	Decay mode	$\mathcal{B}$ [1] (%)	p^* (MeV)	$\lvert\mathcal{A}\rvert$ (10^{-6} GeV)	Rep.	Predicted $\mathcal{B}$ (%)
D^0	$K^-\pi^+$	3.891 ± 0.077	861.1	2.52 ± 0.03	$T+E$	3.905
	$\bar{K}^0\pi^0$	2.380 ± 0.092	860.4	1.97 ± 0.04	$(C-E)/\sqrt{2}$	2.347
	$\bar{K}^0\eta$	0.962 ± 0.060	771.9	1.32 ± 0.04	$C/\sqrt{3}$	1.002
	$\bar{K}^0\eta'$	1.900 ± 0.108	564.9	2.17 ± 0.06	$-(C+3E)/\sqrt{6}$	1.920
D^+	$\bar{K}^0\pi^+$	3.074 ± 0.097	862.4	1.41 ± 0.02	$C+T$	3.090
D_s^+	$\bar{K}^0K^+$	2.98 ± 0.17	850.3	2.12 ± 0.06	$C+A$	2.939
	$\pi^+\eta$	1.84 ± 0.15	902.3	1.62 ± 0.07	$(T-2A)/\sqrt{3}$	1.810
	$\pi^+\eta'$	3.95 ± 0.34	743.2	2.61 ± 0.11	$2(T+A)/\sqrt{6}$	3.603

amplitudes, in units of 10^{-6} GeV, are

$$T = 2.927 \pm 0.022, \tag{4}$$

$$C = (2.337 \pm 0.027)\exp[i(-151.66 \pm 0.63)°], \tag{5}$$

$$E = (1.573 \pm 0.032)\exp[i(120.56 \pm 1.03)°], \tag{6}$$

$$A = (0.33 \pm 0.14)\exp[i(70.47 \pm 10.90)°]. \tag{7}$$

These amplitudes are shown on an Argand diagram in the left-hand panel of Fig. 1. They were extracted by a least χ^2 fit to the data, resulting in $\chi^2 = 1.79$ for 1 degree of freedom, and update those quoted in Refs. [3,6].

While the above analysis works fairly well and yields a low value of χ^2, one could obtain an exact solution ($\chi^2 = 0$) by introducing one more parameter; the number of constraints (known branching ratios) would then equal the number of unknown variables. There are several possible sources of an extra parameter. One might add singlet amplitudes, but they are expected to be much smaller than the nonsinglet ones and would result in too large a parameter space. A plausible new parameter is the angle θ_η describing octet-singlet mixing in η and η', which was fixed in the above analysis to [7] $\theta_\eta = 19.5°$. An exact solution is obtained in this case for $\theta_\eta = 11.7°$. The corresponding amplitudes, in units of 10^{-6} GeV, have been listed below and plotted on an Argand diagram in the right-hand panel of Fig. 1:

$$T = 3.003 \pm 0.023, \tag{8}$$

$$C = (2.565 \pm 0.030)\exp[i(-152.11 \pm 0.57)°], \tag{9}$$

$$E = (1.372 \pm 0.036)\exp[i(123.62 \pm 1.25)°], \tag{10}$$

$$A = (0.452 \pm 0.058)\exp[i(19^{+15}_{-14})°]. \tag{11}$$

Table II lists the corresponding representations of the Cabibbo-favored decay amplitudes as functions of θ_η. In Fig. 2 we show the variation of χ^2 and other parameters as functions of θ_η over the range $\theta_\eta = 9°–22°$. The top left-hand plot shows that χ^2 increases as we move away from the value $\theta_\eta = 11.7°$. Over this range the amplitudes and relative phases show only a slight change ($< 50\%$) in value as observed in the other panels of the same figure,

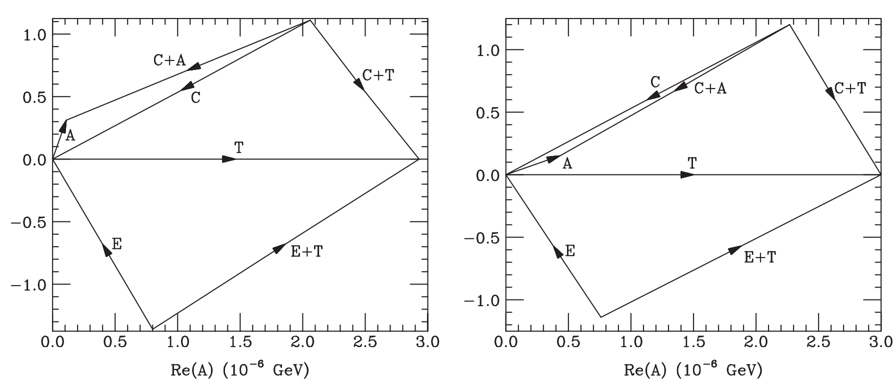

FIG. 1. Construction of Cabibbo-favored amplitudes from observed processes using a least χ^2 fit. The sides $C+T$, $C+A$, and $E+T$ correspond to measured processes; the magnitudes of other amplitudes listed in Table I are also needed to specify T, C, E, and A. These figures correspond to the $|T| > |C|$ solution. Left panel: θ_η fixed at $\arcsin(1/3) = 19.5°$ with $\chi^2 = 1.79$ for 1 degree of freedom. Right panel: exact solution with $\theta_\eta = 11.7°$ and $\chi^2 = 0$.

TABLE II. Branching ratios and invariant amplitudes for Cabibbo-favored decays of charmed mesons to a pair of pseudoscalars with 2 different values of θ_η. ($\phi_1 = 45° - \frac{\phi_2}{2}$ and $\phi_2 = 19.5°$.)

