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Gluon excitations and quark chiral symmetry in the meson spectrum: An einbein solution to the large degeneracy problem of light mesons

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A large approximate degeneracy appears in the light meson spectrum measured at the CERN low energy antiproton ring, suggesting a novel principal quantum number $n + j$ in QCD spectra. We recently showed that the large degeneracy could not be understood with state-of-the-art confining and chiral invariant quark models, derived in a truncated Coulomb gauge. To search for a solution to this problem, here we add the gluon or string degrees of freedom. Although independently the quarks or the gluons would lead to a $2n + j$ or $2n + l$ spectrum, adding them together may lead to the desired $n + j$ pattern.

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To understand the large degeneracy of the light meson spectrum [1–5] observed by the Crystal Barrel Collaboration at the CERN low energy antiproton ring (LEAR), we include by the same token both bosonic and fermionic excitations. This is relevant for the observation of excited mesons in the new generation of experiments, say PANDA at the Facility for Antiproton and Ion Research, and in lattice QCD. We first describe the degeneracy, where the meson mass squared M^2 in the excited meson spectrum is led by the principal quantum number $n + j$. Then we review the status of gluon string excitations with an M^2 possibly of the order of $2\pi\sigma(2n + l)$. Recently we have shown [5] that the quark degrees of freedom are not sufficient to understand the excited light meson spectrum, finding that the chiral invariant and confining quark model has an M^2 of the order of $2\pi\sigma(2n + j)$, with a prefactor which is correct for the angular l or j quantum numbers, but too large for the radial quantum number n. Importantly, both gluon and quark excitations have the same scale, with the string tension σ. Here we show that arriving at the desired $n + j$ pattern is possible if we include the gluon or string degrees of freedom in relativistic quark models with chiral symmetry breaking. For simplicity, in this first comprehensive study of a gluonic and chiral invariant high spectrum, we use einbeins to add the bosonic excitations to the fermionic excitations.

In the report of Bugg [1] a large degeneracy emerges from the spectrum of the angularly and radially excited resonances produced in $p\bar{p}$ annihilation by the Crystal Barrel Collaboration at LEAR [6]. This degeneracy is a larger degeneracy than the chiral degeneracy systematically searched by Glozman *et al.* [4,7,8], Jaffe *et al.* [9], Afonin [10], and retrospectively already present in the light-light spectrum of Le Yaouanc *et al.* [11] and the heavy-light spectrum of Bicudo *et al.* [12,13]. Also notice that, a long time ago, Chew and Frautschi remarked on the existence of linear Regge trajectories [14] for angularly excited mesons. A similar linear aligning of excited resonances was also reported for radial excitations [15]. Presently, the status of the excited meson spectrum approximately generalizes the Regge trajectories for the total angular momentum j,

$$j + n \simeq \alpha_0 + \alpha M^2, \tag{1}$$

where α_0 is the intercept of the $M^2 = 0$ axis, α is the slope, and n is a radial excitation. The slopes for angular and for radial quantum excitations [1], respectively, of $0.877\ \mathrm{GeV}^{-2}$ and $0.855\ \mathrm{GeV}^{-2}$ are almost identical high in the spectrum and similar to $(2\pi\sigma)^{-1} = 0.84\ \mathrm{GeV}^{-2}$ where the string tension is $\sigma = 0.19\ \mathrm{GeV}^2$. This is quite remarkable, since in nature there are few examples of principal quantum numbers, except for the Coulomb nonrelativistic potential, say, in the hydrogen atom, or the harmonic oscillator nonrelativistic potential, say, in the vibrational spectrum of molecules.

We now discuss the gluon or string degrees of freedom in order to include them in the chiral invariant and confining quark model. This is still an open problem; nevertheless, at the end of this paper we will see that it may lead to the observed degeneracy. The hybrid excitations of mesons can be addressed with either the bosonic Nambu-Goto-Polyakov string fluctuations or with a constituent gluon linked to the quarks by fundamental rigid strings [16], as depicted in Fig. 1. Baker and Steinke [17] only consider two transverse modes of the string with rotating quarks fixed to the ends of the string, resulting in the mass spectrum

