

Decay $\eta_b \rightarrow J/\psi J/\psi$ in light cone formalism

 V. V. Braguta^{1,*} and V. Kartvelishvili^{2,†}
¹*Institute for High Energy Physics, Protvino, Russia*
²*Lancaster University, Lancaster, LA1 4YB, United Kingdom*

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The decays of pseudoscalar bottomonium η_b into a pair of vector charmonia, $J/\psi J/\psi$, $J/\psi \psi'$, $\psi' \psi'$, are considered in the light cone formalism. Relativistic and leading logarithmic radiative corrections to the amplitudes of these processes are resummed. It is shown that the small value for the branching ratio of the decay $\eta_b \rightarrow J/\psi J/\psi$ obtained within the leading-order nonrelativistic QCD is a consequence of a fine-tuning between certain parameters, which is broken when relativistic and leading logarithmic radiative corrections are taken into account. As a result, the branching ratio obtained in this paper is enhanced by an order of magnitude.

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I. INTRODUCTION

Ever since the discovery of the Y meson, there have been numerous attempts to observe the lightest pseudoscalar bottomonium state, η_b . However, only recently the first experimental evidence of the existence of this meson was found by the *BABAR* Collaboration, in the radiative decay $Y(3S) \rightarrow \eta_b + \gamma$ [1]. Its mass was found to be $m_{\eta_b} = 9388_{-2.3}^{+3.1}(\text{stat}) \pm 2.7(\text{syst})$ MeV, but our knowledge of its other properties remains rather poor.

In [2] it was proposed to look for the η_b meson in the decay $\eta_b \rightarrow J/\psi J/\psi$, but, despite its clean signature, this process may be hard to observe due to its extremely small branching ratio. Contrary to similar processes, such as the decays $\chi_b \rightarrow J/\psi J/\psi$ [3], the rate of the decay $\eta_b \rightarrow J/\psi J/\psi$ vanishes at leading order of both relative velocity and $1/M_{\eta_b}$ expansions. The calculations made within nonrelativistic QCD (NRQCD) [4] yield $\text{Br}(\eta_b \rightarrow J/\psi J/\psi) \sim 10^{-8}-10^{-7}$ [5,6]; however, in [7] it was shown that the account of final-state interaction effects can enhance it up to about 10^{-5} .

A similar conclusion can be drawn from the comparison of the decays $\eta_b \rightarrow J/\psi J/\psi$, $J/\psi \psi'$, and $\psi' \psi'$ and the processes of double charmonia production at B factories. It is now clear that these processes are greatly affected by radiative and relativistic corrections [8–20]. With the mass of η_b being so close to the energy at which B factories operate, it is natural to expect that the same is true for the decays $\eta_b \rightarrow J/\psi J/\psi$, $J/\psi \psi'$, and $\psi' \psi'$, and hence the consideration of these processes without accounting for radiative and relativistic corrections is unreliable. This was also confirmed by the calculation of radiative corrections within NRQCD, performed in [6].

In this paper, the processes $\eta_b \rightarrow J/\psi J/\psi$, $J/\psi \psi'$, and $\psi' \psi'$ are considered within the light cone (LC) formalism [21]. In this approach, the amplitudes of these processes

are expanded in $(M_{c\bar{c}}/M_{b\bar{b}})^2 \sim 0.1$, which is sufficiently small for the applicability of the method [22].

In the LC formalism, the amplitude of a process under study is decomposed into the perturbative part, dealing with the production of quarks and gluons at small distances, and the large-distance part describing the hadronization of the partons. For hard exclusive processes, the latter can be parametrized by the process-independent distribution amplitudes (DAs), which can be considered as hadrons' wave functions at lightlike separations between the partons inside the hadron. It should be noted that DAs contain information about the structure of mesons and effectively resum relativistic corrections to the amplitude. Moreover, using the renormalization group evolution of DAs, one can take into account the leading logarithmic radiative corrections to the amplitude.

This paper is organized as follows. In Sec. II, DAs for charmonium are defined, and various models for these DAs are discussed. In Sec. III, the amplitude of the decay of η_b into two vector mesons is derived. Finally, in Sec. IV the numerical results and their uncertainties are presented and discussed.