Meson	Decay mode	$\mathcal{B}$ [1] (%)	Rep.	Predicted $\mathcal{B}$ (%) $\theta_\eta = 11.7°$	$\theta_\eta = 19.5°$
D^0	$K^-\pi^+$	3.891 ± 0.077	$T + E$	3.891	3.905
	$\bar{K}^0\pi^0$	2.380 ± 0.092	$(C - E)/\sqrt{2}$	2.380	2.347
	$\bar{K}^0\eta$	0.962 ± 0.060	$\frac{C}{\sqrt{2}}\sin(\theta_\eta + \phi_1) - \frac{\sqrt{3}E}{\sqrt{2}}\cos(\theta_\eta + 2\phi_1)$	0.962	1.002
	$\bar{K}^0\eta'$	1.900 ± 0.108	$-\frac{C}{\sqrt{2}}\cos(\theta_\eta + \phi_1) - \frac{\sqrt{3}E}{\sqrt{2}}\sin(\theta_\eta + 2\phi_1)$	1.900	1.920
D^+	$\bar{K}^0\pi^+$	3.074 ± 0.097	$C + T$	3.074	3.090
D_s^+	$\bar{K}^0K^+$	2.98 ± 0.17	$C + A$	2.980	2.939
	$\pi^+\eta$	1.84 ± 0.15	$T\cos(\theta_\eta + \phi_1) - \sqrt{2}A\sin(\theta_\eta + \phi_1)$	1.840	1.810
	$\pi^+\eta'$	3.95 ± 0.34	$T\sin(\theta_\eta + \phi_1) + \sqrt{2}A\cos(\theta_\eta + \phi_1)$	3.950	3.603

except for the relative phase between T and A. This relative phase (θ_{AT}) varies over a wider range (12° to 88°), as shown in the bottom right-hand plot of Fig. 2.

Another set of solutions, but one with $|T| < |C|$, was also found in the process of minimizing χ^2 as a function of θ_η. This branch has $\chi^2 = 0$ at $\theta_\eta = 18.9°$. Figure 3 shows the Argand diagram plot for the corresponding amplitudes. Figure 4 shows the variation of χ^2 and other parameters as functions of θ_η. In this case the point at $\chi^2 = 0$ lies very close to the frequently quoted [7] value of 19.5°. The amplitudes and relative phases of T, C, E, and A (including the relative phase between T and A) show only slight variations over the range of values considered for θ_η (9°–22°.) However the value of χ^2 in the present case shows a much more rapid increase with a change in θ_η as compared to the $|T| > |C|$ case mentioned earlier. The maximum contribution to the χ^2 comes from the process $D^0 \to \bar{K}^0\eta$ in the $|T| < |C|$ case. To understand this better

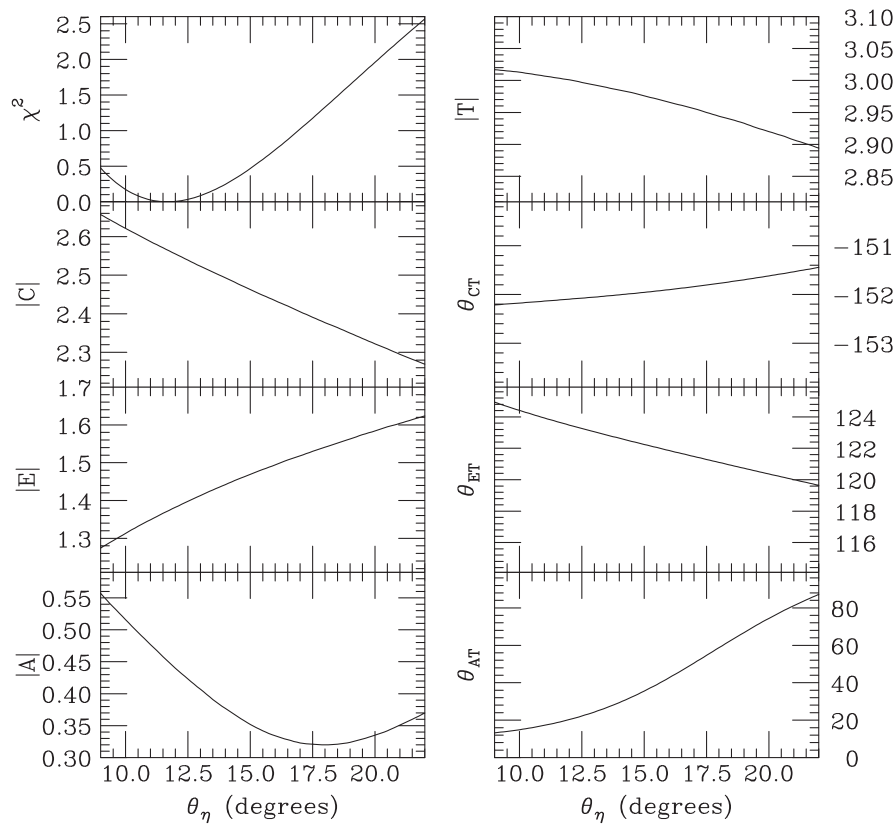

FIG. 2. Behavior of χ^2 and Cabibbo-favored decay amplitudes and relative phases ($|T|$, $|C|$, θ_{CT}, $|E|$, θ_{ET}, $|A|$, θ_{AT}) as functions of θ_η for the $|T| > |C|$ solution.