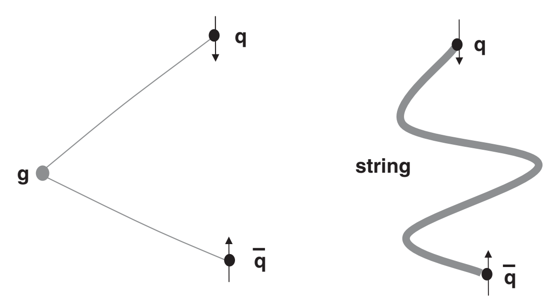

FIG. 1. Gluon (left diagram) and string (right diagram) excitations in a meson.

$$M^2 = 2\pi\sigma\left[n + l - \frac{1}{12} + o\left(\frac{1}{l}\right)\right], \tag{2}$$

led by $n + l$ in units of $2\pi\sigma$, similar to the spectrum of an open string. This result motivated Afonin [3] to propose it as the solution to the large $n + j$ degeneracy; however, this does not yet explain the role of the j quantum number and the chiral partners in the spectrum [18]. Another approach consists in considering a string with static ends, applicable to heavy quarks. In this case the static quark-antiquark potential is extended from the linear potential to

$$V^2_{q\bar{q}}(r) = \sigma^2 r^2 + 2\pi\sigma(2n_g + l_g + \mathcal{E}), \tag{3}$$

where the prefactor 2 of the radial excitation quantum number n is now double the prefactor 1 in Eq. (2) obtained with transverse modes only. $\mathcal{E}$ is a constant. This potential also fits correctly the lattice QCD excitations of the static quark-antiquark potential measured by Juge, Kuti, and Morningstar [19].

We now briefly review how the potential of Eq. (3) has been obtained by Buisseret, Mathieu, and Semay, utilizing a simple [20,21] constituent quark, antiquark, and gluon Hamiltonian,

$$H = \sum_{i=q,\bar{q},g} \sqrt{\mathbf{p}_i^2 + m_i^2} + \sum_{i=q,\bar{q}} \sigma|\mathbf{r}_i - \mathbf{r}_g|, \tag{4}$$

and applying einbeins, or auxiliary fields,

$$\begin{aligned} \sqrt{\mathbf{p}_i^2 + m_i^2} &\simeq \min_{\mu_i}\left[\frac{\mathbf{p}_i^2 + m_i^2}{2\mu_i} + \frac{\mu_i}{2}\right], \\ \sigma|\mathbf{r}_i - \mathbf{r}_g| &\simeq \min_{\nu_j}\left[\frac{\sigma^2(\mathbf{r}_i - \mathbf{r}_g)^2}{2\nu_j} + \frac{\nu_j}{2}\right]. \end{aligned} \tag{5}$$

Einbeins were applied to strings by Morgunov, Nefediev, and Simonov [22] to include the angular momentum of rigid strings in the quark confining potentials. Then Buisseret, Mathieu, and Semay utilized the einbeins to also include the string excitations. The einbeins transform a more difficult problem into a harmonic oscillatorlike Schrödinger equation. Two independent oscillating modes can be separated, one in the usual $\mathbf{r} = \mathbf{r}_q - \mathbf{r}_{\bar{q}}$ relative coordinate and another in the $\mathbf{y} = \frac{\mathbf{r}_q + \mathbf{r}_{\bar{q}}}{2} - \mathbf{r}_g$ orthogonal coordinate. This results in a spectrum, before the einbein minimization, of

$$\begin{aligned} E = {} & \frac{\sigma}{\sqrt{\mu\nu}}(2n_{q\bar{q}} + l_{q\bar{q}} + 3/2) + \sigma\sqrt{\frac{\mu_g + 2\mu}{\mu\nu\mu_g}}(2n_g + l_g + 3/2) \\ & + \frac{m^2}{\mu} + \mu + \frac{m_g^2}{2\mu_g} + \frac{\mu_g}{2} + \nu. \end{aligned} \tag{6}$$

