Alternative auxiliary fields for chiral multiplets

Hitoshi Nishino* and Subhash Rajpoot†

Department of Physics and Astronomy, California State University, 1250 Bellflower Boulevard, Long Beach, California 90840, USA

(Received 7 July 2009; published 1 December 2009)

We study 3-form auxiliary field formulation for chiral multiplets in the Wess-Zumino model. The conventional auxiliary fields F and G are replaced by their Hodge duals $K_{\mu\nu\rho\sigma}$ and $H_{\mu\nu\rho\sigma}$ which are the field strengths of the 3-form potential auxiliary fields $G_{\mu\nu\rho}$ and $F_{\mu\nu\rho}$. Even though duality transformations connect these two formulations, there exist certain differences from the conventional formulation. When boundary conditions are taken into account, the field equations in the 3-form formulation are equivalent to the conventional ones, while our Lagrangian is not. We also show that the new field strengths acquire generalized Chern-Simons terms. The O'Raifeartaigh mechanism works for spontaneous supersymmetry breaking also in the 3-form auxiliary field formulation *via* the boundary conditions on the 3-form auxiliary fields.

DOI: 10.1103/PhysRevD.80.127701 PACS numbers: 12.60.Jv, 11.30.Pb

I. INTRODUCTION

In this paper, we study the so-called 3-form potential field formulation for chiral multiplets in four dimensions (4D). This formulation was first given in [1] as a complex 3-form superfield. In that formulation, the ordinary scalar and pseudoscalar auxiliary fields in the conventional chiral multiplet $(A, B, \chi; F, G)$ [2,3] are replaced by 3-form potential fields. This is easily understood by duality transformations [4]; i.e., regarding the auxiliary fields (F, G) as "zero-rank" field strengths, we perform duality transformations on them into their Hodge dual $(K_{\mu\nu\rho\sigma}, H_{\mu\nu\rho\sigma})$ that are the field strengths of third-rank tensor potential fields: $K_{\mu\nu\rho\sigma} \equiv +4\partial_{[\mu} G_{\nu\rho\sigma]}$, $H_{\mu\nu\rho\sigma} \equiv +4\partial_{[\mu} F_{\nu\rho\sigma]}$.

The usage of alternative representations for auxiliary fields is not limited to this formulation [1]. For example, in [5,6], the D-auxiliary field in the conventional vector multiplet is replaced by its Hodge dual field strength $H_{\mu\nu\rho}$. Another example is $N = 1$ supergravity with two different sets of minimal auxiliary fields [7,8]. Compared with the old minimal set [7], there are certain differences in the new minimal set [8], such as that chiral gauge invariance is required for matter couplings. These examples show that a new multiplet with new auxiliary fields is *not* necessarily equivalent to the old multiplet.

Higher-rank tensor auxiliary fields may well be important in the context of superstring [9] or D-brane theory [10]. For example, the Neveu-Schwarz sector in superstring theory [9] generates the *massless* second-rank antisymmetric tensor $B_{\mu\nu}$ as a moduli field, which is in conflict with low-energy phenomenology. One way out [5,6] is to absorb its field strength $G_{\mu\nu\rho}$ into a third-rank tensor auxiliary field $C_{\mu\nu\rho}$, as a compensator making the latter massive.

In this paper, we study the 3-form auxiliary field formulation first given in [1], and show certain differences from, as well as similarities to, the Lagrangians and field equations of the conventional system [2,3]. Even though a superspace formulation has been given in [1], we give explicit component Lagrangians within 4D. We show that spontaneous supersymmetry breaking can occur, as in the conventional O'Raifeartaigh mechanism [11]. There is no analog of the so-called "F-linear term" in the 3-form auxiliary field system, but the effect of this term is replaced by an integration constant determined by an initial or a boundary condition.

