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Relativistic thermodynamics with an invariant energy scale

Sudipta Das,[1,*] Subir Ghosh,[1,†] and Dibakar Roychowdhury[2,‡]

[1]*Physics and Applied Mathematics Unit, Indian Statistical Institute, 203 B.T.Road, Kolkata 700108, India*

[2]*Department of Physics, University of North Bengal Siliguri, 734013, West Bengal, India*

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A particular framework for quantum gravity is the doubly special relativity (DSR) formalism that introduces a new observer independent scale, the Planck energy. Our aim in this paper is to study the effects of this energy upper bound in relativistic thermodynamics. We have explicitly computed the modified equation of state for an ideal fluid in the DSR framework. In deriving our result we exploited the scheme of treating DSR as a nonlinear representation of the Lorentz group in special relativity.

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I. INTRODUCTION

In recent years doubly special relativity (DSR) [1] has created a lot of interest as a possible framework of quantum fravity. This is mainly due to two basic tenets on which the theory rests: (i) Appearance of a second observer independent scale [1], which can be (Planck) length, (Planck) energy or momentum, apart from the velocity of light, common to special relativity (SR). Incidentally this gives rise to the name DSR. (ii) A naturally emerging Non-Commutative (NC) spacetime [1–4].[1] Both of these features are very close to quantum gravity ideas [6] or the existence of a universal short distance scale that postulates a generalized uncertainty principle [7]. All the models of quantum gravity predict qualitatively different spacetime beyond certain energy (length) scale, generally considered to be the Planck energy (length). Also it is now established [8] that a consistent marriage of ideas of quantum mechanics and gravitation requires NC spacetime to avoid the paradoxical situation of creation of a black hole for an event that is sufficiently localized in spacetime. Quite obviously one would like to have the numerical value of this universal scale to be observer independent, as in DSR.

In this perspective it is very important to study the effects of specific forms of NC spacetime that are relevant to DSR, in particular, the κ-Minkowski spacetime, studied independently [9] and partly motivated by DSR ideas [1,2,10,11]. So far only particle dynamics in DSR framework has been studied, which has revealed many unusual features [2,12]. Some field theory models in DSR spacetime have also been attempted [13]. On the other hand, to our knowledge not much work has been done in studying DSR effects (especially the fact that there exists an upper bound of energy) in the exciting areas of relativistic thermodynamics and eventually in cosmology. In the present paper we have initiated a study along the direction of relativistic thermodynamics in the DSR framework.

Our aim is to follow the prescription of Weinberg [14] where one postulates an explicit form of the energy-momentum tensor (EMT) for a perfect fluid in the Lagrangian framework. The first nontrivial task that we face is the construction of the DSR-covariant EMT. Fortunately we have a powerful tool at our disposal: *DSR kinematics is a manifestation of a nonlinear realization of SR kinematics* [3,11,15]. Throughout the present paper we exploit this principle to develop the fluid EMT for DSR and subsequently study the consequences of the EMT in thermodynamic context. As expected our expressions have a smooth commutative (or equivalently SR) limit, that is, all results reduce to SR results when κ, the effective NC parameter (the energy upper bound) goes to infinity.

The paper is organized as follows: In Sec. II we will provide the explicit nonlinear mapping between the NC DSR variables (expressed as small letters) and commuting (or more precisely canonical) degrees of freedom (expressed as capital letters). The latter obey canonical phase space algebra and SR Lorentz transformations whereas the former satisfy NC κ-Minkowski phase space algebra and DSR-Lorentz transformations. In Sec. III we will construct the DSR compatible EMT. This is one of our main results. In Sec. IV we will explicitly reveal effects of DSR regarding relativistic thermodynamics which constitutes the other major result. We will conclude in Sec. V.

II. NONLINEAR REALIZATION OF LORENTZ GROUP

It has been pointed out by Amelino-Camelia [1] that there is a connection between the appearance of an observer independent scale and the presence of nonlinearity in the corresponding spacetime transformations. Recall that Galilean transformations are completely linear and there are no observer independent parameters in Galilean/Newtonian relativity. With Einstein relativity one finds an observer independent scale—the velocity of light—as well as a nonlinear relation in the velocity addi-

*sudipta jumaths@yahoo.co.in
†sghosh@isical.ac.in
‡dibakar_nbu@yahoo.co.in

[1]NC geometry in general has generated a lot of new ideas in modern physics [5]

tion theorem. In DSR one introduces another observer independent parameter, Planck energy or length, and ushers another level of nonlinearity in which the Lorentz transformation laws become nonlinear. These generalized Lorentz transformation rules, referred to here as DSR-Lorentz transformation, are derivable from basic DSR ideas [1] or in a more systematic way, from integrating small DSR transformations in an NC spacetime scheme [3,16]. Another elegant way of derivation is to interpret DSR laws as a nonlinear realization of SR laws where one can directly exploit the nonlinear map and its inverse, that connects DSR to SR and vice-versa.[2] Obviously to accomplish this one needs the map, which can be constructed by a motivated guess [11,15] or constructed as a form of Darboux map [3].