In the heavy quark case, similar to the static limit, $\mu \simeq m \gg \mu_g$, and the minimization of Eq. (6) leads to the string excitations with static quarks as in Eq. (3). An important technical detail to arrive at Eq. (3) is the prescription of Buisseret, Mathieu, and Semay, $8\sigma \rightarrow 2\pi\sigma$, to correct the einbein enhancement of the radial and angular excitations [20,21], of excited mesons with a linear potential. Solving the relativistic Schrödinger equation, the spectrum is led by $M \rightarrow \sqrt{4\pi\sigma n + 8\sigma l}$ [5]. Morgunov, Nefediev, and Simonov [22] showed that adding to the angular momentum of relativistic quarks the angular momentum of a rigid and relativistic rotating string, the angular momentum contribution is corrected to $M \rightarrow \sqrt{2\pi\sigma(2n + l)}$. On the other hand, the einbein minimization leads to $M \rightarrow \sqrt{8\sigma(2n + l)}$. Therefore, using the einbein minimization together with the $8\sigma \rightarrow 2\pi\sigma$ prescription leads to the spectrum $M \rightarrow \sqrt{2\pi\sigma(2n + l)}$ of the solution of the relativistic Schrödinger equation for a quark, an antiquark, and a nonvibrating string. The problem of the prefactor 2 of the radial excitation remains; nevertheless, that is not a result of the harmonic oscillator appearing with the einbeins, but a result of the linear potential. The difference between the spectra produced with the harmonic oscillator and the linear potentials is the exponent of the principal quantum number in the spectrum. In particular, a linear potential contributes to the spectrum as $M^2 \rightarrow 2\pi\sigma(2n + l)$. This differs from the harmonic oscillator spectra $M \rightarrow \omega(2n + l)$. Also notice that Bicudo [5] tested enlarging the ansatze to a class of different power-law confining potentials, but this did not solve the problem of the principal quantum number, and it also may produce nonlinear Regge trajectories. So it would be better to maintain the linear potential and look for another solution to the prefactor 2 problem.

Importantly, Eqs. (2) and (3), and the Regge behavior $M \rightarrow \sqrt{2\pi\sigma(2n + l)}$ of relativistic (but spinless) quarks suggest that a similar quantitatively correct angular Regge slope $\alpha = (2\pi\sigma)^{-1}$ is possible with both quark and gluonic excitations. However, these equations still lack the quark spin degrees of freedom; in particular, they miss the insensitivity to chiral symmetry breaking observed in the large degeneracy of the excited meson spectrum. To solve this j quantum number problem, it is unavoidable to work with confined and relativistic quarks with spontaneous chiral symmetry breaking. Notice that the spontaneous chiral symmetry breaking does not affect the Regge slopes, but it does correct the intercept; in particular, the π meson is massless in the chiral limit, and the ρ meson mass is reproduced. The ρ sits in the leading Regge trajectory and determines its intercept. High in the spectrum the good angular quantum number is not the orbital one l but the total one j. The necessary techniques were developed by Le Yaouanc, Oliver, Ono, Pene, and Raynal [11], and by Bicudo and Ribeiro [12], and extended to hybrids by Llanes-Estrada, General, and Cotanch [23,24]. But in this paper we neglect the spin contribution of the gluon to the Hamiltonian, since the fine structure of the static potential, also measured in lattice QCD [25], suggests that the spin s_g degree of freedom of the gluon only contributes at small distances, and s_g is not a leading term for excited mesons. The Hamiltonian for the quarks is

$$H = \int d^3x \bigg[\psi^\dagger(x)(m_0\beta - i\vec{\alpha}\cdot\vec{\nabla})\psi(x) + \frac{1}{2}g^2 \int d^4z \bar{\psi}(x)\gamma^\mu \frac{\lambda^a}{2} \times \psi(x) V^{ab}_{\mu\nu}(\mathbf{x}-\mathbf{z})\bar{\psi}(z)\gamma^\nu \frac{\lambda^b}{2}\psi(z)\bigg], \qquad (7)$$

where it is standard in the Coulomb gauge to use a density-density color octet potential $V^{ab}_{\mu\nu}(\mathbf{r}) = \delta_{\mu 0}\delta_{\nu 0}\times \delta^{ab}(-3/16)V(r)$, with a funnel potential for the spatial function $V(r) = \frac{\alpha}{r} + v_0 + \sigma r$. Notice that for excited states we can neglect the Coulomb part $\frac{\alpha}{r}$ of the potential [26], since it would have little effect in the excited spectrum because the quark wave functions have a greater radius mean square. Moreover, when chiral symmetry breaking occurs, the constant term v_0 is canceled in the spectrum by an opposite term present in the quark self-energy. Thus for the study of quark degrees of freedom in excited mesons it is sufficient to consider the linear potential σr only.

