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Pure annihilation type $B_c \to M_2 M_3$ decays in the perturbative QCD approach

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In the standard model the two-body charmless hadronic B_c meson decays can occur via annihilation diagrams only. In this work, we studied the $B_c \to PP$, PV/VP, VV decays by employing the perturbative QCD (pQCD) factorization approach. From our calculations, we find that (a) the pQCD predictions for the branching ratios of the considered B_c decays are in the range of 10^{-6} to 10^{-8}; (b) for $B_c \to PV/VP$, VV decays, the branching ratios of $\Delta S = 0$ decays are much larger than those of $\Delta S = 1$ because the different Cabibbo-Kobayashi-Maskawa (CKM) factors are involved; (c) analogous to $B \to K\eta^{(\prime)}$ decays, we find $\mathrm{Br}(B_c \to K^+\eta') \sim 10 \times \mathrm{Br}(B_c \to K^+\eta)$, which can be understood by the destructive and constructive interference between the η_q and η_s contribution to the $B_c \to K^+\eta$ and $B_c \to K^+\eta'$ decay; (d) the longitudinal polarization fractions of $B_c \to VV$ decays are in the range of 86%–95% and play the dominant role; and (e) there is no CP-violating asymmetries for the considered B_c decays because only one type tree operator is involved.

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I. INTRODUCTION

In 1998, a new stage of B_c physics began because of the first observation of the meson B_c at Tevatron [1]. For B_c meson, one can study the two heavy flavors b and c in a meson simultaneously. From an experimental point of view, more detailed information about B_c meson can be obtained at the Large Hadron Collider (LHC) experiment. The LHC is now running, where the B_c meson could be produced abundantly. The B_c meson decays may provide windows for testing the predictions of the standard model (SM) and can shed light on new physics (NP) scenarios beyond the SM.

From a theoretical point of view [2], the nonleptonic decays of B_c meson are the most complicated decays due to its heavy-heavy nature and the participation of strong interaction, which complicate the extraction of parameters in SM, but they also provide great opportunities to study the perturbative and nonperturbative QCD, final state interactions, etc. The nonleptonic B_c weak decays have been widely studied, for example, in Refs. [2–32] by employing the naive factorization approach (NFA) [33], the QCD factorization approach (QCDF) [34], the perturbative QCD (pQCD) approach [35–37] and other approaches and/or methods.

In this paper we focus on the two-body nonleptonic charmless decays $B_c \to PP$, PV/VP, VV (here P and V stand for the light pseudoscalar and vector mesons), which can occur through the weak annihilation diagrams only. The size of annihilation contributions is an important issue in B physics. Indeed, the two-body charmless B_c decays considered here are rather different from those $B_c \to J/\psi P(V)$ decays where the initial c quark behaves as a spectator.

Recently, the two-body nonleptonic charmless $B_c \to M_2 M_3$[1] decays have been studied by using the $SU(3)$ flavor symmetry or by employing the QCD factorization approach [38]. The authors in Ref. [38] provided two different estimates for nonleptonic charmless B_c decays. But their predictions for the branching ratios of $B_c \to \phi K^+$, $\bar{K}^{*0}K^+$ decays in the QCDF are much smaller (a factor of 10) than those obtained by using the $SU(3)$ flavor symmetry. So large discrepancies among the theoretical predictions for the branching ratios indicate clearly that it is very necessary to make more studies for these kinds of B_c decays by employing different approaches, in order to understand these decays better and provide the theoretical support for the related experimental studies.

In this paper, we will calculate the branching ratios and the polarization fractions of 30 $B_c \to PP$, PV/VP, VV decays by employing the low energy effective Hamiltonian [39] and the pQCD factorization approach. By keeping the transverse momentum k_T of the quarks, the pQCD approach is free of end-point singularity and the Sudakov formalism makes it more self-consistent. It is worth mentioning that one can do the quantitative calculations of the annihilation type diagrams in the pQCD approach. The importance of annihilation contributions

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[1]For the sake of simplicity, we will use M_2 and M_3 to denote the two final state light mesons, respectively, unless otherwise stated.

has already been tested in the previous predictions of branching ratios of pure annihilation $B \to D_s K$ decays [40], direct CP asymmetries of $B^0 \to \pi^+\pi^-$, $K^+\pi^-$ decays [35,36,41], and in the explanation of $B \to \phi K^*$ polarization problem [42,43], which indicate that the pQCD approach is a reliable method to deal with annihilation diagrams.

The paper is organized as follows. In Sec. II, we present the formalism and wave functions of the considered B_c meson decays. Then we perform the perturbative calculations for considered decay channels with pQCD approach in Sec. III. The numerical results and phenomenological analysis are given in Sec. IV. Finally, Sec. V contains the main conclusions and a short summary.

II. FORMALISM AND WAVE FUNCTIONS

A. Formalism

Since the b quark is rather heavy, we work in the frame with the B_c meson at rest, i.e., with the B_c meson momentum $P_1 = (m_{B_c}/\sqrt{2})(1, 1, \mathbf{0}_T)$ in the light-cone coordinates. For the nonleptonic charmless $B_c \to M_2 M_3$ decays, assuming that the M_2 (M_3) meson moves in the plus (minus) z direction carrying the momentum P_2 (P_3) and the polarization vector ϵ_2 (ϵ_3) (if $M_{2(3)}$ are the vector mesons). Then the two final state meson momenta can be written as

$$P_2 = \frac{m_{B_c}}{\sqrt{2}}(1 - r_3^2, r_2^2, \mathbf{0}_T), \qquad P_3 = \frac{m_{B_c}}{\sqrt{2}}(r_3^2, 1 - r_2^2, \mathbf{0}_T), \tag{1}$$

respectively, where $r_2 = m_{M_2}/m_B$, and $r_3 = m_{M_3}/m_B$. When M_2, M_3 are the vector mesons, the longitudinal polarization vectors, ϵ_2^L and ϵ_3^L, can be given by

$$\begin{aligned} \epsilon_2^L &= \frac{m_{B_c}}{\sqrt{2}m_{M_2}}(1 - r_3^2, -r_2^2, \mathbf{0}_T), \\ \epsilon_3^L &= \frac{m_{B_c}}{\sqrt{2}m_{M_3}}(-r_3^2, 1 - r_2^2, \mathbf{0}_T). \end{aligned} \tag{2}$$

The transverse ones are parametrized as $\epsilon_2^T = (0, 0, 1_T)$, and $\epsilon_3^T = (0, 0, 1_T)$. Putting the (light-) quark momenta in B_c, M_2 and M_3 mesons as k_1, k_2, and k_3, respectively, we can choose

$$\begin{aligned} k_1 &= (x_1 P_1^+, 0, \mathbf{k}_{1T}), \qquad k_2 = (x_2 P_2^+, 0, \mathbf{k}_{2T}), \\ k_3 &= (0, x_3 P_3^-, \mathbf{k}_{3T}). \end{aligned} \tag{3}$$

Then, for $B_c \to M_2 M_3$ decays, the integration over k_1^-, k_2^-, and k_3^+ will conceptually lead to the decay amplitudes in the pQCD approach,

$$\begin{aligned} \mathcal{A}(B_c \to M_2 M_3) \sim \int & dx_1 dx_2 dx_3 b_1 db_1 b_2 db_2 b_3 db_3 \\ & \cdot \mathrm{Tr}[C(t)\Phi_{B_c}(x_1, b_1)\Phi_{M_2}(x_2, b_2) \\ & \times \Phi_{M_3}(x_3, b_3)H(x_i, b_i, t)S_t(x_i) \\ & \times e^{-S(t)}], \end{aligned} \tag{4}$$

where b_i is the conjugate space coordinate of k_{iT}, and t is the largest energy scale in function $H(x_i, b_i, t)$. The large logarithms $\ln(m_W/t)$ are included in the Wilson coefficients $C(t)$. The large double logarithms ($\ln^2 x_i$) are summed by the threshold resummation [44], and they lead to $S_t(x_i)$ which smears the end-point singularities on x_i. The last term, $e^{-S(t)}$, is the Sudakov form factor which suppresses the soft dynamics effectively [45]. Thus it makes the perturbative calculation of the hard part H applicable at intermediate scale, i.e., m_{B_c} scale. We will calculate analytically the function $H(x_i, b_i, t)$ for the considered decays at leading order (LO) in α_s expansion and give the convoluted amplitudes in next section.

For these considered decays, the related weak effective Hamiltonian H_{eff} [39] can be written as

$$H_{\mathrm{eff}} = \frac{G_F}{\sqrt{2}}[V_{cb}^* V_{uD}(C_1(\mu)O_1(\mu) + C_2(\mu)O_2(\mu))], \tag{5}$$

with the single tree operators

$$\begin{aligned} O_1 &= \bar{u}_\beta \gamma^\mu (1 - \gamma_5) D_\alpha \bar{c}_\beta \gamma^\mu (1 - \gamma_5) b_\alpha, \\ O_2 &= \bar{u}_\beta \gamma^\mu (1 - \gamma_5) D_\beta \bar{c}_\alpha \gamma^\mu (1 - \gamma_5) b_\alpha, \end{aligned} \tag{6}$$

where V_{cb}, V_{uD} are the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, D denotes the light down quark d or s and $C_i(\mu)$ are Wilson coefficients at the renormalization scale μ. For the Wilson coefficients $C_{1,2}(\mu)$, we will also use the LO expressions, although the next-to-leading order calculations already exist in the literature [39]. This is the consistent way to cancel the explicit μ dependence in the theoretical formulas. For the renormalization group evolution of the Wilson coefficients from higher scale to lower scale, we use the formulas as given in Ref. [36] directly.