II. LAGRANGIAN AND TRANSFORMATION RULE

Consider the plural chiral multiplets $(A^i, B^i, \chi^i; F_{\mu\nu\rho}{}^i, G_{\mu\nu\rho}{}^i)$ $(i, j, \cdots = 1, 2, \cdots, n)$, in the 3-form auxiliary field formulation [1], where the latter two fields are auxiliary fields whose field strengths are Hodge dual to the conventional auxiliary fields G^i and F^i, respectively:

$$K_{\mu\nu\rho\sigma}{}^i \equiv +4\partial_{[\mu} G_{\nu\rho\sigma]}{}^i, \quad H_{\mu\nu\rho\sigma}{}^i \equiv +4\partial_{[\mu} F_{\nu\rho\sigma]}{}^i. \tag{2.1}$$

As in the conventional case [2,3], we have the actions of the kinetic terms $I_{\rm K} \equiv \int d^4x \mathcal{L}_{\rm K}$, mass terms $I_m \equiv \int d^4x \mathcal{L}_m$, and the cubic interaction terms $I_g \equiv \int d^4x \mathcal{L}_g$, all with the x-space component Lagrangians[1]:

$$\mathcal{L}_{\rm K} \equiv -\frac{1}{2}(\partial_\mu A^i)^2 - \frac{1}{2}(\partial_\mu B^i)^2 + \frac{1}{2}(\bar{\chi}^i \not{\partial} \chi^i) - \frac{1}{48}(H_{[4]}{}^i)^2 - \frac{1}{48}(K_{[4]}{}^i)^2, \tag{2.2a}$$

$$\mathcal{L}_m \equiv +m^{ij}[F^i A^j - G^i B^j - \tfrac{1}{2}(\bar{\chi}^i \chi^j)], \tag{2.2b}$$

$$\mathcal{L}_g \equiv g^{ijk}\bigg[\frac{1}{2}F^i(A^j A^k - B^j B^k) - G^i A^j B^k - \frac{1}{2}(\bar{\chi}^i \chi^j)A^k + \frac{i}{2}(\bar{\chi}^i \gamma_5 \chi^j)B^k\bigg], \tag{2.2c}$$

where $m_{ij} = m_{ji}$, $g_{ijk} = g_{(ijk)}$. These component Lagrangians have not been given in [1]. In the "kinetic"

*hnishino@csulb.edu
†rajpoot@csulb.edu

[1]We use the symbol [4] for the totally antisymmetric four-indices, e.g., $H_{[4]}$ is equivalent to $H_{\mu\nu\rho\sigma}$, in order to save space.

term $\mathcal{L}_{\rm K}$, the $(H_{[4]})^2$ and $(K_{[4]})^2$ terms carry *no* kinetic energy, as is well known in 4D. The F and G fields [1] are defined by[2]

$$F^i \equiv +\frac{1}{24}\epsilon^{\mu\nu\rho\sigma}K_{\mu\nu\rho\sigma}{}^i, \qquad G^i \equiv +\frac{1}{24}\epsilon^{\mu\nu\rho\sigma}H_{\mu\nu\rho\sigma}{}^i. \tag{2.3}$$

This symbolism facilitates the comparison with the conventional formulation [2,3]. Note that the Lagrangians *linear* in $H_{[4]}$ or $K_{[4]}$ with the constants e^i and f^i,

$$\begin{aligned}\mathcal{L}_{eK} &\equiv \frac{1}{24}e^i\epsilon^{\mu\nu\rho\sigma}K_{\mu\nu\rho\sigma}{}^i = e^iF^i,\\ \mathcal{L}_{fH} &\equiv \frac{1}{24}f^i\epsilon^{\mu\nu\rho\sigma}H_{\mu\nu\rho\sigma}{}^i = f^iG^i,\end{aligned} \tag{2.4}$$

are total divergences, and do not affect field equations.

