PHYSICAL REVIEW D **81,** 013001 (2010)

TeV scale inverse seesaw model in $SO(10)$ and leptonic nonunitarity effects

P. S. Bhupal Dev and R. N. Mohapatra

Maryland Center for Fundamental Physics and Department of Physics, University of Maryland, College Park, Maryland 20742, USA

(Received 28 October 2009; published 12 January 2010)

We show that a TeV scale inverse seesaw model for neutrino masses can be realized within the framework of a supersymmetric $SO(10)$ model consistent with gauge coupling unification and observed neutrino masses and mixing. We present our expectations for nonunitarity effects in the leptonic mixing matrix, some of which are observable at future neutrino factories as well as the next generation searches for lepton flavor violating processes such as $\mu \to e + \gamma$. The model has TeV scale W_R and Z' bosons which are accessible at the Large Hadron Collider.

DOI: 10.1103/PhysRevD.81.013001 PACS numbers: 14.60.St, 12.10.Dm

I. INTRODUCTION

A precise understanding of the origin of observed neutrino masses and mixing is one of the major goals of particle physics right now. A simple paradigm for understanding the smallness of the masses is the seesaw mechanism [1], where one introduces three standard model (SM) singlet right-handed (RH) neutrinos with Majorana masses M_N, which mix with left-handed (LH) ones via the Yukawa coupling $\bar{L}HN$. The resulting formula for light neutrino masses is given by $\mathcal{M}_\nu = -M_D M_N^{-1} M_D^T$, where M_D is the Dirac neutrino mass. Since the SM does not restrict the Majorana mass M_N, we could choose this to be much larger than the weak scale, thereby providing a natural way to understand the tiny neutrino masses. This is called the type I seesaw mechanism. There are several variations of this mechanism where one replaces the RH neutrino by either a SM triplet Higgs field (type II seesaw mechanism) [2] or a SM triplet of fermions (called type III seesaw mechanism) [3]. A great deal of attention has been devoted to testing these ideas. As far as the type I seesaw mechanism is concerned, the prospects of testing this depends on the scale M_N as well as any associated physics that comes with it at that scale. It can be accessible to current and future collider experiments if the scale is not far above a TeV. A different way to test the type I seesaw mechanism follows from the observation that this mechanism involves the mixing of the LH neutrinos with SM singlet heavy neutrinos, as a result of which there would, in general, be violation of unitarity of the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix that describes only the mixing of the three light neutrinos. One could contemplate searching for such effects in oscillation experiments [4]. However, in the type I seesaw case, the resulting mixing effects are of order $\frac{m_\nu}{M_N}$, and since neutrino masses are in the sub-eV range, such nonunitarity effects are too small to be observable for generic high scale seesaw models. This would also be true with TeV mass RH neutrinos unless there are cancellations to get small neutrino masses from large Dirac masses using symmetries (see, for example, cases in [5]). We note that the nonunitarity effects also exist in the type III seesaw case but not in the type II case even though they have other interesting effects such as lepton flavor violation (LFV) processes [6].

Since the testability of the seesaw mechanism is intimately related to the magnitude of the seesaw scale, a key question of interest is whether there could be any theoretical guidelines for the seesaw scale. In such a case, the searches for seesaw effects in experiments could then be used to test the nature of physics beyond the standard model. It is well known that [7] the simplest grand unified theory (GUT) realizations of the seesaw mechanism are based on the $SO(10)$ group which automatically predicts the existence of the RH neutrinos (along with the SM fermions) required by the seesaw mechanism. An advantage of GUT embedding of the seesaw mechanism is that the constraints of GUT symmetry tends to relate the Dirac neutrino mass M_D to the charged fermion masses, thereby making a prediction for the seesaw scale M_N from observations. For type I seesaw GUT embedding, typical values for the M_N are very high (in the range of 10^{10}–10^{14} GeV). This makes both the collider and nonunitarity probes of the seesaw mechanism impossible. The key feature that leads to such restrictions in the type I seesaw case is the close link between the $B - L$-breaking RH neutrino mass and the smallness of the LH neutrino masses.

A completely different realization of the seesaw mechanism is the so-called inverse seesaw mechanism [8], where instead of one set of three SM singlet fermions, one introduces two sets of them, N_i, S_i ($i = 1, 2, 3$). In the context of $SO(10)$ models, since one of the two sets can be identified with the SM singlet neutrino in the $SO(10)$ **16** representation containing matter, the other would have to be a separate set of three $SO(10)$ singlet fermions. Because of the existence of the second set of singlet fermions [and perhaps additional gauge symmetries, e.g., $SO(10)$], the neutrino mass formula in these models has the form

$$m_\nu \simeq M_D M_N^{-1} \mu (M_N^T)^{-1} M_D^T \equiv F \mu F^T \tag{1}$$

where μ breaks the lepton number. Because of the presence of this new mass scale in this theory, the seesaw scale M_N can be very close to a TeV even for "large" Dirac

masses. This makes the tests of this possibility in colliders much more feasible. In fact, it has recently been argued that [9] the inverse seesaw scenario can also lead to non-negligible nonunitarity effects which can be accessible at the future long-baseline neutrino oscillation experiments. There are also significant LFV effects in these models, as noted many years ago in Ref. [10]. These possibilities have generated a great deal of interest in the inverse seesaw models in recent days [11]. Our effort in this paper focuses on possible grand unification of inverse seesaw models. Similar unification studies have been performed in Ref. [12], but the nonunitarity issues have not been addressed.

An interesting question is whether such models are necessarily compatible with grand unification when the seesaw scale is in the TeV range, and if so, what kind of nonunitarity effects they predict. We find that it is indeed possible to embed the TeV scale inverse seesaw models within a simple $SO(10)$ framework consistent with gauge coupling unification and realistic fermion masses. The $SO(10)$ symmetry helps to reduce the number of parameters in the inverse seesaw matrix, once we require degeneracy of the TeV scale RH neutrinos to have successful resonant leptogenesis. Within this set of assumptions, we present our expectations for the nonunitarity effects as well as consequences for lepton flavor violation, which are in the testable range in future experiments.

This paper is organized as follows: In Sec. II, we describe the general framework of the inverse seesaw model and its embedding into a generic supersymmetric $SO(10)$ theory. In Sec. III, we analyze the nonunitarity predictions of the inverse seesaw model. In Sec. IV, we investigate a specific $SO(10)$-breaking chain and obtain the gauge coupling unification with TeV scale left-right symmetry with a unification scale consistent with proton decay bounds. In Sec. V, we analyze the renormalization group (RG) evolution of the Yukawa couplings and obtain the running masses for quarks and leptons at the unification scale to check that our model leads to realistic fermion masses. In Sec. VI, we determine the Dirac neutrino mass matrix using the results of Sec. V. In Sec. VII, we study the implication of our model on nonunitarity effects and its phenomenological consequences. A brief summary of the results is presented in Sec. VIII. In Appendix A, we have given the expressions for the masses of the $SO(10)$ Higgs multiplets in our model, and in Appendix B, we have derived the RG equations for the quark and lepton masses and the Cabibbo-Kobayashi-Maskawa (CKM) mixing elements in the context of a supersymmetric left-right model.

II. THE INVERSE SEESAW MODEL

The inverse seesaw scheme was originally suggested [8] for theories which lack the representation required to implement the canonical seesaw mechanism, such as the superstring models. As noted in the Introduction, the implementation of the inverse seesaw mechanism requires the addition of three extra SM gauge singlets S_i coupled to the RH neutrinos N_i through the lepton number conserving couplings of the type $\bar{N}S$, while the traditional RH neutrino Majorana mass term is forbidden by extra symmetries. The lepton number is broken only by the self-coupling term μS^2. The mass part of the neutrino sector Lagrangian in the flavor basis is given by

$$\mathcal{L}_{\text{mass}} = (\bar{\nu} M_D N + \bar{N} M_N S + \text{H.c.}) + S\mu S, \qquad (2)$$

where μ is a complex symmetric 3×3 matrix (with dimension of mass), and M_D and M_N are generic 3×3 complex matrices representing the Dirac mass terms in the ν-N and N-S sectors, respectively. In the basis $\{\nu, N, S\}$, the 9×9 neutrino mass matrix becomes

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & M_D & 0 \\ M_D^T & 0 & M_N \\ 0 & M_N^T & \mu \end{pmatrix}. \qquad (3)$$

The LH neutrinos can be made very light (sub-eV scale), as required by the oscillation data, even for a low M_N, much smaller than the unification scale ($M_N \ll M_G$), provided μ is sufficiently small, $\mu \ll M_N$, as the lepton-number-breaking scale μ is decoupled from the RH neutrino mass scale. Assuming $\mu \ll M_\nu^D \ll M_N$ (with $M_N \sim$ TeV), the structure of the light neutrino Majorana mass term at the leading order in $M_D M_N^{-1}$ is given by Eq. (1), where $F = M_D M_N^{-1}$ is a complex 3×3 matrix. We note that in the limit $\mu \to 0$, which corresponds to the unbroken lepton number, we have massless LH neutrinos as in the SM. In reality, a small nonvanishing μ can be viewed as a slight breaking of a global $U(1)$ symmetry; hence, the smallness of μ is natural, in the 't Hooft sense [13], even though there is no dynamical understanding of this smallness.

The generic form of the inverse seesaw matrix in Eq. (3) has more parameters than the usual type I seesaw mechanism. However, if we embed this theory into a grand unified theory such as $SO(10)$, that will help in reducing the parameters as we show below. In order to embed the inverse seesaw mechanism into a generic $SO(10)$ theory, we have to break the $B-L$ symmetry by using a $\mathbf{16}\oplus\overline{\mathbf{16}}$ pair rather than the $\mathbf{126}\oplus\overline{\mathbf{126}}$ pair of the Higgs representation. All the SM fermions are accommodated in a single $\mathbf{16}_F$ representation of $SO(10)$, and we use three copies of $\mathbf{16}_F^i$ for three generations. For each of them, we add a gauge singlet fermion $\mathbf{1}_F^i$ to play the role of S_i. We assume more than one copy of $\mathbf{10}_H$ Higgs multiplets in order to have a realistic fermionic spectrum.

The $SO(10)$ invariant renormalizable Yukawa superpotential is given by

$$W_Y = h_{ij}^a \mathbf{16}_F^i \mathbf{16}_F^j \mathbf{10}_H^a + f_{ijk} \mathbf{16}_F^i \mathbf{1}_F^j \overline{\mathbf{16}}_H^k + \mu_{ij} \mathbf{1}_F^i \mathbf{1}_F^j. \qquad (4)$$

After the $B-L$ symmetry breaking, we get the neutrino mass matrix in Eq. (3) with $M_D = h v_u$ and $M_N = f \bar{v}_R$,

where v_u is the vacuum expectation value (VEV) of one (or a linear combination) of the $\mathbf{10}_H$'s and $\bar{v}_R$ the VEV of the $\overline{\mathbf{16}}_H$. In a typical TeV scale scenario with $v_u \sim 100$ GeV (electroweak scale) and $\bar{v}_R \sim$ TeV, assuming $\mu \ll v_u < \bar{v}_R$, we find the lightest neutrino mass from Eq. (1) in a one generation theory to be

$$m_\nu \simeq \mu\left(\frac{h v_u}{f \bar{v}_R}\right)^2 \tag{5}$$

and the two other heavy eigenstates with mass of order $f\bar{v}_R$. Thus, we can get sub-eV light neutrino mass for $\mu \sim$ keV. Since this is a supersymmetric theory, such small values do not receive radiative corrections and keep the model natural. In the following section, we consider three generations which then result in the nonunitarity effect.

It is important to note that in our model, we do not need to impose a discrete R parity to our matter fermions, unlike the usual $\mathbf{16}_H$ $SO(10)$ models discussed in the literature, in order to prevent fast proton decay via dimension-4 operators of the type $\frac{1}{M}\mathbf{16}_F\mathbf{16}_F\mathbf{16}_F\overline{\mathbf{16}}_H$ because these operators are already suppressed by a factor $\frac{\langle\overline{\mathbf{16}}_H\rangle}{M_{\rm Pl}} \sim 10^{-15}$ for a low-scale $B-L$ breaking with $\langle\overline{\mathbf{16}}_H\rangle \sim$ TeV.

III. NONUNITARITY EFFECTS

The 3×3 light neutrino mass matrix in Eq. (1) can be diagonalized by a unitary transformation:

$$U^\dagger m_\nu U^* = \hat{m}_\nu = \mathrm{diag}(m_1, m_2, m_3), \tag{6}$$

where U is the standard PMNS matrix. Since the above diagonalization of m_ν does not diagonalize the matrices M_N and μ, there will be off-diagonal mixing between the different light neutrinos even after diagonalization of m_ν due to their mixing with the heavy neutrinos. In other words, in the basis where the charged-lepton mass matrix is diagonal, U is only a part of the full mixing matrix responsible for neutrino oscillations. We have to examine the full 9×9 unitary matrix V which diagonalizes the mass matrix $\mathcal{M}_\nu$ given by Eq. (3):

$$V^\dagger \mathcal{M}_\nu V^* = \hat{\mathcal{M}}_\nu = \mathrm{diag}(m_i, m_{N_j}, m_{\tilde{N}_k}) \quad (i, j, k = 1, 2, 3). \tag{7}$$

We can decompose V into the blocks

$$V = \begin{pmatrix} V_{3\times 3} & V_{3\times 6} \\ V_{6\times 3} & V_{6\times 6} \end{pmatrix}. \tag{8}$$

Then the upper-left sub-block $V_{3\times 3}$ will represent the full (nonunitary) PMNS mixing matrix. For a TeV scale M_N and a reasonably small μ, it is sufficient to consider only up to the leading order in F. Then the new PMNS matrix becomes [14]

$$\mathcal{N} \equiv V_{3\times 3} \simeq (1 - \tfrac{1}{2}FF^\dagger)U. \tag{9}$$

In the commonly used parametrization [15], $\mathcal{N} = (1-\eta)U$, and hence, all the nonunitarity effects are determined by the Hermitian matrix $\eta \simeq \frac{1}{2}FF^\dagger$ which depends only on the mass ratio $F = M_D M_N^{-1}$ and not on the parametrization of the PMNS matrix.

The LH neutrinos entering the charged-current interactions of the SM now become superpositions of the nine mass eigenstates $(\hat{\nu}_i, N_i, \tilde{N}_i)$, and at the leading order in F,

$$\nu \simeq \mathcal{N}\hat{\nu} + \mathcal{K}P, \tag{10}$$

where $\mathcal{K} \equiv V_{3\times 6} \simeq (0, F)V_{6\times 6}$ and $P = (N_1, N_2, N_3, \tilde{N}_1, \tilde{N}_2, \tilde{N}_3)$. Then the charged-current Lagrangian in the mass basis is given by

$$\begin{aligned}\mathcal{L}_{\rm CC} &= -\frac{g}{\sqrt{2}}\bar{l}_L\gamma^\mu\nu W_\mu^- + \text{H.c.}\\ &\simeq -\frac{g}{\sqrt{2}}\bar{l}_L\gamma^\mu(\mathcal{N}\hat{\nu} + \mathcal{K}P)W_\mu^- + \text{H.c.}\end{aligned} \tag{11}$$

This mixing between the doublet and singlet components in the charged-current sector has several important phenomenological consequences, as listed below:

(1) The flavor and mass eigenstates of the LH neutrinos are now connected by a nonunitary mixing matrix $\mathcal{N} = (1-\eta)U$, where the nonunitarity effects entering different neutrino oscillation channels are measured by the parameter η. In particular, the CP-violating effects in the leptonic sector will now be governed by the PMNS matrix $\mathcal{N}$ instead of U through the Jarlskog invariant [16]

$$J_{\alpha\beta}^{ij} = \mathrm{Im}(\mathcal{N}_{\alpha i}\mathcal{N}_{\beta j}\mathcal{N}_{\alpha j}^*\mathcal{N}_{\beta i}^*), \tag{12}$$

where the indices $\alpha \neq \beta$ run over e, μ, and τ, while $i \neq j$ can be 1, 2, and 3. In the standard PMNS parametrization of U by the three mixing angles θ_{ij} and the Dirac CP phase δ, one can expand Eq. (12) up to second order in $\eta_{\alpha\beta}$ and $s_{13} \equiv \sin\theta_{13}$ (assuming those to be small) to obtain

$$J_{\alpha\beta}^{ij} \simeq J + \Delta J_{\alpha\beta}^{ij}, \tag{13}$$

where the first term governs the CP-violating effects in the unitary limit and the second term gives the contribution coming from the nonunitarity effect:

$$J = c_{12}c_{13}^2c_{23}s_{12}s_{13}s_{23}\sin\delta, \tag{14}$$

$$\begin{aligned}\Delta J_{\alpha\beta}^{ij} \simeq -\sum_{\gamma=e,\mu,\tau}\mathrm{Im}(&\eta_{\alpha\gamma}U_{\gamma i}U_{\beta j}U_{\alpha j}^*U_{\beta i}^*\\ &+ \eta_{\beta\gamma}U_{\alpha i}U_{\gamma j}U_{\alpha j}^*U_{\beta i}^*\\ &+ \eta_{\alpha\gamma}^*U_{\alpha i}U_{\beta j}U_{\gamma j}^*U_{\beta i}^*\\ &+ \eta_{\beta\gamma}^*U_{\alpha i}U_{\beta j}U_{\alpha j}^*U_{\gamma i}^*).\end{aligned} \tag{15}$$

Note that the unitary term J vanishes if either $s_{13} \to 0$ or $\delta \to 0$. However, $\Delta J_{\alpha\beta}^{ij}$ depends on the off-diagonal elements of η (generally complex) and

does not necessarily vanish even if both s_{13} and δ are zero; in fact, it might even dominate the CP-violating effects in the leptonic sector.