We are working in the DSR2 model of Magueijo and Smolin [11]. Let us start with the all important map [3,11,15],

$$F(X^\mu) \rightarrow x^\mu, \qquad F^{-1}(x^\mu) \rightarrow X^\mu. \tag{1}$$

which in explicit form reads:

$$F(X^\mu) = x^\mu\left(1 - \frac{(np)}{\kappa}\right); \qquad F(P^\mu) = \frac{p^\mu}{(1 - \frac{(np)}{\kappa})}$$
$$F^{-1}(x^\mu) = X^\mu\left(1 + \frac{(nP)}{\kappa}\right); \qquad F^{-1}(p^\mu) = \frac{P^\mu}{(1 + \frac{(nP)}{\kappa})}, \tag{2}$$

where $n_\mu = (1, 0, 0, 0)$ and we use the notation $a_\mu b^\mu = (ab)$, $(np) = p^0$, $(nP) = P^0$. Note that upper case and lower case letters refer to (unphysical) canonical SR variables and (physical) DSR variables, respectively.

As a quick recapitulation let us rederive the DSR-Lorentz transformations ($L_{\rm DSR}$), starting from the familiar (linear) SR Lorentz transformations ($L_{\rm SR}$),

$$\begin{aligned}
&X'^0 = L_{\rm SR}(X^0) = \gamma(X^0 - vX^1),\\
&X'^1 = L_{\rm SR}(X^1) = \gamma(X^1 - vX^0), \qquad X'^2 = X^2,\\
&X'^3 = X^3 \qquad P'^0 = L_{\rm SR}(P^0) = \gamma(P^0 - VP^1),\\
&P'^1 = L_{\rm SR}(P^1) = \gamma(P^1 - VP^0),\\
&P'^2 = P^2, \qquad P'^3 = P^3
\end{aligned} \tag{3}$$

where $\gamma = 1/\sqrt{1 - v^2}$ and the boost is along X^1 direction with velocity $v^i = (v, 0, 0)$. Note that the second line of (3) involves V but following our definition $\frac{dX^i}{dX^0} = \frac{dx^i}{dx^0}$ so that $V = v$. Now the DSR-Lorentz transformation $L_{\rm DSR}$ is formally expressed as,

$$\begin{aligned}
x'^\mu &= L_{\rm DSR}(x^\mu) = F \circ L_{\rm SR} \circ F^{-1}(x^\mu),\\
p'^\mu &= L_{\rm DSR}(p^\mu) = F \circ L_{\rm SR} \circ F^{-1}(p^\mu).
\end{aligned} \tag{4}$$

In explicit form this reads as,

$$\begin{aligned}
x'^0 &= L_{\rm DSR}(x^0) = F \circ L_{\rm SR} \circ F^{-1}(x^0)\\
&= F \circ L_{\rm SR}\left(X^0\left(1 + \frac{P^0}{\kappa}\right)\right)\\
&= F\left(\gamma(X^0 - vX^1)\left(1 + \frac{\gamma}{\kappa}(P^0 - vP^1)\right)\right)\\
&= \gamma\alpha(x^0 - vx^1);\\
p'^0 &= \frac{\gamma}{\alpha}(p^0 - vp^1),
\end{aligned} \tag{5}$$

where $\alpha = 1 + \kappa^{-1}((\gamma - 1)P^0 - \gamma v P^1)$. Similarly for $\mu = 1$ we have the following expressions:

$$x'^1 = \gamma\alpha(x^1 - vx^0), \qquad p'^1 = \frac{\gamma}{\alpha}(p^1 - vp^0). \tag{6}$$

It is important to realize that, in the present formulation, noncommutative effects enter through these generalized (nonlinear) transformation rules.

Note that, in contrast to SR laws (3), components of x^μ, p^μ *transverse* to the frame velocity v are affected,

$$x'^i = \alpha x^i, \qquad p'^i = \frac{p^i}{\alpha}; \qquad i = 2, 3. \tag{7}$$

There are two phase space quantities, invariant under DSR-Lorentz transformation: $\eta_{\mu\nu}p^\mu p^\nu/(1 - p^0/\kappa)^2$ and $\eta_{\mu\nu}x^\mu x^\nu(1 - p^0/\kappa)^2$ with $\eta_{\mu\nu} = {\rm diag}(-1, 1, 1, 1)$. Writing the former as

$$m^2 = \eta_{\mu\nu}p^\mu p^\nu/(1 - p^0/\kappa)^2 \tag{8}$$

yields the well-known Magueijo-Smolin dispersion relation. We interpret the latter invariant to provide an effective metric $\tilde{\eta}_{\mu\nu}$ for DSR:

$$d\tau^2 = \tilde{\eta}_{\mu\nu}dx^\mu dx^\nu = (1 - p^0/\kappa)^2\eta_{\mu\nu}dx^\mu dx^\nu. \tag{9}$$

III. ENERGY-MOMENTUM TENSOR IN κ-MINKOWSKI SPACETIME

In this section our aim is to construct the energy-momentum tensor of a perfect fluid, that will be covariant in the DSR framework. Indeed, this will fit nicely in our future programme of pursuing a DSR based cosmology.

