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Evasion of helicity selection rule in $\chi_{c1} \to VV$ and $\chi_{c2} \to VP$ via intermediate charmed meson loops

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The hadronic decays of $\chi_{c1} \to VV$ and $\chi_{c2} \to VP$ are supposed to be suppressed by the helicity selection rule in the perturbative QCD framework. With an effective Lagrangian method, we show that the intermediate charmed meson loops can provide a mechanism for the evasion of the helicity selection rule, and result in sizeable decay branching ratios in some of those channels. The theoretical predictions can be examined by the forthcoming BES-III data in the near future.

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I. INTRODUCTION

The exclusive decays of heavy quarkonium have been an important platform for studying the nature of strong interactions in the literature [1–6] since the discovery of quantum chromodynamics (QCD). In this energy region a relatively large energy scale ($\sim m_c$, m_b) is involved such that some perturbative QCD (pQCD) asymptotic behaviors can be expected, for instance, the so-called helicity selection rule [2]. By studying the manifestation of the helicity selection rule in heavy quarkonium decays, we expect to gain insights into the property of QCD in the interplay of the perturbative and nonperturbative regime.

We briefly review this powerful method that is elaborated on in Ref. [2]. For a charmonium meson $J_{c\bar{c}}$ decaying into two light mesons h_1 and h_2, the perturbative method gives the asymptotic behavior of the branching ratio as follows:

$$\mathrm{BR}_{J_{c\bar{c}}(\lambda)\to h_1(\lambda_1)h_2(\lambda_2)} \sim \left(\frac{\Lambda^2_{\mathrm{QCD}}}{m_c^2}\right)^{|\lambda_1+\lambda_2|+2}, \tag{1}$$

where λ, λ_1, and λ_2 are the helicities of the corresponding mesons. This is the result of the pQCD method to leading-twist accuracy; i.e. only the valence Fock state (here it is $c\bar{c}$) is considered. It is obvious that the leading contribution comes from the condition when $\lambda_1 + \lambda_2 = 0$, while the helicity configurations that do not satisfy this relation will be suppressed.

An alternative description of this selection rule is to depict it with the quantum number "naturalness," which is defined as $\sigma \equiv P(-1)^J$, where P and J are the parity and spin of the particle, respectively. The selection rule then requires that

$$\sigma^{\mathrm{initial}} = \sigma_1\sigma_2. \tag{2}$$

That is, the initial-state naturalness should be equal to the product of those of the final states. We can comprehend this in such a way that, if $\sigma^{\mathrm{initial}} \neq \sigma_1\sigma_2$, to keep the parity conservation and Lorentz invariance, the amplitude will contain a Levi-Civita tensor $\epsilon_{\mu\nu\alpha\beta}$, which will be contracted with the polarization vectors and momenta of the involved mesons. For instance, for the process $\chi_{c1} \to VV$ where VV represent a pair of vector mesons, in the rest frame of χ_{c1}, the nonvanishing covariant amplitude will require one vector meson be transversely polarized and the other one be longitudinally polarized. This leads to $\lambda_1 + \lambda_2 \neq 0$, which will violate the helicity selection rule and is supposed to be suppressed. Another well-known example is the process $J/\psi \to VP$, where the nonvanishing covariant amplitude also violates the selection rule.

The helicity selection rule has been used for studying some exclusive decay processes of heavy quarkonia. In Refs. [2,3,7], some allowed or suppressed processes are explicitly listed. Interestingly, with the accumulation of the experimental data, more and more observations suggest significant discrepancies between the data and the selection-rule expectations. For instance, the decays of $J/\psi \to VP$ and $\eta_c \to VV$ would be suppressed by this rule. In reality, they are rather important decay channels for J/ψ and η_c, respectively [8]. One possible reason why the perturbative method fails here could be that although the mass of the charm quark is heavy, it is, however, not as heavy as pQCD demands. Therefore, it is not safe to apply the helicity selection rule to charmonium decays, while the situation may be improved in bottomonium decays. Taking into account these issues, it implies that there might be some other mechanisms that can contribute to those helicity-selection-rule-forbidden processes, such as higher order contributions, final-state interactions, or some other long-distance effects. Theoretical discussions on these mechanisms in B-meson or heavy quarkonium decays can be found in the literature [9–19].