(2) The heavy neutrinos N_i and $\tilde{N}_i$ entering the charged-current sector can also mediate the rare lepton decays, $l_\alpha^- \to l_\beta^- \gamma$. Hence, unlike in the canonical seesaw model where this contribution is suppressed by the light neutrino masses [17], in this case it is constrained mainly by the ratio $F = M_D M_N^{-1}$. The LFV decays mediated by these heavy neutrinos have branching ratios [10]

$$\mathrm{BR}(l_\alpha \to l_\beta \gamma) \simeq \frac{\alpha_W^3 s_W^2 m_{l_\alpha}^5}{256\pi^2 M_W^4 \Gamma_\alpha} \times \left| \sum_{i=1}^{6} \mathcal{K}_{\alpha i} \mathcal{K}_{\beta i}^* I\left(\frac{m_{N_i}^2}{M_W^2}\right) \right|^2, \quad (16)$$

where Γ_α is the total decay width of l_α and the function $I(x)$ is defined by

$$I(x) = -\frac{2x^3 + 5x^2 - x}{4(1-x)^3} - \frac{3x^3 \ln x}{2(1-x)^4}. \quad (17)$$

For degenerate RH neutrino masses, a reasonable assumption inspired by resonant leptogenesis [18], the amplitude is proportional to $(\mathcal{K}\mathcal{K}^\dagger)_{\alpha\beta} \sim (FF^\dagger)_{\alpha\beta}$, and hence, for sizable F and a TeV scale RH sector, one could expect appreciable rates in the LFV channels. On the other hand, in the conventional type I seesaw model, one has approximately $\mathcal{K}\mathcal{K}^\dagger = \mathcal{O}(m_\nu M_R^{-1})$, and therefore, the branching ratio $\mathrm{BR}(l_\alpha \to l_\beta \gamma) \propto \mathcal{O}(m_\nu^2)$ is strongly suppressed.

(3) The heavy neutrinos N_i and $\tilde{N}_i$ also couple to the gauge sector of the SM and can be produced on shell, if kinematically accessible, at hadron colliders via the gauge boson exchange diagrams. Because of their pseudo-Dirac nature, the striking lepton number violating LHC signature of the fine-tuned type I and type III scenarios, namely $pp \to l_\alpha^\pm l_\beta^\pm$ + jets, will be suppressed for heavy Majorana states due to cancellation between the graphs with internal lines of the N and $\tilde{N}$ types which have opposite CP-quantum numbers. However, the LFV processes are insensitive to this effect and one can expect to get observable signals at the LHC. The most distinctive signature would be the observation of LFV processes involving three charged leptons in the final state plus missing energy, i.e. $pp \to l_\alpha^\pm l_\beta^\pm l_\gamma^\mp \nu(\bar{\nu})$ + jets [19].

Thus we see that the phenomenology of the inverse seesaw mechanism depends crucially on the mass ratio $F = M_D M_N^{-1}$. As noted earlier, we can choose the RH neutrino masses to be degenerate (with eigenvalue m_N), inspired by resonant leptogenesis. So we are left with a single mass parameter m_N, together with the Dirac mass matrix M_D and the arbitrary small mass parameter μ. In what follows, we explicitly determine the form of M_D in the context of a realistic supersymmetric $SO(10)$ model and then use the present experimental bounds on the elements of the nonunitary parameter $|\eta|$ to get a lower bound on the RH neutrino mass scale m_N. Finally, we fit the observed LH neutrino mass and mixing parameters by the inverse seesaw formula to determine the structure of μ. We then study the phenomenological consequences of our results.

IV. EMBEDDING THE INVERSE SEESAW MECHANISM IN REALISTIC $SO(10)$ GUT

As we have mentioned earlier, in order to embed the inverse seesaw mechanism into a supersymmetric $SO(10)$ theory, we have to break the $B-L$ symmetry by using a $\mathbf{16} \oplus \overline{\mathbf{16}}$ pair rather than a $\mathbf{126} \oplus \overline{\mathbf{126}}$ pair of the Higgs representation. In this context, there are two symmetry-breaking chains that are particularly interesting:

(i) $SO(10) \xrightarrow{M_G} \mathbf{3}_c \mathbf{2}_L \mathbf{2}_R \mathbf{1}_{B-L} \xrightarrow{M_R} \mathbf{3}_c \mathbf{2}_L \mathbf{1}_Y(\mathrm{MSSM}) \xrightarrow{M_{\mathrm{SUSY}}} \mathbf{3}_c \mathbf{2}_L \mathbf{1}_Y(\mathrm{SM}) \xrightarrow{M_Z} \mathbf{3}_c \mathbf{1}_Q$ [20],

(ii) $SO(10) \xrightarrow{M_G} \mathbf{3}_c \mathbf{2}_L \mathbf{2}_R \mathbf{1}_{B-L} \xrightarrow{V_R} \mathbf{3}_c \mathbf{2}_L \mathbf{1}_{I_{3R}} \mathbf{1}_{B-L} \xrightarrow{v_R} \mathbf{3}_c \mathbf{2}_L \mathbf{1}_Y(\mathrm{MSSM}) \xrightarrow{M_{\mathrm{SUSY}}} \mathbf{3}_c \mathbf{2}_L \mathbf{1}_Y(\mathrm{SM}) \xrightarrow{M_Z} \mathbf{3}_c \mathbf{1}_Q$ [21],

where, as an example of our notation, $\mathbf{3}_c$ means $SU(3)_c$. In this paper, we consider only the former (and simpler) case of the $SO(10)$-breaking chain. It was shown in Ref. [20] that it is possible to obtain the gauge coupling unification in this model with a low-energy (TeV scale) $SU(2)_R$ symmetry-breaking scale M_R. However, they considered only one $\mathbf{10}_H$ Higgs field which contains only a single bidoublet [corresponding to the (1, 2, 2, 0) representation of $\mathbf{3}_c \mathbf{2}_L \mathbf{2}_R \mathbf{1}_{B-L}$]. Getting a realistic fermion mass spectrum in this model is difficult (see, however, some recent ideas [22] on how this could be done). Instead, we consider a model with two $\mathbf{10}_H$ at the TeV scale. This requires that we reexamine the unification issue with two Higgs bidoublets. We show that we not only obtain the gauge coupling unification at a scale consistent with the proton decay bounds, but also successfully reproduce the observed fermion masses and mixing.

To study the running of the gauge couplings and the possibility of their unification at a scale $M_G \sim 10^{16}$ GeV, we divide the whole energy range (M_Z, M_G) into three parts, according to the above-mentioned symmetry-breaking chain:

(i) First, we have the well-known SM from the weak scale M_Z to the SUSY-breaking scale M_{SUSY} (which, for practical purposes, we assume to be a little higher than M_Z).

(ii) Then we have the minimal supersymmetric standard model (MSSM) from M_{SUSY} to the $B-L$-breaking

scale M_R (which is assumed to be of order TeV, so that it is of interest for colliders).

(iii) Finally, we have the supersymmetric left-right (SUSYLR) model from M_R to the unification scale M_G (expected to be around 10^{16} GeV).

The running of the gauge couplings at one-loop level is determined by the RG equation

$$\frac{d\alpha_i}{d\ln\tilde{\mu}} = \frac{b_i}{2\pi}\alpha_i^2 \quad \text{or} \quad \alpha_i^{-1}(\tilde{\mu}_1) = \alpha_i^{-1}(\tilde{\mu}_2) + \frac{b_i}{2\pi}\ln\left(\frac{\tilde{\mu}_2}{\tilde{\mu}_1}\right), \tag{18}$$

where $\alpha_i \equiv \frac{g_i^2}{4\pi}$, $\tilde{\mu}$ is the energy scale, and the b_i's are the coefficients of the one-loop β functions. The SM and MSSM β functions are well known [23]:

$$b_i^{\rm SM} = \left(\frac{41}{10}, -\frac{19}{6}, -7\right) \quad \text{and} \quad b_i^{\rm MSSM} = \left(\frac{33}{5}, 1, -3\right), \tag{19}$$

where i stands for $\mathbf{1}_Y$, $\mathbf{2}_L$, and $\mathbf{3}_c$, respectively. Before calculating the β functions for the SUSYLR model, let us first discuss the particle content of this model.

A. Particle content of the SUSYLR model

Here we consider only the doublet implementation of the SUSYLR model [24]; i.e. we use $SU(2)$ doublets of the $\mathbf{16}_H$ Higgs field to break the $B-L$ symmetry. In order to keep the model general, we allow for an arbitrary number of these doublet fields, to be denoted by n_L and n_R, respectively, for $SU(2)_L$ and $SU(2)_R$ doublets. Likewise, we have n_{10} bidoublets of the $\mathbf{10}_H$ Higgs field which, on acquiring VEVs, give masses to the fermions through Yukawa couplings. We also allow for an arbitrary number n_S of singlet fields S^α. This is the minimal set of particles in a generic SUSYLR model.

However, it turns out that, with this minimal set of particles, it is not possible to obtain the gauge coupling unification at a scale higher than $\sim 10^{15}$ GeV as required from current bounds on the proton decay lifetime, $\tau_p \gtrsim 10^{34}$ yr [25]. As we show later in Sec. IV B, unification is possible only after adding the contribution from the color triplets $[(3, 1, \frac{4}{3}) + \text{c.c.}]$, which come from the $\mathbf{45}_H$ representation of the Higgs at the unification scale M_G. It is justified in Appendix A that it is indeed possible to have these color triplets at the TeV scale, while all the other Higgs multiplets are still naturally heavy at the GUT scale.

The particle content and their representations under the $\mathbf{3}_c\mathbf{2}_L\mathbf{2}_R\mathbf{1}_{B-L}$ gauge group are summarized in Table I. Following the notation of Ref. [24], the $SU(2)$ doublets and bidoublets are represented as

$$Q = \begin{pmatrix} u \\ d \end{pmatrix}, \qquad Q^c = \begin{pmatrix} d^c \\ -u^c \end{pmatrix}, \qquad \Phi_a = \begin{pmatrix} \phi^0_{a_d} & \phi^+_{a_u} \\ \phi^-_{a_d} & \phi^0_{a_u} \end{pmatrix}.$$

Other doublet pairs can be written in a similar way as the (Q, Q^c) pair. The charges of the fields must obey the relation

$$Q = I_{3_L} + I_{3_R} + \frac{B-L}{2}. \tag{20}$$

B. Gauge coupling unification

The β function for a general supersymmetric model is given by [23]

$$b_N^{\rm SUSY} = 2n_g - 3N + T(S_N) \tag{21}$$

for n_g generations of fermions, the gauge group $SU(N)$, and the complex Higgs representation parametrized by $T(S_N)$. For the $U(1)$ gauge group, $N = 0$ in Eq. (21) and the gauge coupling is normalized as usual. For the particle content given by Table I, the Higgs contributions in our SUSYLR model are explicitly given by

$$\begin{aligned} &T_{2L} = n_{10} + n_L, \qquad T_{2R} = n_{10} + n_R, \\ &T_{3c} = 1, \quad \text{and} \quad T_{B-L} = 4 + \tfrac{3}{2}(n_L + n_R). \end{aligned} \tag{22}$$

Hence for three fermion generations, we find the β functions for our SUSYLR model to be

$$b_i^{\rm SUSYLR} = (10 + \tfrac{3}{2}n_L + \tfrac{3}{2}n_R, n_{10} + n_L, n_{10} + n_R, -2), \tag{23}$$

where i stands for $\mathbf{1}_{B-L}$, $\mathbf{2}_L$, $\mathbf{2}_R$, and $\mathbf{3}_c$, respectively. Using these β functions, we can now obtain the running of gauge couplings up to the scale M_G, knowing the initial values at $\tilde{\mu} = M_Z$ [26],

$$\begin{aligned} \alpha_{1Y}(M_Z) &= 0.016\,829 \pm 0.000\,017, \\ \alpha_{2L}(M_Z) &= 0.033\,493^{+0.000\,042}_{-0.000\,038}, \\ \alpha_{3c}(M_Z) &= 0.118 \pm 0.003, \end{aligned}$$

TABLE I. The representations of the particles under the $\mathbf{3}_c\mathbf{2}_L\mathbf{2}_R\mathbf{1}_{B-L}$ gauge group in the doublet SUSYLR model. Here $a = 1, \ldots, n_{10}$, $p = 1, \ldots, n_L$, $q = 1, \ldots, n_R$, and $\alpha = 1, \ldots, n_S$. The $B-L$ quantum numbers given here are not GUT renormalized; to do so, we multiply by a factor of $\sqrt{\frac{3}{2}}$ (not $\sqrt{\frac{3}{8}}$ as mentioned in Ref. [24]).

Multiplet	$SU(3)_c$	$SU(2)_L$	$SU(2)_R$	$U(1)_{B-L}$
Q	3	2	1	$+1/3$
Q^c	3	1	2	$-1/3$
L	1	2	1	-1
L^c	1	1	2	$+1$
χ_p	1	2	1	$+1$
χ_q^c	1	1	2	-1
$\bar{\chi}_p$	1	2	1	-1
$\bar{\chi}_q^c$	1	1	2	$+1$
Φ_a	1	2	2	0
S^α	1	1	1	0
δ	3	1	1	$+4/3$
$\bar{\delta}$	$\bar{3}$	1	1	$-4/3$

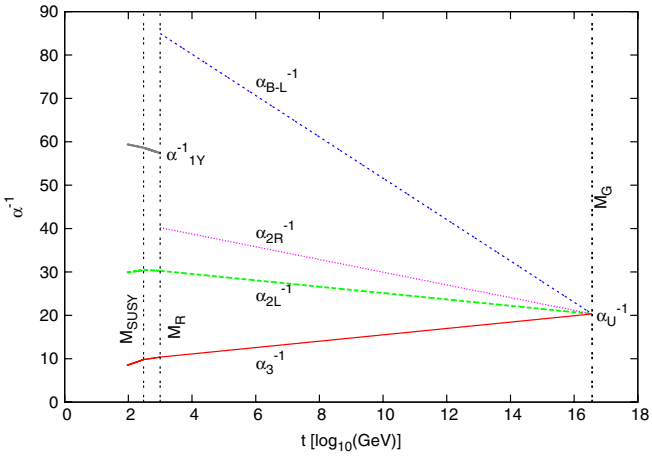

FIG. 1 (color online). Gauge coupling unification in the SUSYLR model. We have used $n_{10} = 2$, $n_L = 0$, $n_R = 2$, $M_{\rm SUSY} = 300$ GeV, and $M_R = 1$ TeV. As the running behavior is mostly controlled by the SUSYLR sector, the values of $M_{\rm SUSY}$ and M_R can be relaxed a little bit, still preserving unification. However, it should be emphasized that the choice of the number of bidoublets and doublets is the only possible choice consistent with both gauge coupling unification and realistic fermion masses. Increasing n_{10} or n_L, or changing n_R in either way, will spoil the unification, and as already noted, reducing n_{10} will not give us a realistic fermion mass spectrum.

and the matching condition [27] at $\tilde{\mu} = M_R$ where the $U(1)_Y$-gauge coupling gets merged into $SU(2)_R \times U(1)_{B-L}$:

$$\alpha_{1Y}^{-1}(M_R) = \tfrac{3}{5}\alpha_{2R}^{-1}(M_R) + \tfrac{2}{5}\alpha_{B-L}^{-1}(M_R). \tag{24}$$

For numerical purposes, we assume $M_{\rm SUSY} = 300$ GeV and $M_R = 1$ TeV. Also, we take the number of Higgs bidoublets, $n_{10} = 2$. However, the number of Higgs doublets can be arbitrary, and we vary these parameters to get the unification. As shown in Fig. 1, we achieve the gauge unification for $n_L = 0$ and $n_R = 2$, with the unification scale parameters

$$M_G \simeq 4 \times 10^{16}\ \text{GeV} \quad \text{and} \quad \alpha_U^{-1}(M_G) \simeq 20.3. \tag{25}$$

Note the asymmetry between n_L and n_R. We show in Appendix A that since the VEV of the $\mathbf{45}_H$ Higgs breaks D parity and decouples it from the $SU(2)_R$-breaking scale [28], it is possible to have only the right-handed doublets and no left-handed ones below the GUT scale. This leads to the asymmetry between α_{2L} and α_{2R}, with $\frac{\alpha_{2L}}{\alpha_{2R}} \simeq 1.3$ in our case.