B. Wave functions

In order to calculate the decay amplitude, we should choose the proper wave functions of the heavy B_c and light mesons. In principle there are two Lorentz structures in the $B_{u,d,s}$ or B_c meson wave function. One should consider both of them in calculations. However, since the contribution induced by one Lorentz structure is numerically small [46,47] and can be neglected approximately, we only consider the contribution from the first Lorentz structure

$$\Phi_{B_c}(x) = \frac{i}{\sqrt{2N_c}}[(\not{P} + M_{B_c})\gamma_5 \phi_{B_c}(x)]_{\alpha\beta}. \tag{7}$$

Since B_c meson consists of two heavy quarks and $m_{B_c} \simeq m_b + m_c$, the distribution amplitude ϕ_{B_c} would be close to $\delta(x - m_c/m_{B_c})$ in the nonrelativistic limit. We therefore adopt the nonrelativistic approximation form of ϕ_{B_c} as [19,28]

$$\phi_{B_c}(x) = \frac{f_{B_c}}{2\sqrt{2N_c}}\delta(x - m_c/m_{B_c}), \tag{8}$$

where f_{B_c} and N_c are the decay constant of B_c meson and the color number, respectively.

For the pseudoscalar meson (P), the wave function can generally be defined as

$$\Phi_P(x) = \frac{i}{\sqrt{2N_c}}\gamma_5\{\not{P}\phi_P^A(x) + m_0^P\phi_P^P(x) + m_0^P(\not{n}\not{v} - 1)\phi_P^T(x)\}_{\alpha\beta}, \tag{9}$$

where $\phi_P^{A,P,T}$ and m_0^P are the distribution amplitudes and chiral scale parameter of the pseudoscalar mesons, respectively, while x denotes the momentum fraction carried by quark in the meson, and $n = (1, 0, \mathbf{0}_T)$ and $v = (0, 1, \mathbf{0}_T)$ are dimensionless lightlike unit vectors.

For the wave functions of vector mesons, one longitudinal (L) and two transverse (T) polarizations are involved, and can be written as

$$\Phi_V^L(x) = \frac{1}{\sqrt{2N_c}}\{M_V\not{\epsilon}_V^{*L}\phi_V(x) + \not{\epsilon}_V^{*L}\not{P}\phi_V^t(x) + M_V\phi_V^s(x)\}_{\alpha\beta}, \tag{10}$$

$$\Phi_V^T(x) = \frac{1}{\sqrt{2N_c}}\{M_V\not{\epsilon}_V^{*T}\phi_V^v(x) + \not{\epsilon}_V^{*T}\not{P}\phi_V^T(x) + M_V i\epsilon_{\mu\nu\rho\sigma}\gamma_5\gamma^\mu\epsilon_T^{*\nu}n^\rho v^\sigma\phi_V^a(x)\}_{\alpha\beta}, \tag{11}$$

where $\epsilon_V^{L(T)}$ denotes the longitudinal (transverse) polarization vector of vector mesons, satisfying $P \cdot \epsilon = 0$ in each polarization. We here adopt the convention $\epsilon^{0123} = 1$ for the Levi-Civita tensor $\epsilon^{\mu\nu\alpha\beta}$. For the distribution amplitudes of pseudoscalar $\phi_P^{A,P,T}$, and longitudinal and transverse polarization, $\phi_V^{,t,s}$ and $\phi_V^{v,T,a}$, which will be presented in Appendix A.

III. PERTURBATIVE CALCULATIONS IN PQCD

From the effective Hamiltonian (5), there are four types of diagrams contributing to the $B_c \to M_2M_3$ decays as illustrated in Fig. 1, which result in the Feynman decay amplitudes $F_{fa}^{M_2M_3}$ and $M_{na}^{M_2M_3}$, where the subscripts fa and na are the abbreviations of factorizable and nonfactorizable annihilation contributions, respectively. Operators $O_{1,2}$ are $(V - A)(V - A)$ currents, and we therefore can combine all contributions from these diagrams and obtain the total decay amplitude as

$$\mathcal{A}(B_c \to M_2M_3) = V_{cb}^* V_{uD}\{f_{B_c}F_{fa}^{M_2M_3}a_1 + M_{na}^{M_2M_3}C_1\}, \tag{12}$$

where $a_1 = C_1/3 + C_2$. In the next three subsections we will give the explicit expressions of $F_{fa}^{M_2M_3}$, $M_{na}^{M_2M_3}$, and the decay amplitude $\mathcal{A}(B_c \to M_2M_3)$ for $B_c \to M_2M_3$ decays: including eight $B_c \to PP$, fifteen $B_c \to PV$ or $B_c \to VP$, and seven $B_c \to VV$ decay modes.

A. $B_c \to PP$ decays

In this section, we will present the factorization formulas for eight nonleptonic charmless $B_c \to PP$ decays. From the first two diagrams of Fig. 1, i.e., (a) and (b), by perturbative QCD calculations, we obtain the decay amplitude for factorizable annihilation contributions as follows:

$$\begin{aligned} F_{fa}^{PP} = &-8\pi C_F m_{B_c}^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \\ &\times\{h_{fa}(1-x_3, x_2, b_3, b_2)E_{fa}(t_a)[x_2\phi_2^A(x_2)\phi_3^A(x_3) \\ &+ 2r_0^2 r_0^3\phi_3^P(x_3)((x_2+1)\phi_2^P(x_2) + (x_2-1)\phi_2^T(x_2))] \\ &+ h_{fa}(x_2, 1-x_3, b_2, b_3)E_{fa}(t_b) \\ &\times[(x_3-1)\phi_2^A(x_2)\phi_3^A(x_3) \\ &+ 2r_0^2 r_0^3\phi_2^P(x_2)((x_3-2)\phi_3^P(x_3) - x_3\phi_3^T(x_3))]\}, \end{aligned} \tag{13}$$

where $\phi_{2(3)}$ corresponds to the distribution amplitudes of mesons $M_{2(3)}$, $r_0^{2(3)} = m_0^{M_2(M_3)}/m_{B_c}$, and $C_F = 4/3$ is a color factor. In Eq. (13), the terms proportional to $(r_0^{2(3)})^2$ have been neglected because they are small indeed, $\max(r_0^{2(3)})^2 \leq 7\%$. The function h_{fa}, the scales t_i and $E_{fa}(t)$ can be found in Appendix B.

For the nonfactorizable diagrams (c) and (d) in Fig. 1, all three meson wave functions are involved. The integration of b_3 can be performed using δ function $\delta(b_3 - b_2)$, leaving only integration of b_1 and b_2. The corresponding decay amplitude is

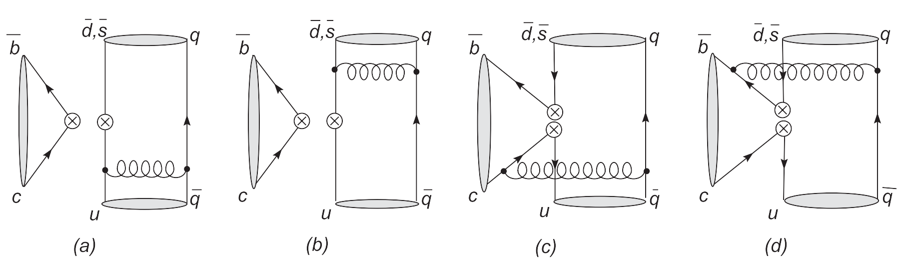

FIG. 1. Typical Feynman diagrams for two-body nonleptonic charmless B_c decays.

$$M_{na}^{PP} = -\frac{16\sqrt{6}}{3}\pi C_F m_{B_c}^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \{h_{na}^c(x_2,x_3,b_1,b_2)E_{na}(t_c)[(r_c - x_3 + 1)\phi_2^A(x_2)\phi_3^A(x_3) + r_0^2 r_0^3(\phi_2^P(x_2)((3r_c + x_2 - x_3 + 1)\phi_3^P(x_3) - (r_c - x_2 - x_3 + 1)\phi_3^T(x_3)) + \phi_2^T(x_2)((r_c - x_2 - x_3 + 1)\phi_3^P(x_3) + (r_c - x_2 + x_3 - 1)\phi_3^T(x_3)))] - E_{na}(t_d)[(r_b + r_c + x_2 - 1)\phi_2^A(x_2)\phi_3^A(x_3) + r_0^2 r_0^3(\phi_2^P(x_2)((4r_b + r_c + x_2 - x_3 - 1)\phi_3^P(x_3) - (r_c + x_2 + x_3 - 1)\phi_3^T(x_3)) + \phi_2^T(x_2)((r_c + x_2 + x_3 - 1)\phi_3^P(x_3) - (r_c + x_2 - x_3 - 1)\phi_3^T(x_3)))]h_{na}^d(x_2,x_3,b_1,b_2)\}, \tag{14}$$

where $r_{b(c)} = m_{b(c)}/m_{B_c}$.

For the $\eta - \eta'$ system, there exist two popular mixing bases: the octet-singlet basis and the quark-flavor basis [48,49]. Here we use the quark-flavor basis [48] and define

$$\eta_q = (u\bar{u} + d\bar{d})/\sqrt{2}, \qquad \eta_s = s\bar{s}. \tag{15}$$

The physical states η and η' are related to η_q and η_s through a single mixing angle ϕ,

$$\begin{pmatrix}\eta\\ \eta'\end{pmatrix} = U(\phi)\begin{pmatrix}\eta_q\\ \eta_s\end{pmatrix} = \begin{pmatrix}\cos\phi & -\sin\phi\\ \sin\phi & \cos\phi\end{pmatrix}\begin{pmatrix}\eta_q\\ \eta_s\end{pmatrix}. \tag{16}$$

We assume that the distribution amplitudes of η_q and η_s are the same as the distribution amplitudes of π, except for the different decay constants and the chiral scale parameters. The three input parameters f_q, f_s, and ϕ in the quark-flavor basis have been extracted from various related experiments [48,49]

$$f_q = (1.07 \pm 0.02)f_\pi, \qquad f_s = (1.34 \pm 0.06)f_\pi, \qquad \phi = 39.3° \pm 1.0°. \tag{17}$$

The chiral enhancement factors are chosen as

$$m_0^{\eta_q} \equiv \frac{m_{qq}^2}{2m_q} = \frac{1}{2m_q}\left[m_\eta^2\cos^2\phi + m_{\eta'}^2\sin^2\phi - \frac{\sqrt{2}f_s}{f_q}(m_{\eta'}^2 - m_\eta^2)\cos\phi\sin\phi\right], \tag{18}$$

$$m_0^{\eta_s} \equiv \frac{m_{ss}^2}{2m_s} = \frac{1}{2m_s}\left[m_{\eta'}^2\cos^2\phi + m_\eta^2\sin^2\phi - \frac{f_q}{\sqrt{2}f_s}(m_{\eta'}^2 - m_\eta^2)\cos\phi\sin\phi\right]. \tag{19}$$

In the numerical calculations, we will use these mixing parameters as inputs. It is worth mentioning that the effects of a possible gluonic component of the η' meson will not be considered here since it is small in size [50–52].