Each of our actions $I_{\rm K}$, I_m, and I_g is invariant under supersymmetry,

$$\delta_Q A^i = +(\bar\epsilon\chi^i), \quad \delta_Q B^i = +i(\bar\epsilon\gamma_5\chi^i), \tag{2.5a}$$

$$\delta_Q\chi^i = -(\gamma^\mu\epsilon)\partial_\mu A^i + i(\gamma_5\gamma^\mu\epsilon)\partial_\mu B^i - \frac{1}{24}(\gamma^{[4]}\epsilon)H_{[4]}{}^i - \frac{i}{24}(\gamma_5\gamma^{[4]}\epsilon)K_{[4]}{}^i, \tag{2.5b}$$

$$\delta_Q F_{\mu\nu\rho}{}^i = +(\bar\epsilon\gamma_{\mu\nu\rho}\chi^i), \quad \delta_Q G_{\mu\nu\rho}{}^i = +i(\bar\epsilon\gamma_5\gamma_{\mu\nu\rho}\chi^i). \tag{2.5c}$$

Equations (2.5b) and (2.5c) can also be rewritten as

$$\begin{aligned}&\delta_Q\chi^i = -(\gamma^\mu\epsilon)\partial_\mu A^i + i(\gamma_5\gamma^\mu\epsilon)\partial_\mu B^i + \epsilon F^i - i(\gamma_5\epsilon)G^i,\\ &\delta_Q F^i = -(\bar\epsilon\partial\!\!\!/\chi^i), \qquad \delta_Q G^i = +i(\bar\epsilon\gamma_5\partial\!\!\!/\chi^i),\end{aligned} \tag{2.6}$$

whose structures are parallel to the conventional formulation [2,3].

The closure of gauge algebra is easily confirmed as $[\delta_Q(\epsilon_1),\delta_Q(\epsilon_2)] = \delta_P(\xi)$ for the translation operator $\delta_P(\xi)$ with the infinitesimal parameter $\xi^\mu \equiv +2(\bar\epsilon_1\gamma^\mu\epsilon_2)$. Crucial identities employed to achieve the closure on $F_{[3]}$ and $G_{[3]}$ are

$$\begin{aligned}i(\bar\epsilon_1\gamma_5\gamma_{[\mu|}{}^{\sigma\tau}\epsilon_1)K_{|\nu\rho]\sigma\tau}{}^i &\equiv 0,\\ i(\bar\epsilon_1\gamma_5\gamma_{[\mu|}{}^{\sigma\tau}\epsilon_1)H_{|\nu\rho]\sigma\tau}{}^i &\equiv 0.\end{aligned} \tag{2.7}$$

III. FIELD EQUATIONS

Even though we have seen the similarity of the 3-form auxiliary field formulation to the conventional one [2,3], there are also certain differences, which we allude to now. Most importantly, the field equations of the 3-form auxiliary fields of the total action $I_{\rm tot} \equiv I_{\rm K} + I_m + I_g$ are

$$\begin{aligned}\frac{\delta\mathcal{L}_{\rm tot}}{\delta F_{\mu\nu\rho}{}^i} &= -4\epsilon^{\mu\nu\rho\sigma}\partial_\sigma\left(\frac{\delta\mathcal{L}}{\delta G^i}\right)\\ &= -4\epsilon^{\mu\nu\rho\sigma}\partial_\sigma[G^i - m^{ij}B^j - g^{ijk}A^jB^k]\\ &\doteq 0,\end{aligned} \tag{3.1a}$$

$$\begin{aligned}\frac{\delta\mathcal{L}_{\rm tot}}{\delta G_{\mu\nu\rho}{}^i} &= -4\epsilon^{\mu\nu\rho\sigma}\partial_\sigma\left(\frac{\delta\mathcal{L}}{\delta F^i}\right)\\ &= -4\epsilon^{\mu\nu\rho\sigma}\partial_\sigma\bigg[F^i + m^{ij}A^j\\ &\qquad + \frac{1}{2}g^{ijk}(A^jA^k - B^jB^k)\bigg] \doteq 0,\end{aligned} \tag{3.1b}$$

where the symbol $\doteq$ is for a field equation or a solution. These field equations are one derivative higher than the conventional F- and G-field equations [2,3], so that the set of solutions for (3.1) is much larger than the conventional set, in which only the quantity *inside* the square brackets is zero. In fact, the most general solutions for (3.1) are

$$\begin{aligned}G^i - m^{ij}B^j - g^{ijk}A^jB^k &\doteq d^i,\\ F^i + m^{ij}A^j + \tfrac{1}{2}g^{ijk}(A^jA^k - B^jB^k) &\doteq c^i,\end{aligned} \tag{3.2}$$

where c^i and d^i are real integration constants determined by the initial or boundary conditions at $|x^\mu| \to \infty$. Note that the absence of the F^i- or G^i-linear terms at the Lagrangian level is compensated by the integration constants c^i and d^i in (3.2).