Here we go beyond these previous studies of mesons with chiral invariant quarks, including in this framework the gluon or string excitations, as in Eq. (4), considering a vanishing current quark mass $m_q = 0$. We also apply the einbein technique to simplify the resulting three-body problem as in Eq. (5). This allows us to separate the two coordinates $\mathbf{r}$ and $\mathbf{y}$. In what concerns the relative quark-antiquark $\mathbf{r}$ coordinate, we address chiral symmetry breaking with the framework of the Bethe-Salpeter equation already developed for mesons. We now review the technical steps of this framework, before including the gluon or string degree of freedom, corresponding to the coordinate $\mathbf{y}$. The bound-state equations are derived by translating the relativistic invariant Dirac-Feynman propagators [11], in the quark and antiquark Bethe-Goldstone propagators [5,27], used in the formalism of nonrelativistic quark models,

$$\mathcal{S}_{\rm Dirac}(k_0,\vec{k}) = \frac{i}{k_0 - \sqrt{k^2 + m_c^2(k)} + i\epsilon}\sum_s u_s u_s^\dagger \beta - \frac{i}{-k_0 - \sqrt{k^2 + m_c^2(k)} + i\epsilon}\sum_s v_s v_s^\dagger \beta,$$

$$u_s(\mathbf{k}) = \bigg[\sqrt{\frac{1+S}{2}} + \sqrt{\frac{1-S}{2}}\hat{k}\cdot\vec{\sigma}\gamma_5\bigg]u_s(0),$$

$$v_s(\mathbf{k}) = \bigg[\sqrt{\frac{1+S}{2}} - \sqrt{\frac{1-S}{2}}\hat{k}\cdot\vec{\sigma}\gamma_5\bigg]v_s(0), \qquad (8)$$

where it is convenient to define different functions of the constituent quark mass $m_c(k)$, $\varphi(k) = \arctan\frac{m_c(k)}{k}$, $S(k) = \sin\varphi(k)$, $C = \cos\varphi(k)$, and $\mathcal{G}(k) = 1 - S(k)$. In the non-condensed vacuum, $m_c(k)$ equals the bare quark mass m_0. In the physical vacuum, the constituent quark mass $m_c(k)$ is a variational function which is determined by the mass gap equation. But here we do not detail it since the constituent mass is only crucial for the ground-state mesons, and not for the higher states in the spectrum where it can be neglected when compared to the momentum of the quark. The Salpeter-RPA equations for a meson (a color singlet quark-antiquark bound state) can be derived from the Lippman-Schwinger equations for a quark and an antiquark, or by replacing the propagator of Eq. (8) in the Bethe-Salpeter equation. For a detailed derivation of the bound-state equations, see Ref. [27]. One gets

$$R^+(k,P) = \frac{u^\dagger(k_1)\chi(k,P)v(k_2)}{+M(P) - E(k_1) - E(k_2)},$$

$$R^{-t}(k,P) = \frac{v^\dagger(k_1)\chi(k,P)u(k_2)}{-M(P) - E(k_1) - E(k_2)},$$

$$\chi(k,P) = \int \frac{d^3k'}{(2\pi)^3}V(k-k')[u(k_1')R^+(k',P)v^\dagger(k_2') + v(k_1')R^{-t}(k',P)u^\dagger(k_2')] \qquad (9)$$

where $V(k)$ is the Fourier transform of the potential in Eq. (7), and where $k_1 = k + \frac{P}{2}$, $k_2 = k - \frac{P}{2}$, and P is the total momentum of the meson. Equation (9) includes both the Salpeter-RPA equations of Bicudo *et al.* [12] and of Llanes-Estrada *et al.* [28] and the Salpeter equations of Le Yaouanc *et al.* [11]. Substituting χ we get the equation for the positive energy R^+ and negative energy R^- radial wave functions. This results in four potentials $V^{\alpha\beta}$ with $\alpha = \pm$, respectively, coupling $\rho^\alpha = kR^\alpha$ to ρ^β, in the bound-state Salpeter equation,

$$(2T + V^{++})\rho^+ + V^{+-}\rho^- = M\rho^+,$$
$$V^{-+}\rho^+ + (2T + V^{--})\rho^- = -M\rho^-. \qquad (10)$$