In Refs. [15,16], the role played by the intermediate charmed meson loops in the understanding of the so-called ρ-π puzzle and $\psi(3770)$ non-$D\bar{D}$ decays into VP was studied. It shows that the long-distance interactions via the intermediate meson loops provide a possible mecha-

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nism for understanding the deviations from pQCD expectations. In the present work, we will study the P-wave charmonium decays, i.e. $\chi_{c1} \to VV$ and $\chi_{c2} \to VP$, which are suppressed by the helicity selection rule but may be enhanced by the intermediate charmed meson loop transitions. The branching ratio of $\chi_{c1} \to K^{*0}\bar{K}^{*0}$ at order of 10^{-3} has been measured by experiment [8]. According to the SU(3) flavor symmetry, it is justified to predict some other channels as $\rho\rho$, $\omega\omega$, and $\phi\phi$, of which the branching ratios may also be sizeable. Therefore, it is interesting to investigate the intermediate meson loop transitions as a mechanism for evading the helicity selection rule. We also note that the process $\chi_{c2} \to VP$ will be further suppressed by the approximate G parity or isospin (U spin for strange mesons) conservation, and the χ_{c2} decays into neutral VP with fixed C parities are forbidden by C-parity conservation. By comparing the results with the SU(3) flavor symmetry expectation, we anticipate that the nonperturbative mechanism can be highlighted.

It is worth noting that for the P-wave charmonium exclusive decays, the next higher Fock state $c\bar{c}g$, i.e. the so-called color-octet state, will contribute at the same order as the color-singlet state $c\bar{c}$ in the framework of perturbative method [7,20–23]. This scenario may share the same intrinsic physics with the intermediate meson loop transitions, but based on different view angles. Namely, one description is at the quark-gluon level, while the other is at the hadron level. Taking into account the relatively low energy scale, it is not easy to handle these processes with the factorization method for the quark-gluon interactions. Some qualitative discussions can be found in [11,13,14] and references therein.

This paper is arranged as follows: In Sec. II, the formulas for the intermediate meson loops as a long-distance effect in the charmonium decays will be given by an effective Lagrangian method. The numerical results and discussions will be given in Sec. III. The conclusion will be drawn in Sec. IV.

II. LONG-DISTANCE CONTRIBUTION VIA INTERMEDIATE MESON LOOPS

In Figs. 1–3, the intermediate meson loops are plotted for $\chi_{c1} \to VV$, $\chi_{c2} \to \rho\pi$, and $\chi_{c2} \to K^*\bar{K}$, respectively. The relevant effective Lagrangians that will be used are based on the heavy quark symmetry and SU(3) symmetry [13,14,24]. The spin multiplet for these four P-wave charmonium states are expressed as

$$P^{\mu}_{c\bar{c}} = \left(\frac{1+\not{v}}{2}\right)\left(\chi_{c2}^{\mu\alpha}\gamma_\alpha + \frac{1}{\sqrt{2}}\epsilon_{\mu\nu\alpha\beta}v^\alpha\gamma^\beta\chi_{c1}^{\nu} + \frac{1}{\sqrt{3}}(\gamma^\mu - v^\mu)\chi_{c0} + h_c^\mu\gamma_5\right)\left(\frac{1-\not{v}}{2}\right). \quad (3)$$

The charmed and anticharmed meson triplets read

$$H_{1i} = \left(\frac{1+\not{v}}{2}\right)[\mathcal{D}_i^{*\mu}\gamma_\mu - \mathcal{D}_i\gamma_5], \quad (4)$$

$$H_{2i} = [\bar{\mathcal{D}}_i^{*\mu}\gamma_\mu - \bar{\mathcal{D}}_i\gamma_5]\left(\frac{1-\not{v}}{2}\right), \quad (5)$$

where $\mathcal{D}$ and $\mathcal{D}^*$ denote the pseudoscalar and vector charmed meson fields, respectively, i.e. $\mathcal{D}^{(*)} = (D^{0(*)}, D^{+(*)}, D_s^{+(*)})$. Then the effective Lagrangian that describes the interaction between the P-wave charmonium and the charmed mesons reads

$$\mathcal{L}_1 = ig_1 \,\mathrm{Tr}[P^{\mu}_{c\bar{c}}\bar{H}_{2i}\gamma_\mu\bar{H}_{1i}] + \mathrm{H.c.} \quad (6)$$

The effective Lagrangians that describe the couplings of charmed mesons to light hadrons read

$$\mathcal{L}_{\mathcal{D}\mathcal{D}\mathcal{V}} = -ig_{DDV}\bar{\mathcal{D}}_i\overleftrightarrow{\partial}_\mu\mathcal{D}_j(\mathcal{V}^\mu)_{ij}, \quad (7)$$

$$\mathcal{L}_{\mathcal{D}^*\mathcal{D}\mathcal{V}} = -2f_{D^*DV}\epsilon_{\mu\nu\alpha\beta}(\partial^\mu\mathcal{V}^\nu)_{ij}(\bar{\mathcal{D}}_i\overleftrightarrow{\partial}^\alpha\mathcal{D}_j^{*\beta} - \bar{\mathcal{D}}_i^{*\beta}\overleftrightarrow{\partial}^\alpha\mathcal{D}_j), \quad (8)$$