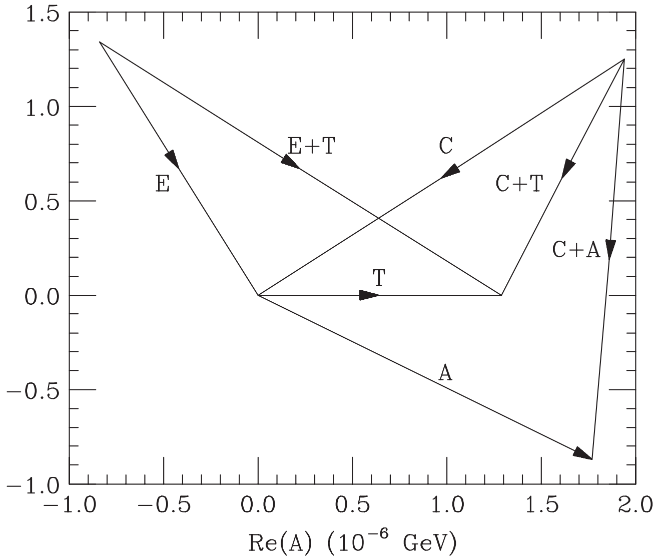

FIG. 3. Construction of Cabibbo-favored amplitudes from observed processes using a least χ^2 fit. The sides $C + T$, $C + A$, and $E + T$ correspond to measured processes; the magnitudes of other amplitudes listed in Table I are also needed to specify T, C, E, and A. These figures correspond to the $|T| < |C|$ solution with $\chi^2 = 0$ and $\theta_\eta = 18.9°$.

let us look at the representation for this amplitude (listed in Table II). The part of this representation that depends on E is proportional to $\sin(\theta_\eta - \phi_2)$ which takes the value zero at $\theta_\eta = \phi_2 = 19.5°$. Hence this term does not vary appreciably over the range of θ_η values we consider. The other term in this representation that depends on C is proportional to $\sin(\theta_\eta + \phi_1)$ which is of the order of unity in this range of θ_η values. Now the $|C|$ vs θ_η plot in Fig. 4 shows that as we decrease θ_η, $|C|$ decreases so that the amplitude of the $D^0 \to \bar{K}^0\eta$ process decreases. This leads to a 3–4σ variation in the branching ratio and hence a high χ^2 contribution. In the $|T| > |C|$ case, however, Fig. 2 shows an increase in $|C|$ as we reduce θ_η from 19.5° to 9°. This leads to a reasonably stable value of the branching ratio and hence a very small contribution to the relevant χ^2.

IV. SINGLY CABIBBO-SUPPRESSED DECAYS

A. SCS decays involving pions and kaons

Assuming the relative hierarchy between Cabibbo-favored and singly Cabibbo-suppressed decay amplitudes described in Sec. I, and using $\lambda = \tan\theta_C = 0.2317$ we find, in units of 10^{-7} GeV,

$$T' = 6.96, \tag{12}$$

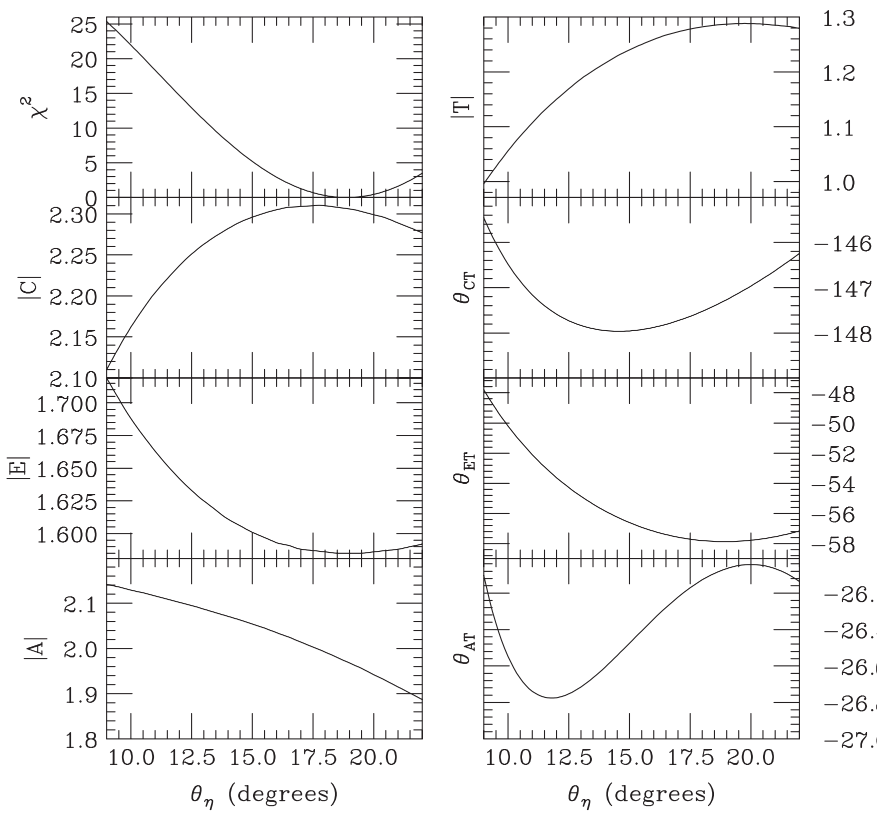

FIG. 4. Behavior of χ^2 and Cabibbo-favored decay amplitudes and relative phases ($|T|$, $|C|$, θ_{CT}, $|E|$, θ_{ET}, $|A|$, θ_{AT}) as functions of θ_η for the $|T| < |C|$ solution.

TABLE III. Branching ratios and invariant amplitudes for singly Cabibbo-suppressed decays of charmed mesons to pions and kaons.