A. Fluid in SR theory

A perfect fluid can be considered as a system of structureless point particles, experiencing only spatially localized interactions among themselves. The idea is to consider boosts in a passive transformation framework. In this way one can ascertain the structure of energy-momentum tensor

[2] It needs to be stressed that even though there exists an explicit map between SR and DSR variables, the two theories will not lead to the same physics, (in particular upon quantization), due to the essential nonlinearity involved in the map. According to DSR the physical degrees of freedom live in a noncanonical phase space and the canonically mapped phase space is to be used only as a convenient intermediate step.

in an arbitrary inertial frame (laboratory frame) by boosting the expression valid in the fluid rest frame (comoving frame). In a comoving Lorentz frame, from spherical symmetry, the energy-momentum tensor $\tilde{T}^{\mu\nu}$ of a perfect fluid becomes diagonal and the components are explicitly written as,

$$\tilde{T}^{ii} = P; \qquad \tilde{T}^{00} = D; \qquad \tilde{T}^{0i} = \tilde{T}^{i0} = 0. \tag{10}$$

The thermodynamic quantities P and D represent pressure and energy density of the fluid. The components of the canonical energy-momentum tensor transform under SR Lorentz transformation $L_{\rm SR}$ as a second rank tensor and in an arbitrary inertial frame it assumes the form [14]

$$\begin{aligned} T^{00} &= L_c(\tilde{T}^{00}) = \gamma^2(D + Pv^2),\\ T^{i0} &= L_c(\tilde{T}^{i0}) = \gamma^2(D + P)v^i\\ T^{ij} &= L_c(\tilde{T}^{ij}) = \gamma^2(D + P)v^i v^j + P\delta^{ij}. \end{aligned} \tag{11}$$

The above set of equations can be integrated into a single SR covariant tensor,

$$T^{\mu\nu} = (P + D)U^\mu U^\nu + P\eta^{\mu\nu} \tag{12}$$

where the velocity 4-vector U^μ is defined as $U^0 = \gamma$, $U^i = \gamma v^i$ with $U^\mu U_\mu = -1$.

We can derive this result in another way, the so called Lagrangian formalism, which will be useful later. Let us treat the fluid as a collection of noninteracting particles, the latter having in general, an energy-momentum tensor of the form [14]

$$T^{\mu\nu} = \sum_i \frac{P_i^\mu P_i^\nu}{P_i^0}\delta^3(\mathrm{X} - \mathrm{X_i}), \tag{13}$$

where P_i^μ is the energy-momentum four-vector associated with the i-th particle located at $\mathrm{X_i}$. Once again in the comoving frame it will reduce to the diagonal form:

$$\begin{aligned} \tilde{T}^{ii} &= P = \frac{1}{3}\sum_i \frac{\mathbf{P_i^2}}{P_i^0}\delta^3(X - X_i);\\ \tilde{T}^{00} &= D = \sum_i P_i^0 \delta^3(X - X_i);\\ \tilde{T}^{i0} &= \tilde{T}^{0i} = 0. \end{aligned} \tag{14}$$

In the above relations $\mathbf{P_i}$ and P_i^0 respectively stand for the momentum three-vector and the energy of the i-th fluid particle. The thermodynamic quantities P and D represent pressure and energy density of the fluid. The particle number density is naturally defined as

$$N = \sum_i \delta^3(X - X_i). \tag{15}$$

The Lorentz transformation equation for $T^{\mu\nu}$ is

$$T^{\mu\nu} = L_{\rm SR}(\tilde{T}^{\mu\nu}) = \Lambda^\mu_\alpha \Lambda^\nu_\beta T^{\alpha\beta}, \tag{16}$$

where Λ is the Lorentz transformation matrix. For $\mu = \nu = 0$ we have

$$T^{00} = (\Lambda^0_0)^2\tilde{T}^{00} + (\Lambda^0_i)^2\tilde{T}^{ii} = \gamma^2\tilde{T}^{00} + \gamma^2 v^2\tilde{T}^{11}. \tag{17}$$

We put the summation expressions for $\tilde{T}^{00}$ and $\tilde{T}^{ii}$ from (14) in the above equation instead of putting D and P. Then using the relation

$$T^\alpha_\alpha = -\sum_i \frac{m^2}{P_i^0}\delta^3(X - X_i) = -D + 3P \tag{18}$$

we can easily verify that the final expression for T^{00} is exactly the same as in (11). Similarly the other relations will follow.