$$\mathcal{L}_{\mathcal{D}^*\mathcal{D}^*\mathcal{V}} = ig_{D^*D^*V}\bar{\mathcal{D}}_i^{*\nu}\overleftrightarrow{\partial}_\mu\mathcal{D}^*_{j\nu}(\mathcal{V}^\mu)_{ij} + 4if_{D^*D^*V}\bar{\mathcal{D}}_i^{*\mu}(\partial_\mu\mathcal{V}_\nu - \partial_\nu\mathcal{V}_\mu)_{ij}\mathcal{D}_j^{*\nu}, \quad (9)$$

$$\mathcal{L}_{\mathcal{D}^*\mathcal{D}\mathcal{P}} = -ig_{D^*DP}(\bar{\mathcal{D}}_i\partial_\mu\mathcal{P}_{ij}\mathcal{D}_j^{*\mu} - \bar{\mathcal{D}}_i^{*\mu}\partial_\mu\mathcal{P}_{ij}\mathcal{D}_j), \quad (10)$$

$$\mathcal{L}_{\mathcal{D}^*\mathcal{D}^*\mathcal{P}} = \tfrac{1}{2}g_{D^*D^*P}\epsilon_{\mu\nu\alpha\beta}\bar{\mathcal{D}}_i^{*\mu}\partial^\nu\mathcal{P}_{ij}\overleftrightarrow{\partial}^\alpha\mathcal{D}_j^{*\beta}, \quad (11)$$

where $\mathcal{V}$ and $\mathcal{P}$ are the pseudoscalar octet and vector nonet, and we take the following conventions

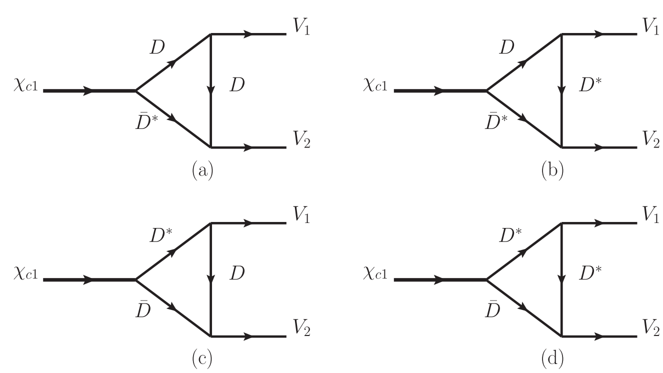

FIG. 1. Triangle loop diagrams that describe the long-distance contributions in $\chi_{c1} \to V_1V_2$, where V_1V_2 represent $K^*\bar{K}^*$, $\rho\rho$, $\omega\omega$, and $\phi\phi$, respectively.

$$\mathcal{V} = \begin{pmatrix} \frac{1}{\sqrt{2}}(\rho^0+\omega) & \rho^+ & K^{*+} \\ \rho^- & \frac{1}{\sqrt{2}}(-\rho^0+\omega) & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}, \quad (12)$$

$$\mathcal{P} = \begin{pmatrix} \frac{1}{\sqrt{2}}(\pi^0+\eta) & \pi^+ & K^+ \\ \pi^- & \frac{1}{\sqrt{2}}(-\pi^0+\eta) & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}. \quad (13)$$

A. $\chi_{c1} \to VV$

Figure 1 illustrates the long-distance contributions for $\chi_{c1} \to VV$. One notices that there is no such a vertex of $\chi_{c1}D^*\bar{D}^*$ in Fig. 1. It is because to conserve parity and keep Lorentz invariance, the relative angular momentum between $D^*\bar{D}^*$ should be $L = 2$, and the total spin $S = 2$. But such kind of coupling is not present in the expansion of the effective Lagrangian $\mathcal{L}_1$. For Figs. 1(a) and 1(b), we take the convention of the momenta as $\chi_{c1}(p) \to D(q_1)\bar{D}^*(q_2)[D^{(*)}(q)] \to V_1(p_1)V_2(p_2)$, where the exchanged particle between D and $\bar{D}^*$ is indicated in the square bracket. Figures 1(c) and 1(d) will give the same contribution as Figs. 1(a) and 1(b). For simplicity, we just write down the amplitude $\mathcal{M}_{1a}$ and $\mathcal{M}_{1b}$ and take the final states $\rho^+\rho^-$ as an example:

$$\mathcal{M}_{1a} = 2ig_{DD^*\chi_{c1}}g_{DDV}f_{D^*DV}\epsilon_\lambda^{\chi_{c1}}\epsilon_1^{*\sigma}\epsilon_2^{*\tau} \times \int \frac{d^4q}{(2\pi)^4}(q_{1\sigma}+q_\sigma)\epsilon_{\mu\tau\alpha\beta}p_2^\mu(q^\alpha - q_2^\alpha) \times \frac{g^{\lambda\beta} - q_2^\lambda q_2^\beta/m_{D^*}^2}{D_aD_1D_2}\mathcal{F}(q^2), \quad (14)$$

$$\mathcal{M}_{1b} = 2ig_{DD^*\chi_{c1}}g_{DDV}f_{D^*DV}\epsilon_\lambda^{\chi_{c1}}\epsilon_1^{*\sigma}\epsilon_2^{*\tau} \times \int \frac{d^4q}{(2\pi)^4}\epsilon_{\mu\sigma\alpha\beta}p_1^\mu(q_1^\alpha + q^\alpha)[g_{D^*D^*V}(q_{2\tau}-q_\tau) \times g_{\gamma\delta} + 4f_{D^*D^*V}(p_{2\delta}g_{\tau\gamma} - p_{2\gamma}g_{\delta\tau})] \times (g^{\beta\gamma} - q^\beta q^\gamma/m_{D^*}^2)(g^{\lambda\delta} - q_2^\lambda q_2^\delta/m_{D^*}^2) \times \frac{1}{D_bD_1D_2}\mathcal{F}(q^2), \quad (15)$$

where $D_a = q^2 - m_D^2$, $D_b = q^2 - m_{D^*}^2$, $D_1 = q_1^2 - m_D^2$, and $D_2 = q_2^2 - m_{D^*}^2$. The results of other channels can be obtained similarly.

Since the mass of χ_{c1} is under the threshold of $D\bar{D}^*$, intermediate mesons D and $\bar{D}^*$ cannot be on-shell simultaneously. We phenomenally introduce a form factor $\mathcal{F}(q^2)$ as has been done in Refs. [13,14],

$$\mathcal{F}(q^2) = \prod_i\left(\frac{m_i^2 - \Lambda_i^2}{q_i^2 - \Lambda_i^2}\right), \quad (16)$$

where $q_i = q, q_1, q_2$. The cutoff energy is chosen as $\Lambda_i = m_i + \alpha\Lambda_{\rm QCD}$, and m_i is the mass of the corresponding exchanged particle. This is somehow different from the form factor adopted in Refs. [13,14]. We should mention that the form factor is necessary for killing the divergence of the loop integrals, although it will also give rise to the model-dependent aspects of the calculations. We will discuss this in detail later.

B. $\chi_{c2} \to VP$

As mentioned previously, the decay $\chi_{c2} \to VP$ suffers not only from the suppression of the helicity selection rule but also from the approximate G-parity or isospin/U-spin conservation. However, because of the relatively large mass difference between the u/d quark and s quark, the intermediate meson loops may still bring in sizeable branching ratios for $\chi_{c2} \to K\bar{K}^*$ + c.c. Next, we will consider $\chi_{c2} \to \rho^+\pi^-$ + c.c. and $\chi_{c2} \to K^*\bar{K}$ + c.c. The χ_{c2} decays into neutral VP with fixed C parity, such as $\rho^0\pi^0$ and $\omega\eta$, etc., forbidden by C-parity conservation. The loop diagrams for these two processes are presented in Figs. 2 and 3, respectively. The convention of the momenta follows that of Fig. 1.

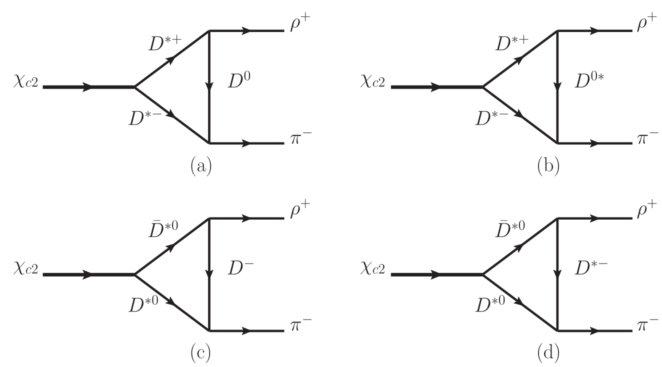

FIG. 2. Triangle loop diagrams that describe the long-distance contributions in $\chi_{c2} \to \rho^+\pi^-$. The diagrams for $\chi_{c2} \to \rho^-\pi^+$ are implicated.