II. DISTRIBUTION AMPLITUDES FOR CHARMONIUM

The amplitude of the process $\eta_b \rightarrow V_1 V_2$, with $V_{1,2}$ standing for either J/ψ or ψ' , can be parametrized with a single form factor F :

$$M = F e_{\mu\nu\sigma\rho} p_1^\mu p_2^\nu \epsilon_1^\sigma \epsilon_2^\rho, \quad (1)$$

where p_1 , p_2 and ϵ_1 , ϵ_2 are the momenta and polarization vectors of V_1 and V_2 , respectively. Hence, the width of the decay $\eta_b \rightarrow V_1 V_2$ can be written in the form

$$\Gamma[\eta_b \rightarrow V_1 V_2] = |F|^2 \frac{|\mathbf{p}|^3}{4\pi}, \quad (2)$$

where \mathbf{p} is the 3-momentum of a final meson in the η_b rest

*braguta@mail.ru

†V.Kartvelishvili@lancaster.ac.uk

frame. If the final mesons are identical, $V_1 = V_2$, the width Γ should be divided by $2!$.

In the LC formalism, the amplitude of a hard exclusive process is expanded in the inverse powers of the hard energy scale E_h , which for the decay $\eta_b \rightarrow V_1 V_2$ can be identified as M_{η_b} . The leading-order contribution in this expansion requires the two vector mesons to be produced with polarizations $\lambda_1 = \lambda_2 = 0$ [21], but in this case the amplitude (1) vanishes. In order to obtain a nonzero result, both vector mesons need to be transversely polarized, which in turn means that the helicities of the quarks in both mesons must be flipped twice, and hence leads to a suppression factor $\sim 1/(M_{\eta_b})^2$ [5]. Therefore, the decay

$\eta_b \rightarrow V_1 V_2$ is a next-to-next-to-leading-order twist process, and in order for the calculations to be consistent one needs DAs up to twist-4. In general, twist-4 DAs should contain terms corresponding to higher Fock states in addition to the ‘‘valence’’ charm quark-antiquark state. For instance, one can expect the contribution of four-quark states or quark-quark-gluon bound states. However, since charmonium can be considered as a nonrelativistic meson, such higher states in charmonium are suppressed by higher powers of relative velocity [4], and in the following we will neglect their contribution.

The DAs for a vector meson V with momentum p and polarization vector ϵ can be defined as follows [23,24]:

$$\begin{aligned} \langle V(p, \epsilon) | \bar{c}(x) \gamma_\rho [x, -x] c(-x) | 0 \rangle &= f_V M_V \left[\frac{(\epsilon x)}{(px)} p_\rho \int_{-1}^1 d\xi e^{i\xi(px)} \left(\varphi_1(\xi, \mu) + \frac{M_V^2 x^2}{4} \varphi_2(\xi, \mu) \right) \right. \\ &\quad \left. + \left(\epsilon_\rho - p_\rho \frac{(\epsilon x)}{(px)} \right) \int_{-1}^1 d\xi e^{i\xi(px)} \varphi_3(\xi, \mu) - \frac{1}{2} x_\rho \frac{(\epsilon x)}{(px)^2} M_V^2 \int_{-1}^1 d\xi e^{i\xi(px)} \varphi_4(\xi, \mu) \right], \\ \langle V(p, \epsilon) | \bar{c}(x) \sigma_{\rho\lambda} [x, -x] c(-x) | 0 \rangle &= f_T(\mu) \left[(\epsilon_\rho p_\lambda - \epsilon_\lambda p_\rho) \int_{-1}^1 d\xi e^{i\xi(px)} \left(\chi_1(\xi, \mu) + \frac{M_V^2 x^2}{4} \chi_2(\xi, \mu) \right) + (p_\rho x_\lambda - p_\lambda x_\rho) \right. \\ &\quad \left. \times \frac{(\epsilon x)}{(px)^2} M_V^2 \int_{-1}^1 d\xi e^{i\xi(px)} \chi_3(\xi, \mu) + \frac{1}{2} (\epsilon_\rho x_\lambda - \epsilon_\lambda x_\rho) \frac{M_V^2}{(px)} \int_{-1}^1 d\xi e^{i\xi(px)} \chi_4(\xi, \mu) \right], \\ \langle V(p, \epsilon) | \bar{c}(x) \gamma_\rho \gamma_5 [x, -x] c(-x) | 0 \rangle &= f_A(\mu) e_{\rho\lambda\alpha\beta} \epsilon^\lambda p^\alpha x^\beta \int_{-1}^1 d\xi e^{i\xi(px)} \Phi_1(\xi, \mu), \\ \langle V(p, \epsilon) | \bar{c}(x) [x, -x] c(-x) | 0 \rangle &= -i f_S(\mu) (\epsilon x) \int_{-1}^1 d\xi e^{i\xi(px)} \Phi_2(\xi, \mu). \end{aligned} \quad (3)$$