We now apply some educated approximations to arrive at an analytical expression for the excited meson spectrum. As in Eq. (5), we use einbeins, and this avoids the problem of computing the matrix elements of the linear potential. Actually, we only need to know the matrix elements of the harmonic oscillator potential $V^{\alpha\beta}_{\rm HO}$ as shown [5] in Table I, and to minimize each energy in the spectrum with regards to μ, ν, and μ_g. Importantly, the potentials $V^{++} = V^{--}$ and $V^{+-} = V^{-+}$ include the usual spin-tensor potentials [5,27], produced by the Pauli $\vec{\sigma}$ matrices in the spinors of Eq. (8). They are detailed explicitly in Table I. Moreover, we are interested in highly excited states, where both $\langle r\rangle$ and $\langle k\rangle$ are large due to the virial theorem, and thus we consider the limit where $\frac{m_c}{k}\to 0$ as in Bicudo, Cardoso, Van Cauteren, and Llanes-Estrada [29]. This implies that in the potentials listed in Table I, we may simplify $\varphi'(k)\to 0$, $C(k)\to 1$, and $\mathcal{G}(k) = 1 - S(k)\to 1$. We also notice that the relativistic equal time equations have twice as many

TABLE I. The positive and negative energy spin-independent, spin-spin, spin-orbit, and tensor potentials, computed exactly in the framework of the simple density-density harmonic oscillator. For a detailed derivation, see Refs. [5,27].

	$V_{\rm HO}^{++} = V_{\rm HO}^{--}$
Spin-independent spin-spin	$-\frac{d^2}{dk^2} + \frac{\mathbf{L}^2}{k^2} + \frac{1}{4}(\varphi_q'^2 + \varphi_{\bar q}'^2) + \frac{1}{k^2}(\mathcal{G}_q + \mathcal{G}_{\bar q}) - U\frac{4}{3k^2}\mathcal{G}_q\mathcal{G}_{\bar q}\mathbf{S}_q\cdot\mathbf{S}_{\bar q}$
Spin-orbit tensor	$\frac{1}{k^2}[(\mathcal{G}_q + \mathcal{G}_{\bar q})(\mathbf{S}_q + \mathbf{S}_{\bar q}) + (\mathcal{G}_q - \mathcal{G}_{\bar q})(\mathbf{S}_q - \mathbf{S}_{\bar q})]\cdot\mathbf{L} - \frac{2}{k^2}\mathcal{G}_q\mathcal{G}_{\bar q}[(\mathbf{S}_q\cdot\hat k)(\mathbf{S}_{\bar q}\cdot\hat k) - \frac{1}{3}\mathbf{S}_q\cdot\mathbf{S}_{\bar q}]$
	$V_{\rm HO}^{+-} = V_{\rm HO}^{-+}$
Spin-independent spin-spin	$0 - \frac{4}{3}[\frac{1}{2}\varphi_q'\varphi_{\bar q}' + \frac{1}{k^2}C_qC_{\bar q}]\mathbf{S}_q\cdot\mathbf{S}_{\bar q}$
Spin-orbit tensor	$[-2\varphi_q'\varphi_{\bar q}' + \frac{2}{k^2}C_qC_{\bar q}][(\mathbf{S}_q\cdot\hat k)(\mathbf{S}_{\bar q}\cdot\hat k) - \frac{1}{3}\mathbf{S}_q\cdot\mathbf{S}_{\bar q}]$

coupled equations as does the Schrödinger equation. But the negative energy component ρ^- is smaller than the positive energy component ρ^+ by a factor of the order of $\sqrt{\sigma}/M$. Thus when the meson mass M is large, and this is the case for the excited mesons, the negative energy components can be neglected and the Salpeter equation simplifies to a Schrödinger equation, where the relevant potential V^{++} becomes

$$\begin{aligned} V_{\rm HO}^{++} &= -\frac{d^2}{dk^2} + \frac{\mathbf{L}^2}{k^2} + \frac{2}{k^2} + \frac{2}{k^2}(\mathbf{S}_q + \mathbf{S}_{\bar q})\cdot\mathbf{L} \\ &\quad + \frac{4}{3k^2}\mathbf{S}_q\cdot\mathbf{S}_{\bar q} - \frac{2}{k^2}\left(\mathbf{S}_q\cdot\hat k\mathbf{S}_{\bar q}\cdot\hat k - \frac{1}{3}\mathbf{S}_q\cdot\mathbf{S}_{\bar q}\right) \\ &= -\frac{d^2}{dk^2} + \frac{\mathbf{J}^2}{k^2} + \frac{\mathcal{E}'}{k^2} \end{aligned} \tag{11}$$

and the dependence on $\mathbf{L}^2$ is traded by a dependence on $\mathbf{J}^2 = \mathbf{L}^2 + 2\mathbf{L}\cdot\mathbf{S} + \mathbf{S}^2$. The other remaining potentials are the spin-spin and tensor potentials, both much smaller than k^2 or than J^2, and they simply contribute to a Coulomb-like potential $\frac{\mathcal{E}'}{k^2}$.