V. RG EVOLUTION OF THE FERMION MASSES AND MIXING

The RG evolution of the fermion masses and mixing has been extensively studied for both the SM and the MSSM cases [29], but not for the SUSYLR model, even though the analytical expressions for the Yukawa couplings have already been derived in Ref. [24]. Here we present a detailed RG analysis in our SUSYLR model and obtain the quark and lepton masses and the CKM matrix elements at the unification scale M_G.

The superpotential relevant for the RG evolution of the Yukawa couplings in the SUSYLR model is given by [24]

$$\begin{aligned} W \supset\ & i h_a Q^T \tau_2 \Phi_a Q^c + i h'_a L^T \tau_2 \Phi_a L^c + i\lambda_{apq} \chi_p^T \tau_2 \Phi_a \chi_q^c \\ & + i\bar{\lambda}_{apq} \bar{\chi}_p^T \tau_2 \Phi_a \bar{\chi}_q^c + \mu^{\Phi}_{\alpha ab} S^\alpha \,\mathrm{Tr}(\Phi_a^T \tau_2 \Phi_b \tau_2) \\ & + i\mu^L_{\alpha p} S^\alpha L^T \tau_2 \chi_p + i\mu^{L^c}_{\alpha q} S^\alpha L^{c^T} \tau_2 \chi_q^c, \end{aligned} \tag{26}$$

where we have suppressed the generational and $SU(2)$ indices. Also, we have ignored all nonrenormalizable terms in the superpotential, as their contributions to the renormalization group equations (RGEs) are suppressed by M_R/M_G. We note that the superpotential given by Eq. (26) has two additional terms of the form $SL\chi$ and $SL^c\chi^c$ (as required by the inverse seesaw model) as compared to that given in Ref. [24]. Also, note that since the δ, $\bar{\delta}$ fields do not couple to any of the matter fields, they do not affect the renormalization group running except through their effect on the color gauge coupling evolution.

We have seen from the previous section that the gauge coupling unification requires that we take two $SU(2)_R$ doublets and no $SU(2)_L$ doublets from the Higgs fields. Hence, dropping the χ, $\bar{\chi}$ terms from the superpotential of Eq. (26), we have

$$\begin{aligned} W \supset\ & i h_a Q^T \tau_2 \Phi_a Q^c + i h'_a L^T \tau_2 \Phi_a L^c + i\mu^{L^c}_{\alpha q} S^\alpha L^{c^T} \tau_2 \chi_q^c \\ & + \mu^{\Phi}_{\alpha ab} S^\alpha \,\mathrm{Tr}(\Phi_a^T \tau_2 \Phi_b \tau_2), \end{aligned} \tag{27}$$

where $a = 1, 2$; $q = 1, 2$; and $\alpha = 1, 2, 3$, corresponding to the two bidoublets, two RH doublets, and three fermion singlets, respectively.

The RGEs for the Yukawa couplings h_a and h'_a in Eq. (27) are given by (with $t = \ln\tilde{\mu}$)

$$\begin{aligned} 16\pi^2 \frac{dh_a}{dt} = h_a\Big[& 2h_b^\dagger h_b - \frac{16}{3} g_3^2 - 3g_{2L}^2 - 3g_{2R}^2 \\ & - \frac{1}{6} g_{B-L}^2 \Big] + h_b[\mathrm{Tr}(3h_b^\dagger h_a + h_b'^\dagger h'_a) \\ & + 2h_b^\dagger h_a + 4(\mu_\alpha^{\Phi\dagger} \mu_\alpha^{\Phi})_{ba}], \end{aligned} \tag{28}$$

$$\begin{aligned} 16\pi^2 \frac{dh'_a}{dt} = h'_a\Big[& 2h_b'^\dagger h'_b - 3g_{2L}^2 - 3g_{2R}^2 - \frac{3}{2} g_{B-L}^2 \Big] \\ & + h'_b[\mathrm{Tr}(3h_b^\dagger h_a + h_b'^\dagger h'_a) + 2h_b'^\dagger h'_a \\ & + 4(\mu_\alpha^{\Phi\dagger} \mu_\alpha^{\Phi})_{ba} + (\mu^{L^c})^\dagger_{\alpha q} \mu^{L^c}_{\alpha q} \delta_{ba}], \end{aligned} \tag{29}$$

where the repeated indices are summed over and $a, b = 1, 2$; $q = 1, 2$; and $\alpha = 1, 2, 3$, corresponding to the two Higgs bidoublets, two $SU(2)_R$ doublets, and three fermion singlets, respectively. Note that we have an additional contribution to the RGE of the lepton Yukawa coupling h'_a as compared to those given in Ref. [24] which comes

from the $S\chi^c L^c$ term in the superpotential. Note also the presence of the h_b terms in the second line in both the Yukawa runnings even for $a \neq b$, which are characteristic of left-right models and are absent in the case of the MSSM, arising from the Higgs self-energy effects.

The fermion masses arise through the Yukawa couplings h_a and h'_a when the two Higgs bidoublets $\Phi_{1,2}$ acquire VEVs. In general, a linear combination of h_1 and h_2 will give masses to the up-type quarks, and similarly different linear combinations for the other masses. The dynamics of the superpotential can be chosen in such a way that the bidoublets acquire VEVs in the following simple manner:

$$\langle\Phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} v_d & 0 \\ 0 & 0 \end{pmatrix}, \qquad \langle\Phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 \\ 0 & v_u \end{pmatrix}, \tag{30}$$

and we identify the ratio $v_u/v_d \equiv \tan\beta$ (MSSM) with $\sqrt{v_u^2 + v_d^2} = 246$ GeV. For numerical purposes, we use $\tan\beta(\mathrm{MSSM}) = 10$. To obtain the RGEs for the mass matrices, we choose the most frequently used renormalization scheme [29], where the Yukawa couplings and the Higgs VEVs run separately. The RGEs for the Higgs VEVs are obtained from the gauge and scalar self-energy contributions:

$$16\pi^2 \frac{dv_u}{dt} = v_u\left[\frac{3}{2}g_{2L}^2 + \frac{3}{2}g_{2R}^2 - \mathrm{Tr}(3h_2^\dagger h_2 + h_2'^\dagger h_2') - 4(\mu_\alpha^{\Phi\dagger}\mu_\alpha^{\Phi})_{22}\right], \tag{31}$$

$$16\pi^2 \frac{dv_d}{dt} = v_d\left[\frac{3}{2}g_{2L}^2 + \frac{3}{2}g_{2R}^2 - \mathrm{Tr}(3h_1^\dagger h_1 + h_1'^\dagger h_1') - 4(\mu_\alpha^{\Phi\dagger}\mu_\alpha^{\Phi})_{11}\right]. \tag{32}$$

Using Eqs. (28) and (29) for $\dot{h}_a$, $\dot{h}'_a$ and Eqs. (31) and (32) for $\dot{v}_u$, $\dot{v}_d$, we have derived the RGEs for the physical fermion masses and the quark mixing in our SUSYLR model in Appendix B. Using the initial values for the mass and mixing parameters at $\tilde{\mu} = M_Z$ [26],

$$\begin{aligned}
m_u(M_Z) &= 2.33^{+0.42}_{-0.45}\ \mathrm{MeV}, \\
m_c(M_Z) &= 677^{+56}_{-61}\ \mathrm{MeV}, \\
m_t(M_Z) &= 181 \pm 13\ \mathrm{GeV}, \\
m_d(M_Z) &= 4.69^{+0.60}_{-0.66}\ \mathrm{MeV}, \\
m_s(M_Z) &= 93.4^{+11.8}_{-13.0}\ \mathrm{MeV}, \\
m_b(M_Z) &= 3.00 \pm 0.11\ \mathrm{GeV}, \\
m_e(M_Z) &= 0.486\,847\,27 \pm 0.000\,000\,14\ \mathrm{MeV}, \\
m_\mu(M_Z) &= 102.751\,38 \pm 0.000\,33\ \mathrm{MeV}, \\
m_\tau(M_Z) &= 1.746\,69^{+0.000\,30}_{-0.000\,27}\ \mathrm{GeV},
\end{aligned}$$

and with the quark-sector mixing parameters $\theta_{12} = 13.04° \pm 0.05°$, $\theta_{13} = 0.201° \pm 0.011°$, $\theta_{23} = 2.38° \pm 0.06°$, and $\delta_{13} = 1.20 \pm 0.08$,

$$\begin{aligned}
V_{\mathrm{CKM}}(M_Z) &= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \\
&= \begin{pmatrix} 0.9742 & 0.2256 & 0.0013 - 0.0033i \\ -0.2255 - 0.0001i & 0.9734 & 0.0415 \\ 0.0081 - 0.0032i & -0.0407 - 0.0007i & 0.9991 \end{pmatrix},
\end{aligned}$$

and the SM and MSSM Yukawa RGEs [29], we numerically solve the SUSYLR RGEs given in Appendix B to obtain the running quark and lepton masses and the CKM matrix elements at the unification scale M_G:

$$\begin{aligned}
&m_u(M_G) = 0.0017\ \mathrm{GeV}, \quad m_c(M_G) = 0.1910\ \mathrm{GeV}, \quad m_t(M_G) = 77.8035\ \mathrm{GeV}; \quad m_d(M_G) = 0.0013\ \mathrm{GeV}, \\
&m_s(M_G) = 0.0263\ \mathrm{GeV}, \quad m_b(M_G) = 1.7092\ \mathrm{GeV}; \quad m_e(M_G) = 0.0004\ \mathrm{GeV}, \quad m_\mu(M_G) = 0.0911\ \mathrm{GeV}, \\
&m_\tau(M_G) = 1.7096\ \mathrm{GeV} \quad V_{\mathrm{CKM}}(M_G) = \begin{pmatrix} 0.9793 & 0.2023 + 0.0018i & 0.0005 - 0.0057i \\ -0.2023 + 0.0016i & 0.9791 & 0.0240 \\ 0.0044 - 0.0056i & -0.0236 - 0.0013i & 0.9997 \end{pmatrix}.
\end{aligned} \tag{33}$$

We also have a mild running for $\tan\beta$ with $\tan\beta(M_G) = 7$ from $\tan\beta(M_R) = 10$.

Figure 2 shows the running of the quark and charged-lepton masses up to the unification scale M_G. Note that we are able to generate the fermion mass spectrum at the GUT scale with

$$\frac{m_b^0}{m_\tau^0} \simeq 1, \qquad \frac{m_\mu^0}{m_s^0} \simeq 3, \qquad \frac{m_e^0}{m_d^0} \simeq \frac{1}{3}. \tag{34}$$

Figure 3 shows the running of the CKM elements involving the third generation. Note that in addition to the significant running for the third-generation CKM elements $V_{ub,cb,td,ts}$,

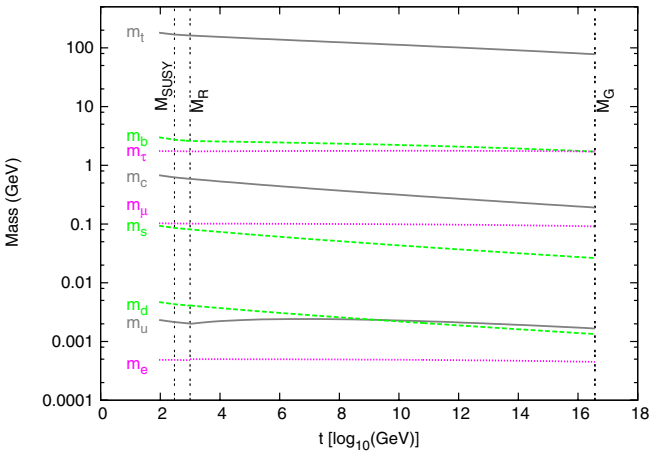

FIG. 2 (color online). Running of fermion masses to the GUT scale in our model for $M_{\rm SUSY} = 300$ GeV and $M_R = 1$ TeV. Note the $b - \tau$ unification which is a generic feature of GUT models.

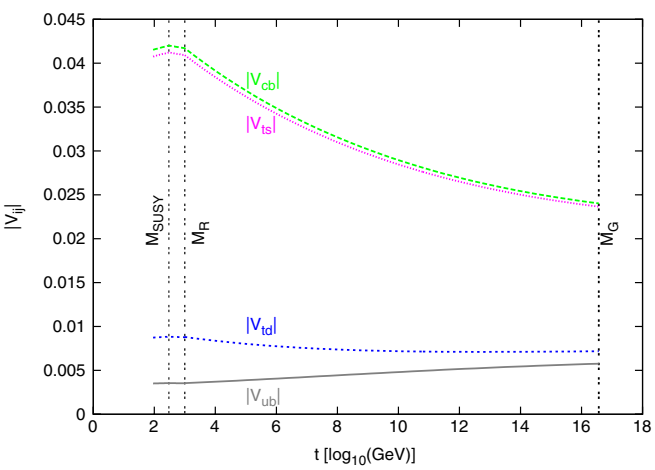

FIG. 3 (color online). Running of the CKM mixing elements involving the third generation to the GUT scale in our model for $M_{\rm SUSY} = 300$ GeV and $M_R = 1$ TeV. The running of other CKM elements, being small, is not shown here.

we have a relatively milder running for the other elements as well [cf. Eq. (33)], even in the third-generation dominance approximation. This is a characteristic of the left-right model, in contrast to the MSSM case where in the third-generation dominance, the first- and second-generation elements do not run at the one-loop level.

VI. THE DIRAC MASS FOR NEUTRINOS IN A SPECIFIC $SO(10)$ MODEL

As discussed in Sec. II, in order to implement the inverse seesaw mechanism, we have to use the class of $SO(10)$ models in which the $B - L$ subgroup is broken by a $\mathbf{16}_H \oplus \overline{\mathbf{16}}_H$ pair. We also need at least two $\mathbf{10}_H$ and a $\mathbf{45}_H$ to have a realistic fermion spectrum. With this minimum set of Higgs multiplets $\{\mathbf{10}_H, \mathbf{16}_H, \overline{\mathbf{16}}_H, \mathbf{45}_H\}$, several $SO(10)$ models have been constructed [30]. All these models require various dimension-5 operators to get the right fermion masses: In principle, they are also present in our model. However, most of them, e.g. $\frac{h_{ij}}{M}\mathbf{16}_i\mathbf{16}_j\mathbf{16}_H\mathbf{16}_H$, are suppressed by the factor $\frac{M_R}{M_{\rm Pl}} \sim 10^{-15}$, as the $\mathbf{16}_H$ Higgs acquires only TeV scale VEVs. The only other dimension-5 operator that can make a significant contribution to fermion masses is $\frac{h'_{ij}}{M}\mathbf{16}_i\mathbf{16}_j\mathbf{10}_H\mathbf{45}_H$; we assume its effects to be small in our model and keep the dimension-6 operator

$$\frac{f_{ij}}{M^2}\mathbf{16}_i\mathbf{16}_j\mathbf{10}_H\mathbf{45}_H\mathbf{45}_H. \tag{35}$$

This operator is suppressed only by $(\frac{M_G}{M_{\rm Pl}})^2 \sim 10^{-4}$, as the $\mathbf{45}_H$ acquires a VEV at the scale M_G and plays an important role in the fermion mass fitting given below.