Here $[x, -x]$ is the gluon string which makes the matrix element gauge invariant, ξ is a dimensionless variable describing the relative motion of the charmed quark and antiquark inside the meson, μ is the energy scale at which the DAs are defined, while the constants f_V and $f_T(\mu)$ are defined by

$$\begin{aligned} \langle V(p, \epsilon) | \bar{c}(0) \gamma_\mu c(0) | 0 \rangle &= f_V M_V \epsilon_\mu, \\ \langle V(p, \epsilon) | \bar{c}(0) \sigma_{\mu\nu} c(0) | 0 \rangle &= f_T(\mu) (\epsilon_\mu p_\nu - \epsilon_\nu p_\mu). \end{aligned} \quad (4)$$

The constants $f_A(\mu)$, $f_S(\mu)$ can be expressed through f_V , f_T as follows:

$$\begin{aligned} f_A(\mu) &= \frac{1}{2} \left(f_V - f_T(\mu) \frac{2m_c(\mu)}{M_V} \right) M_V, \\ f_S(\mu) &= \left(f_T(\mu) - f_V \frac{2m_c(\mu)}{M_V} \right) M_V^2, \end{aligned} \quad (5)$$

where $m_c(\mu)$ is the running mass of the c quark. Below it will be shown that the amplitude of the process under study contains only terms proportional to the constants f_A , f_S . According to Eqs. (5) these constants are related to the constants f_V , f_T . These relations are fixed by equations of motion and due to these relations the amplitude obtained within the light cone formalism is in agreement with NRQCD prediction.

Equations (3) contain 10 independent DAs, but only 4 of these are relevant for the calculation of the $\eta_b \rightarrow V_1 V_2$

decay rate: $\varphi_1(\xi)$, $\chi_1(\xi)$, $\Phi_1(\xi)$, and $\Phi_2(\xi)$ (see below). For the first two, $\varphi_1(\xi)$ and $\chi_1(\xi)$, we will use models proposed in [25–28]. In [20] it was shown that, if the higher Fock states are ignored, the functions $\Phi_1(\xi)$ and $\varphi_3(\xi)$ can be unambiguously determined from the equations of motion. The same is true for the functions $\Phi_2(\xi)$ and $\chi_3(\xi)$.

In the remainder of this section, a relation between $\Phi_2(\xi)$, $\chi_3(\xi)$ and $\varphi_1(\xi)$, $\chi_1(\xi)$ will be derived. The functions $\Phi_2(\xi)$ and $\chi_3(\xi)$ can be expanded into a series of Gegenbauer polynomials [23]:

$$\begin{aligned} \chi_3(x, \mu) &= \frac{1}{2} \left[1 + \sum_{n=2,4,\dots} c_n(\mu) C_n^{1/2}(2x-1) \right], \\ \Phi_2(x, \mu) &= \frac{3}{4} (1 - \xi^2) \left[1 + \sum_{n=2,4,\dots} d_n(\mu) C_n^{3/2}(2x-1) \right]. \end{aligned} \quad (6)$$