As for the A- and B-field equations, they are

$$\partial_\mu^2 A^i + m^{ij}F^j + g^{ij}(F^jA^k - G^jB^k) - \tfrac{1}{2}g^{ijk}(\bar\chi^j\chi^k) \doteq 0, \tag{3.3a}$$

$$\partial_\mu^2 B^i - m^{ij}G^j - g^{ijk}(F^jB^k + G^jA^k) + \frac{i}{2}g^{ijk}(\bar\chi^j\gamma_5\chi^k) \doteq 0. \tag{3.3b}$$

We can eliminate the F^i and G^i fields using their general *solutions* (3.2):

$$\begin{aligned}&\partial_\mu^2 A^i - (m^2)^{ij}A^j - \tfrac{1}{2}m^{ij}g^{jkl}(A^lA^m - B^lB^m) - g^{ikj}m^{jl}(A^kA^l + B^kB^l) - \tfrac{1}{2}g^{ikj}g^{jlm}A^k(A^lA^m - B^lB^m)\\ &\qquad - g^{ikj}g^{jlm}B^kA^lB^m + m^{ij}c^j + g^{ijk}(c^jA^k - d^jB^k) - \tfrac{1}{2}g^{ijk}(\bar\chi^j\chi^k) \doteq 0,\end{aligned} \tag{3.4a}$$

$$\begin{aligned}&\partial_\mu^2 B^i - (m^2)^{ij}B^j - m^{ij}g^{jkl}A^kB^l + g^{ikj}m^{jl}B^kA^l - g^{ikj}m^{jl}A^kB^l + \frac{1}{2}g^{ikj}g^{jlm}B^k(A^lA^m - B^lB^m)\\ &\qquad - g^{ikj}g^{jlm}A^kB^mA^l - m^{ij}d^j - g^{ijk}(c^jB^k + d^jA^k) + \frac{i}{2}g^{ijk}(\bar\chi^j\gamma_5\chi^k) \doteq 0.\end{aligned} \tag{3.4b}$$

In the conventional formulation [2,3], one can eliminate auxiliary fields by substituting their algebraic field equations into the original Lagrangian. In the 3-form auxiliary field system, however, the "auxiliary" field equations (3.1) have

[2]Note that the potential $F_{[3]}{}^i$ (or $G_{[3]}{}^i$) is parity even (or odd), and $H_{[4]}{}^i$ (or $K_{[4]}{}^i$) is parity even (or odd), so that G^i (or F^i) is parity odd (or even). Namely, the corresponding fields (F^i, G^i) and $(G_{[3]}{}^i, F_{[3]}{}^i)$ flip parities because of the duality (2.3) with the ϵ tensor.

higher derivatives, so that such substitutions lead to erroneous results. Instead, what we can rely on is the consistency of field equations (3.2) and (3.3) or (3.4).

This feature is also elucidated by rewriting our total Lagrangian $\mathcal{L}_{\rm tot} \equiv \mathcal{L}_{\rm K} + \mathcal{L}_m + \mathcal{L}_g$[3]:

$$\begin{aligned}\mathcal{L}_{\rm tot} = & -\tfrac{1}{2}(\partial_\mu A^i)^2 - \tfrac{1}{2}(\partial_\mu B^i)^2 + \tfrac{1}{2}(\bar{\chi}^i \partial\!\!\!/ \chi^i) + \tfrac{1}{2}m^{ij}(\bar{\chi}^i\chi^j) \\ & + \tfrac{1}{2}[F^i + m^{ij}A^j + \tfrac{1}{2}g^{ijk}(A^jA^k - B^jB^k)]^2 \\ & + \tfrac{1}{2}(G^i - m^{ij}B^j - g^{ijk}A^jB^k)^2 \\ & - \tfrac{1}{2}[m^{ij}A^j + \tfrac{1}{2}g^{ijk}(A^jA^k - B^jB^k)]^2 \\ & - \tfrac{1}{2}(m^{ij}B^j + g^{ijk}A^jB^k)^2. \end{aligned} \tag{3.5}$$