Based on Eqs. (12)–(14), we can write down the total decay amplitudes for eight $B_c \to PP$ decays easily,

$$\mathcal{A}(B_c \to \pi^+\pi^0) = V_{cb}^* V_{ud}\{[f_{B_c}F_{fa}^{\pi^+\pi^0_{\bar{u}u}}a_1 + M_{na}^{\pi^+\pi^0_{\bar{u}u}}C_1] - [f_{B_c}F_{fa}^{\pi^0_{\bar{d}d}\pi^+}a_1 + M_{na}^{\pi^0_{\bar{d}d}\pi^+}C_1]\} = 0, \tag{20}$$

$$\mathcal{A}(B_c \to \pi^+\eta) = V_{cb}^* V_{ud}\{[f_{B_c}F_{fa}^{\pi^+\eta_{\bar{u}u}}a_1 + M_{na}^{\pi^+\eta_{\bar{u}u}}C_1] + [f_{B_c}F_{fa}^{\eta_{\bar{d}d}\pi^+}a_1 + M_{na}^{\eta_{\bar{d}d}\pi^+}C_1]\}\cos\phi, \tag{21}$$

$$\mathcal{A}(B_c \to \pi^+\eta') = V_{cb}^* V_{ud}\{[f_{B_c}F_{fa}^{\pi^+\eta_{\bar{u}u}}a_1 + M_{na}^{\pi^+\eta_{\bar{u}u}}C_1] + [f_{B_c}F_{fa}^{\eta_{\bar{d}d}\pi^+}a_1 + M_{na}^{\eta_{\bar{d}d}\pi^+}C_1]\}\sin\phi, \tag{22}$$

$$\mathcal{A}(B_c \to \bar{K}^0K^+) = V_{cb}^* V_{ud}\{f_{B_c}F_{fa}^{\bar{K}^0K^+}a_1 + M_{na}^{\bar{K}^0K^+}C_1\}, \tag{23}$$

$$\mathcal{A}(B_c \to K^+\pi^0) = V_{cb}^* V_{us}\{f_{B_c}F_{fa}^{K^+\pi^0}a_1 + M_{na}^{K^+\pi^0}C_1\}, \tag{24}$$

$$\mathcal{A}(B_c \to K^0\pi^+) = \sqrt{2}\mathcal{A}(B_c \to K^+\pi^0), \tag{25}$$

$$\mathcal{A}(B_c \to K^+\eta) = V_{cb}^* V_{us}\{f_{B_c}[F_{fa}^{K^+\eta_q}\cos\phi - F_{fa}^{\eta_s K^+}\sin\phi]a_1 + [M_{na}^{K^+\eta_q}\cos\phi - M_{na}^{\eta_s K^+}\sin\phi]C_1\}, \tag{26}$$

$$\mathcal{A}(B_c \to K^+\eta') = V_{cb}^* V_{us}\{f_{B_c}[F_{fa}^{K^+\eta_q}\sin\phi + F_{fa}^{\eta_s K^+}\cos\phi]a_1 + [M_{na}^{K^+\eta_q}\sin\phi + M_{na}^{\eta_s K^+}\cos\phi]C_1\}. \tag{27}$$

B. $B_c \to PV, VP$ decays

By following the same procedure as stated in the above subsection, we can obtain the analytic decay amplitudes for $B_c \to PV, VP$ decays,

$$F_{fa}^{PV} = 8\pi C_F m_{B_c}^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \{h_{fa}(1-x_3, x_2, b_3, b_2) E_{fa}(t_a)[x_2 \phi_2^A(x_2)\phi_3(x_3) - 2r_0^2 r_3 \phi_3^s(x_3)((x_2+1)\phi_2^P(x_2) + (x_2-1)\phi_2^T(x_2))] + h_{fa}(x_2, 1-x_3, b_2, b_3) E_{fa}(t_b)[(x_3-1)\phi_2^A(x_2)\phi_3(x_3) - 2r_0^2 r_3 \phi_2^P(x_2)((x_3-2)\phi_3^s(x_3) - x_3\phi_3^t(x_3))]\}, \tag{28}$$

$$M_{na}^{PV} = \frac{16\sqrt{6}}{3}\pi C_F m_{B_c}^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \{h_{na}^c(x_2, x_3, b_1, b_2) E_{na}(t_c)[(r_c - x_3 + 1)\phi_2^A(x_2)\phi_3(x_3) - r_0^2 r_3(\phi_2^P(x_2)((3r_c + x_2 - x_3 + 1)\phi_3^s(x_3) - (r_c - x_2 - x_3 + 1)\phi_3^t(x_3)) + \phi_2^T(x_2)((r_c - x_2 - x_3 + 1)\phi_3^s(x_3) + (r_c - x_2 + x_3 - 1)\phi_3^t(x_3)))] - E_{na}(t_d)[(r_b + r_c + x_2 - 1)\phi_2^A(x_2)\phi_3(x_3) - r_0^2 r_3(\phi_2^P(x_2)((4r_b + r_c + x_2 - x_3 - 1)\phi_3^s(x_3) - (r_c + x_2 + x_3 - 1)\phi_3^t(x_3)) + \phi_2^T(x_2)((r_c + x_2 + x_3 - 1)\phi_3^s(x_3) - (r_c + x_2 - x_3 - 1)\phi_3^t(x_3)))] h_{na}^d(x_2, x_3, b_1, b_2)\}, \tag{29}$$

$$F_{fa}^{VP} = 8\pi C_F m_{B_c}^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \{h_{fa}(1-x_3, x_2, b_3, b_2) E_{fa}(t_a)[x_2 \phi_2(x_2)\phi_3^A(x_3) + 2r_2 r_0^3 \phi_3^P(x_3)((x_2+1)\phi_2^s(x_2) + (x_2-1)\phi_2^t(x_2))] + h_{fa}(x_2, 1-x_3, b_2, b_3) E_{fa}(t_b)[(x_3-1)\phi_2(x_2)\phi_3^A(x_3) + 2r_2 r_0^3 \phi_2^s(x_2)((x_3-2)\phi_3^P(x_3) - x_3\phi_3^T(x_3))]\}, \tag{30}$$

$$M_{na}^{VP} = \frac{16\sqrt{6}}{3}\pi C_F m_{B_c}^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \{h_{na}^c(x_2, x_3, b_1, b_2) E_{na}(t_c)[(r_c - x_3 + 1)\phi_2(x_2)\phi_3^A(x_3) + r_2 r_0^3(\phi_2^s(x_2)((3r_c + x_2 - x_3 + 1)\phi_3^P(x_3) - (r_c - x_2 - x_3 + 1)\phi_3^T(x_3)) + \phi_2^t(x_2)((r_c - x_2 - x_3 + 1)\phi_3^P(x_3) + (r_c - x_2 + x_3 - 1)\phi_3^T(x_3)))] - E_{na}(t_d)[(r_b + r_c + x_2 - 1)\phi_2(x_2)\phi_3^A(x_3) + r_2 r_0^3(\phi_2^s(x_2)((4r_b + r_c + x_2 - x_3 - 1)\phi_3^P(x_3) - (r_c + x_2 + x_3 - 1)\phi_3^T(x_3)) + \phi_2^t(x_2)((r_c + x_2 + x_3 - 1)\phi_3^P(x_3) - (r_c + x_2 - x_3 - 1)\phi_3^T(x_3)))] h_{na}^d(x_2, x_3, b_1, b_2)\}. \tag{31}$$

The total decay amplitudes of the 15 $B_c \to PV$, VP decays can therefore be written as,

$$\mathcal{A}(B_c \to \pi^+\rho^0) = V_{cb}^* V_{ud}\{[f_{B_c} F_{fa}^{\pi^+\rho^0_{\bar{u}u}} a_1 + M_{na}^{\pi^+\rho^0_{\bar{u}u}} C_1] - [f_{B_c} F_{fa}^{\rho^0_{\bar{d}d}\pi^+} a_1 + M_{na}^{\rho^0_{\bar{d}d}\pi^+} C_1]\}, \tag{32}$$

$$\mathcal{A}(B_c \to \pi^+\omega) = V_{cb}^* V_{ud}\{[f_{B_c} F_{fa}^{\pi^+\omega_{\bar{u}u}} a_1 + M_{na}^{\pi^+\omega_{\bar{u}u}} C_1] + [f_{B_c} F_{fa}^{\omega_{\bar{d}d}\pi^+} a_1 + M_{na}^{\omega_{\bar{d}d}\pi^+} C_1]\}, \tag{33}$$

$$\mathcal{A}(B_c \to \bar{K}^0 K^{*+}) = V_{cb}^* V_{ud}\{f_{B_c} F_{fa}^{\bar{K}^0 K^{*+}} a_1 + M_{na}^{\bar{K}^0 K^{*+}} C_1\}, \tag{34}$$

$$\mathcal{A}(B_c \to K^+\rho^0) = V_{cb}^* V_{us}\{f_{B_c} F_{fa}^{K^+\rho^0} a_1 + M_{na}^{K^+\rho^0} C_1\}, \tag{35}$$