The coefficients $c_n(\mu)$ and $d_n(\mu)$ are related to the moments of the functions $\varphi_1(\xi)$, $\chi_1(\xi)$ through the equations of motion [23],

$$\begin{aligned} \frac{n+2}{2} \langle \xi^n \rangle_\chi &= \langle \xi^n \rangle_T + \frac{n(n-1)}{2} \\ &\quad \times (1 - \delta(\mu)) \langle \xi^{n-2} \rangle_\Phi, \\ (n+1)(1 - \delta(\mu)) \langle \xi^n \rangle_\Phi &= \langle \xi^n \rangle_\chi - \delta(\mu) \langle \xi^n \rangle_L, \end{aligned} \quad (7)$$

where $\langle \xi^n \rangle_{L,T,\chi,\Phi}$ denote the moments of the DAs $\phi_1(\xi)$, $\chi_1(\xi)$, $\chi_3(\xi)$, and $\Phi_2(\xi)$, respectively, while $\delta(\mu) = 2f_V/f_T(\mu)(m_c(\mu)/M_V)$. By solving Eqs. (7) recursively, one can determine the functions $\Phi_2(\xi)$ and $\chi_3(\xi)$. In [25] it was shown that there is a fine-tuning of the coefficients of the Gegenbauer expansion at the scale $\mu \sim \bar{m}_c \equiv m_c(\mu = m_c)$. Without this fine-tuning the DAs of a nonrelativistic system would show an unphysical relativistic tail already at the scale $\mu \sim \bar{m}_c$. In order to get rid of this tail in the DAs $\Phi_2(\xi)$ and $\chi_3(\xi)$, fine-tuning is required between the coefficients c_n, d_n and the parameter δ , which is related to the wave functions $\phi_1(\xi), \chi_1(\xi)$ [20]:

$$\delta(\bar{m}_c) = \frac{\int_{-1}^1 \frac{d\xi}{1-\xi^2} \chi_1(\xi, \mu \sim \bar{m}_c)}{\int_{-1}^1 \frac{d\xi}{(1-\xi^2)^2} \varphi_1(\xi, \mu \sim \bar{m}_c)}. \quad (8)$$

III. THE AMPLITUDE OF THE PROCESS $\eta_b \rightarrow V_1 V_2$

The diagrams that contribute to the amplitude of the process under study at leading order in the α_s expansion are shown in Fig. 1. The procedure of calculating the amplitude is described in detail in [21]. This is a lengthy but straightforward exercise, yielding a result which looks remarkably simple:

$$\begin{aligned} F = & \int d\xi_1 d\xi_2 H(\xi_1, \xi_2, \mu) \\ & \times (f_{V_1} f_{A_2}(\mu) M_{V_1} \varphi_1(\xi_1, \mu) \Phi_1(\xi_2, \mu) \\ & + f_{V_2} f_{A_1}(\mu) M_{V_2} \varphi_1(\xi_2, \mu) \Phi_1(\xi_1, \mu) \\ & + f_{S_1}(\mu) f_{T_2}(\mu) \chi_1(\xi_2, \mu) \Phi_2(\xi_1, \mu) \\ & + f_{S_2}(\mu) f_{T_1}(\mu) \chi_1(\xi_1, \mu) \Phi_2(\xi_2, \mu)). \quad (9) \end{aligned}$$

Here the function $H(\xi_1, \xi_2, \mu)$ represents the hard part of the amplitude,

$$\begin{aligned} H(\xi_1, \xi_2, \mu) = & \frac{1024\pi^2 \alpha_s^2(\mu)}{27} f_{\eta_b} \frac{1}{M_{\eta_b}^6} \\ & \times \frac{1}{(1-\xi_1^2)(1-\xi_2^2)(1+\xi_1\xi_2)}, \quad (10) \end{aligned}$$

with the decay constant f_{η_b} defined by

$$\langle 0 | \bar{b}(0) \gamma_\rho \gamma_5 b(0) | \eta_b(p) \rangle = i f_{\eta_b} p_\rho. \quad (11)$$

At this point, some comments are in order.

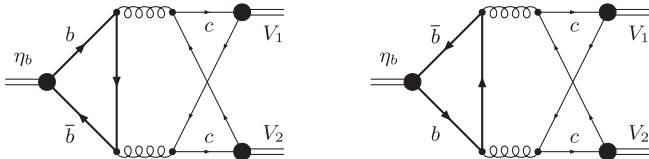


FIG. 1. The diagrams contributing to the amplitude of the process $\eta_b \rightarrow J/\psi J/\psi$ at leading order in α_s .