Meson	Decay mode	$\mathcal{B}$ [1] (10^{-3})	p^* (MeV)	$\lvert\mathcal{A}\rvert$ (10^{-7} GeV)	Rep.	Predicted $\mathcal{B}$ (10^{-3}) $\lvert T\rvert < \lvert C\rvert$	$\lvert T\rvert > \lvert C\rvert$
D^0	$\pi^+\pi^-$	1.45 ± 0.05	921.9	4.70 ± 0.08	$-(T' + E')$	2.24	2.24
	$\pi^0\pi^0$	0.81 ± 0.05	922.6	3.51 ± 0.11	$-(C' - E')/\sqrt{2}$	1.36	1.35
	K^+K^-	4.07 ± 0.10	791.0	8.49 ± 0.10	$(T' + E')$	1.92	1.93
	$K^0\bar{K}^0$	0.32 ± 0.02	788.5	2.39 ± 0.14	0	0	0
D^+	$\pi^+\pi^0$	1.18 ± 0.06	924.7	2.66 ± 0.07	$-(T' + C')/\sqrt{2}$	0.88	0.89
	$K^+\bar{K}^0$	6.12 ± 0.22	792.6	6.55 ± 0.12	$(T' - A')$	0.73	6.15
D_s^+	π^+K^0	2.52 ± 0.27	915.7	5.94 ± 0.32	$-(T' - A')$	0.37	3.08
	π^0K^+	0.62 ± 0.23	917.1	2.94 ± 0.55	$-(C' + A')/\sqrt{2}$	0.86	0.85

$$C' = -5.25 - 2.78i, \tag{13}$$

$$E' = -1.76 + 2.65i, \tag{14}$$

$$A' = 0.99 + 0.34i, \tag{15}$$

where we have considered the $|T| > |C|$ solution for $\theta_\eta = 11.7°$. These amplitudes may then be used to predict the branching ratios for SCS D decays. In Table III we summarize the measured and predicted amplitudes of SCS decays to pions and kaons. As was noted in [3], we predict $\mathcal{B}(D^0 \to \pi^+\pi^-)$ larger than observed and $\mathcal{B}(D^0 \to K^+K^-)$ smaller than observed. This deviation from flavor SU(3) symmetry is due at least in part to the ratios of decay constants $f_K/f_\pi = 1.2$ and form factors $f_+(D \to K)/f_+(D \to \pi) > 1$. Other predictions for singly Cabibbo-suppressed decays involving pions and kaons are consistent with those in Ref. [3]. Table III also includes branching ratios predicted using the $|T| < |C|$ ($\theta_\eta = 18.9°$) solution that was obtained in the previous section. However the predictions for the branching ratios of $D^+ \to K^+\bar{K}^0$ and $D_s^+ \to \pi^+K^0$, in this case are an order of magnitude smaller than their measured values.

B. SCS decays involving η, η'

In Table IV we quote the branching ratios and extracted amplitudes for singly Cabibbo-suppressed D-meson decays involving η and η' as reported in [1]. The values of C' and E' obtained in the previous section may be used to determine the relevant parts of the amplitudes for D^0 decays involving η and η'. Additional "disconnected" flavor-singlet diagrams SE' and SA' [8] are required. In Table V we show the representations of the above amplitudes as a function of θ_η using Eqs. (1) and (2).

In Tables VI ($\theta_\eta = 19.5°$) and VII ($\theta_\eta = 11.7°$) we write the amplitudes so that the coefficient of SE' or SA' is always 1. As explained in [3,6] this information may be used to determine SE' and SA' using a plotting technique.

In Fig. 5 we show the construction technique to find the singlet amplitudes for $\theta_\eta = 19.5°$. A χ^2 minimization fit was used to obtain the amplitudes. We find both a small and a large solution for SE' with a small value for χ^2 of around 0.215 for both solutions. In units of 10^{-7} GeV, they are $SE' = -(0.37 \pm 0.20) - (0.56 \pm 0.31)i$ and $SE' = (5.25 \pm 0.29) - (3.43 \pm 0.21)i$. The second is less likely, as it has too large an amplitude when compared to the connected amplitudes. For SA', only a large solution

TABLE IV. Branching ratios and amplitudes for singly Cabibbo-suppressed decays of D^0, D^+, and D_s^+ involving η and η'. Here we use the representations quoted in Eq. (3).

Meson	Decay mode	$\mathcal{B}$ [1] (10^{-4})	p^* (MeV)	$\lvert\mathcal{A}\rvert$ (10^{-7} GeV)	Rep.
D^0	$\pi^0\eta$	6.80 ± 0.70	846.2	3.36 ± 0.17	$-\frac{1}{\sqrt{6}}(2E' - C' + SE')$
	$\pi^0\eta'$	9.10 ± 1.27	678.0	4.34 ± 0.30	$\frac{1}{\sqrt{3}}(E' + C' + 2SE')$
	$\eta\eta$	16.7 ± 1.8	754.7	5.57 ± 0.30	$\frac{2\sqrt{2}}{3}(C' + SE')$
	$\eta\eta'$	10.5 ± 2.6	536.9	5.24 ± 0.65	$-\frac{1}{3\sqrt{2}}(C' + 6E' + 7SE')$
D^+	$\pi^+\eta$	35.4 ± 2.1	848.4	4.82 ± 0.14	$\frac{1}{\sqrt{3}}(T' + 2C' + 2A' + SA')$
	$\pi^+\eta'$	46.8 ± 3.0	680.5	6.18 ± 0.20	$-\frac{1}{\sqrt{6}}(T' - C' + 2A' + 4SA')$
D_s^+	$K^+\eta$	17.6 ± 3.6	835.0	5.20 ± 0.53	$\frac{1}{\sqrt{3}}(T' + 2C' - SA')$
	$K^+\eta'$	18.0 ± 5.0	646.1	5.98 ± 0.83	$\frac{1}{\sqrt{6}}(2T' + C' + 3A' + 4SA')$

[$SA' = -(5.79 \pm 0.14) + (1.61 \pm 0.17)i$ with $\chi^2 = 2.6$] is found using a χ^2 minimization fit. A similar exercise was carried out for $\theta_\eta = 11.7°$. The corresponding construction is shown in Fig. 6. In this case we once again find two solutions for SE' (one large and one small in magnitude) and only one solution for SA'. These are $SE' = -(0.20 \pm 0.20) - (0.81 \pm 0.48)i$ ($\chi^2 = 0.5$), $SE' = (4.25^{+0.37}_{-0.47}) - (3.67 \pm 0.21)i$ ($\chi^2 = 0.5$), and $SA' = -(6.52 \pm 0.13) + (1.13 \pm 0.24)i$ ($\chi^2 = 4.9$). Quali-

TABLE V. Representations for singly Cabibbo-suppressed decays of D^0, D^+, and D_s^+ involving η and η' for an arbitrary $\eta - \eta'$ mixing angle θ_η. Here we use the representations quoted in Eqs. (1) and (2). ($\phi_1 = 45° - \frac{\phi_2}{2}$ and $\phi_2 = 19.5°$.)