$$\mathcal{A}(B_c \to K^0\rho^+) = \sqrt{2}\mathcal{A}(B_c \to K^+\rho^0), \tag{36}$$

$$\mathcal{A}(B_c \to K^+\omega) = V_{cb}^* V_{us}\{f_{B_c} F_{fa}^{K^+\omega} a_1 + M_{na}^{K^+\omega} C_1\}, \tag{37}$$

$$\mathcal{A}(B_c \to \rho^+\pi^0) = V_{cb}^* V_{ud}\{[f_{B_c} F_{fa}^{\rho^+\pi^0_{\bar{u}u}} a_1 + M_{na}^{\rho^+\pi^0_{\bar{u}u}} C_1] - [f_{B_c} F_{fa}^{\pi^0_{\bar{d}d}\rho^+} a_1 + M_{na}^{\pi^0_{\bar{d}d}\rho^+} C_1]\}, \tag{38}$$

$$\mathcal{A}(B_c \to \rho^+\eta) = V_{cb}^* V_{ud}\{[f_{B_c} F_{fa}^{\rho^+\eta_{\bar{u}u}} a_1 + M_{na}^{\rho^+\eta_{\bar{u}u}} C_1] + [f_{B_c} F_{fa}^{\eta_{\bar{d}d}\rho^+} a_1 + M_{na}^{\eta_{\bar{d}d}\rho^+} C_1]\}\cos\phi, \tag{39}$$

$$\mathcal{A}(B_c \to \rho^+\eta') = V_{cb}^* V_{ud}\{[f_{B_c} F_{fa}^{\rho^+\eta_{\bar{u}u}} a_1 + M_{na}^{\rho^+\eta_{\bar{u}u}} C_1] + [f_{B_c} F_{fa}^{\eta_{\bar{d}d}\rho^+} a_1 + M_{na}^{\eta_{\bar{d}d}\rho^+} C_1]\}\sin\phi, \tag{40}$$

$$\mathcal{A}(B_c \to \bar{K}^{*0} K^+) = V_{cb}^* V_{ud}\{f_{B_c} F_{fa}^{\bar{K}^{*0} K^+} a_1 + M_{na}^{\bar{K}^{*0} K^+} C_1\}, \tag{41}$$

$$\mathcal{A}(B_c \to K^{*+}\pi^0) = V_{cb}^* V_{us}\{f_{B_c} F_{fa}^{K^{*+}\pi^0} a_1 + M_{na}^{K^{*+}\pi^0} C_1\}, \tag{42}$$

$$\mathcal{A}(B_c \to K^{*0}\pi^+) = \sqrt{2}\mathcal{A}(B_c \to K^{*+}\pi^0), \tag{43}$$

$$\begin{aligned}\mathcal{A}(B_c \to K^{*+}\eta) = V_{cb}^* V_{us}\{f_{B_c}[F_{fa}^{K^{*+}\eta_q}\cos\phi \\ - F_{fa}^{\eta_s K^{*+}}\sin\phi]a_1 + [M_{na}^{K^{*+}\eta_q}\cos\phi \\ - M_{na}^{\eta_s K^{*+}}\sin\phi]C_1\},\end{aligned} \tag{44}$$

$$\begin{aligned}\mathcal{A}(B_c \to K^{*+}\eta') = V_{cb}^* V_{us}\{f_{B_c}[F_{fa}^{K^{*+}\eta_q}\sin\phi \\ + F_{fa}^{\eta_s K^{*+}}\cos\phi]a_1 + [M_{na}^{K^{*+}\eta_q}\sin\phi \\ + M_{na}^{\eta_s K^{*+}}\cos\phi]C_1\},\end{aligned} \tag{45}$$

$$\mathcal{A}(B_c \to \phi K^+) = V_{cb}^* V_{us}\{f_{B_c}F_{fa}^{\phi K^+}a_1 + M_{na}^{\phi K^+}C_1\}. \tag{46}$$

C. $B_c \to VV$ decays

There are three kinds of polarizations of a vector meson, namely, longitudinal (L), normal (N), and transverse (T). The amplitudes for a B_c meson decay to two vector mesons are also characterized by the polarization states of these vector mesons. The decay amplitudes $\mathcal{M}^{(\sigma)}$ in terms of helicities, for $B_c \to V(P_2, \epsilon_2^*)V(P_3, \epsilon_3^*)$ decays, can be generally described by

$$\begin{aligned}\mathcal{M}^{(\sigma)} &= \epsilon_{2\mu}^*(\sigma)\epsilon_{3\nu}^*(\sigma)\Big[ag^{\mu\nu} + \frac{b}{m_{M_2}m_{M_3}}P_1^\mu P_1^\nu \\ &\quad + i\frac{c}{m_{M_2}m_{M_3}}\epsilon^{\mu\nu\alpha\beta}P_{2\alpha}P_{3\beta}\Big], \\ &\equiv m_{B_c}^2\mathcal{M}_L + m_{B_c}^2\mathcal{M}_N\epsilon_2^*(\sigma = T)\cdot\epsilon_3^*(\sigma = T) \\ &\quad + i\mathcal{M}_T\epsilon^{\alpha\beta\gamma\rho}\epsilon_{2\alpha}^*(\sigma)\epsilon_{3\beta}^*(\sigma)P_{2\gamma}P_{3\rho},\end{aligned} \tag{47}$$

where the superscript σ denotes the helicity states of the two vector mesons with $L(T)$ standing for the longitudinal (transverse) component. And the definitions of the amplitudes $\mathcal{M}_i$ ($i = L, N, T$) in terms of the Lorentz-invariant amplitudes a, b, and c are

$$\begin{aligned}m_{B_c}^2\mathcal{M}_L &= a\epsilon_2^*(L)\cdot\epsilon_3^*(L) + \frac{b}{m_{M_2}m_{M_3}}\epsilon_2^*(L)\cdot P_3\epsilon_3^*(L)\cdot P_2, \\ m_{B_c}^2\mathcal{M}_N &= a, \quad m_{B_c}^2\mathcal{M}_T = \frac{c}{r_2r_3}.\end{aligned} \tag{48}$$

We therefore will evaluate the helicity amplitudes $\mathcal{M}_L$, $\mathcal{M}_N$, $\mathcal{M}_T$ based on the pQCD factorization approach, respectively.

For every component of the polarization, the corresponding Feynman amplitude can be written as the following form:

$$\begin{aligned}F_{fa}^L &= 8\pi C_F m_{B_c}^2\int_0^1 dx_2dx_3\int_0^\infty b_2db_2b_3db_3\{[x_2\phi_2(x_2)\phi_3(x_3) - 2r_2r_3((x_2+1)\phi_2^s(x_2) \\ &\quad + (x_2-1)\phi_2^t(x_2))\phi_3^s(x_3)]E_{fa}(t_a)h_{fa}(1-x_3, x_2, b_3, b_2) \\ &\quad + E_{fa}(t_b)h_{fa}(x_2, 1-x_3, b_2, b_3)[(x_3-1)\phi_2(x_2)\phi_3(x_3) - 2r_2r_3\phi_2^s(x_2)((x_3-2)\phi_3^s(x_3) - x_3\phi_3^t(x_3))]\},\end{aligned} \tag{49}$$

$$\begin{aligned}M_{na}^L &= \frac{16\sqrt{6}}{3}\pi C_F m_{B_c}^2\int_0^1 dx_2dx_3\int_0^\infty b_1db_1b_2db_2\{E_{na}(t_c)[(r_c - x_3 + 1)\phi_2(x_2)\phi_3(x_3) \\ &\quad - r_2r_3(\phi_2^s(x_2)((3r_c + x_2 - x_3 + 1)\phi_3^s(x_3) - (r_c - x_2 - x_3 + 1)\phi_3^t(x_3)) + \phi_2^t(x_2)((r_c - x_2 - x_3 + 1)\phi_3^s(x_3) \\ &\quad + (r_c - x_2 + x_3 - 1)\phi_3^t(x_3)))]h_{na}^c(x_2, x_3, b_1, b_2) - h_{na}^d(x_2, x_3, b_1, b_2)E_{na}(t_d)[(r_b + r_c + x_2 - 1)\phi_2(x_2)\phi_3(x_3) \\ &\quad - r_2r_3(\phi_2^s(x_2)((4r_b + r_c + x_2 - x_3 - 1)\phi_3^s(x_3) - (r_c + x_2 + x_3 - 1)\phi_3^T(x_3)) \\ &\quad + \phi_2^t(x_2)((r_c + x_2 + x_3 - 1)\phi_3^s(x_3) - (r_c + x_2 - x_3 - 1)\phi_3^t(x_3)))]\},\end{aligned} \tag{50}$$

$$\begin{aligned}F_{fa}^N &= 8\pi C_F m_{B_c}^2\int_0^1 dx_2dx_3\int_0^\infty b_2db_2b_3db_3r_2r_3\{h_{fa}(1-x_3, x_2, b_3, b_2)E_{fa}(t_a)[(x_2+1)(\phi_2^a(x_2)\phi_3^a(x_3) + \phi_2^v(x_2)\phi_3^v(x_3)) \\ &\quad + (x_2-1)(\phi_2^v(x_2)\phi_3^a(x_3) + \phi_2^a(x_2)\phi_3^v(x_3))] + E_{fa}(t_b)h_{fa}(x_2, 1-x_3, b_2, b_3)[(x_3-2)(\phi_2^a(x_2)\phi_3^a(x_3) \\ &\quad + \phi_2^v(x_2)\phi_3^v(x_3)) - x_3(\phi_2^a(x_2)\phi_3^v(x_3) + \phi_2^v(x_2)\phi_3^a(x_3))]\},\end{aligned} \tag{51}$$