The fermion mass splitting is obtained by the completely antisymmetric combination of the operator given by the expression (35), i.e. in the notation of Ref. [31],

$$\langle \psi_+^* | B[\Gamma_i\Gamma_j\Gamma_k\Gamma_l\Gamma_m] A_{ij}A_{kl}\Phi_m | \psi_+ \rangle \tag{36}$$

with $B = \prod_{\mu=\rm odd}\Gamma_\mu$ and $[\ldots]$ denoting the completely antisymmetric combination. Here Φ and A denote the $\mathbf{10}_H$ and $\mathbf{45}_H$ fields, respectively. When the following VEVs are nonzero:

$$\langle \Phi_{9,10} \rangle \neq 0, \qquad \langle A_{12,34,56} \rangle \neq 0, \tag{37}$$

this antisymmetric combination acts as an effective $\mathbf{126}_H$ operator which gives the mass relation $m_e = -3m_d$ and $m_\nu = -3m_u$ due to the VEVs $\langle A_{ij} \rangle$, while m_u and m_d are split in the usual manner by the two $\mathbf{10}_H$ VEVs, $\langle \phi_{9,10} \rangle$. To obtain a realistic fermion mass spectrum, we construct the following model using the Higgs multiplets $\{\mathbf{10}_H, \mathbf{45}_H, \mathbf{54}_H\}$. The $SO(10)$ symmetry breaking to $\mathbf{3}_c\mathbf{2}_L\mathbf{2}_R\mathbf{1}_{B-L}$ is obtained by a combination of the $\mathbf{45}_H$ and $\mathbf{54}_H$, with the following VEVs in an $SU(5)$ basis:

$$\langle \mathbf{45} \rangle \propto \mathrm{diag}(a, a, a, 0, 0),$$
$$\langle \mathbf{54} \rangle \propto \mathrm{diag}(2a, 2a, 2a, 2a, 2a, -3a, -3a, -3a, -3a). \tag{38}$$

In this model, the fermion mass matrices at the GUT scale have the following form:

$$\begin{aligned} M_u &= \tilde{h}_u + \tilde{f}, \qquad & M_d &= \tilde{h}_d + \tilde{f}, \\ M_e &= \tilde{h}_d - 3\tilde{f}, \qquad & M_D &= \tilde{h}_u - 3\tilde{f}, \end{aligned} \tag{39}$$

where the $h_{u,d}$ matrices come from the usual Yukawa terms $h_{ij}\mathbf{16}_i\mathbf{16}_j\mathbf{10}_H(\mathbf{10}'_H)$ and the f matrix comes from the $\mathbf{45}_H$ contribution given by the expression (35), where we have assumed the same coupling for both the $\mathbf{10}_H$ fields. The tildes denote the normalized couplings with mass dimensions where the VEVs have been absorbed. We know the nine eigenvalues of the quark and charged-lepton mass

matrices at the scale M_G from our RG analysis in Sec. V; however, we have 18 unknowns (for three Hermitian matrices) to fit into Eq. (39). Hence a unique fit is not possible; we just give here one sample fit that is consistent with all the masses and mixing at the GUT scale obtained from the RGEs.

We work in a basis in which the charged-lepton mass matrix is diagonal, i.e.

$$M_e = \begin{pmatrix} m_e(M_G) & 0 & 0 \\ 0 & m_\mu(M_G) & 0 \\ 0 & 0 & m_\tau(M_G) \end{pmatrix} = \begin{pmatrix} 0.0004 & 0 & 0 \\ 0 & 0.0911 & 0 \\ 0 & 0 & 1.7096 \end{pmatrix} \text{GeV}.$$

This immediately implies from Eq. (39) that

$$\tilde{h}_{d,ij} = 3\tilde{f}_{ij}, \qquad \forall\ i \neq j. \tag{40}$$

For simplicity, let us choose the $\tilde{f}$ matrix to be diagonal. Then Eq. (40) implies that $\tilde{h}_d$ is also a diagonal matrix. We also have the following relations:

$$\tilde{h}_{d,\alpha\alpha} + \tilde{f}_{\alpha\alpha} = m_\alpha, \qquad \tilde{h}_{d,\beta\beta} - 3\tilde{f}_{\beta\beta} = m_\beta, \tag{41}$$

where $m_\alpha = (m_d, m_s, m_b)$ are the eigenvalues of M_d and $m_\beta = (m_e, m_\mu, m_\tau)$ the eigenvalues of M_e. These six equations (41) now fix the h_d and f matrices completely:

$$\tilde{f} = \begin{pmatrix} \frac{1}{4}(m_d - m_e) & 0 & 0 \\ 0 & \frac{1}{4}(m_s - m_\mu) & 0 \\ 0 & 0 & \frac{1}{4}(m_b - m_\tau) \end{pmatrix} = \begin{pmatrix} 2.25\times 10^{-4} & 0 & 0 \\ 0 & -0.0162 & 0 \\ 0 & 0 & -0.0001 \end{pmatrix} \text{GeV},$$

$$\tilde{h}_d = \begin{pmatrix} \frac{1}{4}(3m_d + m_e) & 0 & 0 \\ 0 & \frac{1}{4}(3m_s + m_\mu) & 0 \\ 0 & 0 & \frac{1}{4}(3m_b + m_\tau) \end{pmatrix} = \begin{pmatrix} 0.0011 & 0 & 0 \\ 0 & 0.0425 & 0 \\ 0 & 0 & 1.7093 \end{pmatrix} \text{GeV}. \tag{42}$$

The $\tilde{h}_u$ matrix can now be determined by fitting to M_u, which, in this basis, is given by

$$M_u = V_{\text{CKM}} M_u^{\text{diag}} V_{\text{CKM}}^\dagger = \begin{pmatrix} 0.0120 & 0.0384 - 0.0103i & 0.038 - 0.4433i \\ 0.0384 + 0.0103i & 0.2280 & 1.8623 + 0.0002i \\ 0.038 + 0.4433i & 1.8623 - 0.0002i & 77.7569 \end{pmatrix} \text{GeV}. \tag{43}$$

Then from Eq. (39) the $\tilde{h}_u$ matrix is given by

$$\tilde{h}_u = \begin{pmatrix} 0.0118 & 0.0384 - 0.0103i & 0.038 - 0.4433i \\ 0.0384 + 0.0103i & 0.2442 & 1.8623 + 0.0002i \\ 0.038 + 0.4433i & 1.8623 - 0.0002i & 77.757 \end{pmatrix} \text{GeV}. \tag{44}$$

Hence the Dirac neutrino mass matrix is given by

$$M_D = \begin{pmatrix} 0.0111 & 0.0384 - 0.0103i & 0.038 - 0.4433i \\ 0.0384 + 0.0103i & 0.2928 & 1.8623 + 0.0002i \\ 0.038 + 0.4433i & 1.8623 - 0.0002i & 77.7573 \end{pmatrix} \text{GeV}. \tag{45}$$

It may be noted here that even though the specific form of the Dirac neutrino mass matrix may depend on the choice of the particular basis we have chosen, the individual values of the matrix elements are more or less fixed by the up-type quark mass values, due to the mass relation (39), and hence, do not depend on the basis so much. Therefore, all the predictions of the model that follow from the form of M_D given by Eq. (45) will be independent of the initial choice of our basis, up to a few percent.

VII. NONUNITARITY EFFECTS IN THE LEPTON MIXING MATRIX

In this section we obtain the nonunitarity parameter η using the structure of the Dirac neutrino mass matrix obtained in Eq. (45) and discuss the phenomenological consequences of our results.

A. Bounds on $|\eta|$

As discussed in Sec. III, the nonunitarity parameter is given by

$$\eta \simeq \tfrac{1}{2} F F^\dagger \quad \text{with} \quad F = M_D M_N^{-1}. \tag{46}$$

For simplicity, choosing M_N to be diagonal, and motivated by resonant leptogenesis, assuming degenerate eigenvalues for M_N equal to m_N, we have

$$\eta \simeq \frac{1}{2m_N^2} M_D M_D^\dagger. \tag{47}$$

With the form of M_D derived in the last section after extrapolation to the weak scale, we can readily calculate the elements of η:

$$\eta \simeq \frac{1\ \mathrm{GeV}^2}{m_N^2}\begin{pmatrix} 0.1 & 0.0412 - 0.4144i & 1.5134 - 17.247i \\ 0.0412 + 0.4144i & 1.78 & 72.6794 - 0.0005i \\ 1.5134 + 17.247i & 72.6794 + 0.0005i & 3024.93 \end{pmatrix}. \tag{48}$$

This is to be compared with the present bounds on $|\eta_{ij}|$ (at the 90% C.L.) [32]:

$$|\eta| < \begin{pmatrix} 2.0\times 10^{-3} & 3.5\times 10^{-5} & 8.0\times 10^{-3} \\ 3.5\times 10^{-5} & 8.0\times 10^{-4} & 5.1\times 10^{-3} \\ 8.0\times 10^{-3} & 5.1\times 10^{-3} & 2.7\times 10^{-3} \end{pmatrix}. \tag{49}$$

This gives a lower bound on the mass of the RH neutrino:

$$m_N \gtrsim 1.06\ \mathrm{TeV}, \tag{50}$$

which should be kinematically accessible at the LHC to be produced on shell. Note that the right-handed neutrinos are pseudo-Dirac fermions in our model (with a small Majorana component), which is distinct from the type I seesaw models where they are pure Majorana. As a result, the like-sign dilepton final states which are the "smoking gun" collider signals of the type I seesaw mechanism are suppressed in our model; however, the trilepton signals can be used in this case for testing these models [19].

With this lower bound on m_N, we get the following improved bounds on $|\eta_{\alpha\beta}|$:

$$\begin{aligned} &|\eta_{ee}| < 8.9\times 10^{-8}, \qquad |\eta_{e\mu}| < 3.7\times 10^{-7}, \\ &|\eta_{e\tau}| < 1.5\times 10^{-5}, \qquad |\eta_{\mu\mu}| < 1.6\times 10^{-6}, \\ &|\eta_{\mu\tau}| < 6.5\times 10^{-5}. \end{aligned} \tag{51}$$

At least one of these bounds, namely $|\eta_{e\mu}|$, is reachable at future neutrino factories from the improved branching ratio of $\mu \to e\gamma$ down to 10^{-18} [33]. Similar sensitivities are also reachable in the PRISM/PRIME project [34]. We note that relaxing the condition of degenerate RH neutrinos but fitting the neutrino masses affects the values of $\eta_{\alpha\beta}$; we present these results in Table II. It appears that $|\eta_{e\mu}|$ values are all accessible to the future $\mu \to e + \gamma$ searches. The largest value of $|\eta_{\mu\tau}|$ in this table may also be accessible to neutrino oscillation experiments, preferably with short baselines ($L \lesssim 100$ km).

B. Fitting the neutrino oscillation data

The structure of the small mass parameter μ can be obtained using the inverse seesaw formula, Eq. (1):

$$\mu = F^{-1} m_\nu (F^T)^{-1}, \tag{52}$$

where m_ν is diagonalized by the new PMNS matrix $\mathcal{N} = (1-\eta)U$ instead of U in Eq. (6):

$$m_\nu = \mathcal{N}\hat{m}_\nu \mathcal{N}^T. \tag{53}$$

The form of U is obtained from the standard PMNS parametrization using the 2σ results from neutrino oscillation data [35]:

$$\begin{aligned} \Delta m^2_\odot &= 7.67(1^{+0.044}_{-0.047})\times 10^{-5}\ \mathrm{eV}^2, \\ \Delta m^2_{\mathrm{atm}} &= 2.39(1^{+0.113}_{-0.084})\times 10^{-3}\ \mathrm{eV}^2, \\ \sin^2\theta_{12} &= 0.312(1^{+0.128}_{-0.109}), \\ \sin^2\theta_{23} &= 0.466(1^{+0.292}_{-0.215}). \end{aligned} \tag{54}$$

Here we assume $\theta_{13} = 0$. Now using the form of η obtained in Eq. (48) and taking $m_N = 1.1$ TeV for its lower bound value, we get the new PMNS matrix $\mathcal{N} = (1-\eta)U$. For illustration, let us assume normal hierarchy for neutrino masses with $m_1 = 10^{-3}$ eV. Then we obtain from Eq. (52)

$$\mu \simeq \begin{pmatrix} -1.5934 + 0.0283i & 0.2244 - 0.0063i & -0.0044 + 0.0092i \\ 0.2244 - 0.0063i & -0.0322 + 0.0012i & 0.0006 - 0.0013i \\ -0.0044 + 0.0092i & 0.0006 - 0.0013i & 4.0\times 10^{-5} + 5.1\times 10^{-5}i \end{pmatrix}\ \mathrm{GeV}. \tag{55}$$

TABLE II. Predictions for the nonunitarity parameter $|\eta_{\alpha\beta}|$ for the above choice of parameters in the model including RH neutrino masses (given in GeVs).

m_{N_1}	m_{N_2}	m_{N_3}	$\lvert\eta_{e\mu}\rvert$	$\lvert\eta_{e\tau}\rvert$	$\lvert\eta_{\mu\tau}\rvert$
1100	1100	1100	3.7×10^{-7}	1.5×10^{-5}	6.5×10^{-5}
100	100	1100	7.9×10^{-7}	1.6×10^{-5}	8.9×10^{-5}
50	50	1200	2.5×10^{-6}	2.2×10^{-5}	1.6×10^{-4}
30	30	2100	6.7×10^{-6}	4.4×10^{-5}	3.2×10^{-4}

C. *CP*-violation effects

The CP-violation effects due to nonunitarity are measured by the Jarlskog invariant $\Delta J^{ij}_{\alpha\beta}$ given by Eq. (13). Note that $\Delta^{ij}_{\alpha\beta}$ is nonzero in our case, as η is a complex matrix (the phases arising from the Dirac neutrino sector). Using the values of θ_{ij} obtained from neutrino oscillation data given by Eqs. (54) and the structure of η determined in Eq. (48) with $m_N = 1.1$ TeV, we obtain the following values for $\Delta J^{ij}_{\alpha\beta}$:

$$\Delta J^{12}_{e\mu} \simeq -2.4 \times 10^{-6}, \tag{56}$$

$$\Delta J^{23}_{e\mu} \simeq -2.7 \times 10^{-6}, \tag{57}$$

$$\Delta J^{23}_{\mu\tau} \simeq 2.7 \times 10^{-6}, \tag{58}$$

$$\Delta J^{31}_{\mu\tau} \simeq 2.7 \times 10^{-6}, \tag{59}$$

$$\Delta J^{12}_{\tau e} \simeq 7.1 \times 10^{-6}, \tag{60}$$

and $\Delta J^{23}_{e\mu} = \Delta J^{31}_{e\mu} = -\Delta J^{12}_{\mu\tau} = \Delta J^{23}_{\tau e} = \Delta J^{31}_{\tau e}$. Note that these values are just 1 order of magnitude smaller than the quark-sector value, $J_{\rm CKM} = (3.05^{+0.19}_{-0.20}) \times 10^{-5}$ [26], and can be the dominant source of CP violation in the leptonic sector for vanishing θ_{13}, thus leading to distinctive CP-violating effects in neutrino oscillations [36,37]. For instance, the transition probability for the "golden channel" $\nu_\mu \to \nu_\tau$ with nonunitarity effects is given by [36]

$$P_{\mu\tau} \simeq 4|\eta_{\mu\tau}|^2 + 4s^2_{23}c^2_{23}\sin^2\left(\frac{\Delta m^2_{31}L}{4E}\right) - 4|\eta_{\mu\tau}|\sin\delta_{\mu\tau}s_{23}c_{23}\sin\left(\frac{\Delta m^2_{31}L}{2E}\right), \tag{61}$$

where the last term is CP odd due to the phase $\delta_{\mu\tau}$ of the element $\eta_{\mu\tau}$ which, in our model, is $\sim 7 \times 10^{-6}$ [cf. Eq. (48)]. Hence, the CP-violating effects should be pronounced for long-baseline neutrino factories.

D. LFV decay rates

Lepton flavor violating decays such as $\mu \to e\gamma$, $\tau \to e\gamma$, and $\tau \to \mu\gamma$ are often a signature of seesaw models for neutrino masses. In this model, they can arise from the nonunitarity effects and can be obtained using Eq. (16) which, for degenerate RH neutrinos, becomes

$$\mathrm{BR}\,(l_\alpha \to l_\beta\gamma) \simeq \frac{\alpha^3_W s^2_W m^5_{l_\alpha}}{256\pi^2 M^4_W \Gamma_\alpha}\left|(\mathcal{K}\mathcal{K}^\dagger)_{\alpha\beta} I\left(\frac{m^2_N}{M^2_W}\right)\right|^2, \tag{62}$$

with $\mathcal{K} = V_{3\times 6}$ and $I(x)$ defined in Eq. (17). Now that we know all the three 3×3 mass matrices entering the inverse seesaw formula given by Eq. (3), we can easily determine the structure of the full unitary matrix V by diagonalizing the 9×9 neutrino mass matrix $\mathcal{M}_\nu$, and hence, we obtain $V_{3\times 6}$.