Meson	Decay mode	Representation
D^0	$\pi^0\eta$	$\frac{C'}{\sqrt{2}}\cos(\theta_\eta + \phi_1) - E'\sin(\theta_\eta + \phi_1) - \frac{\sqrt{3}SE'}{\sqrt{2}}\sin\theta_\eta$
	$\pi^0\eta'$	$\frac{C'}{\sqrt{2}}\sin(\theta_\eta + \phi_1) + E'\cos(\theta_\eta + \phi_1) + \frac{\sqrt{3}SE'}{\sqrt{2}}\cos\theta_\eta$
	$\eta\eta$	$\frac{\sqrt{3}C'}{\sqrt{2}}\cos\theta_\eta\sin(\theta_\eta + \phi_1) - \frac{3E'}{\sqrt{2}}\cos\theta_\eta\cos(\theta_\eta + 2\phi_1) + \frac{3SE'}{2}\sin(2\theta_\eta)$
	$\eta\eta'$	$-\frac{\sqrt{3}C'}{2}\cos(2\theta_\eta + \phi_1) + \frac{3E'}{2}\sin(2\theta_\eta + 2\phi_1) - \frac{3SE'}{\sqrt{2}}\cos(2\theta_\eta)$
D^+	$\pi^+\eta$	$\frac{T'}{\sqrt{2}}\sin(\theta_\eta + \phi_1) + \frac{\sqrt{3}C'}{\sqrt{2}}\cos\theta_\eta + \sqrt{2}A'\sin(\theta_\eta + \phi_1) + \sqrt{3}SA'\sin\theta_\eta$
	$\pi^+\eta'$	$-\frac{T'}{\sqrt{2}}\cos(\theta_\eta + \phi_1) + \frac{\sqrt{3}C'}{\sqrt{2}}\sin\theta_\eta - \sqrt{2}A'\cos(\theta_\eta + \phi_1) - \sqrt{3}SA'\cos\theta_\eta$
D_s^+	$K^+\eta$	$T'\cos(\theta_\eta + \phi_1) + \frac{\sqrt{3}C'}{\sqrt{2}}\cos\theta_\eta + \frac{\sqrt{3}A'}{\sqrt{2}}\cos(\theta_\eta + 2\phi_1) - \sqrt{3}SA'\sin\theta_\eta$
	$K^+\eta'$	$T'\sin(\theta_\eta + \phi_1) + \frac{\sqrt{3}C'}{\sqrt{2}}\sin\theta_\eta - \frac{\sqrt{3}A'}{\sqrt{2}}\sin(\theta_\eta + 2\phi_1) + \sqrt{3}SA'\cos\theta_\eta$

TABLE VI. Real and imaginary parts of amplitudes for singly Cabibbo-suppressed decays of D^0, D^+, and D_s^+ involving η and η' ($\theta_\eta = 19.5°$), in units of 10^{-7} GeV, used to generate plots in Fig. 5.

Amplitude ($\mathcal{A}$)	Expression	Re	Im	$\lvert\mathcal{A}_{\rm exp}\rvert$
$-\sqrt{6}\mathcal{A}(D^0 \to \pi^0\eta)$	$2E' - C' + SE'$	1.06	8.85	8.22 ± 0.42
$\frac{\sqrt{3}}{2}\mathcal{A}(D^0 \to \pi^0\eta')$	$\frac{1}{2}(C' + E') + SE'$	−3.31	0.28	3.76 ± 0.26
$\frac{3}{2\sqrt{2}}\mathcal{A}(D^0 \to \eta\eta)$	$C' + SE'$	−4.77	−2.57	5.91 ± 0.32
$-\frac{3\sqrt{2}}{7}\mathcal{A}(D^0 \to \eta\eta')$	$\frac{1}{7}(C' + 6E') + SE'$	−2.27	2.32	3.17 ± 0.39
$\sqrt{3}\mathcal{A}(D^+ \to \pi^+\eta)$	$T' + 2C' + 2A' + SA'$	−2.24	−3.70	8.34 ± 0.25
$-\frac{\sqrt{6}}{4}\mathcal{A}(D^+ \to \pi^+\eta')$	$\frac{1}{4}(T' - C' + 2A') + SA'$	3.01	1.00	3.79 ± 0.12
$-\sqrt{3}\mathcal{A}(D_s^+ \to \eta K^+)$	$-(T' + 2C') + SA'$	2.75	5.14	9.00 ± 0.92
$\frac{\sqrt{6}}{4}\mathcal{A}(D_s^+ \to \eta' K^+)$	$\frac{1}{4}(2T' + C' + 3A') + SA'$	2.39	−0.10	3.66 ± 0.51

TABLE VII. Real and imaginary parts of amplitudes for singly Cabibbo-suppressed decays of D^0, D^+, and D_s^+ involving η and η'($\theta_\eta = 11.7°$), in units of 10^{-7} GeV, used to generate plots in Fig. 6.