$$\begin{aligned}M_{na}^N &= \frac{32\sqrt{6}}{3}\pi C_F m_{B_c}^2\int_0^1 dx_2dx_3\int_0^\infty b_1db_1b_2db_2r_2r_3\{r_c[\phi_2^a(x_2)\phi_3^a(x_3) + \phi_2^v(x_2)\phi_3^v(x_3)]E_{na}(t_c)h_{na}^c(x_2, x_3, b_1, b_2) \\ &\quad - r_b[\phi_2^a(x_2)\phi_3^a(x_3) + \phi_2^v(x_2)\phi_3^v(x_3)]E_{na}(t_d)h_{na}^d(x_2, x_3, b_1, b_2)\},\end{aligned} \tag{52}$$

$$F^T_{fa} = 16\pi C_F m^2_{B_c} \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 r_2 r_3 \{h_{fa}(1-x_3, x_2, b_3, b_2) E_{fa}(t_a)[(x_2+1)(\phi^a_2(x_2)\phi^v_3(x_3) + \phi^v_2(x_2)\phi^a_3(x_3)) + (x_2-1)(\phi^a_2(x_2)\phi^a_3(x_3) + \phi^v_2(x_2)\phi^v_3(x_3))] + h_{fa}(x_2, 1-x_3, b_2, b_3) E_{fa}(t_b)[(x_3-2) \times (\phi^a_2(x_2)\phi^v_3(x_3) + \phi^v_2(x_2)\phi^a_3(x_3)) - x_3(\phi^a_2(x_2)\phi^a_3(x_3) + \phi^v_2(x_2)\phi^v_3(x_3))]\}, \quad (53)$$

$$M^T_{na} = \frac{64\sqrt{6}}{3}\pi C_F m^2_{B_c} \int_0^1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 r_2 r_3 \{r_c[\phi^a_2(x_2)\phi^v_3(x_3) + \phi^v_2(x_2)\phi^a_3(x_3)] E_{na}(t_c) h^c_{na}(x_2, x_3, b_1, b_2) - r_b[\phi^a_2(x_2)\phi^v_3(x_3) + \phi^v_2(x_2)\phi^a_3(x_3)] E_{na}(t_d) h^d_{na}(x_2, x_3, b_1, b_2)\}. \quad (54)$$

For seven $B_c \to VV$ decays, considering all the polarization ($H = L, N, T$) contributions and the Feynman decay amplitudes as shown in Eqs. (49)–(54), the total decay amplitude of these channels can be obtained directly,

$$\mathcal{M}^H(B_c \to \rho^+\rho^0) = V^*_{cb}V_{ud}\{[f_{B_c}F^{\rho^+\rho^0_{\bar{u}u}}_{fa;H}a_1 + M^{\rho^+\rho^0_{\bar{u}u}}_{na;H}C_1] - [f_{B_c}F^{\rho^0_{\bar{d}d}\rho^+}_{fa;H}a_1 + M^{\rho^0_{\bar{d}d}\rho^+}_{na;H}C_1]\} = 0, \quad (55)$$

$$\mathcal{M}^H(B_c \to \rho^+\omega) = V^*_{cb}V_{ud}\{[f_{B_c}F^{\rho^+\omega_{\bar{u}u}}_{fa;H}a_1 + M^{\rho^+\omega_{\bar{u}u}}_{na;H}] + [f_{B_c}F^{\omega_{\bar{d}d}\rho^+}_{fa;H}a_1 + M^{\omega_{\bar{d}d}\rho^+}_{na;H}]C_1\}, \quad (56)$$

$$\mathcal{M}^H(B_c \to \bar{K}^{*0}K^{*+}) = V^*_{cb}V_{ud}\{f_{B_c}F^{\bar{K}^{*0}K^{*+}}_{fa;H}a_1 + M^{\bar{K}^{*0}K^{*+}}_{na;H}C_1\}, \quad (57)$$

$$\mathcal{M}^H(B_c \to \phi K^{*+}) = V^*_{cb}V_{us}\{f_{B_c}F^{\phi K^{*+}}_{fa;H}a_1 + M^{\phi K^{*+}}_{na;H}C_1\}, \quad (58)$$

$$\mathcal{M}^H(B_c \to K^{*+}\rho^0) = V^*_{cb}V_{us}\{f_{B_c}F^{K^{*+}\rho^0}_{fa;H}a_1 + M^{K^{*+}\rho^0}_{na;H}C_1\}, \quad (59)$$

$$\mathcal{M}^H(B_c \to K^{*0}\rho^+) = \sqrt{2}\mathcal{M}^H(B_c \to K^{*+}\rho^0), \quad (60)$$

$$\mathcal{M}^H(B_c \to K^{*+}\omega) = V^*_{cb}V_{us}\{f_{B_c}F^{K^{*+}\omega}_{fa;H}a_1 + M^{K^{*+}\omega}_{na;H}C_1\}. \quad (61)$$

IV. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we will calculate the branching ratios (and polarization fractions, relative phases) for those considered 30 $B_c \to M_2M_3$ decay modes. The input parameters and the wave functions to be used are given in Appendix A. In numerical calculations, central values of input parameters will be used implicitly unless otherwise stated.

For $B_c \to PP$, PV, VP decays, the decay rate can be written as

$$\Gamma = \frac{G_F^2 m^3_{B_c}}{32\pi}|\mathcal{A}(B_c \to M_2M_3)|^2, \quad (62)$$

where the corresponding decay amplitudes $\mathcal{A}$ have been given explicitly in Eqs. (20)–(27) and (32)–(46). Using the decay amplitudes obtained in last section, it is straightforward to calculate the branching ratios with uncertainties as presented in Tables I, II, and III.

For $B_c \to VV$ decays, the decay rate can be written explicitly as

$$\Gamma = \frac{G_F^2|\mathbf{P_c}|}{16\pi m^2_{B_c}} \sum_{\sigma=L,T} \mathcal{M}^{(\sigma)\dagger}\mathcal{M}^{(\sigma)}, \quad (63)$$

where $|\mathbf{P_c}| \equiv |\mathbf{P_{2z}}| = |\mathbf{P_{3z}}|$ is the momentum of either of the outgoing vector mesons.

TABLE I. The pQCD predictions of branching ratios (BRs) for $B_c \to PP$ modes. The dominant errors are induced from charm quark mass $m_c = 1.5 \pm 0.15$ GeV, combined Gegenbauer moments a_i of related meson distribution amplitudes (see Appendix A explicitly), and the chiral enhancement factors $m_0^\pi = 1.4 \pm 0.3$ GeV and $m_0^K = 1.6 \pm 0.1$ GeV, respectively.

Decay modes ($\Delta S = 0$)	BRs (10^{-8})	Decay modes ($\Delta S = 1$)	BRs (10^{-8})
$B_c \to \pi^+\pi^0$	0	$B_c \to \pi^+K^0$	$4.0^{+1.0}_{-0.6}(m_c)^{+2.3}_{-1.6}(a_i)^{+0.5}_{-0.3}(m_0)$
$B_c \to \pi^+\eta$	$22.8^{+6.9}_{-4.6}(m_c)^{+7.2}_{-4.5}(a_i)^{+3.4}_{-4.2}(m_0)$	$B_c \to K^+\eta$	$0.6^{+0.0}_{-0.0}(m_c)^{+0.6}_{-0.5}(a_i)^{+0.2}_{-0.1}(m_0)$
$B_c \to \pi^+\eta'$	$15.3^{+4.6}_{-3.1}(m_c)^{+4.8}_{-3.0}(a_i)^{+2.2}_{-2.8}(m_0)$	$B_c \to K^+\eta'$	$5.7^{+0.9}_{-0.9}(m_c)^{+1.0}_{-1.6}(a_i)^{+0.0}_{-0.3}(m_0)$
$B_c \to K^+\bar{K}^0$	$24.0^{+2.4}_{-0.0}(m_c)^{+7.3}_{-6.0}(a_i)^{+6.8}_{-5.8}(m_0)$	$B_c \to K^+\pi^0$	$2.0^{+0.5}_{-0.3}(m_c)^{+1.2}_{-0.8}(a_i)^{+0.3}_{-0.1}(m_0)$

TABLE II. Same as Table I but for $B_c \to PV$ modes.

Decay modes ($\Delta S = 0$)	BRs (10^{-7})	Decay modes ($\Delta S = 1$)	BRs (10^{-8})
$B_c \to \pi^+ \rho^0$	$1.7^{+0.1}_{-0.0}(m_c)^{+0.1}_{-0.2}(a_i)^{+0.6}_{-0.3}(m_0)$	$B_c \to K^+ \rho^0$	$3.1^{+0.6}_{-0.8}(m_c)^{+1.2}_{-1.5}(a_i)^{+0.1}_{-0.2}(m_0)$
$B_c \to \bar{K}^0 K^{*+}$	$1.8^{+0.7}_{-0.1}(m_c)^{+4.1}_{-2.1}(a_i)^{+0.1}_{-0.0}(m_0)$	$B_c \to K^0 \rho^+$	$6.1^{+1.3}_{-1.5}(m_c)^{+2.5}_{-2.9}(a_i)^{+0.2}_{-0.3}(m_0)$
$B_c \to \pi^+ \omega$	$5.8^{+1.4}_{-2.2}(m_c)^{+1.1}_{-1.3}(a_i)^{+0.4}_{-1.2}(m_0)$	$B_c \to K^+ \omega$	$2.3^{+1.1}_{-0.3}(m_c)^{+1.8}_{-1.2}(a_i) \pm 0.1(m_0)$

Based on the helicity amplitudes (48), we can define the transversity amplitudes,

$$\mathcal{A}_L = -\xi m_{B_c}^2 \mathcal{M}_L, \qquad \mathcal{A}_\parallel = \xi \sqrt{2} m_{B_c}^2 \mathcal{M}_N,$$
$$\mathcal{A}_\perp = \xi m_{B_c}^2 \sqrt{2(r^2-1)} \mathcal{M}_T, \tag{64}$$

for the longitudinal, parallel, and perpendicular polarizations, respectively, with the normalization factor $\xi = \sqrt{G_F^2 \mathbf{P_c}/(16\pi m_{B_c}^2 \Gamma)}$ and the ratio $r = P_2 \cdot P_3/(m_{M_2} \cdot m_{M_3})$. These amplitudes satisfy the relation

$$|\mathcal{A}_L|^2 + |\mathcal{A}_\parallel|^2 + |\mathcal{A}_\perp|^2 = 1 \tag{65}$$

following the summation in Eq. (63).