- (1) In Eq. (9) there is a clear separation of large- and small-distance contributions. While $H(\xi_1, \xi_2, \mu)$ describes the hard part of the amplitude, the large-distance part is parametrized by the combination of the DAs, which effectively include resummation of the relativistic corrections to the amplitude. A discussion of this point can be found in [20,22].
- (2) In Eq. (9) the dependence of the hard part of the amplitude, the constants, and the DAs on the scale μ is explicitly shown. If the process in question were a leading-twist process, one could perform an exact resummation of all leading-twist radiative corrections to the amplitude, $\sim \alpha_s \log(M_{\eta_b}^2/M_{J/\psi}^2)$, simply by taking $\mu \sim M_{\eta_b}$ [21]. Indeed, for a leading-twist process, one would use the axial gauge, in which double-logarithmic and logarithmic corrections only appear in the self-energy diagrams and rescattering of final particles. The double-logarithmic corrections are canceled since final particles are colorless objects, while the logarithmic corrections lead to the renormalization of the DAs themselves. Although the decay $\eta_b \rightarrow V_1 V_2$ is a next-to-next-to-leading-twist process, all the arguments given above still seem to be applicable. Note also that in Eq. (9) there is no divergence in the end-point region, $|\xi| \sim 1$, indicating that all logarithms are collected. These arguments allow us to believe that Eq. (9) includes the exact resummation of leading logarithmic radiative corrections to all loops.
- (3) Whenever NRQCD and LC approaches are used to describe the same process, one should expect some kind of duality between the two results. For the process $\eta_b \rightarrow VV$ this duality can be checked at the leading-order approximation in relative velocity of the c -quark-antiquark pair inside charmonia. In particular, by taking infinitely narrow DAs and the constants f_T, f_V and masses $M_V, 2m_c$ at the next-to-leading-order approximation in relative velocity [29],

$$\frac{f_T}{f_V} = 1 - \frac{\langle v^2 \rangle}{3}, \quad \frac{M_V}{2m_c} = 1 + \frac{\langle v^2 \rangle}{2}, \quad (12)$$

and by neglecting all radiative corrections, one gets from Eq. (9):

$$F = \frac{256\pi^2 \alpha_s^2}{81} \frac{1}{m_b^6} f_{\eta_b} f_V^2 m_c^2 \langle v^2 \rangle, \quad (13)$$

which coincides with the result obtained in [5]. In these formulas, $\langle v^2 \rangle$ is the NRQCD matrix element, defined as

$$\langle v^2 \rangle = -\frac{1}{m_c^2} \frac{\langle 0 | \chi^+ (\vec{\sigma} \cdot \vec{\epsilon}) (\vec{\mathbf{D}})^2 \varphi | V(\epsilon) \rangle}{\langle 0 | \chi^+ (\vec{\sigma} \cdot \vec{\epsilon}) \varphi | V(\epsilon) \rangle}. \quad (14)$$

As noted in [20,22], the duality between NRQCD and LC allows us to estimate the size of power

corrections. The idea is that if one expands the NRQCD result in powers of $1/M_{\eta_b}$, then the first term coincides with the LC prediction and the second term gives an estimate of power corrections to the LC result. Thus, power corrections to the amplitude of the $\eta_b \rightarrow VV$ decay can be estimated as $\sim 4v^2 M_V^2/M_{\eta_b}^2$.

Now we have all the ingredients needed to calculate the rates of the decays $\eta_b \rightarrow V_1 V_2$.

IV. NUMERICAL RESULTS AND DISCUSSION

A. Input parameters

In order to obtain numerical results for the branching ratios of the decays $\eta_b \rightarrow J/\psi J/\psi$, $J/\psi \psi'$, and $\psi' \psi'$ the following input parameters were used:

- (1) The strong coupling constant $\alpha_s(\mu)$ is taken at the one loop,

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \log(\mu^2/\Lambda^2)}, \quad (15)$$

with $\Lambda = 0.2 \text{ GeV}$, $\beta_0 = 25/3$.