The last two lines *seem* to correspond to the positive-definite "potential"

$$\begin{aligned} V \equiv & +\tfrac{1}{2}[m^{ij}A^j + \tfrac{1}{2}g^{ijk}(A^jA^k - B^jB^k)]^2 \\ & + \tfrac{1}{2}(m^{ij}B^j + g^{ijk}A^jB^k)^2. \end{aligned} \tag{3.6}$$

However, this is *not* the case, because the variations of V by the A^i and B^i fields do *not* yield the terms consistent with their field equations (3.4). In fact, the c^i- and d^i-dependent terms in the latter are entirely absent in (3.6).

The second and third lines in (3.5) can be interpreted as the "kinetic terms" for $G_{[3]}$ and $F_{[3]}$, where their field strengths now have *generalized* Chern-Simons (CS) terms:

$$\begin{aligned}\tilde{H}_{\mu\nu\rho\sigma}{}^i \equiv & +4\partial_{[\mu}F_{\mu\nu\rho]}{}^i \\ & - \epsilon^{\mu\nu\rho\sigma}[m^{ij}A^j + \tfrac{1}{2}g^{ijk}(A^jA^k - B^jB^k)],\end{aligned} \tag{3.7a}$$

$$\tilde{K}_{\mu\nu\rho\sigma}{}^i \equiv +4\partial_{[\mu}G_{\mu\nu\rho]}{}^i + \epsilon_{\mu\nu\rho\sigma}(m^{ij}B^j + g^{ijk}A^jB^k). \tag{3.7b}$$

The first terms on the right-hand side are the *unmodified* field strengths $H_{[4]}{}^i$ and $K_{[4]}{}^i$. The remaining terms are generalized CS terms, such that the Bianchi identities vanish, *modulo* field strengths. In fact, due to their maximal ranks, their Bianchi identities automatically vanish:

$$\partial_{[\mu}\tilde{H}_{\nu\rho\sigma\tau]}{}^i \equiv 0, \qquad \partial_{[\mu}\tilde{K}_{\nu\rho\sigma\tau]}{}^i \equiv 0. \tag{3.8}$$

This feature again clarifies the incorrectness of the "elimination" of F^i or G^i in (3.5). This is because F^i and G^i are *not* fundamental fields, and moreover the field redefinitions of the fundamental fields $F_{[3]}$ and $G_{[3]}$ *cannot* absorb the generalized CS terms in (3.7).

We have seen that the field equations (3.1) and (3.3) are different from the conventional formulation [2,3], due to (3.1) with derivatives. However, when the general *solutions* (3.2) are considered, (3.2) and (3.4) are equivalent to the conventional field equations [2,3]. The constant c^i and d^i correspond to the F- and G-linear Lagrangian terms in the conventional formulation [2,3]. Despite this equivalence, the difference at the Lagrangian level is caused by the higher derivatives for the potential fields $F_{[3]}$ and $G_{[3]}$.

IV. SPONTANEOUS SUPERSYMMETRY BREAKING

We now consider possible spontaneous supersymmetry breaking, using O'Raifeartaigh's mechanism. We stress that this was not discussed in [1]. First, (2.6) shows that F_i and G_i[4] enter linearly in $\delta_Q\chi_i$. When they develop nonzero vacuum expectation values, the χ_i field becomes a Nambu-Goldstone boson. As in the conventional case [2,3,11], we can use $\langle F_i\rangle \neq 0$ as a criterion for parity-preserving spontaneous supersymmetry breaking. However, the difference is that we have to satisfy the field equations (3.1) or (3.2) and (3.3), instead of "minimizing the potential" (3.6).