Since the transverse-helicity contributions manifest themselves in polarization observables, we therefore define two kinds of polarization observables, i.e., polarization fractions $(f_L, f_\parallel, f_\perp)$ and relative phases $(\phi_\parallel, \phi_\perp)$ as [53]

$$f_{L(\parallel,\perp)} = \frac{|\mathcal{A}_{L(\parallel,\perp)}|^2}{|\mathcal{A}_L|^2 + |\mathcal{A}_\parallel|^2 + |\mathcal{A}_\perp|^2},$$
$$\phi_{\parallel(\perp)} \equiv \arg\frac{\mathcal{A}_{\parallel(\perp)}}{\mathcal{A}_L}. \tag{66}$$

It should be noted that a phase of π should be added to the relative phase $\phi_{\parallel(\perp)}$ as defined in Eq. (66), in order to cancel the additional minus sign in the definition of $\mathcal{A}_L$ in Eq. (64).

We also define another two quantities reflecting the effects of CP-violating asymmetries indirectly [53,54],

$$\Delta\phi_\parallel = \frac{\bar{\phi}_\parallel - \phi_\parallel}{2}, \qquad \Delta\phi_\perp = \frac{\bar{\phi}_\perp - \phi_\perp - \pi}{2}, \tag{67}$$

where $\bar{\phi}_\parallel$ and $\bar{\phi}_\perp$ are the CP-conjugated relative phases corresponding to $\phi_\parallel$ and $\phi_\perp$, respectively.

With the complete decay amplitudes, by employing Eq. (63) and the input parameters and wave functions as given in Appendix A, we will present the pQCD predictions for CP-averaged branching ratios, longitudinal polarization fractions, and relative phases of the considered decays with errors as shown in Tables IV and V.

TABLE III. Same as Table I but for $B_c \to VP$ modes.

Decay modes ($\Delta S = 0$)	BRs (10^{-7})	Decay modes ($\Delta S = 1$)	BRs (10^{-8})
$B_c \to \rho^+ \pi^0$	$0.5^{+0.1}_{-0.1}(m_c)^{+0.3}_{-0.2}(a_i)^{+0.2}_{-0.3}(m_0)$	$B_c \to K^{*0} \pi^+$	$3.3^{+0.7}_{-0.2}(m_c)^{+0.4}_{-0.4}(a_i)^{+0.2}_{-0.1}(m_0)$
$B_c \to \rho^+ \eta$	$5.4^{+2.1}_{-1.2}(m_c)^{+0.9}_{-1.4}(a_i) \pm 0.0(m_0)$	$B_c \to K^{*+} \pi^0$	$1.6^{+0.4}_{-0.1}(m_c)^{+0.3}_{-0.1}(a_i)^{+0.1}_{-0.0}(m_0)$
$B_c \to \rho^+ \eta'$	$3.6^{+1.4}_{-0.8}(m_c)^{+0.6}_{-0.9}(a_i) \pm 0.0(m_0)$	$B_c \to K^{*+} \eta$	$0.9^{+0.1}_{-0.0}(m_c)^{+0.6}_{-0.2}(a_i) \pm 0.0(m_0)$
$B_c \to \bar{K}^{*0} K^+$	$10.0^{+0.5}_{-0.6}(m_c)^{+1.7}_{-3.3}(a_i)^{+0.0}_{-0.2}(m_0)$	$B_c \to K^{*+} \eta'$	$3.8 \pm 1.1(m_c)^{+1.0}_{-0.6}(a_i) \pm 0.0(m_0)$
		$B_c \to \phi K^+$	$5.6^{+1.1}_{-0.0}(m_c)^{+1.2}_{-0.9}(a_i)^{+0.3}_{-0.0}(m_0)$

TABLE IV. The pQCD predictions of branching ratios (BRs) and longitudinal polarization fractions (LPFs) for $B_c \to VV$ modes.

Decay modes	BRs (10^{-7})	LPFs (%)
$B_c \to \rho^+ \rho^0$	0	-
$B_c \to \rho^+ \omega$	$10.6^{+3.2}_{-0.2}(m_c)^{+2.1}_{-0.2}(a_i)$	$92.9^{+1.6}_{-0.1}(m_c)^{+1.2}_{-0.1}(a_i)$
$B_c \to \bar{K}^{*0} K^{*+}$	$10.0^{+0.6}_{-0.4}(m_c)^{+8.1}_{-4.8}(a_i)$	$92.0^{+0.5}_{-0.4}(m_c)^{+3.6}_{-7.1}(a_i)$
$B_c \to K^{*0} \rho^+$	$0.6^{+0.0}_{-0.0}(m_c)^{+0.2}_{-0.1}(a_i)$	$94.9^{+0.2}_{-0.2}(m_c)^{+2.0}_{-1.4}(a_i)$
$B_c \to K^{*+} \rho^0$	$0.3^{+0.0}_{-0.0}(m_c)^{+0.1}_{-0.1}(a_i)$	$94.9^{+0.2}_{-0.2}(m_c)^{+1.3}_{-1.4}(a_i)$
$B_c \to K^{*+} \omega$	$0.3^{+0.0}_{-0.0}(m_c)^{+0.0}_{-0.2}(a_i)$	$94.8^{+0.3}_{-0.2}(m_c)^{+1.1}_{-1.2}(a_i)$
$B_c \to \phi K^{*+}$	$0.5^{+0.0}_{-0.1}(m_c)^{+0.1}_{-0.3}(a_i)$	$86.4^{+0.0}_{-1.4}(m_c)^{+4.9}_{-9.0}(a_i)$

TABLE V. The pQCD predictions of relative phases for $B_c \to VV$ modes.

Decay modes	$\phi_\parallel$ (rad)	$\phi_\perp$ (rad)	$\Delta\phi_\parallel$	$\Delta\phi_\perp$
$B_c \to \rho^+\rho^0$				
$B_c \to \rho^+\omega$	$3.86^{+0.31}_{-0.26}(m_c)^{+0.25}_{-0.19}(a_i)$	$4.43^{+0.16}_{-0.17}(m_c)^{+0.25}_{-0.19}(a_i)$	0	$-\pi/2$
$B_c \to \bar{K}^{*0}K^{*+}$	$3.68^{+0.18}_{-0.13}(m_c)^{+0.48}_{-0.21}(a_i)$	$3.76^{+0.16}_{-0.00}(m_c)^{+0.48}_{-0.20}(a_i)$	0	$-\pi/2$
$B_c \to K^{*0}\rho^+$	$4.11^{+0.17}_{-0.20}(m_c)^{+0.30}_{-0.20}(a_i)$	$4.20^{+0.14}_{-0.05}(m_c)^{+0.30}_{-0.21}(a_i)$	0	$-\pi/2$
$B_c \to K^{*+}\rho^0$	$4.11^{+0.17}_{-0.20}(m_c)^{+0.30}_{-0.20}(a_i)$	$4.20^{+0.14}_{-0.05}(m_c)^{+0.30}_{-0.21}(a_i)$	0	$-\pi/2$
$B_c \to K^{*+}\omega$	$4.15^{+0.13}_{-0.25}(m_c)^{+0.25}_{-0.25}(a_i)$	$4.23^{+0.11}_{-0.09}(m_c)^{+0.26}_{-0.24}(a_i)$	0	$-\pi/2$
$B_c \to \phi K^{*+}$	$3.80^{+0.25}_{-0.34}(m_c)^{+0.44}_{-0.20}(a_i)$	$3.89^{+0.22}_{-0.19}(m_c)^{+0.43}_{-0.21}(a_i)$	0	$-\pi/2$

Based on the pQCD predictions as given in Tables I, II, III, IV, and V, we have the following remarks:

(i) Among considered pure annihilation $B_c \to PV/VP$, VV decays, the pQCD predictions for the CP-averaged branching ratios for those $\Delta S = 0$ processes are much larger than those of $\Delta S = 1$ channels (one of the two final state mesons is the $K^{(*)}$ meson), which are mainly due to the large CKM factor $|V_{ud}/V_{us}|^2 \sim 19$. For $B_c \to \pi^+\pi^0$, $\rho^+\rho^0$ decays, the contributions from $\bar{u}u$ and $\bar{d}d$ components cancel each other exactly and result in the zero branching ratios. In fact, these two channels are forbidden, even with final state interactions. Simply, two pions cannot form an s wave isospin 1 state, because of Bose-Einstein statics. Any other nonzero data for these two channels may indicate the effects of exotic new physics.

(ii) There is no CP violation for all these decays within the standard model, since there is only one kind of tree operator involved in the decay amplitude of all considered B_c decays, which can be seen from Eq. (12).

(iii) The pQCD predictions for the branching ratios of considered B_c decays vary in the range of 10^{-6} (for $B_c \to \bar{K}^{*0}K^+$, $\bar{K}^{*0}K^{*+}$, and $\rho^+\omega$ decays) to 10^{-8} (for most $\Delta S = 1$ B_c decays). The B_c decays with the branching ratio of 10^{-6} can be measured at the LHC experiment [38].

(iv) As mentioned in the introduction, the authors of Ref. [38] studied many pure annihilation B_c decays by employing the $SU(3)$ flavor symmetry and the one-gluon exchange (OGE) model, respectively, and presented their numerical estimates for the branching ratios of $B_c \to \phi K^+$, $\bar{K}^0K^+$, $\bar{K}^{*0}K^+$, and $\bar{K}^{*0}K^{*+}$ decays. As a comparison, we show in Table VI the pQCD predictions and the results as given in Ref. [38] for relevant channels. From Table VI, one can see easily that the pQCD predictions basically agree with the results obtained based on the $SU(3)$ flavor symmetry.