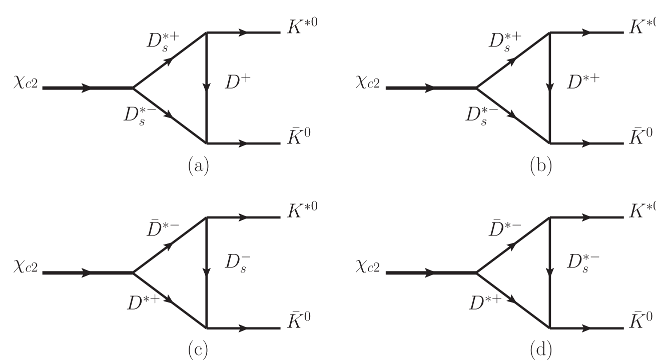

FIG. 3. Triangle loop diagrams that describe the long-distance contributions in $\chi_{c2} \to K^{*0}\bar{K}^0$. The diagrams for $\chi_{c2} \to \bar{K}^{*0}K^0$ are implicated.

The relative signs between the following amplitudes are opposite, i.e. $\mathcal{M}_{2a}$ and $\mathcal{M}_{2c}$, $\mathcal{M}_{2b}$ and $\mathcal{M}_{2d}$, $\mathcal{M}_{3a}$ and $\mathcal{M}_{3c}$, and $\mathcal{M}_{3b}$ and $\mathcal{M}_{3d}$. This leads to destructive interferences between these amplitudes, and is a reflection of the approximate isospin or U-spin invariance. We explicitly list the amplitudes, $\mathcal{M}_{2a}$ and $\mathcal{M}_{2b}$,

$$\mathcal{M}_{2a} = 2ig_{D^*D^*\chi_{c2}}f_{D^*DV}g_{D^*DP}\epsilon^{\chi_{c2}}_{\xi\eta}\epsilon^{\nu}_{\rho^+} \times \int\frac{d^4q}{(2\pi)^4}\epsilon_{\mu\nu\alpha\beta}p_1^\mu(q_1^\alpha + q^\alpha) \times p_2^\lambda(g^{\xi\beta} - q_1^\xi q_1^\beta/m^2_{D^*})(g^{\eta\lambda} - q_2^\eta q_2^\lambda/m^2_{D^*}) \times \frac{1}{D_aD_1D_2}\mathcal{F}(q^2), \tag{17}$$

$$\mathcal{M}_{2b} = -\frac{1}{2}ig_{D^*D^*\chi_{c2}}g_{D^*D^*P}\epsilon^{\chi_{c2}}_{\xi\eta}\epsilon^{\tau}_{\rho^+}\int\frac{d^4q}{(2\pi)^4}\epsilon_{\rho\sigma\alpha\beta} \times p_2^\sigma(q^\alpha - q_2^\alpha)[-g_{D^*D^*V}(q_{1\tau} + q_\tau)g^{\gamma\delta} - 4f_{D^*D^*V}(p_1^\gamma g_\tau^\delta - p_1^\delta g_\tau^\gamma)](g^{\xi\gamma} - q_1^\xi q_1^\gamma/m^2_{D^*}) \times (g^{\eta\beta} - q_2^\eta q_2^\beta/m^2_{D^*})(g^{\delta\rho} - q^\delta q^\rho/m^2_{D^*}) \times \frac{1}{D_bD_1D_2}\mathcal{F}(q^2), \tag{18}$$

while the others can be obtained similarly. As for the polarization sums of $\epsilon^{\chi_{c2}}_{\xi\eta}$, we follow the expressions in Ref. [25].

III. NUMERICAL RESULTS

Before proceeding to the numerical results, we first discuss the parameters, such as the coupling constants, in the formulation. In the chiral and heavy quark limit, the following relations can be obtained [13,24]:

$$g_{DDV} = g_{D^*D^*V} = \frac{\beta g_V}{\sqrt{2}}, \qquad f_{D^*DV} = \frac{f_{D^*D^*V}}{m_{D^*}} = \frac{\lambda g_V}{\sqrt{2}}, \qquad g_V = \frac{m_\rho}{f_\pi}, \qquad g_{D^*D^*\pi} = \frac{g_{D^*D\pi}}{\sqrt{m_D m_{D^*}}} = \frac{2}{f_\pi}g, \qquad g_{D^*D_sK} = \sqrt{\frac{m_{D_s}}{m_D}}g_{D^*D\pi}, \qquad g_{D_s^*DK} = \sqrt{\frac{m_{D_s^*}}{m_{D^*}}}, \tag{19}$$

where β and λ are commonly taken as $\beta = 0.9$, $\lambda = 0.56\ \mathrm{GeV}^{-1}$ [13,17,26], while f_π is the pion decay constant. With the measured branching ratio of $D^* \to D\pi$ by CLEO-c, the coupling g is determined as $g = 0.59$ [27]. In the heavy quark limit, the expansion of the effective Lagrangian $\mathcal{L}_1$ leads to relations for the P-wave charmonium couplings to the charmed mesons as follows:

$$g_{DD^*\chi_{c1}} = 2\sqrt{2}g_1\sqrt{m_D m_{D^*} m_{\chi_{c1}}}, \qquad g_{D^*D^*\chi_{c2}} = 4g_1 m_{D^*}\sqrt{m_{\chi_{c2}}}, \qquad g_1 = -\sqrt{\frac{m_{\chi_{c0}}}{3}}\frac{1}{f_{\chi_{c0}}}, \tag{20}$$

where g_1 has been related to the χ_{c0} decay constant $f_{\chi_{c0}}$. It can be approximately determined by the QCD sum rule method, i.e. $f_{\chi_{c0}} \simeq 0.51$ GeV [28].

TABLE I. Branching ratios for $\chi_{c1} \to VV$ predicted by the intermediate meson loop transitions in the range $\alpha = 0.3 \sim 0.33$ corresponding to the measured lower and upper bound of BR$(\chi_{c2} \to K^{*0}\bar{K}^{*0})$ [8]. Two results from the SU(3) flavor symmetry relation are presented with $R = 1$ and $R = 0.838$. The dots mean that experimental data are unavailable.

BR ($\times 10^{-4}$)	$K^{*0}\bar{K}^{*0}$	$\rho\rho$	$\omega\omega$	$\phi\phi$
Experimental data	16 ± 4	...	...	...
Meson loop	$12 \sim 20$	$26 \sim 54$	$8.7 \sim 18$	$2.7 \sim 4.6$
SU(3) ($R = 1$)	16.0	26.8	8.8	6.8
SU(3) ($R = 0.838$)	16.0	32.0	10.6	4.0

The form factor parameter α generally cannot be determined from the first principle. It is usually taken to be order of unity, and depends on the particular process. In this work, since the branching ratio of $\chi_{c1} \to K^{*0}\bar{K}^{*0}$ has been measured in experiment [8], we will adopt the data to constrain the form factor parameter.

The loop integrals are calculated with the software package LOOPTOOLS [29]. In Tables I and II, we display the numerical results for the branching ratios of $\chi_{c1} \to VV$ and $\chi_{c2} \to VP$ at $\alpha = 0.3 \sim 0.33$, which correspond to the lower and upper bounds of BR$(\chi_{c1} \to K^{*0}\bar{K}^{*0}) = (1.6 \pm 0.4) \times 10^{-3}$ [8].

For $\chi_{c1} \to VV$, the branching ratio of the $\rho\rho$ channel is significant, while the $\omega\omega$ and $\phi\phi$ channels are relatively small. We also include the predictions from the SU(3) flavor symmetry as a comparison. The parametrization is given in the Appendix. It is interesting to see that the results from the SU(3) flavor symmetry are basically compatible with those given by the intermediate meson loops except that the branching ratio for $\phi\phi$ is larger. It should be an indication for the SU(3) flavor symmetry breaking. With the SU(3) symmetry breaking parameter $R \simeq f_\pi/f_K = 0.838$ [8], the branching ratios agree with the intermediate meson loop results pretty well.

Some insights into the transition mechanisms can be gained here:

(i) The SU(3) flavor symmetry breaking $R = 0.838$ is consistent with the loop transition behavior. Note that the intermediate D_s and D_s^* pair has higher mass threshold, and the production of the $\phi\phi$ will be relatively suppressed in comparison with the nonstrange $\rho\pi$ and $\omega\omega$ apart from the final-state phase space differences. This corresponds to the flavor symmetry breaking at leading order.

(ii) The consistencies support the idea that the intermediate meson loops provide a natural mechanism for the evasion of the helicity selection rule as a long-distance transition. Since the $\chi_{c0,1,2}$ are P-wave states, the short-distance transition probes the first

TABLE II. Branching ratios for $\chi_{c2} \to VP$ predicted by the intermediate meson loop transitions in the same range of $\alpha = 0.3 \sim 0.33$. The dots mean that experimental data are unavailable.

BR ($\times 10^{-5}$)	$K^{*0}\bar{K}^0$ + c.c.	$K^{*+}K^-$ + c.c.	$\rho^+\pi^-$ + c.c.
Meson loop	$4.0 \sim 6.7$	$4.0 \sim 6.7$	$(1.2 \sim 2.0) \times 10^{-2}$
Experimental data	...	...	...

derivative of the $c\bar{c}$ wave function at the origin, which, however, would be suppressed in $\chi_{c1} \to VV$ and $\chi_{c2} \to VP$ due to the helicity selection rule. By annihilating the $c\bar{c}$ at long distance via the intermediate meson loops, the helicity selection rule is then evaded.