- (2) The mass of the c quark in the $\overline{\text{MS}}$ scheme, $\bar{m}_c = 1.2 \text{ GeV}$.
- (3) The leptonic decay constants of the J/ψ and ψ' mesons $f_V^{J/\psi}$, $f_V^{\psi'}$ were determined directly from experimental data, while the constants $f_T^{J/\psi}$ and $f_T^{\psi'}$ were calculated within NRQCD in [29]:

$$\begin{aligned} (f_V^{J/\psi})^2 &= 0.173 \pm 0.004 \text{ GeV}^2, \\ (f_V^{\psi'})^2 &= 0.092 \pm 0.002 \text{ GeV}^2, \\ (f_T^{J/\psi}(M_{J/\psi}))^2 &= 0.144 \pm 0.016 \text{ GeV}^2, \\ (f_T^{\psi'}(M_{J/\psi}))^2 &= 0.068 \pm 0.022 \text{ GeV}^2. \end{aligned} \quad (16)$$

- (4) We assume that the total decay width of the η_b meson $\Gamma_{\text{tot}}(\eta_b)$ can be approximated by its two-gluon decay width $\Gamma(\eta_b \rightarrow gg)$ which, at leading order in relative velocity and α_s , is equal to

$$\Gamma_{\text{tot}}(\eta_b) = \Gamma(\eta_b \rightarrow gg) = \frac{8\pi}{9} \frac{\alpha_s^2}{M_{\eta_b}} f_{\eta_b}^2. \quad (17)$$

- (5) The leading-twist DAs needed for the calculations are taken from models developed in [25–28].

B. Estimation of uncertainties

The most important uncertainties come from the following sources:

- (1) *Model dependence of the DAs.*—These uncertainties can be estimated by varying the parameters of these models (see [25–28] for more details). The calculations show that for the processes $\eta_b \rightarrow J/\psi J/\psi$, $J/\psi \psi'$, and $\psi' \psi'$ these uncertainties are no larger

than $\sim 5\%$, 13% , and 30% , respectively. In fact, these uncertainties are expected to be rather low, due to the property that the precision of any DA model improves with evolution [25].

- (2) *Radiative corrections.*—Within the approach used in this paper, the leading logarithmic radiative corrections due to the evolution of the DAs and the strong coupling constant were effectively resummed. Although we argued above that this is also true for all leading logarithmic radiative corrections, there is no strict proof of this statement. For this reason, we estimate the uncertainty due to the radiative corrections as $\sim \alpha_s(M_{\eta_b}/2) \times \log(M_{\eta_b}^2/(4M_{J/\psi}^2)) \sim 50\%$.

- (3) *Power corrections.*—As mentioned above, this source of uncertainty can be estimated as $\sim 4\langle v^2 \rangle M_V^2/M_{\eta_b}^2$, which is the largest for the decay $\eta_b \rightarrow \psi' \psi'$, reaching $\sim 4\langle v^2 \rangle_{\psi'} M_{\psi'}^2/M_{\eta_b}^2 \sim 20\%$.

- (4) *Relativistic corrections.*—This source of uncertainty appears because we treated the η_b meson at the leading-order approximation in relative velocity. It can be estimated as $\sim v_{\eta_b}^2 \sim 10\%$.

- (5) *The uncertainties in the values of constants (16).*—For the three processes $\eta_b \rightarrow J/\psi J/\psi$, $J/\psi \psi'$, and $\psi' \psi'$ these errors are estimated to be $\sim 16\%$, 27% , and 49% , respectively.

- (6) *Higher Fock states.*—It can be argued that at the scale μ relevant to the η_b decay process, only a small fraction of quarkonium momentum is carried by the quark-gluon sea, typically $\sim 5\%$ – 10% [30]. Hence, we expect the effects of higher Fock states to be negligible, compared to other uncertainties considered here.

The overall uncertainties of our calculations were obtained by adding the above errors in quadrature.