(v) For $B_c \to (\pi^+, \rho^+)(\eta, \eta')$ decays, the relevant final state mesons contain the same component $\bar{u}u + \bar{d}d$, they therefore have the similar branching ratios. The small differences among their branching ratios mainly come from the different mixing coefficients, i.e., $\cos\phi$ and $\sin\phi$.

(vi) For $B_c \to K^+\eta^{(\prime)}$ decays, however, one finds that $\mathrm{Br}(B_c \to K^+\eta') \sim 10 \times \mathrm{Br}(B_c \to K^+\eta)$, which is rather different from the pattern of $\mathrm{Br}(B_c \to \pi^+\eta) \sim \mathrm{Br}(B_c \to \pi^+\eta')$ and $\mathrm{Br}(B_c \to \rho^+\eta) \sim \mathrm{Br}(B_c \to \rho^+\eta')$. This large difference can be understood as follows: For the $\Delta S = 1$ processes, both η_q and η_s will contribute to $B_c \to K^+\eta$ and $K^+\eta'$ decays but with an opposite sign for η_q and η_s term, as well as different coefficients. This results in a destructive interference between η_q and η_s component for $B_c \to K^+\eta$, but a constructive interference for $B_c \to K^+\eta'$. This situation is very similar to that for the $B \to K\eta$ and $K\eta'$ decays [55–57].

(vii) Unlike $B_c \to K^+\eta^{(\prime)}$ decays, $\mathrm{Br}(B_c \to K^{*+}\eta') \approx 4\mathrm{Br}(B_c \to K^{*+}\eta) \sim 3.8 \times 10^{-8}$. The reason is that both of them are mainly determined by the factorizable contributions of η_s term.

TABLE VI. The pQCD predictions of branching ratios for $B_c \to \phi K^+$ and $B_c \to \bar{K}^{(*)0}K^{(*)+}$ modes. As a comparison, the numerical results as given in Ref. [38] are also listed in the last two columns.

Channels	pQCD predictions	$SU(3)$ symmetry	OGE model
$\mathrm{Br}(B_c \to \phi K^+)$	$5.6^{+1.1}_{-0.0}(m_c)^{+1.2}_{-0.9}(a_i) \times 10^{-8}$	$\mathcal{O}(10^{-7} \sim 10^{-8})$	5×10^{-9}
$\mathrm{Br}(B_c \to \bar{K}^0K^+)$	$2.4^{+0.2}_{-0.0}(m_c)^{+0.7}_{-0.6}(a_i) \times 10^{-7}$	$\mathcal{O}(10^{-6})$	6.3×10^{-8}
$\mathrm{Br}(B_c \to \bar{K}^0K^{*+})$	$1.8^{+0.7}_{-0.1}(m_c)^{+4.1}_{-2.1}(a_i) \times 10^{-7}$		
$\mathrm{Br}(B_c \to \bar{K}^{*0}K^+)$	$1.0 \pm 0.1(m_c)^{+0.2}_{-0.3}(a_i) \times 10^{-6}$	$\mathcal{O}(10^{-6})$	9.0×10^{-8}
$\mathrm{Br}(B_c \to \bar{K}^{*0}K^{*+})$	$1.0^{+0.1}_{-0.0}(m_c)^{+0.8}_{-0.5}(a_i) \times 10^{-6}$	$\mathcal{O}(10^{-6})$	9.1×10^{-8}

(viii) For $B_c \to VV$ decays, we can find that (a) the branching ratios are in order of $\mathcal{O}(10^{-8} \sim 10^{-7})$ except for $\mathrm{Br}(B_c \to \bar{K}^{*0} K^{*+})$ and $\mathrm{Br}(B_c \to \rho^+ \omega) \sim 10^{-6}$; and (b) the longitudinal polarization fractions are around 95% within the theoretical errors except for $B_c \to \phi K^{*+}$ ($\sim 86\%$) and play the dominant role.

(ix) According to the discussions in Ref. [38], there are some simple relations among some decay channels in the limit of exact $SU(3)$ flavor symmetry. For $B_c \to PP$ decays, such relations are

$$\begin{aligned} A(B_c \to K^0 \pi^+) &= \sqrt{2} A(B_c \to K^+ \pi^0) \\ &= \lambda A(B_c \to K^+ \bar{K}^0), \end{aligned} \tag{68}$$

where $\lambda = V_{us}/V_{ud} \approx 0.2$. For $B_c \to VP/PV$ and $B_c \to VV$ decays, the relations read

$$\begin{aligned} A(B_c \to K^{*0} \pi^+) &= \sqrt{2} A(B_c \to K^{*+} \pi^0) \\ &= \lambda A(B_c \to \bar{K}^{*0} K^+), \end{aligned} \tag{69}$$

$$\begin{aligned} A(B_c \to \rho^+ K^0) &= \sqrt{2} A(B_c \to \rho^0 K^+) \\ &= \lambda A(B_c \to K^{*+} \bar{K}^0), \end{aligned} \tag{70}$$

$$\begin{aligned} (-1)^\ell A(B_c^+ \to \rho^+ K^{*0}) &= (-1)^\ell \sqrt{2} A(B_c^+ \to \rho^0 K^{*+}) \\ &= \lambda A(B_c \to K^{*+} \bar{K}^0), \end{aligned} \tag{71}$$

where $\ell = 0, 1, 2$.[2] From our pQCD calculations, we notice that the first equality of each of the above relations (68)–(71) is valid in isospin symmetry. They hold exactly in our numerical calculations. The second equality of each relation is only valid at exact $SU(3)$ symmetry thus they are violated at the order of $SU(3)$ breaking effect in our calculations.

(x) Since the LHC experiment can measure the B_c decays with a branching ratio at 10^{-6} level, our pQCD predictions for the branching ratios of $B_c \to \bar{K}^{*0} K^+$, $\bar{K}^{*0} K^{*+}$, and $\rho^+ \omega$ decays could be tested in the forthcoming LHC experiments.

(xi) For most considered pure annihilation B_c decays, it is hard to observe them even in LHC due to their tiny decay rate. Their observation at LHC, however, would mean a large nonperturbative contribution or a signal for new physics beyond the SM.

(xii) It is worth stressing that the theoretical predictions in the pQCD approach still have large theoretical errors induced by the still large uncertainties of many input parameters. Any progress in reducing the error of input parameters, such as the Gegenbauer moments a_i and the charm quark mass m_c, will help us to improve the precision of the pQCD predictions.

V. SUMMARY

In short, we studied the two-body charmless hadronic $B_c \to PP$, PV/VP, VV decays by employing the pQCD factorization approach based on the k_T factorization theorem. These considered decay channels can occur only via the annihilation diagram and they will provide an important testing ground for the magnitude of the annihilation contribution.

The pQCD predictions for CP-averaged branching ratios, longitudinal polarization fractions, and relative phases are displayed in Tables I, II, III, IV, and V. From our numerical evaluations and phenomenological analysis, we found the following results:

(i) The pQCD predictions for the branching ratios vary in the range of 10^{-6} to 10^{-8} and basically agree with the predictions obtained by using the exact $SU(3)$ flavor symmetry. The $B_c \to \bar{K}^{*0} K^+$ and other decays with a decay rate at 10^{-6} or larger could be measured at the LHC experiment.

(ii) For $B_c \to PV/VP$, VV decays, the branching ratios of $\Delta S = 0$ processes are basically larger than those of $\Delta S = 1$ ones. Such differences are mainly induced by the CKM factors involved: $V_{ud} \sim 1$ for the former decays while $V_{us} \sim 0.22$ for the latter ones.

(iii) Analogous to $B \to K\eta^{(\prime)}$ decays, we find $\mathrm{Br}(B_c \to K^+ \eta') \sim 10 \times \mathrm{Br}(B_c \to K^+ \eta)$. This large difference can be understood by the destructive and constructive interference between the η_q and η_s contribution to the $B_c \to K^+ \eta$ and $B_c \to K^+ \eta'$ decay.

(iv) For $B_c \to VV$ decays, the longitudinal polarization fractions are around 95% except for $B_c \to \phi K^{*+}$ ($f_L \sim 86\%$) and play the dominant role.

(v) Because only tree operators are involved, the CP-violating asymmetries for these considered B_c decays are absent naturally.

(vi) The pQCD predictions still have large theoretical uncertainties, induced by the uncertainties of input parameters.

(vii) We here calculated the branching ratios and other physical observables of the pure annihilation B_c decays by employing the pQCD approach. We do not consider the possible long-distance contributions, such as the rescattering effects, although they may be large and affect the theoretical predictions. It is beyond the scope of this work.

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[2]Here, since the longitudinal contributions dominate the $B_c \to K^{*0} \rho^+$ decay, we use its longitudinal part (i.e., $\ell = 0$) to compare with the decay amplitude of $B_c \to K^{*+} \bar{K}^0$ decay.

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APPENDIX A: INPUT PARAMETERS AND DISTRIBUTION AMPLITUDES

The masses (GeV), decay constants (GeV), QCD scale (GeV) and B meson lifetime are

$$\begin{aligned}
&\Lambda_{\overline{\rm MS}}^{(f=4)} = 0.250, \quad m_W = 80.41, \quad m_{B_c} = 6.286,\\
&f_{B_c} = 0.489, \quad m_\phi = 1.02, \quad f_\phi = 0.231,\\
&f_\phi^T = 0.200, \quad m_{K^*} = 0.892, \quad f_{K^*} = 0.217,\\
&f_{K^*}^T = 0.185, \quad m_\rho = 0.770, \quad f_\rho = 0.209,\\
&f_\rho^T = 0.165, \quad m_\omega = 0.782, \quad f_\omega = 0.195,\\
&f_\omega^T = 0.145, \quad m_0^\pi = 1.4, \quad m_0^K = 1.6,\\
&m_0^{\eta_q} = 1.08, \quad m_0^{\eta_s} = 1.92, \quad m_b = 4.8,\\
&f_\pi = 0.131, \quad f_K = 0.16, \quad \tau_{B_c^+} = 0.46\ \text{ps}.
\end{aligned} \tag{A1}$$

For the CKM matrix elements, here we adopt the Wolfenstein parametrization for the CKM matrix, and take $A = 0.814$ and $\lambda = 0.2257$, $\bar{\rho} = 0.135$ and $\bar{\eta} = 0.349$ [55].