Including the gluonic coordinate $\mathbf{y}$ in Eq. (7) and Fourier transforming it as well, removing the Coulomb terms unaffecting the excited spectrum in Eq. (11), we finally get a simple harmonic oscillator Hamiltonian for the quarks and gluon coordinates. Substituting the harmonic oscillator matrix elements, we arrive at the meson spectrum,

$$\begin{aligned} M &= \frac{\sigma}{\sqrt{\mu\nu}}(2n_{q\bar q} + j_{q\bar q} + 3/2) + \sigma\sqrt{\frac{\mu_g + 2\mu}{\mu\nu\mu_g}} \\ &\quad \times (2n_g + l_g + 3/2) + \mu + \frac{m_g^2}{2\mu_g} + \frac{\mu_g}{2} + \nu, \end{aligned} \tag{12}$$

before minimization, similar to Eq. (6) except that the quark orbital angular momentum $l_{q\bar q}$ is replaced by the total quark angular momentum $j_{q\bar q}$. Equation (12) is hard to minimize analytically, except in two different limits, the limit of a large effective gluon mass and the limit of a small number of gluon excitations. Then we get

$$E \simeq \sqrt{2\pi\sigma(\mathcal{N}_{q\bar q} + \mathcal{N}_g)} + m_g, \tag{13}$$

where we denote $\mathcal{N}_{q\bar q} = 2n_{q\bar q} + j_{q\bar q} + 3/2$ and $\mathcal{N}_g = 2n_g + l_g + 3/2$. In the case of a vanishing effective gluon mass or of a significant $\mathcal{N}_g$, we have to resort to a numerical minimization of Eq. (12) (again with the $8\sigma \to 2\pi\sigma$ prescription), and we get the fit

$$\begin{aligned} M \simeq \sqrt{2\pi\sigma(\mathcal{N}_{q\bar q} + \mathcal{N}_g)}\bigg[& 1.29 + 0.18\frac{\mathcal{N}_{q\bar q} - \mathcal{N}_g}{\mathcal{N}_{q\bar q} + \mathcal{N}_g} \\ & + 0.08\left(\frac{\mathcal{N}_{q\bar q} - \mathcal{N}_g}{\mathcal{N}_{q\bar q} + \mathcal{N}_g}\right)^2\bigg], \end{aligned} \tag{14}$$

in terms of a dominant principal number $\mathcal{N}_{q\bar q} + \mathcal{N}_g$, and a small contribution from a second number $\mathcal{N}_{q\bar q} - \mathcal{N}_g$. Notice that the dominant principal quantum number is, in general, much larger than the second number, due to the zero point energy of the harmonic oscillator. The simplest of all these cases is the one of a massless gluon and of few gluonic excitations, producing

$$M \simeq \sqrt{2\pi\sigma(\mathcal{N}_{q\bar q} + \mathcal{N}_g)}, \tag{15}$$

also leading to linear Regge trajectories.

Thus in any of the cases considered we find, as a good approximation, that a principal quantum number $\mathcal{N}_{q\bar q} + \mathcal{N}_g$ dominates the excited light meson spectrum. We now show that the principal quantum number $\mathcal{N}_{q\bar q} + \mathcal{N}_g = 2n_{q\bar q} + j_{q\bar q} + 2n_g + l_g + 6$ can be simplified to a form $n + j$. This is quite interesting since it allows us to understand the experimental spectrum with both the quark quantum numbers and the gluon quantum numbers. Obviously, the trajectories in $\mathbf{J}_{\rm total} = \mathbf{J}_{q\bar q} + \mathbf{L}_g$ have the correct slope of $(2\pi\sigma)^{-1}$. Thus we only need to show that the radial-like trajectories, i.e. the ones that maintain $j_{\rm total}$ and the parity P, also have the same slope, even in the less favorable case where the gluonic radial excitation prefactor is 2 and not 1 as in Eq. (2). As in the radial trajectories of Bugg [1], let us consider a radial trajectory starting with mesons which already have angular quark excitations $j_{q\bar q} > 0$. Because of the chiral symmetry insensitivity, we start with a degenerate parity doublet, since the quark-antiquark pair may have both $P_{q\bar q} = +$ and $P_{q\bar q} = -$ with the same energy. The ground state, in the leading trajectory, has no hybrid excitation; i.e. it has $n_g = 0$ and $l_g = 0$. Then, increasing l_g by one unit and summing it to the quarks' angular