Amplitude	Coefficients in the expression						Re	Im	$\lvert\mathcal{A}_{\rm exp}\rvert$
	T'	C'	E'	A'	SE'	SA'			
$-4.03\mathcal{A}(D^0 \to \pi^0\eta)$	0	−1.95	2.95	0	1	0	5.04	13.22	13.54 ± 0.70
$0.83\mathcal{A}(D^0 \to \pi^0\eta')$	0	0.43	0.57	0	1	0	−3.27	0.31	3.62 ± 0.25
$1.68\mathcal{A}(D^0 \to \eta\eta)$	0	1.47	−0.47	0	1	0	−6.91	5.35	9.37 ± 0.51
$-0.51\mathcal{A}(D^0 \to \eta\eta')$	0	0.23	0.77	0	1	0	−2.57	1.39	2.69 ± 0.33
$2.85\mathcal{A}(D^+ \to \pi^+\eta)$	1.47	3.42	0	2.95	0	1	−4.80	−8.50	13.74 ± 0.41
$0.59\mathcal{A}(D^+ \to \pi^+\eta')$	−0.28	0.15	0	−0.57	0	1	3.31	0.60	3.65 ± 0.12
$-2.85\mathcal{A}(D_s^+ \to \eta\eta)$	−1.95	−3.42	0	1.81	0	1	6.21	10.13	14.83 ± 1.52
$0.59\mathcal{A}(D_s^+ \to \eta\eta')$	2.84	−0.19	0	0.62	0	1	2.84	−0.19	3.52 ± 0.49

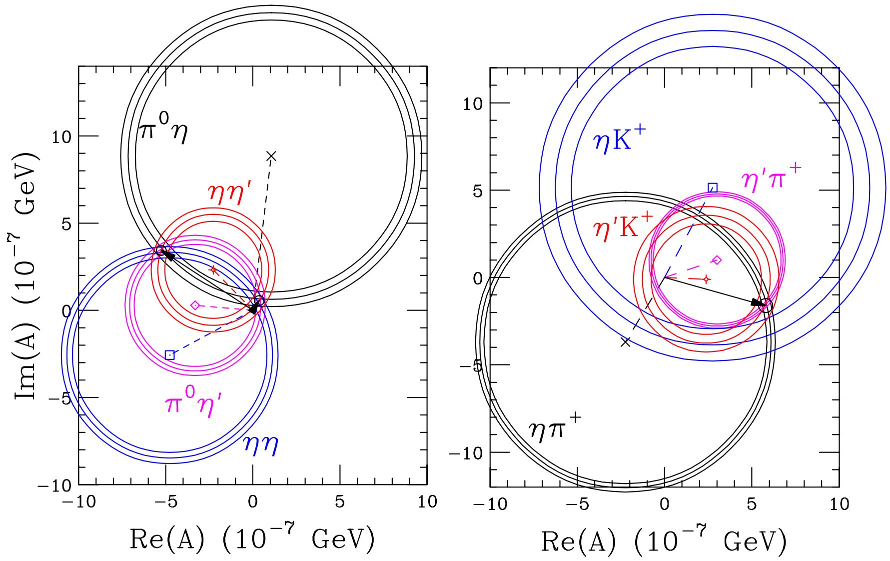

FIG. 5 (color online). Determination of the disconnected singlet annihilation amplitudes SE' (left panel) and SA' (right panel) from SCS charmed meson decays involving η and η' in the solutions with $|T| > |C|$ and $\theta_\eta = 19.5°$. Left panel: D^0 decays to final states as shown; right panel: D^+ or D_s^+ decays to final states as shown. The small black circles show the solution regions. Arrows pointing to them denote the complex amplitudes $-SE'$ (left panel) and $-SA'$ (right panel).

tatively similar conclusions were reached in [3] where we used only the value $\theta_\eta = 19.5°$.

C. Sum rules for $D^0 \to (\pi^0\pi^0, \pi^0\eta, \eta\eta, \pi^0\eta', \eta\eta')$

Let us consider the singly Cabibbo-suppressed decays of the D^0, where the final pseudoscalars are π^0, η, or η'. In Tables III and IV we list the flavor-topology representations for all such decays. It is interesting to note that there are 5 such amplitudes all of which depend only on C', E', and SE'. This means we can algebraically relate two or more of these amplitudes through sum rules, such as

$$\sqrt{2}\mathcal{A}(D^0 \to \pi^0\pi^0) + \sqrt{3}\sin\theta_\eta \mathcal{A}(D^0 \to \pi^0\eta') + \sqrt{3}\cos\theta_\eta \mathcal{A}(D^0 \to \pi^0\eta) = 0, \tag{16}$$

$$(1 - 3\sin^2\theta_\eta)\mathcal{A}(D^0 \to \pi^0\pi^0) + \mathcal{A}(D^0 \to \eta\eta) - \sqrt{6}\sin\theta_\eta \mathcal{A}(D^0 \to \pi^0\eta') = 0, \tag{17}$$

$$(2\cos^2\theta_\eta \cos(2\theta_\eta) + \sin^2(2\theta_\eta))\mathcal{A}(D^0 \to \pi^0\pi^0) + 2\cos(2\theta_\eta)\mathcal{A}(D^0 \to \eta\eta) + \sqrt{2}\sin(2\theta_\eta)\mathcal{A}(D^0 \to \eta\eta') = 0. \tag{18}$$