The total decay width Γ_α entering Eq. (62) is given by $\hbar/\tau_\alpha$, where the mean lifetimes for μ and τ are, respectively [26],

$$\tau_\mu = (2.197\,019 \pm 0.000\,021) \times 10^{-6}\ \text{sec},$$
$$\tau_\tau = (290.6 \pm 1.0) \times 10^{-15}\ \text{sec}.$$

Using these values, we obtain the following branching ratios for the rare LFV decays:

$$\mathrm{BR}\,(\mu \to e\gamma) \simeq 3.5 \times 10^{-16}, \tag{63}$$

$$\mathrm{BR}\,(\tau \to e\gamma) \simeq 1.1 \times 10^{-13}, \tag{64}$$

$$\mathrm{BR}\,(\tau \to \mu\gamma) \simeq 2.0 \times 10^{-12}. \tag{65}$$

We have estimated the contribution to the $\mu \to e + \gamma$ branching ratio from the off-diagonal Dirac Yukawa coupling contribution to slepton masses and have found that for universal scalar mass of 500 GeV and $\tan\beta \simeq 5$, it is comparable to this value or less. Such values for the $\mu \to e\gamma$ branching ratio are accessible to future experiments [33,34] capable of reaching sensitivities down to 10^{-18}. They can be used to test the model.

In our model we assume that squark and slepton masses are above a TeV so that their contribution to the flavor changing neutral current effects are negligible. The predictions for $\mu \to 3e$ and $\mu \to e$ conversion [38] for a TeV scale slepton mass, as in our model, are much smaller than what can be probed in planned experiments.

VIII. SUMMARY

In conclusion, we have presented a TeV scale realistic inverse seesaw scenario that arises from a supersymmetric $SO(10)$ model consistent with gauge coupling unification and a fermion mass spectrum. This required us to carry out an extrapolation of quark masses and mixing to the GUT scale with a TeV scale SUSYLR rather than a MSSM. This appears to be the first time that such an extrapolation has been carried out. Implementation of the inverse seesaw mechanism within the $SO(10)$ helps to reduce the number of parameters, making the model predictive. We present our expectations for the nonunitarity of the PMNS leptonic mixing matrix with the choice of parameters and its other phenomenological consequences. The heavy RH neutrinos which are pseudo-Dirac fermions have TeV scale mass and can be produced in colliders, thus giving rise to distinctive signatures. We also give our predictions (with our choice of parameters) for the nonunitarity contribution to the branching ratios for the rare LFV decays of muons and taus. The model can also be tested by the production of W_R and Z' bosons which are at the TeV scale. Of these, the branching ratio $\mu \to e + \gamma$ could be testable in future experiments. Some of the elements of the nonunitarity matrix $|\eta|$ pre-

dicted by our model may be accessible to the next-generation neutrino factories too.

ACKNOWLEDGMENTS

This work is supported by the NSF under Grant No. PHY-0652363. B. D. would like to thank Aleksandr Azatov for discussions.

APPENDIX A: MASSES OF THE $SO(10)$ HIGGS MULTIPLETS

As discussed in Sec. IV B, we obtain the gauge coupling unification at an acceptable scale only after including the contribution from the color triplets $\delta(3, 1, 1, +\frac{4}{3})$, $\bar{\delta}(\bar{3}, 1, 1, -\frac{4}{3})$. This pair of Higgs fields is contained in the **45** representation of the Higgs in a generic $SO(10)$ model. However, in principle, there could be other light gauge multiplets of **45** and/or **54** that might contribute to the gauge coupling running as well. Here we argue that in a generic $SO(10)$ model with only $\mathbf{45}_H$ and $\mathbf{54}_H$ representations of the Higgs (apart from the essential $\mathbf{10}_H$ and $\mathbf{16}_H$), it is possible to have only the δ's as light states (TeV scale), whereas all the other states are very heavy at the GUT scale and, hence, do not contribute to the RG running. It turns out that we need to have at least two $\mathbf{45}_H$'s in our model in order to have these light color triplets.

The most general Higgs superpotential with two $\mathbf{A} \equiv \mathbf{45}$'s and an $\mathbf{E} \equiv \mathbf{54}$ Higgs field is given by

$$W_H = \tfrac{1}{2}m_1\mathbf{A}^2 + \tfrac{1}{2}m_1'\mathbf{A}'^2 + \tfrac{1}{2}m_2\mathbf{E}^2 + \lambda_1\mathbf{E}^3 + \lambda_2\mathbf{E}\mathbf{A}^2 + \lambda_2'\mathbf{E}\mathbf{A}'^2 + \lambda_3\mathbf{E}\mathbf{A}\mathbf{A}', \tag{A1}$$

where we have absorbed the $\mathbf{A}\mathbf{A}'$ term by a redefinition of the fields. The Higgs fields $\mathbf{A}$, $\mathbf{A}'$, and $\mathbf{E}$ contain three directions of singlets (with $\mathbf{A}$ and $\mathbf{A}'$ VEVs parallel) under the SM subgroup $\mathbf{3}_c\mathbf{2}_L\mathbf{1}_Y$ [39]. The corresponding VEVs are defined by

$$\langle\mathbf{A}\rangle = \sum_{i=1}^{2} A_i\hat{A}_i, \qquad \langle\mathbf{A}'\rangle = \sum_{i=1}^{2} A_i'\hat{A}_i', \qquad \langle\mathbf{E}\rangle = E\hat{E}, \tag{A2}$$

where in the notation of Ref. [39], the unit directions $\hat{A}_i$ and $\hat{E}$ in the Y-diagonal basis are given by

$$\begin{aligned}
\hat{A}_1 &= \hat{A}^{(1,1,0)}_{(1,1,3)} = \frac{i}{2}[78 + 90] = \hat{A}_1',\\
\hat{A}_2 &= \hat{A}^{(1,1,0)}_{(15,1,1)} = \frac{i}{\sqrt{6}}[12 + 34 + 56] = \hat{A}_2',\\
\hat{E} &= \hat{E}^{(1,1,0)}_{(1,1,1)} = \frac{1}{\sqrt{60}}(-2\times[12 + 34 + 56]\\
&\quad + 3\times[78 + 90]),
\end{aligned} \tag{A3}$$

where the upper and lower indices denote the $\mathbf{3}_c\mathbf{2}_L\mathbf{1}_Y$ and $\mathbf{4}_c\mathbf{2}_L\mathbf{2}_R$ quantum numbers, respectively. The unit directions in Eq. (A2) satisfy the orthonormality relations

$$\hat{A}_i\cdot\hat{A}_j = \delta_{ij} \quad \text{and} \quad \hat{E}\cdot\hat{E} = 1. \tag{A4}$$

The superpotential of Eq. (A1) calculated at the VEVs in Eq. (A2) is given by

$$\begin{aligned}
\langle W_H\rangle &= \frac{1}{2}m_1\langle A\rangle^2 + \frac{1}{2}m_1'\langle A'\rangle^2 + \frac{1}{2}m_2\langle E\rangle^2 + \lambda_1\langle E\rangle^3\\
&\quad + \lambda_2\langle E\rangle\langle A\rangle^2 + \lambda_2'\langle E\rangle\langle A'\rangle^2 + \lambda_3\langle E\rangle\langle A\rangle\langle A'\rangle\\
&= \frac{1}{2}m_1(A_1^2 + A_2^2) + \frac{1}{2}m_1'(A_1'^2 + A_2'^2) + \frac{1}{2}m_2E^2\\
&\quad + \frac{\lambda_1}{2\sqrt{15}}E^3 + \frac{E}{2\sqrt{15}}[\lambda_2(3A_1^2 - 2A_2^2)\\
&\quad + \lambda_2'(3A_1'^2 - 2A_2'^2) + \lambda_3(3A_1A_1' - 2A_2A_2')]
\end{aligned} \tag{A5}$$

using the definitions in Eqs. (A3) and the orthonormality relations given by Eqs. (A4). The VEVs are determined by the minimization of the superpotential with respect to the fields:

$$\left\{\frac{\partial}{\partial A_1}, \frac{\partial}{\partial A_2}, \frac{\partial}{\partial A_1'}, \frac{\partial}{\partial A_2'}, \frac{\partial}{\partial E}\right\}\langle W_H\rangle = 0. \tag{A6}$$

This yields a set of five equations for A_1, A_2, A_1', A_2', and E:

$$\begin{aligned}
0 &= m_1A_1 + \frac{3}{\sqrt{15}}\lambda_2EA_1 + \frac{3}{2\sqrt{15}}\lambda_3EA_1',\\
0 &= m_1A_2 - \frac{2}{\sqrt{15}}\lambda_2EA_2 - \frac{2}{2\sqrt{15}}\lambda_3EA_2',\\
0 &= m_1'A_1' + \frac{3}{\sqrt{15}}\lambda_2'EA_1' + \frac{3}{2\sqrt{15}}\lambda_3EA_1,\\
0 &= m_1'A_2' - \frac{2}{\sqrt{15}}\lambda_2'EA_2' - \frac{2}{2\sqrt{15}}\lambda_3EA_2,\\
0 &= m_2E + \frac{1}{2\sqrt{15}}[3\lambda_1E^2 + \lambda_2(3A_1^2 - 2A_2^2)\\
&\quad + \lambda_2'(3A_1'^2 - 2A_2'^2) + \lambda_3(3A_1A_1' - 2A_2A_2')].
\end{aligned} \tag{A7}$$

As in our model, the $SO(10)$ symmetry is broken by the **45** and **54** VEVs to the $\mathbf{3}_c\mathbf{2}_L\mathbf{2}_R\mathbf{1}_{B-L}$ gauge group at the scale M_G, and we are interested in the $\mathbf{3}_c\mathbf{2}_L\mathbf{2}_R\mathbf{1}_{B-L}$ symmetry solutions [39]

$$A_1 = A_1' = 0, \qquad A_2 \neq 0, \qquad A_2' \neq 0, \qquad E \neq 0.$$

Hence it follows from Eqs. (A7) that

$$m_1 - \frac{2\lambda_2E}{\sqrt{15}} = \frac{\lambda_3E}{\sqrt{15}}\frac{A_2'}{A_2}, \qquad m_1' - \frac{2\lambda_2'E}{\sqrt{15}} = \frac{\lambda_3E}{\sqrt{15}}\frac{A_2}{A_2'}. \tag{A8}$$

In order to study the mass matrices, it is convenient to decompose the Higgs representations under the SM gauge group $\mathbf{3}_c\mathbf{2}_L\mathbf{1}_Y$. In Table III we present the explicit decompositions of all the Higgs representations under the chain of subgroups

TABLE III. Decomposition of the **10**, **16**, **45**, and **54** Higgs representations under the chain of subgroups $\mathbf{4}_c\mathbf{2}_L\mathbf{2}_R \supset \mathbf{3}_c\mathbf{2}_L\mathbf{2}_R\mathbf{1}_{B-L} \supset \mathbf{3}_c\mathbf{2}_L\mathbf{1}_Y$.

$SO(10)$	$\mathbf{4}_c$, $\mathbf{2}_L$, $\mathbf{2}_R$	$\mathbf{3}_c$, $\mathbf{2}_L$, $\mathbf{2}_R$, $\mathbf{1}_{B-L}$	$\mathbf{3}_c$, $\mathbf{2}_L$, $\mathbf{1}_Y$
10	(1, 2, 2)	(1, 2, 2, 0)	(1, 2, ± 1)
	(6, 1, 1)	(3, 1, 1, $-\frac{2}{3}$) ($\bar{3}$, 1, 1, $\frac{2}{3}$)	(3, 1, $-\frac{2}{3}$) ($\bar{3}$, 1, $\frac{2}{3}$)
16	(4, 2, 1)	(3, 2, 1, $\frac{1}{3}$) (1, 2, 1, -1)	(3, 2, $\frac{1}{3}$) (1, 2, -1)
	($\bar{4}$, 1, 2)	($\bar{3}$, 1, 2, $-\frac{1}{3}$)	($\bar{3}$, 1, $\frac{2}{3}$) ($\bar{3}$, 1, $-\frac{4}{3}$)
		(1, 1, 2, 1)	(1, 1, 2) (1, 1, 0)
45	(1, 1, 3)	(1, 1, 3, 0)	(1, 1, 2) (1, 1, 0) (1, 1, -2)
	(1, 3, 1)	(1, 3, 1, 0)	(1, 3, 0)
	(6, 2, 2)	(3, 2, 2, $-\frac{2}{3}$)	(3, 2, $-\frac{1}{3}$) (3, 2, $-\frac{5}{3}$)
		($\bar{3}$, 2, 2, $\frac{2}{3}$)	($\bar{3}$, 2, $\frac{5}{3}$) ($\bar{3}$, 2, $-\frac{1}{3}$)
	(15, 1, 1)	(1, 1, 1, 0) (3, 1, 1, $\frac{4}{3}$) ($\bar{3}$, 1, 1, $-\frac{4}{3}$) (8, 1, 1, 0)	(1, 1, 0) (3, 1, $\frac{4}{3}$) ($\bar{3}$, 1, $-\frac{4}{3}$) (8, 1, 0)
54	(1, 1, 1)	(1, 1, 1, 0)	(1, 1, 0)
	(1, 3, 3)	(1, 3, 3, 0)	(1, 3, 2) (1, 3, 0) (1, 3, -2)
	(6, 2, 2)	(3, 2, 2, $-\frac{2}{3}$)	(3, 2, $\frac{1}{3}$) (3, 2, $-\frac{5}{3}$)
		($\bar{3}$, 2, 2, $\frac{2}{3}$)	($\bar{3}$, 2, $\frac{5}{3}$) ($\bar{3}$, 2, $-\frac{1}{3}$)
	(20′, 1, 1)	(6, 1, 1, $-\frac{4}{3}$) ($\bar{6}$, 1, 1, $\frac{4}{3}$) (8, 1, 1, 0)	(6, 1, $-\frac{4}{3}$) ($\bar{6}$, 1, $\frac{4}{3}$) (8, 1, 0)

$$\mathbf{4}_c\mathbf{2}_L\mathbf{2}_R \supset \mathbf{3}_c\mathbf{2}_L\mathbf{2}_R\mathbf{1}_{B-L} \supset \mathbf{3}_c\mathbf{2}_L\mathbf{1}_Y.$$

Using the Clebsch-Gordan coefficients given in Ref. [39], we obtain the masses of these multiplets as follows. The basis designating columns (c) of the mass matrices is given in the same way as in Table III, while rows (r) are designated by the corresponding complex conjugated $\mathbf{3}_c\mathbf{2}_L\mathbf{1}_Y$ multiplets.

First, we obtain the masses of the multiplet $[(3, 1, \frac{4}{3}) + \text{c.c.}]$ in the basis

$$\text{c}: \hat{A}^{(3,1,4/3)}_{(15,1,1)}, \hat{A}'^{(3,1,4/3)}_{(15,1,1)}; \qquad \text{r}: \hat{A}^{(\bar{3},1,-4/3)}_{(15,1,1)}, \hat{A}'^{(\bar{3},1,-4/3)}_{(15,1,1)};$$

$$M_\delta^{(3,1,4/3)} = \begin{pmatrix} m_1 - \frac{2\lambda_2 E}{\sqrt{15}} & -\frac{\lambda_3 E}{\sqrt{15}} \\ -\frac{\lambda_3 E}{\sqrt{15}} & m_1' - \frac{2\lambda_2' E}{\sqrt{15}} \end{pmatrix} = \frac{\lambda_3 E}{\sqrt{15}} \begin{pmatrix} \frac{A_2'}{A_2} & -1 \\ -1 & \frac{A_2}{A_2'} \end{pmatrix} \tag{A9}$$

using Eq. (A8). It is obvious that $\det(M_\delta) = 0$, and hence, one of the two eigenvalues is zero while the other eigenvalue is given by

$$\mathrm{Tr}\,(M_\delta) = \frac{\lambda_3 E}{\sqrt{15}}\left(\frac{A_2'}{A_2} + \frac{A_2}{A_2'}\right). \tag{A10}$$

The zero eigenvalues (six in total) are easily identified as the longitudinal Nambu-Goldstone modes as the $SU(4)_c$ gauge group breaks to $SU(3)_c \times U(1)_{B-L}$ and they acquire mass of order M_G by the usual Higgs mechanism once the $\mathbf{45}_H$ gets VEVs at the GUT scale. We keep the other six eigenvalues given by Eq. (A10) at the TeV scale by fine-tuning the coupling λ_3. In what follows, we explicitly calculate the mass eigenvalues for all the other multiplets given by Table III and show that it is possible to have only the above six massive δ's at the TeV scale while all the other states of **45** and **54** are heavy at the GUT scale.