Based upon this criterion, we mimic the conventional O'Raifeartaigh mechanism for three chiral multiplets [3,11]. We first put $n = 3$, and impose the ansätze

$$m_{12} = m_{21} \equiv m \neq 0, \qquad g_{113} = g_{131} = g_{311} \equiv g \neq 0$$
$$(\text{other } g_{ijk} \text{ and } m_{ij} \text{ are } 0),$$
$$c_3 \equiv c \neq 0, \qquad c_1 = c_2 = d^i = 0. \tag{4.1}$$

Accordingly, we have in (3.2) that

$$F_1 \doteq -mA_2 - g(A_1A_3 - B_1B_3), \tag{4.2a}$$
$$F_2 \doteq -mA_1, \tag{4.2b}$$
$$F_3 \doteq -\tfrac{1}{2}gA_1^2 + \tfrac{1}{2}gB_1^2 + c, \tag{4.2c}$$
$$G_1 \doteq +mB_2 + g(A_1B_3 + A_3B_1), \tag{4.2d}$$
$$G_2 \doteq +mB_1, \tag{4.2e}$$
$$G_3 \doteq +gA_1B_1. \tag{4.2f}$$

Similarly to [11], F_2, F_3, and G_2 *cannot* be simultaneously zero, due to $c \neq 0$. In other words, $c \neq 0$ causes spontaneous supersymmetry breaking. This is because the linear F or G terms are simply replaced by the initial or boundary conditions with c_i and d_i in (3.2).

The A- and B-field equations can be obtained from (3.4), as

$$m^2A_1 + gmA_2A_3 + gmB_2B_3 + \tfrac{1}{2}g^2A_1(A_1^2 - B_1^2) + g^2A_3(A_1A_3 - B_1B_3) + g^2A_1B_1^2 + g^2A_1B_3^2 + g^2A_3B_1B_3 - gcA_1 \doteq 0, \tag{4.3a}$$
$$m^2A_2 + mg(A_1A_3 - B_1B_3) \doteq 0, \quad m^2B_2 + mgA_1B_3 + mgA_3B_1 \doteq 0, \tag{4.3b}$$
$$mgA_1A_2 + gmB_1B_2 + g^2A_1(A_1A_3 - B_1B_3) + g^2A_1B_1B_3 - g^2A_3B_1^2 \doteq 0, \tag{4.3c}$$
$$mgB_1A_2 - gmA_1B_2 + g^2(A_1A_3 - B_1B_3)B_1 - g^2A_1^2B_3 - g^2A_1A_3B_1 \doteq 0, \tag{4.3d}$$
$$mgB_1A_2 - mgA_1B_2 + g^2B_1(A_1A_3 - B_1B_3) - g^2A_1^2B_3 - g^2A_1A_3B_1 \doteq 0. \tag{4.3e}$$

[3]Although we use the same symbols F^i and G^i as in the conventional formulation, ours are defined by (2.3) with higher derivatives.

[4]We use *subscripts* for *i, j, ... only* in this section to avoid confusing expressions, such as $(A^1)^2$.

The significant difference from the conventional case [11] is that the crucial value $c \neq 0$ is *not* from the Lagrangian, potential or action, but is an integration constant determined at $|x^\mu| \rightarrow \infty$. Hence in the 3-form auxiliary system, spontaneous supersymmetry breaking is controlled by the integration constants c^i.

V. CONCLUDING REMARKS

We have studied the alternative field representations [1] for the conventional auxiliary fields F and G for chiral multiplets [2,3]. We used the field strengths $H_{[4]}$ and $K_{[4]}$ for the third-rank potential fields $F_{[3]}$ and $G_{[3]}$.

There are both differences and similarities between this 3-form auxiliary formulation [1] and the conventional one [2,3]. The most important difference is that the field equations for the auxiliary fields $F_{\mu\nu\rho}$ and $G_{\mu\nu\rho}$ have one derivative higher than the conventional ones [2]. The difference is also crystallized as the general solutions (3.2) to (3.1) with c^i and d^i determined by the initial or boundary conditions at $|x^\mu| \rightarrow \infty$. These subtle and nontrivial features are regarded as progress since [1].

Although the field equations (3.1) have higher derivatives than the conventional ones [2,3], the general solutions (3.2) to (3.1) combined with (3.4) are equivalent to the conventional set. The terms (2.4) analogous to the conventional "F- and G-linear terms" are total divergences in the 3-form auxiliary field system, while their roles are played by the integration constants c^i and d^i.