A sum rule relating three amplitudes can be represented by

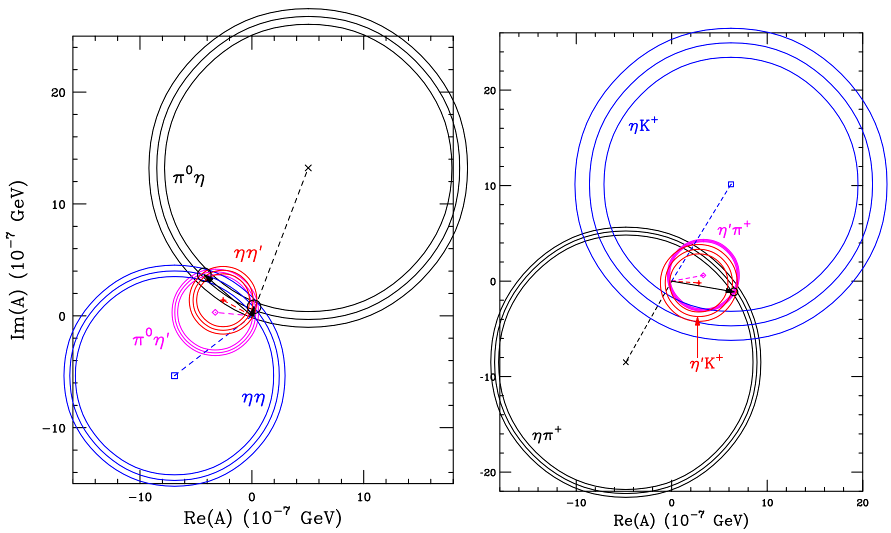

FIG. 6 (color online). Same as Fig. 5 except $\theta_\eta = 11.7°$.

a triangle whose sides are the magnitudes of the corresponding amplitudes. As in [3], these triangles have angles not equal to zero or 180°, showing that the amplitudes have nonzero relative strong phases.

The sum rule in [3] relating squares of magnitudes of amplitudes,

$$\begin{aligned} &8|\mathcal{A}(D^0 \to \pi^0\eta')|^2 + 16|\mathcal{A}(D^0 \to \pi^0\pi^0)|^2 \\ &\quad = 16|\mathcal{A}(D^0 \to \pi^0\eta)|^2 + 9|\mathcal{A}(D^0 \to \eta\eta)|^2, \end{aligned} \tag{19}$$

may be reevaluated using the data from Tables III and IV. We find, in units of 10^{-14} GeV^2,

$$8|\mathcal{A}(D^0 \to \pi^0\eta')|^2 + 16|\mathcal{A}(D^0 \to \pi^0\pi^0)|^2 = 348 \pm 24, \tag{20}$$

$$16|\mathcal{A}(D^0 \to \pi^0\eta)|^2 + 9|\mathcal{A}(D^0 \to \eta\eta)|^2 = 460 \pm 35. \tag{21}$$

The deviation from equality by about 2.6σ indicates the degree of flavor-SU(3) breaking.

The above sum rule refers to the special case of $\theta_\eta = 19.5°$. A more general sum rule, valid for arbitrary values of θ_η, is

$$\begin{aligned} &81\sin^2\theta_\eta\cos^2\theta_\eta|\mathcal{A}(D^0 \to \pi^0\eta')|^2 \\ &\quad + 27\cos^2\theta_\eta(3\cos^2\theta_\eta - 2)|\mathcal{A}(D^0 \to \pi^0\pi^0)|^2 \\ &= 27\cos^2\theta_\eta(3\cos^2\theta_\eta - 2)|\mathcal{A}(D^0 \to \pi^0\eta)|^2 \\ &\quad + 9|\mathcal{A}(D^0 \to \eta\eta)|^2, \end{aligned} \tag{22}$$

where we have kept the normalization of the previous sum rule. The left-hand side of this sum rule, in units of 10^{-14} GeV^2, is 340 ± 19, while the right-hand side is 535 ± 40. Here the discrepancy rises to 4.4σ. Allowing θ_η to be a free parameter does not improve the description of SCS decays.

V. DOUBLY CABIBBO-SUPPRESSED DECAYS

Table VIII contains a list of doubly Cabibbo-suppressed decay branching ratios, amplitudes, and their representation in terms of $\tilde{T}$, $\tilde{C}$, $\tilde{E}$, and $\tilde{A}$. The magnitudes of these are obtained by multiplying the corresponding Cabibbo-favored amplitudes listed in Sec. III by $-\lambda^2$. Using $\lambda = 0.2317$ we find $\mathcal{B}(D^0 \to K^+\pi^-) = 1.12 \times 10^{-4}$ and $\mathcal{B}(D^+ \to K^+(\pi^0, \eta, \eta')) = (1.49, 1.06, 1.16) \times 10^{-4}$, where we have used $\theta_\eta = 11.7°$ and the $|T| > |C|$ solution. While the experimental branching ratio for $D^0 \to K^+\pi^-$ remains 29% above the prediction from flavor SU(3), the measured branching ratio for $D^+ \to K^+\pi^0$ matches the predicted value to around 1σ. In Fig. 7 we plot the branching ratios of the predicted doubly Cabibbo-suppressed decays $D^0 \to K^-\pi^+$ and $D^+ \to K^+(\pi^0, \eta, \eta')$, which are ones for which measurements or upper bounds exist, as functions of θ_η.

As described in [3], in general the decays $D \to (K\pi, \bar{K}\pi)$ are related to each other by either a simple U-spin interchange $s \leftrightarrow d$, or by interchanging "tree" and "annihilation" amplitudes. In the former case, the extent of SU(3) breaking in the prediction for the decay asymmetry is expected to be very small.

In D^0 decays the asymmetry is predicted as a consequence of the U spin:

$$R(D^0) \equiv \frac{\Gamma(D^0 \to K_S\pi^0) - \Gamma(D^0 \to K_L\pi^0)}{\Gamma(D^0 \to K_S\pi^0) + \Gamma(D^0 \to K_L\pi^0)} \tag{23}$$

$$= 2\lambda^2 = 0.107, \tag{24}$$

and is indeed consistent with the observed value [9]

$$R(D^0) = 0.108 \pm 0.025 \pm 0.024. \tag{25}$$

Similarly for the D^+ and D_s^+ decays one may construct the following quantities and predict

TABLE VIII. Branching ratios and amplitudes for doubly Cabibbo-suppressed decays of D^0, D^+, and D_s^+.