We note that once we assume λ_3 to be small, the effect of the second $\mathbf{45}_H$ multiplet becomes negligible and we can drop the primed terms in the superpotential as well. For simplicity, we also assume that $A_2 = E \sim M_G$. Then the VEV conditions given by Eqs. (A7) yield

$$m_1 \simeq \frac{2\lambda_2 E}{\sqrt{15}}, \qquad m_2 \simeq \frac{E}{\sqrt{15}}\left(\lambda_2 - \frac{3}{2}\lambda_1\right). \tag{A11}$$

We list below the mass eigenvalues for all the multiplets given in Table III.

(i) (1, 1, 0): We have three such states, and the mass matrix is given by

$$\text{c}: \hat{A}^{(1,1,0)}_{(1,1,3)}, \hat{A}^{(1,1,0)}_{(15,1,1)}, \hat{E}^{(1,1,0)}_{(1,1,1)};$$
$$\text{r}: \hat{A}^{(1,1,0)}_{(1,1,3)}, \hat{A}^{(1,1,0)}_{(15,1,1)}, \hat{E}^{(1,1,0)}_{(1,1,1)};$$
$$\begin{pmatrix} m_1 + \frac{3\lambda_2 E}{\sqrt{15}} & 0 & \frac{3\lambda_2 A_1}{\sqrt{15}} \\ 0 & m_1 - \frac{2\lambda_2 E}{\sqrt{15}} & -\frac{2\lambda_2 A_2}{\sqrt{15}} \\ \frac{3\lambda_2 A_1}{\sqrt{15}} & -\frac{2\lambda_2 A_2}{\sqrt{15}} & m_2 + \frac{3\lambda_1 E}{\sqrt{15}} \end{pmatrix} = \frac{E}{\sqrt{15}} \begin{pmatrix} 5\lambda_2 & 0 & 0 \\ 0 & 0 & -2\lambda_2 \\ 0 & -2\lambda_2 & \lambda_2 + \frac{3}{2}\lambda_1 \end{pmatrix}.$$

So the mass eigenvalues are

$$M_1^{(1,1,0)} = \frac{5E}{\sqrt{15}}\lambda_2 \neq 0,$$
$$M_2^{(1,1,0)} = \frac{E}{2\sqrt{15}}\left[\left(\lambda_2 + \frac{3}{2}\lambda_1\right) + \sqrt{\left(\lambda_2 + \frac{3}{2}\lambda_1\right)^2 + 16\lambda_2^2}\right] \neq 0,$$
$$M_3^{(1,1,0)} = \frac{E}{2\sqrt{15}}\left[\left(\lambda_2 + \frac{3}{2}\lambda_1\right) - \sqrt{\left(\lambda_2 + \frac{3}{2}\lambda_1\right)^2 + 16\lambda_2^2}\right] \neq 0. \quad \text{(A12)}$$

(ii) $[(1, 1, 2) + \text{c.c.}]$: There is only one such multiplet, and its mass is

$$\text{c}: \hat{A}^{(1,1,2)}_{(1,1,3)}, \qquad \text{r}: \hat{A}^{(1,1,-2)}_{(1,1,3)},$$
$$M^{(1,1,2)} = m_1 + \frac{3}{\sqrt{15}}\lambda_2 E = \frac{5E}{\sqrt{15}}\lambda_2 \neq 0. \quad \text{(A13)}$$

(iii) $[(3, 2, -\frac{5}{3}) + \text{c.c.}]$: There are two such multiplets, and the mass matrix is

$$\text{c}: \hat{A}^{(3,2,-\frac{5}{3})}_{(6,2,2)}, \hat{E}^{(3,2,-\frac{5}{3})}_{(6,2,2)}; \qquad \text{r}: \hat{A}^{(\bar{3},2,\frac{5}{3})}_{(6,2,2)}, \hat{E}^{(\bar{3},2,\frac{5}{3})}_{(6,2,2)};$$
$$\begin{pmatrix} m_1 + \frac{\lambda_2 E}{2\sqrt{15}} & -\frac{\lambda_2 A_1}{2} - \frac{\lambda_2 A_2}{\sqrt{6}} \\ -\frac{\lambda_2 A_1}{2} - \frac{\lambda_2 A_2}{\sqrt{6}} & m_2 + \frac{3\lambda_1 E}{2\sqrt{15}} \end{pmatrix} = \frac{\lambda_2 E}{\sqrt{15}}\begin{pmatrix} 3 & -\sqrt{\frac{5}{2}} \\ -\sqrt{\frac{5}{2}} & 1 \end{pmatrix}$$

with the eigenvalues

$$M_{1,2}^{(3,2,-5/3)} = \frac{E\lambda_2}{2\sqrt{15}}[4 \pm \sqrt{14}] \neq 0. \quad \text{(A14)}$$

(iv) $[(3, 2, \frac{1}{3}) + \text{c.c.}]$: There are two of them, and the mass matrix is

$$\text{c}: \hat{A}^{(3,2,1/3)}_{(6,2,2)}, \hat{E}^{(3,2,1/3)}_{(6,2,2)}; \qquad \text{r}: \hat{A}^{(\bar{3},2,-1/3)}_{(6,2,2)}, \hat{E}^{(\bar{3},2,-1/3)}_{(6,2,2)};$$
$$\begin{pmatrix} m_1 + \frac{\lambda_2 E}{2\sqrt{15}} & \frac{\lambda_2 A_1}{2} - \frac{\lambda_2 A_2}{\sqrt{6}} \\ \frac{\lambda_2 A_1}{2} - \frac{\lambda_2 A_2}{\sqrt{6}} & m_2 + \frac{3\lambda_1 E}{2\sqrt{15}} \end{pmatrix} = \frac{\lambda_2 E}{\sqrt{15}}\begin{pmatrix} 3 & -\sqrt{\frac{5}{2}} \\ -\sqrt{\frac{5}{2}} & 1 \end{pmatrix}$$

with the same eigenvalues as the previous one:

$$M_{1,2}^{(3,2,(1/3))} = \frac{E\lambda_2}{2\sqrt{15}}[4 \pm \sqrt{14}] \neq 0. \quad \text{(A15)}$$

(v) (1, 3, 0): There are also two of them, and the mass matrix is

$$\text{c}: \hat{A}^{(1,3,0)}_{(1,3,1)}, \hat{E}^{(1,3,0)}_{(1,3,3)}; \qquad \text{r}: \hat{A}^{(1,3,0)}_{(1,3,1)}, \hat{E}^{(1,3,0)}_{(1,3,3)};$$
$$\begin{pmatrix} m_1 + \frac{3\lambda_2 E}{\sqrt{15}} & \lambda_2 A_1 \\ \lambda_2 A_1 & m_2 + \frac{9\lambda_1 E}{\sqrt{15}} \end{pmatrix} = \frac{E}{\sqrt{15}}\begin{pmatrix} 5\lambda_2 & 0 \\ 0 & \lambda_2 + \frac{15}{2}\lambda_1 \end{pmatrix}.$$

So the mass eigenvalues are

$$M_1^{(1,3,0)} = \frac{5E}{\sqrt{15}}\lambda_2 \neq 0,$$
$$M_2^{(1,3,0)} = \frac{E}{\sqrt{15}}\left(\lambda_2 + \frac{15}{2}\lambda_1\right) \neq 0. \quad \text{(A16)}$$

(vi) $[(1, 3, 2) + \text{c.c.}]$: There is only one such multiplet whose eigenvalue is given by

$$\text{c}: \hat{E}^{(1,3,2)}_{(1,3,3)}, \qquad \text{r}: \hat{E}^{(1,3,-2)}_{(1,3,3)},$$
$$M^{(1,3,2)} = m_2 + \frac{9}{\sqrt{15}}\lambda_1 E = \frac{E}{\sqrt{15}}\left(\lambda_2 + \frac{15}{2}\lambda_1\right) \neq 0. \quad \text{(A17)}$$

(vii) $[(6, 1, -\frac{4}{3}) + \text{c.c.}]$: Its eigenvalue is

$$\text{c}: \hat{E}^{(6,1,-4/3)}_{(20',1,1)}, \qquad \text{r}: \hat{E}^{(6,1,4/3)}_{(20',1,1)},$$
$$M^{(6,1,-4/3)} = m_2 - \frac{6}{\sqrt{15}}\lambda_1 E = \frac{E}{\sqrt{15}}\left(\lambda_2 - \frac{15}{2}\lambda_1\right) \neq 0 \quad \text{(A18)}$$

unless $\lambda_2 = \frac{15}{2}\lambda_1$ (which we assume not to be the case).

(viii) (8, 1, 0): There are two of them, and the mass matrix is

$$\text{c}: \hat{A}^{(8,1,0)}_{(15,1,1)}, \hat{E}^{(8,1,0)}_{(20',1,1)}; \qquad \text{r}: \hat{A}^{(8,1,0)}_{(15,1,1)}, \hat{E}^{(8,1,0)}_{(20',1,1)};$$
$$\begin{pmatrix} m_1 - \frac{2\lambda_2 E}{\sqrt{15}} & \sqrt{\frac{2}{3}}\lambda_2 A_2 \\ \sqrt{\frac{2}{3}}\lambda_2 A_2 & m_2 - \frac{6\lambda_1 E}{\sqrt{15}} \end{pmatrix} = \frac{E}{\sqrt{15}}\begin{pmatrix} 0 & \sqrt{\frac{45}{2}}\lambda_2 \\ \sqrt{\frac{45}{2}}\lambda_2 & \lambda_2 - \frac{15}{2}\lambda_1 \end{pmatrix}$$

with the mass eigenvalues

$$M_{1,2}^{(8,1,0)} = \frac{E}{2\sqrt{15}}\left[\left(\lambda_2 - \frac{15}{2}\lambda_1\right)^2 \pm \sqrt{\left(\lambda_2 - \frac{15}{2}\lambda_1\right)^2 + 90\lambda_2^2}\right] \neq 0. \tag{A19}$$

Thus we see that all the other multiplets have nonzero masses, and moreover, all these masses are of order $E \sim M_G$. Hence, none of these multiplets will contribute to the running of the gauge coupling up to the unification scale M_G except the color triplets since these color triplets have masses of order of the SUSY-breaking scale.

Note that the $\mathbf{10}_H$-Higgs field also has a color triplet pair $[(3, 1, -\frac{2}{3}) + \text{c.c.}]$ under the SM gauge group, apart from the TeV scale bidoublet fields $\Phi_{1,2}$ used in the SUSYLR model in Sec. IV which reduce to $(1, 2, \pm 1)$ under the SM gauge group. At the GUT scale, the $\mathbf{H} \equiv \mathbf{10}_H$ field interacts with the $\mathbf{E} \equiv \mathbf{54}_H$ field by the following term in the superpotential:

$$W_{10} = \tfrac{1}{2}m_3\mathbf{H}^2 + \lambda_3\mathbf{E}\mathbf{H}^2. \tag{A20}$$

After the $\mathbf{54}_H$ acquires a VEV, this gives rise to the color triplet mass

$$\text{c}: \hat{H}^{(3,1,-2/3)}_{(6,1,1)}, \qquad \text{r}: \hat{H}^{(\bar{3},1,2/3)}_{(6,1,1)}, \qquad M^{(3,1,2/3)} = m_3 - \frac{2\lambda_3 E}{\sqrt{15}}, \tag{A21}$$

while the doublet mass is

$$\text{c}: \hat{H}^{(1,2,1)}_{(1,2,2)}, \qquad \text{r}: \hat{H}^{(1,2,-1)}_{(1,2,2)}, \qquad M^{(1,2,1)} = m_3 + \sqrt{\tfrac{3}{5}}\lambda_3 E. \tag{A22}$$

We see that the $(1, 2, \pm 1)$ field can be made light by fine-tuning $m_3 + \sqrt{\frac{3}{5}}\lambda_3 E \sim$ TeV, which still leaves the $(3, 1, \frac{2}{3})$ field heavy (of order M_G).

Finally, let us discuss how only the right-handed doublet fields $(\chi^c, \bar{\chi}^c)$ from $\mathbf{16}_H$-Higgs fields (ψ_H) remain massless at the GUT scale. Note that in the left-right language, the fields in $\mathbf{16}$ are $Q_H(3, 2, 1, \frac{1}{3}) \oplus Q_H^c(\bar{3}, 1, 2, -\frac{1}{3})$ and $\chi(1, 2, 1, -1) \oplus \chi^c(1, 1, 2, +1)$, and similarly for the $\overline{\mathbf{16}}_H \equiv \bar{\psi}_H$ field. The superpotential involving these fields is

$$W_{16} = M_{16}\bar{\psi}_H\psi_H + \lambda\bar{\psi}_H A\psi_H. \tag{A23}$$

The second coupling has been worked out explicitly in Ref. [40]. On substituting the VEV of the $\mathbf{45}$-Higgs field (A), we get the following masses for the $Q_H(3, 2, 1, \frac{1}{3}) \oplus Q_H^c(\bar{3}, 1, 2, -\frac{1}{3})$ and $\chi(1, 2, 1, -1) \oplus \chi^c(1, 1, 2, +1)$ fields:

$$M_{Q_H-\bar{Q}_H} = M_{16} + \lambda A_2; \qquad M_{Q_H^c-\bar{Q}_H^c} = M_{16} - \lambda A_2;$$
$$M_{\chi-\bar{\chi}} = M_{16} - 3\lambda A_2; \qquad M_{\chi^c-\bar{\chi}^c} = M_{16} + 3\lambda A_2. \tag{A24}$$

From this we see that to get only the χ^c fields light, we have to fine-tune $M_{16} + 3\lambda A_2 \sim$ TeV. With this assumption, all other fields remain heavy at the GUT scale.

APPENDIX B: RGES FOR FERMION MASSES AND MIXING

Given the form of the bidoublet VEVs as in Eq. (30), it immediately follows from the first two terms of the superpotential Eq. (27) that the fermion mass matrices can be written as

$$M_u = \frac{1}{\sqrt{2}}v_u h_2, \qquad M_d = \frac{1}{\sqrt{2}}v_d h_1,$$
$$M_e = \frac{1}{\sqrt{2}}v_d h_1', \quad \text{and} \quad M_D = \frac{1}{\sqrt{2}}v_u h_2'. \tag{B1}$$

Henceforth, for clarity, we will denote the Yukawa couplings as

$$h_U \equiv h_2, \qquad h_D \equiv h_1, \qquad h_E \equiv h_1', \qquad h_N \equiv h_2'.$$

Then using Eqs. (28), (29), (31), and (32), the RGEs for the fermion mass matrices can be written as

$$16\pi^2\frac{dM_u}{dt} = M_u\left[4h_U^\dagger h_U + 2h_D^\dagger h_D - \sum_i \tilde{C}_i^{(q)} g_i^2\right] + M_d\tan\beta[\text{Tr}(3h_D^\dagger h_U + h_E^\dagger h_N) + 2h_D^\dagger h_U + C_{12}^\Phi], \tag{B2}$$

$$16\pi^2\frac{dM_d}{dt} = M_d\left[4h_D^\dagger h_D + 2h_U^\dagger h_U - \sum_i \tilde{C}_i^{(q)} g_i^2\right] + \frac{M_u}{\tan\beta} \times [\text{Tr}(3h_U^\dagger h_D + h_N^\dagger h_E) + 2h_U^\dagger h_D + C_{21}^\Phi], \tag{B3}$$

$$16\pi^2\frac{dM_e}{dt} = M_e\left[4h_E^\dagger h_E + 2h_N^\dagger h_N + C^\chi - \sum_i \tilde{C}_i^{(l)} g_i^2\right] + \frac{M_D}{\tan\beta}[\text{Tr}(3h_U^\dagger h_D + h_N^\dagger h_E) + 2h_N^\dagger h_E + C_{21}^\Phi], \tag{B4}$$

$$16\pi^2 \frac{dM_D}{dt} = M_D\left[4h_N^\dagger h_N + 2h_E^\dagger h_E + C^\chi - \sum_i \tilde{C}_i^{(l)} g_i^2\right] + M_e \tan\beta[\mathrm{Tr}(3h_D^\dagger h_U + h_E^\dagger h_N) + 2h_E^\dagger h_N + C_{12}^\Phi], \tag{B5}$$

where $C_{ab}^\Phi = 4(\mu_\alpha^{\Phi\dagger}\mu_\alpha^\Phi)_{ab}$, $C^\chi = (\mu^{L^c})_{\alpha q}^\dagger \mu_{\alpha q}^{L^c}$, and for $i = \mathbf{3}_c, \mathbf{2}_L, \mathbf{2}_R, \mathbf{1}_{B-L}$,

$$\tilde{C}_i^{(q)} = \left(\frac{16}{3}, \frac{3}{2}, \frac{3}{2}, \frac{1}{6}\right), \qquad \tilde{C}_i^{(l)} = \left(0, \frac{3}{2}, \frac{3}{2}, \frac{3}{2}\right). \tag{B6}$$

Note that the second line in Eq. (B3) and the second and third lines in Eqs. (B2), (B4), and (B5) are characteristic of the left-right models, and do not appear in the MSSM.