(iii) This phenomenon is slightly different from the S-wave charmonium decays, such as $J/\psi \to \rho\pi$, etc., where it is not easy to separate out the long-distance transitions from the short-distance ones arising from a possible anomalous component of wave function [15,30,31].

For $\chi_{c2} \to VP$, since there are no data available at this moment, our predictions are based on the same form factor parameter for $\chi_{c1} \to VV$. As listed in Table II, the accessible channels are only $K^*\bar{K}$ + c.c. and $\rho^+\pi^-$ + c.c. Interestingly, the branching ratio for $K^*\bar{K}$ + c.c. turns out to be sizeable, and the charged $\rho\pi$ is found to be much smaller than the $K^*\bar{K}$ + c.c. channel. Qualitatively, this is because that the cancellations between (a) and (c) [and similarly between (b) and (d)] in Figs. 2 and 3 are rather different. Namely, the $K^*\bar{K}$ + c.c. channel experiences large U-spin symmetry breakings due to the significant mass difference between the d and s quark. In contrast, the $\rho\pi$ channel is originated from the u-d mass difference. This unique result can be examined by BES-III as a test of our model.

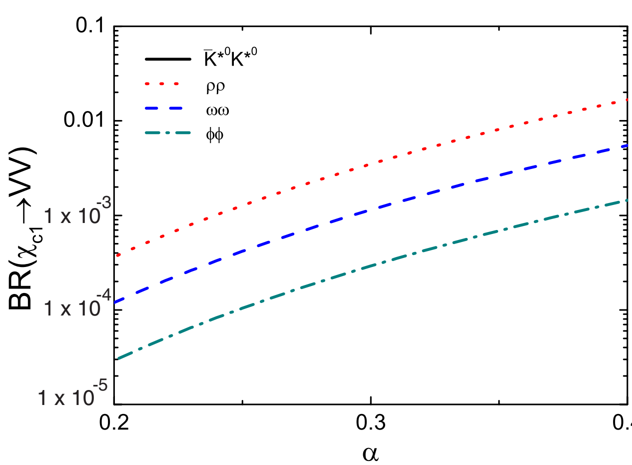

FIG. 4 (color online). α dependence of the calculated branching ratios for $\chi_{c1} \to VV$.

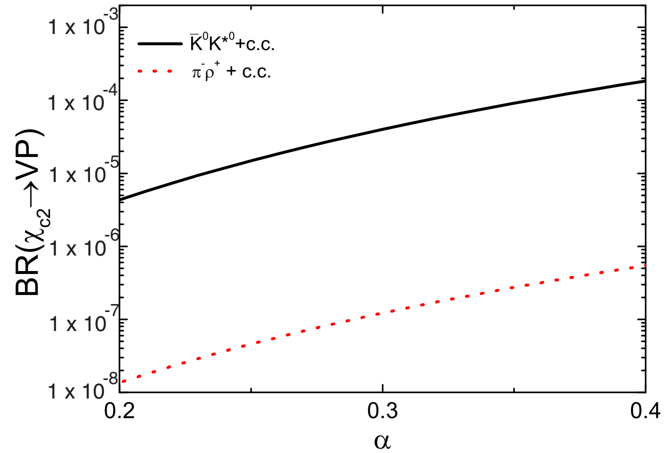

FIG. 5 (color online). α dependence of the calculated branching ratios for $\chi_{c2} \to VP$.

The sensitivities of the calculation results to the form factor parameter α are presented in Figs. 4 and 5. One notices that the values for α in this work are relatively smaller than those adopted in some other works [13,17,18]. This is acceptable since in this work the form factor takes care of off-shell effects arising from those three intermediate mesons, instead of only the exchanged one in the rescattering [13,17,18]. Given the inevitable model dependence introduced by the form factor, what turns out to be relatively stable and less model-dependent is the relative branching ratio fractions among those decay channels within the adopted range of α. As a consequence, the theoretical predictions for other decay channels can be better controlled by the data for $\chi_{c1} \to K^{*0}\bar{K}^{*0}$.

IV. CONCLUSION

In this work, we have discussed how the long-distance transitions via the intermediate meson loops would contribute to the processes $\chi_{c1} \to VV$ and $\chi_{c2} \to VP$, which are supposed to be suppressed according to the helicity selection rule in QCD. With an effective Lagrangian method with heavy quark and chiral symmetry, this helicity-selection-rule evading mechanism is quantified. Although there are still relatively large uncertainties arising from the form factor parameter, we argue that the fewer sensitivities of the branching ratio fractions among the accessible channels would provide a better control of the theory predictions. Reasonable ranges of the predictions are obtained.