The 3-form auxiliary field formulation [1] uses the *maximal-rank* field strengths in 4D. In our paper in 2006 [12], we have shown a vanishing cosmological constant mechanism using maximal-rank tensors. Therefore, the 3-form auxiliary field formulation may well be associated also with the cosmological constant problem.

The situation with the constants c^i and d^i determined by initial conditions resembles the so-called "unimodular" (super)gravity [13,14] with the cosmological constant determined as an initial condition. In our model, the integration constants c^i and d^i determined at $|x^\mu| \rightarrow \infty$ control the magnitudes of supersymmetry breaking. To our knowledge, such a mechanism of supersymmetry breaking has never been presented in the past.

Even though we have not introduced interactions with other multiplets, such as that of supergravity, there is no reason to believe that our system is entirely equivalent to the conventional chiral multiplet [2,3]. This is analogous to the new minimal auxiliary fields in $N = 1$ supergravity [8].

We have seen lots of nontrivial features for the 3-form auxiliary field formulation [1] for chiral multiplets [2,3]. These are also reflected in spontaneous supersymmetry breaking mechanisms. Hence, it is naturally expected that considerable nontrivial differences from the conventional formulation [2,3] are yet to be uncovered in future studies.

[1] S. J. Gates, Jr. and W. Siegel, Nucl. Phys. **B187**, 389 (1981); S. J. Gates, Jr., M. T. Grisaru, M. Roček, and W. Siegel, Front. Phys. **58**, 1 (1983), Section 4.4.d.2.
[2] J. Wess and B. Zumino, Nucl. Phys. **B70**, 39 (1974).
[3] For an excellent review, see J. Wess and J. Bagger, *Superspace and Supergravity* (Princeton University Press, Princeton, NJ, 1992); see also P. Fayet and S. Ferrara, Phys. Rep. **32**, 249 (1977); M. Sohnius, Phys. Rep. **128**, 39 (1985).
[4] H. Nicolai and P. K. Townsend, Phys. Lett. **98B**, 257 (1981).
[5] H. Nishino and S. Rajpoot, Phys. Rev. D **76**, 065004 (2007).
[6] H. Nishino and S. Rajpoot, Phys. Rev. D **78**, 125006 (2008).
[7] K. S. Stelle and P. C. West, Phys. Lett. **74B**, 330 (1978); S. Ferrara and P. van Nieuwenhuizen, Phys. Lett. **74B**, 333 (1978).
[8] M. Sohnius and P. C. West, Nucl. Phys. **B198**, 493 (1982).
[9] M. Green, J. H. Schwarz, and E. Witten, *Superstring Theory* (Cambridge University Press, Cambridge, England, 1986), Vols. I–II.
[10] K. Becker, M. Becker, and J. H. Schwarz, *String Theory and M-Theory: A Modern Introduction* (Cambridge University Press, Cambridge, England, 2007).
[11] L. O'Raifeartaigh, Nucl. Phys. **B96**, 331 (1975).
[12] H. Nishino and S. Rajpoot, Mod. Phys. Lett. A **21**, 127 (2006).
[13] J. J. van der Bij, H. van Dam, and Y. J. Ng, Physica A (Amsterdam) **116**, 307 (1982); F. Wilczek, Phys. Rep. **104**, 143 (1984); A. Zee, in *Proceedings of the Twentieth Annual Orbis Scientiae on High Energy Physics*, edited by S. L. Mintz and A. Perlmutter (Plenum, New York, 1985); W. Buchmller and N. Dragon, Phys. Lett. B **207**, 292 (1988); S. Weinberg, Rev. Mod. Phys. **61**, 1 (1989); W. G. Unruh, Phys. Rev. D **40**, 1048 (1989); E. Alvarez, J. High Energy Phys. 03 (2005) 002; M. Henneaux and C. Teitelboim, Phys. Lett. B **222**, 195 (1989).
[14] H. Nishino and S. Rajpoot, Phys. Lett. B **528**, 259 (2002).