The twist-2 pseudoscalar meson distribution amplitude ϕ_P^A ($P = \pi, K$), and the twist-3 ones ϕ_P^P and ϕ_P^T have been parametrized as [58–60]

$$\phi_P^A(x) = \frac{f_P}{2\sqrt{2N_c}} 6x(1-x)[1 + a_1^P C_1^{3/2}(2x-1) + a_2^P C_2^{3/2}(2x-1) + a_4^P C_4^{3/2}(2x-1)], \tag{A2}$$

$$\phi_P^P(x) = \frac{f_P}{2\sqrt{2N_c}}\left[1 + \left(30\eta_3 - \frac{5}{2}\rho_P^2\right)C_2^{1/2}(2x-1) - 3\left\{\eta_3\omega_3 + \frac{9}{20}\rho_P^2(1+6a_2^P)\right\}C_4^{1/2}(2x-1)\right], \tag{A3}$$

$$\phi_P^T(x) = \frac{f_P}{2\sqrt{2N_c}}(1-2x)\left[1 + 6\left(5\eta_3 - \frac{1}{2}\eta_3\omega_3 - \frac{7}{20}\rho_P^2 - \frac{3}{5}\rho_P^2 a_2^P\right)(1 - 10x + 10x^2)\right], \tag{A4}$$

with the Gegenbauer moments $a_1^\pi = 0$, $a_1^K = 0.17 \pm 0.17$, $a_2^P = 0.115 \pm 0.115$, $a_4^P = -0.015$, the mass ratio $\rho_{\pi(K)} = m_{\pi(K)}/m_0^{\pi(K)}$ and $\rho_{\eta_{q(s)}} = 2m_{q(s)}/m_{qq(ss)}$, and the Gegenbauer polynomials $C_n^\nu(t)$,

$$\begin{aligned}
&C_2^{1/2}(t) = \frac{1}{2}(3t^2 - 1), \qquad C_4^{1/2}(t) = \frac{1}{8}(3 - 30t^2 + 35t^4),\\
&C_1^{3/2}(t) = 3t, \qquad C_2^{3/2}(t) = \frac{3}{2}(5t^2 - 1),\\
&C_4^{3/2}(t) = \frac{15}{8}(1 - 14t^2 + 21t^4).
\end{aligned} \tag{A5}$$

In the above distribution amplitudes for kaon, the momentum fraction x is carried by the s quark. For both the pion and kaon, we choose $\eta_3 = 0.015$ and $\omega_3 = -3$ [58,59].

The twist-2 distribution amplitudes for the longitudinally and tranversely polarized vector meson can be parametrized as

$$\phi_V(x) = \frac{3f_V}{\sqrt{6}} x(1-x)[1 + a_{1V}^{\parallel} C_1^{3/2}(2x-1) + a_{2V}^{\parallel} C_2^{3/2}(2x-1)], \tag{A6}$$

$$\phi_V^T(x) = \frac{3f_V^T}{\sqrt{6}} x(1-x)[1 + a_{1V}^{\perp} C_1^{3/2}(2x-1) + a_{2V}^{\perp} C_2^{3/2}(2x-1)]. \tag{A7}$$

Here f_V and f_V^T are the decay constants of the vector meson with longitudinal and transverse polarization, respectively. The Gegenbauer moments have been studied extensively in the literature [61,62]; here we adopt the following values from the recent updates [63–65]:

$$\begin{aligned}
&a_{1K^*}^{\parallel} = 0.03 \pm 0.02, \qquad a_{2\rho}^{\parallel} = a_{2\omega}^{\parallel} = 0.15 \pm 0.07,\\
&a_{2K^*}^{\parallel} = 0.11 \pm 0.09, \qquad a_{2\phi}^{\parallel} = 0.18 \pm 0.08
\end{aligned} \tag{A8}$$

$$\begin{aligned}
&a_{1K^*}^{\perp} = 0.04 \pm 0.03, \qquad a_{2\rho}^{\perp} = a_{2\omega}^{\perp} = 0.14 \pm 0.06,\\
&a_{2K^*}^{\perp} = 0.10 \pm 0.08, \qquad a_{2\phi}^{\perp} = 0.14 \pm 0.07.
\end{aligned} \tag{A9}$$

The asymptotic forms of the twist-3 distribution amplitudes $\phi_V^{t,s}$ and $\phi_V^{v,a}$ are [42]

$$\phi_V^t(x) = \frac{3f_V^T}{2\sqrt{6}}(2x-1)^2, \qquad \phi_V^s(x) = -\frac{3f_V^T}{2\sqrt{6}}(2x-1), \tag{A10}$$

$$\begin{aligned}
&\phi_V^v(x) = \frac{3f_V}{8\sqrt{6}}(1 + (2x-1)^2),\\
&\phi_V^a(x) = -\frac{3f_V}{4\sqrt{6}}(2x-1).
\end{aligned} \tag{A11}$$

APPENDIX B: RELATED HARD FUNCTIONS

In this appendix, we group the functions which appear in the factorization formulas.

The functions h in the decay amplitudes consist of two parts: one is the jet function $S_t(x_i)$ derived by the threshold resummation [44], the other is the propagator of virtual quark and gluon. They are defined by

$$h_{fa}(x_3, x_2, b_3, b_2) = \left(\frac{i\pi}{2}\right)^2 S_t(x_2)[\theta(b_3 - b_2)H_0^{(1)}(\sqrt{x_2}M_{B_c}b_3)J_0(\sqrt{x_2}M_{B_c}b_2) + \theta(b_2 - b_3)H_0^{(1)}(\sqrt{x_2}M_{B_c}b_2)J_0(\sqrt{x_2}M_{B_c}b_3)]H_0^{(1)}(\sqrt{x_2x_3}M_{B_c}b_3), \tag{B1}$$

$$h_{na}^{c(d)}(x_2, x_3, b_1, b_2) = \frac{i\pi}{2}[\theta(b_1 - b_2)H_0^{(1)}(\sqrt{x_2(1-x_3)}M_{B_c}b_1)J_0(\sqrt{x_2(1-x_3)}M_{B_c}b_2) + \theta(b_2 - b_1)H_0^{(1)}(\sqrt{x_2(1-x_3)}M_{B_c}b_2)J_0(\sqrt{x_2(1-x_3)}M_{B_c}b_1)]\begin{cases} \frac{i\pi}{2}H_0^{(1)}(\sqrt{|F^2_{c(d)}|}M_{B_c}b_1), & F_{c(d)} < 0 \\ K_0(\sqrt{F_{c(d)}}M_{B_c}b_1), & F_{c(d)} > 0 \end{cases}, \tag{B2}$$

where

$$F_c = (r_c - x_2)(1 - x_3) + r_c^2, \qquad F_d = r_b^2 - (1 - r_c - x_2)x_3, \tag{B3}$$

and $H_0^{(1)}(z) = J_0(z) + iY_0(z)$.

The hard scales are chosen as

$$t_a = \max\{\sqrt{x_2}M_{B_c}, 1/b_2, 1/b_3\}, \tag{B4}$$

$$t_b = \max\{\sqrt{1-x_3}M_{B_c}, 1/b_2, 1/b_3\}, \tag{B5}$$

$$t_c = \max\{\sqrt{x_2(1-x_3)}M_{B_c}, \sqrt{|(r_c - x_2)(1-x_3) + r_c^2|}M_{B_c}, 1/b_1, 1/b_2\}, \tag{B6}$$

$$t_d = \max\{\sqrt{x_2(1-x_3)}M_{B_c}, \sqrt{|r_b^2 - (1 - r_c - x_2)x_3|}M_{B_c}, 1/b_1, 1/b_2\}. \tag{B7}$$

They are given as the maximum energy scale appearing in each diagram to kill the large logarithmic radiative corrections.

The S_t resums the threshold logarithms $\ln^2 x$ appearing in the hard kernels to all orders and it has been parametrized as

$$S_t(x) = \frac{2^{1+2c}\Gamma(3/2 + c)}{\sqrt{\pi}\Gamma(1 + c)}[x(1-x)]^c, \tag{B8}$$

with $c = 0.4 \pm 0.1$. In the nonfactorizable contributions, $S_t(x)$ gives a very small numerical effect to the amplitude [66]. Therefore, we drop $S_t(x)$ in h_{na}.

The evolution factors E_{fa} and E_{na} entering in the expressions for the matrix elements (see Sec. III) are given by

$$E_{fa}(t) = \alpha_s(t)\exp[-S_2(t) - S_3(t)], \tag{B9}$$

$$E_{na}(t) = \alpha_s(t)\exp[-S_B(t) - S_2(t) - S_3(t)]|_{b_2=b_3}, \tag{B10}$$

in which the Sudakov exponents are defined as

$$S_B(t) = s\left(r_c\frac{M_{B_c}}{\sqrt{2}}, b_1\right) + \frac{5}{3}\int_{1/b_1}^{t}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_q(\alpha_s(\bar{\mu})), \tag{B11}$$

$$S_2(t) = s\left(x_2\frac{M_{B_c}}{\sqrt{2}}, b_2\right) + s\left((1-x_2)\frac{M_{B_c}}{\sqrt{2}}, b_2\right) + 2\int_{1/b_2}^{t}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_q(\alpha_s(\bar{\mu})), \tag{B12}$$

with the quark anomalous dimension $\gamma_q = -\alpha_s/\pi$. Replacing the kinematic variables of M_2 to M_3 in S_2, we can get the expression for S_3. The explicit forms for the function $s(Q, b)$ are defined in Appendix A in Ref. [36].

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