Meson	Decay mode	$\mathcal{B}$ [1] (10^{-4})	p^* (MeV)	Representation	Predicted $\mathcal{B}$ $\theta_\eta = 11.7°$
D^0	$K^+\pi^-$	1.45 ± 0.04	861.1	$\tilde{T} + \tilde{E}$	1.12
	$K^0\pi^0$		860.4	$(\tilde{C} - \tilde{E})/\sqrt{2}$	0.69
	$K^0\eta$		771.9	$\frac{\tilde{C}}{\sqrt{2}}\sin(\theta_\eta + \phi_1) - \frac{\sqrt{3}\tilde{E}}{\sqrt{2}}\cos(\theta_\eta + 2\phi_1)$	0.28
	$K^0\eta'$		564.9	$-\frac{\tilde{C}}{\sqrt{2}}\cos(\theta_\eta + \phi_1) - \frac{\sqrt{3}\tilde{E}}{\sqrt{2}}\sin(\theta_\eta + 2\phi_1)$	0.55
D^+	$K^0\pi^+$		862.6	$\tilde{C} + \tilde{A}$	2.01
	$K^+\pi^0$	1.72 ± 0.19	864.0	$(\tilde{T} - \tilde{A})/\sqrt{2}$	1.49
	$K^+\eta$	<1.3	775.8	$-\frac{\tilde{T}}{\sqrt{2}}\sin(\theta_\eta + \phi_1) - \frac{\sqrt{3}\tilde{A}}{\sqrt{2}}\cos(\theta_\eta + 2\phi_1)$	1.06
	$K^+\eta'$	<1.8	570.8	$\frac{\tilde{T}}{\sqrt{2}}\cos(\theta_\eta + \phi_1) + \frac{\sqrt{3}\tilde{A}}{\sqrt{2}}\sin(\theta_\eta + 2\phi_1)$	1.16
D_s^+	K^0K^+		850.3	$\tilde{T} + \tilde{C}$	0.38

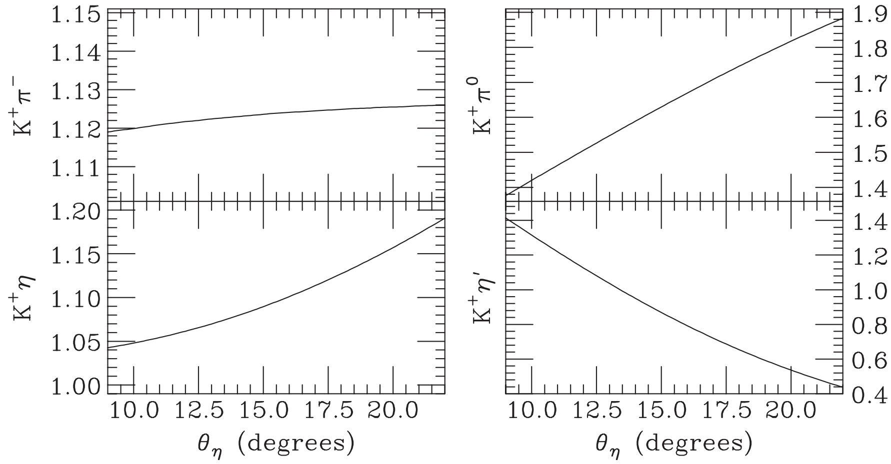

FIG. 7. Branching ratios in units of 10^{-4} of doubly Cabibbo-suppressed decays $D^0 \to K^+\pi^-$, $D^0 \to K^0(\pi^0, \eta, \eta')$, and $D^+ \to K^+(\eta, \eta')$ as functions of θ_η for the $|T| > |C|$ solution.

$$R(D^+) \equiv \frac{\Gamma(D^+ \to K_S\pi^+) - \Gamma(D^+ \to K_L\pi^+)}{\Gamma(D^+ \to K_S\pi^+) + \Gamma(D^+ \to K_L\pi^+)} \tag{26}$$

$$= 2\lambda^2 \,\mathrm{Re}\frac{C + A}{T + C} = -0.005 \pm 0.013, \tag{27}$$

$$R(D_s^+) \equiv \frac{\Gamma(D_s^+ \to K_SK^+) - \Gamma(D_s^+ \to K_LK^+)}{\Gamma(D_s^+ \to K_SK^+) + \Gamma(D_s^+ \to K_LK^+)} \tag{28}$$

$$= 2\lambda^2 \,\mathrm{Re}\frac{C + T}{A + C} = -0.0022 \pm 0.0087. \tag{29}$$

Here we have made use of the amplitudes obtained from the χ^2 minimum solution that satisfies $|T| > |C|$ and allows for $\theta_\eta = 11.7°$. These represent similarity with respect to the predictions in Ref. [3], $R(D^+) = -0.006^{+0.033}_{-0.028}$ and $R(D_s^+) = -0.003^{+0.019}_{-0.017}$. Experimentally only the first asymmetry is measured [9]:

$$R(D^+) = 0.022 \pm 0.016 \pm 0.018, \tag{30}$$

in agreement with the new prediction just as with the earlier one.

VI. CONCLUSIONS

We have reanalyzed the decays of charmed mesons to pairs of light pseudoscalars using flavor SU(3) in the light of new experimental determinations of branching ratios [1] with experimental errors often smaller than previous world averages [2]. The main difference with respect to a previous analysis [3] is that the annihilation amplitude A, found previously to have a phase of almost 180° with respect to the ''exchange'' amplitude E, now has a phase of $\sim(100 \pm 10)°$ with respect to E for the preferred set of amplitudes and phases. While consequences for singly Cabibbo-suppressed decays are qualitatively similar to those in [3], similar decay asymmetries involving $D^+ \to K_{(S,L)}\pi^+$ and $D_s^+ \to K_{(S,L)}\pi^+$ are predicted; both are in good agreement with the observed value as well as the ones obtained previously.

ACKNOWLEDGMENTS

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