We also compare the results from the intermediate meson loops with the expectations of the SU(3) flavor symmetry, and find that they are in a good agreement with each other. In particular, the SU(3) flavor symmetry breaking would lead to a suppression of $\chi_{c1} \to \phi\phi$ in comparison with the $\omega\omega$. It can be well understood by the heavier mass of the intermediate D_s (D_s^*) than the D (D^*) state. Namely, the mass threshold of the $D_s\bar{D}_s^*$ + c.c. is higher than

$D\bar{D}^* +$ c.c. In $\chi_{c2} \to VP$, the predicted branching ratio for the $K^*\bar{K}$ + c.c. channel is at the order of 10^{-5}, while the charged $\rho\pi$ channel is small. The suppression on the charged $\rho\pi$ (in comparison with the $K^*\bar{K}$ + c.c.) can be comprehended as a consequence of the larger effects due to the U-spin symmetry breaking rather than the isospin symmetry breaking, i.e. $m_s - m_d \gg m_d - m_u$.

In brief, we emphasize that the P-wave charmonium decay should be ideal for examining the evading mechanisms of the helicity selection rule. Our predictions can be examined by the high-statistics χ_{cJ} production in the BES-III experiment [6,32].

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APPENDIX

We adopt a simple parametrization [33–35] to study $\chi_{c1} \to VV$ based on the SU(3) flavor symmetry. By assuming that $\hat{H}$ represents the potential for the $c\bar{c}$ annihilating into gluons and then hadronizing into $(q\bar{q})_{V_1}(q\bar{q})_{V_2}$, we define the transition amplitude strength as

$$g_0 \equiv \langle (q\bar{q})_{V_1}(q\bar{q})_{V_2}|\hat{H}|\chi_{c1}\rangle, \quad \text{(A1)}$$

where q ($\bar{q}$) is a nonstrange quark (antiquark). Considering the SU(3) flavor symmetry breaking, we introduce parameter R which describes

$$R \equiv \langle (q\bar{s})_{V_1}(s\bar{q})_{V_2}|\hat{H}|\chi_{c1}\rangle / \langle (q\bar{q})_{V_1}(q\bar{q})_{V_2}|\hat{H}|\chi_{c1}\rangle, \quad \text{(A2)}$$

and

$$R^2 \equiv \langle (s\bar{s})_{V_1}(s\bar{s})_{V_2}|\hat{H}|\chi_{c1}\rangle / \langle (q\bar{q})_{V_1}(q\bar{q})_{V_2}|\hat{H}|\chi_{c1}\rangle, \quad \text{(A3)}$$

where the exchange of V_1 and V_2 is implicated, and the SU(3) flavor symmetry is recognized by $R = 1$. This parameter can be related to the ratio of the π and K meson decay constant, i.e.

$$R \simeq f_\pi / f_K, \quad \text{(A4)}$$

which gives $R \simeq 0.838$ [8].

A commonly adopted vertex form factor is also applied,

$$\mathcal{F}^2(\mathbf{p}_1) \equiv |\mathbf{p}_1|^{2L} \exp(-\mathbf{p}_1^2/8\beta^2), \quad \text{(A5)}$$

where $\mathbf{p}_1$ is the three-vector momentum of the final-state meson V_1 in the rest frame of χ_{c1}, and L is the relative orbital angular momentum between V_1 and V_2. As discussed earlier, $L = 2$ is required by parity conservation and Lorentz invariance. We adopt $\beta = 0.5$ GeV, which is the same as in Refs. [15,33–36]. The partial decay widths are given as follows:

$$\begin{aligned}
\Gamma_{\rho^+\pi^-} &= \Gamma_{\rho^-\pi^+} = \frac{|\mathbf{p}_1|}{24\pi M_{\chi_{c1}}^2} g_0^2 \mathcal{F}^2(\mathbf{p}_1), \\
\Gamma_{K^{*0}\bar{K}^{*0}} &= \Gamma_{\bar{K}^{*0}K^{*0}} = \Gamma_{K^{*+}K^{*-}} = \Gamma_{K^{*-}K^{*+}} \\
&= \frac{|\mathbf{p}_1|}{24\pi M_{\chi_{c1}}^2} g_0^2 R^2 \mathcal{F}^2(\mathbf{p}_1), \\
\Gamma_{\omega\omega} &= \frac{|\mathbf{p}_1|}{24\pi M_{\chi_{c1}}^2} g_0^2 \mathcal{F}^2(\mathbf{p}_1), \\
\Gamma_{\phi\phi} &= \frac{|\mathbf{p}_1|}{24\pi M_{\chi_{c1}}^2} g_0^2 R^4 \mathcal{F}^2(\mathbf{p}_1).
\end{aligned} \quad \text{(A6)}$$

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