Not all the parameters of the Yukawa matrices are physical. Under an arbitrary unitary transformation on the left (right)-handed fermion fields, $\mathcal{F}_{L(R)} \to L(R)_f \mathcal{F}_{L(R)}$ (where $\mathcal{F} = U, D, E, N$), the Yukawa matrices undergo a bi-unitary transformation, $h_f \to L_f h_f R_f^\dagger$, and the charged current becomes off-diagonal, with the CKM mixing matrix $L_U L_D^\dagger$. We will also have a leptonic counterpart of the CKM matrix that represents the mixing between the charged lepton and the Dirac neutrino sector. However, as the running of lepton masses is very mild and we are working only to the one-loop order, we can safely ignore this mixing in the leptonic sector. Moreover, if we assume the CP phase in the Higgs VEV to be zero, then the mass matrices are Hermitian and $L_f = R_f$ (manifest left-right). Thus we may perform scale-dependent unitary transformations $L_f(\mu)$ on the fermion bases so as to diagonalize the Yukawa matrices, and hence the mass matrices, at each scale:

$$\hat{h}_f(\mu) = L_f(\mu) h_f(\mu) L_f^\dagger(\mu) \quad \text{and} \quad \hat{M}_f = L_f(\mu) M_f(\mu) L_f^\dagger(\mu), \tag{B7}$$

where $\hat{h}_f$ and $\hat{M}_f$ denote the diagonalized Yukawa and mass matrices, respectively.

The RGEs for the physically relevant quantities, namely, the mass eigenvalues $\hat{M}_f(\mu)$ and the scale-dependent CKM matrix $V_{\mathrm{CKM}}(\mu) = L_U(\mu) L_D^\dagger(\mu)$, are both contained in the RGEs of $\hat{M}_f^2(\mu) = L_f^\dagger(\mu) M_f(\mu) M_f^\dagger(\mu) L_f(\mu)$:

$$\frac{d}{dt}(\hat{M}_u^2) = [\dot{L}_U L_U^\dagger, \hat{M}_u^2] + \frac{1}{16\pi^2}\left[4\hat{h}_U^2 + 2\hat{h}_D^2 - \sum_i \tilde{C}_i^{(q)} g_i^2\right] 2\hat{M}_u^2 + \frac{1}{16\pi^2}\tan\beta[\{\mathrm{Tr}(3V_{\mathrm{CKM}}\hat{h}_D V_{\mathrm{CKM}}^\dagger \hat{h}_U) + C_{12}^\Phi\}(V_{\mathrm{CKM}}\hat{M}_d V_{\mathrm{CKM}}^\dagger \hat{M}_u) + 2V_{\mathrm{CKM}}\hat{M}_d \hat{h}_D V_{\mathrm{CKM}}^\dagger \hat{h}_U \hat{M}_u + \mathrm{H.c.}], \tag{B8}$$

$$\frac{d}{dt}(\hat{M}_d^2) = [\dot{L}_D L_D^\dagger, \hat{M}_d^2] + \frac{1}{16\pi^2}\left[4\hat{h}_D^2 + 2\hat{h}_U^2 - \sum_i \tilde{C}_i^{(q)} g_i^2\right] 2\hat{M}_d^2 + \frac{1}{16\pi^2}\frac{1}{\tan\beta}[\{\mathrm{Tr}(3V_{\mathrm{CKM}}\hat{h}_D V_{\mathrm{CKM}}^\dagger \hat{h}_U) + C_{12}^\Phi\}(\hat{M}_d V_{\mathrm{CKM}}^\dagger \hat{M}_u V_{\mathrm{CKM}}) + 2\hat{M}_d \hat{h}_D V_{\mathrm{CKM}}^\dagger \hat{h}_U \hat{M}_u V_{\mathrm{CKM}} + \mathrm{H.c.}], \tag{B9}$$

$$\frac{d}{dt}(\hat{M}_e^2) = [\dot{L}_E L_E^\dagger, \hat{M}_e^2] + \frac{1}{16\pi^2}\left[4\hat{h}_E^2 + 2\hat{h}_N^2 + \mathrm{Re}(C^\chi) - \sum_i \tilde{C}_i^{(l)} g_i^2\right] 2\hat{M}_e^2, \tag{B10}$$

$$\frac{d}{dt}(\hat{M}_D^2) = [\dot{L}_N L_N^\dagger, \hat{M}_D^2] + \frac{1}{16\pi^2}\left[4\hat{h}_N^2 + 2\hat{h}_E^2 + \mathrm{Re}(C^\chi) - \sum_i \tilde{C}_i^{(l)} g_i^2\right] 2\hat{M}_D^2, \tag{B11}$$

where $\dot{L} \equiv \frac{dL}{dt}$ and $\mathrm{Re}(C^\chi)$ denotes the real part of C^χ. The commutator $[\dot{L}_f L_f^\dagger, \hat{M}_f^2]$ has vanishing diagonal elements because $\hat{M}_f^2$ is diagonal. Thus the RGEs for the mass eigenvalues m_f^2 follow immediately from the diagonal entries of Eqs. (B8)–(B11). Using the dominance of Yukawa couplings of the third generation over the first two, i.e.

$$h_t^2 \gg h_c^2 \gg h_u^2, \qquad h_b^2 \gg h_s^2 \gg h_d^2, \qquad h_\tau^2 \gg h_\mu^2 \gg h_e^2, \qquad h_{N_3}^2 \gg h_{N_2}^2 \gg h_{N_1}^2,$$

we obtain the following RGEs for the mass eigenvalues of the fermions:

$$
\begin{aligned}
16\pi^2\frac{dm_u}{dt} &\simeq \left(4h_u^2 + 2h_d^2 - \sum_i \tilde{C}_i^{(q)} g_i^2\right) m_u + \tan\beta[3|V_{tb}|^2 h_b h_t + r_q] \sum_{j=d,s,b} |V_{uj}|^2 m_j, \\
16\pi^2\frac{dm_c}{dt} &\simeq \left(4h_c^2 + 2h_s^2 - \sum_i \tilde{C}_i^{(q)} g_i^2\right) m_c + \tan\beta[3|V_{tb}|^2 h_b h_t + r_q] \sum_{j=d,s,b} |V_{cj}|^2 m_j, \\
16\pi^2\frac{dm_t}{dt} &\simeq \left(4h_t^2 + 2h_b^2 - \sum_i \tilde{C}_i^{(q)} g_i^2\right) m_t + \tan\beta[(3|V_{tb}|^2 + 2) h_b h_t + r_q] |V_{tb}|^2 m_b, \\
16\pi^2\frac{dm_d}{dt} &\simeq \left(4h_d^2 + 2h_u^2 - \sum_i \tilde{C}_i^{(q)} g_i^2\right) m_d + \frac{1}{\tan\beta}[3|V_{tb}|^2 h_b h_t + r_q] \sum_{j=u,c,t} |V_{jd}|^2 m_j, \\
16\pi^2\frac{dm_s}{dt} &\simeq \left(4h_s^2 + 2h_c^2 - \sum_i \tilde{C}_i^{(q)} g_i^2\right) m_s + \frac{1}{\tan\beta}[3|V_{tb}|^2 h_b h_t + r_q] \sum_{j=u,c,t} |V_{js}|^2 m_j, \\
16\pi^2\frac{dm_b}{dt} &\simeq \left(4h_b^2 + 2h_t^2 - \sum_i \tilde{C}_i^{(q)} g_i^2\right) m_b + \frac{1}{\tan\beta}[(3|V_{tb}|^2 + 2) h_b h_t + r_q] |V_{tb}|^2 m_t, \\
16\pi^2\frac{dm_e}{dt} &\simeq \left(4h_e^2 + 2h_{N_1}^2 + r_l - \sum_i \tilde{C}_i^{(l)} g_i^2\right) m_e, \\
16\pi^2\frac{dm_\mu}{dt} &\simeq \left(4h_\mu^2 + 2h_{N_2}^2 + r_l - \sum_i \tilde{C}_i^{(l)} g_i^2\right) m_\mu, \\
16\pi^2\frac{dm_\tau}{dt} &\simeq \left(4h_\tau^2 + 2h_{N_3}^2 + r_l - \sum_i \tilde{C}_i^{(l)} g_i^2\right) m_\tau, \\
16\pi^2\frac{dm_{N_1}}{dt} &\simeq \left(4h_{N_1}^2 + 2h_e^2 + r_l - \sum_i \tilde{C}_i^{(l)} g_i^2\right) m_{N_1}, \\
16\pi^2\frac{dm_{N_2}}{dt} &\simeq \left(4h_{N_2}^2 + 2h_\mu^2 + r_l - \sum_i \tilde{C}_i^{(l)} g_i^2\right) m_{N_2}, \\
16\pi^2\frac{dm_{N_3}}{dt} &\simeq \left(4h_{N_3}^2 + 2h_\tau^2 + r_l - \sum_i \tilde{C}_i^{(l)} g_i^2\right) m_{N_3},
\end{aligned}
\tag{B12}
$$

where $r_q = \mathrm{Re}(C_{12}^{\Phi})$ and $r_l = \mathrm{Re}(C^{\chi})$.

The VEV RGEs, Eqs. (31) and (32), for third-generation dominance become

$$
16\pi^2\frac{dv_u}{dt} \simeq v_u\left[\frac{3}{2}g_{2L}^2 + \frac{3}{2}g_{2R}^2 - 3h_t^2 - h_{N_3}^2 - C_{22}^{\Phi}\right], \tag{B13}
$$

$$
16\pi^2\frac{dv_d}{dt} \simeq v_d\left[\frac{3}{2}g_{2L}^2 + \frac{3}{2}g_{2R}^2 - 3h_b^2 - h_\tau^2 - C_{11}^{\Phi}\right]. \tag{B14}
$$

The RGE for the CKM matrix $V_{\mathrm{CKM}} = L_U L_D^\dagger$ is given by

$$
\frac{d}{dt}V_{\mathrm{CKM}} = \dot{L}_U L_D^\dagger + L_U \dot{L}_D^\dagger = \dot{L}_U L_U^\dagger V_{\mathrm{CKM}} - V_{\mathrm{CKM}} \dot{L}_D L_D^\dagger
$$

$$
\text{or} \quad \frac{d}{dt}V_{\alpha\beta} = \sum_{\gamma=u,c,t} (\dot{L}_U L_U^\dagger)_{\alpha\gamma} V_{\gamma\beta} - \sum_{\gamma=d,s,b} V_{\alpha\gamma} (\dot{L}_D L_D^\dagger)_{\gamma\beta}. \tag{B15}
$$

However, the diagonal elements of $\dot{L}_{U,D} L_{U,D}^\dagger$ are not determined by Eqs. (B8) and (B9). This is because Eq. (B7) determines $L_{U,D}$ only up to right multiplication by a diagonal matrix of scale-dependent phases. These undetermined phases contribute arbitrary imaginary functions to the diagonal elements of $\dot{L}_{U,D} L_{U,D}^\dagger$. But the off-diagonal elements are unambiguously determined because they receive no contribution from the phases. We can, nevertheless, make the diagonal entries of $\dot{L}_{U,D} L_{U,D}^\dagger$, which are manifestly imaginary, vanish by an appropriate choice of phases. With this choice of phases, we can then obtain the RGEs for the CKM matrix elements using Eq. (B15):

$$\frac{d}{dt}V_{\alpha\beta} = \sum_{\substack{\gamma=u,c,t\\ \gamma\neq\alpha}}(\dot{L}_U L_U^\dagger)_{\alpha\gamma}V_{\gamma\beta} - \sum_{\substack{\gamma=d,s,b\\ \gamma\neq\beta}}V_{\alpha\gamma}(\dot{L}_D L_D^\dagger)_{\gamma\beta}$$
$$= \frac{1}{16\pi^2}\Bigg(\sum_{\substack{\gamma=u,c,t\\ \gamma\neq\alpha}}\Bigg[\frac{\tan\beta}{m_\alpha - m_\gamma}\{\mathrm{Tr}(3V\hat{h}_D V^\dagger \hat{h}_U) + r_q\}(V\hat{M}_d V^\dagger)_{\alpha\gamma} + \frac{4}{v_d^2}\frac{m_\alpha^2 + m_\gamma^2}{m_\alpha^2 - m_\gamma^2}(V\hat{M}_d^2 V^\dagger)_{\alpha\gamma}\Bigg]V_{\gamma\beta}$$
$$- \sum_{\substack{\gamma=d,s,b\\ \gamma\neq\beta}}V_{\alpha\gamma}\Bigg[\frac{1}{\tan\beta(m_\gamma - m_\beta)}\{\mathrm{Tr}(3V\hat{h}_D V^\dagger \hat{h}_U) + r_q\}(V^\dagger\hat{M}_u V)_{\gamma\beta} + \frac{4}{v_u^2}\frac{m_\gamma^2 + m_\beta^2}{m_\gamma^2 - m_\beta^2}(V^\dagger\hat{M}_u^2 V)_{\gamma\beta}\Bigg]\Bigg). \tag{B16}$$

As before, we use the third-generation dominance and get the following RGEs for $V_{\alpha\beta}$:

$$16\pi^2\frac{d}{dt}V_{ud} \simeq -\tan\beta(3|V_{tb}|^2 h_b h_t + r_q)\Bigg[\frac{(V\hat{M}_d V^\dagger)_{uc}V_{cd}}{m_c} + \frac{(V\hat{M}_d V^\dagger)_{ut}V_{td}}{m_t}\Bigg] - \frac{4}{v_d^2}[(V\hat{M}_d^2 V^\dagger)_{uc}V_{cd} + (V\hat{M}_d^2 V^\dagger)_{ut}V_{td}]$$
$$- \frac{1}{\tan\beta}(3|V_{tb}|^2 h_b h_t + r_q)\Bigg[\frac{V_{us}(V^\dagger\hat{M}_u V)_{sd}}{m_s} + \frac{V_{ub}(V^\dagger\hat{M}_u V)_{bd}}{m_b}\Bigg] - \frac{4}{v_u^2}[V_{us}(V^\dagger\hat{M}_u^2 V)_{sd} + V_{ub}(V^\dagger\hat{M}_u^2 V)_{bd}],$$

$$16\pi^2\frac{d}{dt}V_{us} \simeq -\tan\beta(3|V_{tb}|^2 h_b h_t + r_q)\Bigg[\frac{(V\hat{M}_d V^\dagger)_{uc}V_{cs}}{m_c} + \frac{(V\hat{M}_d V^\dagger)_{ut}V_{ts}}{m_t}\Bigg] - \frac{4}{v_d^2}[(V\hat{M}_d^2 V^\dagger)_{uc}V_{cs} + (V\hat{M}_d^2 V^\dagger)_{ut}V_{ts}]$$
$$- \frac{1}{\tan\beta}(3|V_{tb}|^2 h_b h_t + r_q)\Bigg[-\frac{V_{ud}(V^\dagger\hat{M}_u V)_{ds}}{m_s} + \frac{V_{ub}(V^\dagger\hat{M}_u V)_{bs}}{m_b}\Bigg]$$
$$- \frac{4}{v_u^2}[-V_{ud}(V^\dagger\hat{M}_u^2 V)_{ds} + V_{ub}(V^\dagger\hat{M}_u^2 V)_{bs}],$$

$$16\pi^2\frac{d}{dt}V_{ub} \simeq -\tan\beta(3|V_{tb}|^2 h_b h_t + r_q)\Bigg[\frac{(V\hat{M}_d V^\dagger)_{uc}V_{cb}}{m_c} + \frac{(V\hat{M}_d V^\dagger)_{ut}V_{tb}}{m_t}\Bigg] - \frac{4}{v_d^2}[(V\hat{M}_d^2 V^\dagger)_{uc}V_{cb} + (V\hat{M}_d^2 V^\dagger)_{ut}V_{tb}]$$
$$+ \frac{1}{m_b\tan\beta}(3|V_{tb}|^2 h_b h_t + r_q)[V_{ud}(V^\dagger\hat{M}_u V)_{db} + V_{us}(V^\dagger\hat{M}_u V)_{sb}] + \frac{4}{v_u^2}[V_{ud}(V^\dagger\hat{M}_u^2 V)_{db} + V_{us}(V^\dagger\hat{M}_u^2 V)_{sb}],$$