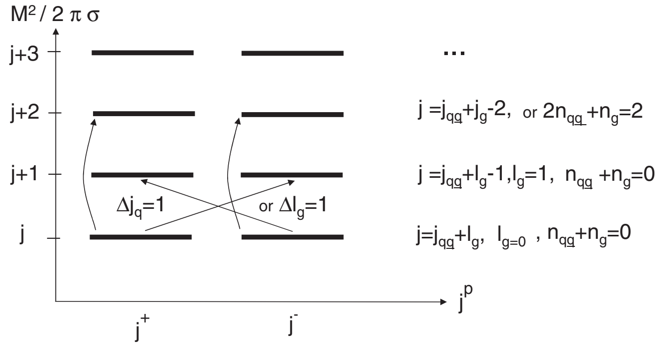

FIG. 2. Mass squared M^2, as a function of j^P, of radial-like excitations in $2\pi\sigma$ units starting from the ground states in the total angular momentum j shell.

momentum $j_{q\bar{q}}$ with the triangular relation $|j_{q\bar{q}} - l_g| < j_{\text{total}} < j_{q\bar{q}} + l_g$, it is clearly possible to maintain the initial $j_{\text{total}} = j_{q\bar{q}}$. Although the gluon angular momentum does invert the parity of the initial state, as depicted in Fig. 2, since we start with a parity doublet, this gluonic excitation,

$$\Delta j = 0, \qquad \Delta M^2 = 2\pi\sigma, \tag{16}$$

leads to another parity doublet. This doublet is a radial-like excitation of the one we started from. But, importantly, the M^2 splitting for this radial-like excitation is only $2\pi\sigma$. The next excitation, with $\Delta M^2 = 2 \times 2\pi\sigma$, is easier to get since it can be intrinsically radial, with $\Delta(2n_{q\bar{q}} + cn_g) = 2$. And so, with the interplay of $\mathbf{J}_{q\bar{q}}$ and $\mathbf{L}_g$ we are able to produce new radial-like excitations, in the middle of the purely radial excitations. Thus the Regge slope for the radial-like excitations $\alpha = (2\pi\sigma)^{-1}$ is the same one as the angular excitations.

To conclude, we address a novel large degeneracy observed in quantum physics, with a principal quantum number $n + j$ in the excited light meson spectrum. With the interplay of quark excitations insensitive to chiral symmetry and of gluon string excitations, we show that the degeneracy may be explained. We simplify the problem of a quark with strings to a three-body problem with two chiral invariant quarks and a gluon. An exact numerical solution of this three-body problem would require an effort similar to the one necessary to study other three-body systems of chiral invariant quarks and gluons [29,30], e.g. nine-dimensional spatial integrals, the sum of six spin indices, and the use of a variational basis with hundreds of multidimensional orthogonal functions, implying man-years of coding and debugging together with the use of a supercomputer for several months. Thus for this first study, and to arrive at simple and clear analytical mass formulas, we use the einbein technique. As a by-product of our research, we find that the einbein technique is applicable to the quark models with chiral symmetry breaking. For instance, Eq. (15), with the quark degrees of freedom only, should provide a good approximation to the exact numerical solution of Llanes-Estrada and Cotanch [28] and of Glozman and Wagenbrunn [7].

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Note added.—Very recently Bugg reanalyzed the Crystal Barrel data at LEAR [31], and his spectrum shows clearly the large degeneracy, where the angular excitations have the same scale as the radial-like excitations. However, he found no evidence for parity doublets in the leading Regge trajectory, but only in daughter trajectories. More experimental, computational, and theoretical works on the light meson spectrum are urgent.

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