C. Results and discussion

By substituting the expressions for DAs and the necessary constants into Eqs. (2) and (9), we get the following values for the three branching ratios:

$$\begin{aligned} \text{Br}(\eta_b \rightarrow J/\psi J/\psi) &= (6.2 \pm 3.5) \times 10^{-7}, \\ \text{Br}(\eta_b \rightarrow J/\psi \psi') &= (10 \pm 6) \times 10^{-7}, \\ \text{Br}(\eta_b \rightarrow \psi' \psi') &= (3.7 \pm 2.8) \times 10^{-7}. \end{aligned} \quad (18)$$

It is interesting to compare these results with previous calculations. In particular, within the leading-order NRQCD, one has [5]:

$$\text{Br}(\eta_b \rightarrow J/\psi J/\psi) = (2.4_{-1.9}^{+4.2}) \times 10^{-8}, \quad (19)$$

which is roughly 20 times smaller than our result shown above. The reason of this suppression can be traced to the expression for the amplitude (9), where all terms are in fact proportional to the constants f_A and f_S , which, in turn, are

expressed through f_V and f_T [see Eq. (5)]. In the absence of relativistic and radiative corrections, the fine-tuning between f_V , f_T and the masses, clearly visible in Eqs. (12), guarantees that f_A , f_S and hence the form factor F are proportional to $\langle v^2 \rangle$, which is small for nonrelativistic systems. Taking relativistic and leading logarithmic radiative corrections to the constants f_A and f_S into account breaks the fine-tuning, thus leading to a considerable enhancement of the branching ratio. To illustrate the above argument numerically, we take an infinitely narrow approximation for the DAs, parameters with fine-tuning given by Eqs. (12), and $\langle v^2 \rangle = 0.25$, to obtain $\text{Br}(\eta_b \rightarrow J/\psi J/\psi) \simeq 2 \times 10^{-8}$, in agreement with the leading-order NRQCD result [5]. Next, we take into account relativistic and leading logarithmic radiative corrections to the constants f_A and f_S , but still use an infinitely narrow approximation for the DAs. In this case fine-tuning is broken, and we get $\sim 3 \times 10^{-7}$, an order-of-magnitude increase compared to the NRQCD value. By including renormalization group evolution and relativistic motion into the DAs, we get a further increase of the branching ratio by a factor ~ 2 .

In [6] the authors took into account one-loop radiative corrections and obtained

$$\text{Br}(\eta_b \rightarrow J/\psi J/\psi) = (2.1-18.6) \times 10^{-8}. \quad (20)$$

Although this number seems to be compatible with ours shown in Eq. (18), we do not believe that the two results are in agreement with each other. In particular, the analytical form of the form factor F obtained in [6] contains logarithmic terms:

$$\begin{aligned} \text{Re } F &\sim \frac{19}{32} \log^2 \frac{M_{\eta_b}^2}{M_{J/\psi}^2} + \dots, \\ \text{Im } F &\sim \pi \frac{19}{16} \log \frac{M_{\eta_b}^2}{M_{J/\psi}^2} + \dots. \end{aligned} \quad (21)$$

In the LC approach used in our calculation, all double logarithms cancel as the final particles are colorless objects [31]. Moreover, there are only two reasons why a general

QCD amplitude may contain large logarithms: renormalization and collinear divergences [31,32]. Clearly, the imaginary part of F is not renormalized at one loop; hence the large logarithm in Eq. (21) must be due to a collinear divergence. However, it is known that collinear divergences can be factored out and do not have an imaginary part [32]. In light of these arguments, the result obtained in [6] looks strange.

The authors of [6] believe that there is no need for renormalization in their calculation of the radiative corrections, since the counterterms are proportional to the leading-order contribution, which vanishes at leading order in both α_s and v_c . We do not think that this statement is correct, since the expansion is done in operators which are not multiplicatively renormalizable. Therefore, the ultraviolet divergences may arise at leading order in v_c due to the v_c -suppressed operators. This effect violates NRQCD velocity scaling rules and is discussed in detail in [20,25].

Yet another estimate for the same branching ratio was obtained in [7], where the final-state interaction effects due to a different decay mechanism were taken into account, yielding

$$\text{Br}(\eta_b \rightarrow J/\psi J/\psi) = (0.5 \times 10^{-8} - 1.2 \times 10^{-5}). \quad (22)$$

In conclusion, we have calculated the branching fractions of the decays $\eta_b \rightarrow J/\psi J/\psi$, $J/\psi \psi'$, and $\psi' \psi'$ in the framework of the light cone formalism. The uncertainties of our calculation have also been assessed. Our results, presented in Eqs. (18), are more than an order of magnitude larger than those obtained within NRQCD.

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