$$16\pi^2\frac{d}{dt}V_{cd} \simeq -\tan\beta(3|V_{tb}|^2 h_b h_t + r_q)\Bigg[-\frac{(V\hat{M}_d V^\dagger)_{cu}V_{ud}}{m_c} + \frac{(V\hat{M}_d V^\dagger)_{ct}V_{td}}{m_t}\Bigg] - \frac{4}{v_d^2}[-(V\hat{M}_d^2 V^\dagger)_{cu}V_{ud} + (V\hat{M}_d^2 V^\dagger)_{ct}V_{td}]$$
$$- \frac{1}{\tan\beta}(3|V_{tb}|^2 h_b h_t + r_q)\Bigg[\frac{V_{cs}(V^\dagger\hat{M}_u V)_{sd}}{m_s} + \frac{V_{cb}(V^\dagger\hat{M}_u V)_{bd}}{m_b}\Bigg] - \frac{4}{v_u^2}[V_{cs}(V^\dagger\hat{M}_u^2 V)_{sd} + V_{cb}(V^\dagger\hat{M}_u^2 V)_{bd}],$$

$$16\pi^2\frac{d}{dt}V_{cs} \simeq -\tan\beta(3|V_{tb}|^2 h_b h_t + r_q)\Bigg[-\frac{(V\hat{M}_d V^\dagger)_{cu}V_{us}}{m_c} + \frac{(V\hat{M}_d V^\dagger)_{ct}V_{ts}}{m_t}\Bigg] - \frac{4}{v_d^2}[-(V\hat{M}_d^2 V^\dagger)_{cu}V_{us} + (V\hat{M}_d^2 V^\dagger)_{ct}V_{ts}]$$
$$- \frac{1}{\tan\beta}(3|V_{tb}|^2 h_b h_t + r_q)\Bigg[-\frac{V_{cd}(V^\dagger\hat{M}_u V)_{ds}}{m_s} + \frac{V_{cb}(V^\dagger\hat{M}_u V)_{bs}}{m_b}\Bigg] - \frac{4}{v_u^2}[-V_{cd}(V^\dagger\hat{M}_u^2 V)_{ds} + V_{cb}(V^\dagger\hat{M}_u^2 V)_{bs}],$$

$$16\pi^2\frac{d}{dt}V_{cb} \simeq -\tan\beta(3|V_{tb}|^2 h_b h_t + r_q)\Bigg[-\frac{(V\hat{M}_d V^\dagger)_{cu}V_{ub}}{m_c} + \frac{(V\hat{M}_d V^\dagger)_{ct}V_{tb}}{m_t}\Bigg] - \frac{4}{v_d^2}[-(V\hat{M}_d^2 V^\dagger)_{cu}V_{ub} + (V\hat{M}_d^2 V^\dagger)_{ct}V_{tb}]$$
$$+ \frac{1}{m_b\tan\beta}(3|V_{tb}|^2 h_b h_t + r_q)[V_{cd}(V^\dagger\hat{M}_u V)_{db} + V_{cs}(V^\dagger\hat{M}_u V)_{sb}] + \frac{4}{v_u^2}[V_{cd}(V^\dagger\hat{M}_u^2 V)_{db} + V_{cs}(V^\dagger\hat{M}_u^2 V)_{sb}],$$

$$16\pi^2\frac{d}{dt}V_{td} \simeq \frac{\tan\beta}{m_t}(3|V_{tb}|^2 h_b h_t + r_q)[(V\hat{M}_d V^\dagger)_{tu}V_{ud} + (V\hat{M}_d V^\dagger)_{tc}V_{cd}] + \frac{4}{v_d^2}[(V\hat{M}_d^2 V^\dagger)_{tu}V_{ud} + (V\hat{M}_d^2 V^\dagger)_{tc}V_{cd}]$$
$$- \frac{1}{\tan\beta}(3|V_{tb}|^2 h_b h_t + r_q)\Bigg[\frac{V_{ts}(V^\dagger\hat{M}_u V)_{sd}}{m_s} + \frac{V_{tb}(V^\dagger\hat{M}_u V)_{bd}}{m_b}\Bigg] - \frac{4}{v_u^2}[V_{ts}(V^\dagger\hat{M}_u^2 V)_{sd} + V_{tb}(V^\dagger\hat{M}_u^2 V)_{bd}],$$

$$16\pi^2 \frac{d}{dt} V_{ts} \simeq \frac{\tan\beta}{m_t}(3|V_{tb}|^2 h_b h_t + r_q)[(V\hat{M}_d V^\dagger)_{tu} V_{us} + (V\hat{M}_d V^\dagger)_{tc} V_{cs}] + \frac{4}{v_d^2}[(V\hat{M}_d^2 V^\dagger)_{tu} V_{us} + (V\hat{M}_d^2 V^\dagger)_{tc} V_{cs}]$$
$$- \frac{1}{\tan\beta}(3|V_{tb}|^2 h_b h_t + r_q)\left[-\frac{V_{td}(V^\dagger \hat{M}_u V)_{ds}}{m_s} + \frac{V_{tb}(V^\dagger \hat{M}_u V)_{bs}}{m_b}\right] - \frac{4}{v_u^2}[-V_{td}(V^\dagger \hat{M}_u^2 V)_{ds} + V_{tb}(V^\dagger \hat{M}_u^2 V)_{bs}],$$

$$16\pi^2 \frac{d}{dt} V_{tb} \simeq \frac{\tan\beta}{m_t}(3|V_{tb}|^2 h_b h_t + r_q)[(V\hat{M}_d V^\dagger)_{tu} V_{ub} + (V\hat{M}_d V^\dagger)_{tc} V_{cb}] + \frac{4}{v_d^2}[(V\hat{M}_d^2 V^\dagger)_{tu} V_{ub} + (V\hat{M}_d^2 V^\dagger)_{tc} V_{cb}]$$
$$+ \frac{1}{m_b \tan\beta}(3|V_{tb}|^2 h_b h_t + r_q)[V_{td}(V^\dagger \hat{M}_u V)_{db} + V_{ts}(V^\dagger \hat{M}_u V)_{sb}] + \frac{4}{v_u^2}[V_{td}(V^\dagger \hat{M}_u^2 V)_{db} + V_{ts}(V^\dagger \hat{M}_u^2 V)_{sb}]. \tag{B17}$$

We have presented the results for these RGEs, even though they look quite messy, because we believe this is the first time such an analysis has been carried out in the SUSYLR model, and these analytical results at the one-loop level may be useful later for future work in this direction.

In order to solve these mass and mixing RGEs numerically, we need to know the initial values for all the 23 variables (12 masses, 9 CKM elements, and 2 VEVs). We know the experimental values at $\tilde{\mu} = M_Z$ for all of them except for the Dirac neutrino masses m_{N_i}. We fix these values by iterations using the GUT scale predicted values, $m_{N_i}(M_G)$, which, in turn, are determined completely in terms of the other fermion masses at the GUT scale in $SO(10)$ GUT models. Here we note that adjusting the GUT scale values of m_{N_i} to fit the $SO(10)$ model prediction does not change the other fermion masses at this scale significantly, even though they are all coupled equations, because of the mild running of the neutrino masses. Hence the mass and mixing values given in Eqs. (33) can be considered as generic and independent of the specific $SO(10)$ model chosen.

We also have the free parameters r_q and r_l corresponding to the couplings μ_α^Φ and $\mu_{\alpha q}^{L^c}$. Assuming the couplings μ_α to be the same, $\forall\ \alpha = 1, 2, 3$, we have

$$C_{ab}^\Phi = 4(\mu_\alpha^{\Phi\dagger}\mu_\alpha^\Phi)_{ab} = 12(\mu^{\Phi\dagger}\mu^\Phi)_{ab} = 12\sum_{c=1}^{2}\mu_{ca}^{\Phi *}\mu_{cb}^\Phi,$$
$$C^\chi = (\mu^{L^c})_{\alpha q}^\dagger \mu_{\alpha q}^{L^c} = 3[(\mu^{L^c})_q^* \mu_q^{L^c}].$$

Further assuming $\mu_{ab}^\Phi = \mu_\phi \quad \forall\ a, b = 1, 2$ and $\mu_q^{L^c} = \mu_l \quad \forall\ q = 1, 2$, we have

$$r_q = 24|\mu_\phi|^2, \qquad r_l = 6|\mu_l|^2,$$

where μ_ϕ and μ_l can take values between 0 and 1 (for the theory to remain perturbative).

For the running behavior shown in Fig. 2, we have chosen $\mu_\phi = 0.01$ and $\mu_l = 0.46$ (requiring $b - \tau$ unification) and the initial values of the Dirac neutrino masses

$$m_{N_1}(M_R) = 0.0031\ \text{GeV}, \qquad m_{N_2}(M_R) = 0.2825\ \text{GeV},$$
$$m_{N_3} = 71.86\ \text{GeV}$$

such that the masses evaluated at the GUT scale, $m_{N_i}(M_G)$, agree with those predicted by the specific $SO(10)$ model described in Sec. VI. For a consistency check, we note that the $SO(10)$ model predicted eigenvalues of M_D given by Eq. (45),

$$m_{N_i}^{\text{predicted}} = (0.0028, 0.2538, 77.8046)\ \text{GeV},$$

agree quite well with those obtained from the RGEs,

$$m_{N_i}^{\text{RG}}(M_G) = (0.0028, 0.2538, 77.8106)\ \text{GeV}.$$

[1] P. Minkowski, Phys. Lett. **67B**, 421 (1977); M. Gell-Mann, P. Ramond, and R. Slansky, *Supergravity*, edited by P. van Nieuwenhuizen *et al.* (North Holland, Amsterdam, 1980), p. 315; T. Yanagida, in *Proceedings of the Workshop on the Unified Theory and the Baryon Number in the Universe*, edited by O. Sawada and A. Sugamoto (KEK, Tsukuba, Japan, 1979), p. 95; S. L. Glashow, in *Proceedings of the 1979 Cargèse Summer Institute on Quarks and Leptons*, edited by M. Lévy *et al.* (Plenum Press, New York, 1980), p. 687; R. N. Mohapatra and G. Senjanović, Phys. Rev. Lett. **44**, 912 (1980).

[2] G. Lazarides, Q. Shafi, and C. Wetterich, Nucl. Phys. **B181**, 287 (1981); R. N. Mohapatra and G. Senjanović, Phys. Rev. D **23**, 165 (1981); J. Schecter and J. W. F. Valle, Phys. Rev. D **22**, 2227 (1980).

[3] R. Foot, H. Lew, X. G. He, and G. C. Joshi, Z. Phys. C **44**,

441 (1989).
[4] S. Antusch, C. Biggio, E. Fernández-Martínez, M. Belen Gavela, and J. López-Pavón, J. High Energy Phys. 10 (2006) 084.
[5] J. Kersten and A. Y. Smirnov, Phys. Rev. D **76**, 073005 (2007).
[6] A. Abada, C. Biggio, F. Bonnet, M. B. Gavela, and T. Hambye, J. High Energy Phys. 12 (2007) 061.
[7] R. N. Mohapatra and A. Y. Smirnov, Annu. Rev. Nucl. Part. Sci. **56**, 569 (2006).
[8] R. N. Mohapatra, Phys. Rev. Lett. **56**, 561 (1986); R. N. Mohapatra and J. W. F. Valle, Phys. Rev. D **34**, 1642 (1986).
[9] M. Malinský, T. Ohlsson, and H. Zhang, Phys. Rev. D **79**, 073009 (2009).
[10] A. Ilakovac and A. Pilaftsis, Nucl. Phys. **B437**, 491 (1995).
[11] F. Deppisch, T. S. Kosmas, and J. W. F. Valle, Nucl. Phys. **B752**, 80 (2006); J. Garayoa, M. C. Gonzalez-Garcia, and N. Rius, J. High Energy Phys. 02 (2007) 021; C. Arina, F. Bazzocchi, N. Fornengo, J. C. Romao, and J. W. F. Valle, Phys. Rev. Lett. **101**, 161802 (2008); M. Malinský, T. Ohlsson, Z.-z. Xing, and H. Zhang, Phys. Lett. B **679**, 242 (2009); E. Ma, Mod. Phys. Lett. A **24**, 2491 (2009); M. B. Gavela, T. Hambye, D. Hernandez, and P. Hernandez, J. High Energy Phys. 09 (2009) 038; M. Hirsch, T. Kernreiter, J. C. Romao, and A. Villanova del Moral, arXiv:0910.2435.
[12] T. Fukuyama, A. Ilakovac, T. Kikuchi, and K. Matsuda, J. High Energy Phys. 06 (2005) 016.
[13] G. 't Hooft, in *Proceedings of the 1979 Cargése Summer Institute on Recent Developments in Gauge Theories*, edited by G.'t Hooft *et al.* (Plenum Press, New York, 1980).
[14] K. Kanaya, Prog. Theor. Phys. **64**, 2278 (1980).
[15] G. Altarelli and D. Meloni, Nucl. Phys. **B809**, 158 (2009).
[16] C. Jarlskog, Phys. Rev. Lett. **55**, 1039 (1985).
[17] F. Deppisch and J. W. F. Valle, Phys. Rev. D **72**, 036001 (2005).
[18] S. Blanchet, Z. Chacko, S. S. Granor, and R. N. Mohapatra, arXiv:0904.2174.
[19] F. del Aguila and J. A. Aguilar-Saavedra, Phys. Lett. B **672**, 158 (2009); F. del Aguila, J. A. Aguilar-Saavedra, and J. de Blas, arXiv:0910.2720.
[20] N. G. Deshpande, E. Keith, and T. G. Rizzo, Phys. Rev. Lett. **70**, 3189 (1993).
[21] M. Malinsky, J. C. Romão, and J. W. F. Valle, Phys. Rev. Lett. **95**, 161801 (2005).
[22] B. Dutta, Y. Mimura, and R. N. Mohapatra, Phys. Rev. D **80**, 095021 (2009).
[23] S. P. Martin and M. T. Vaughn, Phys. Rev. D **50**, 2282 (1994).
[24] N. Setzer and S. Spinner, Phys. Rev. D **71**, 115010 (2005).
[25] H. Nishino *et al.* (Super-Kamiokande Collaboration), Phys. Rev. Lett. **102**, 141801 (2009).
[26] C. Amsler *et al.* (Particle Data Group), Phys. Lett. B **667**, 1 (2008).
[27] R. N. Mohapatra, *Unification and Supersymmetry* (Springer-Verlag, New York, 2003), 3rd ed., p. 185.
[28] D. Chang, R. N. Mohapatra, and M. K. Parida, Phys. Rev. Lett. **52**, 1072 (1984).
[29] C. R. Das and M. K. Parida, Eur. Phys. J. C **20**, 121 (2001).
[30] C. H. Albright and S. M. Barr, Phys. Rev. D **62**, 093008 (2000); K. S. Babu, J. C. Pati, and F. Wilczek, Nucl. Phys. **B566**, 33 (2000); X. Ji, Y. Li, and R. N. Mohapatra, Phys. Lett. B **633**, 755 (2006), and references therein.
[31] R. N. Mohapatra and B. Sakita, Phys. Rev. D **21**, 1062 (1980).
[32] S. Antusch, J. P. Baumann, and E. Fernández-Martínez, Nucl. Phys. **B810**, 369 (2009).
[33] A. van der Schaaf, J. Phys. G **29**, 2755 (2003).
[34] Y. Kuno (PRIME Working Group), Nucl. Phys. B, Proc. Suppl. **149**, 376 (2005).
[35] G. L. Fogli, E. Lisi, A. Marrone, A. Palazzo, and A. M. Rotunno, Nucl. Phys. B, Proc. Suppl. **188**, 27 (2009).
[36] E. Fernández-Martínez, M. B. Gavela, J. López-Pavón, and O. Yasuda, Phys. Lett. B **649**, 427 (2007).
[37] G. Altarelli and D. Meloni, Nucl. Phys. **B809**, 158 (2009); S. Antusch, M. Blennow, and E. Fernández-Martínez, Phys. Rev. D **80**, 033002 (2009).
[38] A. Ilakovac and A. Pilaftsis, Phys. Rev. D **80**, 091902 (2009).
[39] T. Fukuyama, A. Ilakovac, T. Kikuchi, S. Meljanac, and N. Okada, J. Math. Phys. (N.Y.) **46**, 033505 (2005).
[40] C. S. Aulakh and A. Girdhar, Int. J. Mod. Phys. A **20**, 865 (2005).