B. Fluid in DSR theory

In order to derive the expression for the DSR EMT ($t^{\mu\nu}$) we shall exploit the same approach as above for SR EMT. Spatial rotational invariance remains intact in DSR allowing us to postulate a similar diagonal form for DSR EMT in the comoving frame. The next step (in principle) is to apply the $L_{\rm DSR}$ to obtain the general form of EMT in DSR. We first define the nonlinear mapping for the energy-momentum tensor of a perfect fluid in a comoving frame. In the second step we shall apply the Lorentz boost ($L_{\rm SR}$) on our mapped variable and finally arrive at the desired expression in the DSR spacetime through an inverse mapping. But we will see that when we try to introduce the fluid variables in the DSR EMT in arbitrary frame we face a nontrivial problem unless we make some simplifying assumptions, which, however, will still introduce DSR corrections pertaining to the Planck scale cutoff.

As the spherical symmetry remains intact in the DSR theory [3] we define the respective components of energy-momentum tensor $\tilde{t}^{\mu\nu}$ in the NC framework analogous to (14) and (15) as,

$$\begin{aligned} \tilde{t}^{ii} &= p = \frac{1}{3}\sum_i \frac{\mathbf{p_i^2}}{p_i^0}\delta^3(x - x_i);\\ \tilde{t}^{00} &= \rho = \sum_i p_i^0 \delta^3(x - x_i);\\ n &= \sum_i \delta^3(x - x_i), \end{aligned} \tag{19}$$

where $\mathbf{p_i}$ and p_i^0 are, respectively, the momentum three-vector and the energy of the i-th fluid particle in the DSR spacetime. Using (2) and using the scaling properties of Dirac-δ function we obtain the following results,

$$F^{-1}(p) = \frac{1}{3}\sum_i \frac{\mathbf{P_i^2}}{P_i^0(1 + P_i^0/\kappa)^4}\delta^3(X - X_i), \tag{20}$$

$$F^{-1}(\rho) = \sum_i \frac{P_i^0}{(1 + P_i^0/\kappa)^4}\delta^3(X - X_i), \tag{21}$$

$$F^{-1}(n) = \sum_i \frac{N}{(1+\frac{P_i^0}{\kappa})^3}\delta^3(X - X_i). \tag{22}$$

In a combined form, we can write down the following nonlinear mapping (and its inverse) as,

$$\begin{aligned} F^{-1}(\tilde{t}^{\mu\nu}) &= \sum_i \frac{P_i^\mu P_i^\nu}{P_i^0(1+P_i^0/\kappa)^4}\delta^3(X - X_i), \\ F(\tilde{T}^{\mu\nu}) &= \sum_i \frac{p_i^\mu p_i^\nu}{p_i^0(1+p_i^0/\kappa)^4}\delta^3(x - x_i). \end{aligned} \tag{23}$$

The way we have defined the DSR EMT it is clear that comoving form of EMT also receives DSR corrections. But problem crops up when, in analogy to SR EMT [14], we attempt to boost the $\tilde{t}^{\mu\nu}$ to a laboratory frame with an arbitrary velocity v. Recall that for a single particle DSR boosts involve its energy and momentum. Since p and ρ (for $\tilde{t}^{\mu\nu}$) denote composite variables it is not clear which energy or momentum will come into play. To proceed further in the DSR boost we put in a single energy $\bar{p}^0$ and momentum $\bar{p}^i$ that signifies the average energy and momentum (modulus) of the whole fluid. In fact this simplification is not very artificial since we are obviously considering equilibrium systems (however see [17]). This allows us to use the mappings:

$$\begin{aligned} F^{-1}(p) &= \frac{P}{(1+\bar{P}^0/\kappa)^4}, \\ F^{-1}(\rho) &= \frac{D}{(1+\bar{P}^0/\kappa)^4}, \\ F^{-1}(n) &= \frac{N}{(1+\bar{P}^0/\kappa)^4}. \end{aligned} \tag{24}$$

In a covariant form the mapping and its inverse between $\tilde{t}^{\mu\nu}$ and $\tilde{T}^{\mu\nu}$ are,

$$F^{-1}(\tilde{t}^{\mu\nu}) = \frac{\tilde{T}^{\mu\nu}}{(1+\bar{P}^0/\kappa)^4}, \qquad F(\tilde{T}^{\mu\nu}) = \frac{\tilde{t}^{\mu\nu}}{(1-\bar{p}^0/\kappa)^4}. \tag{25}$$

Finally we can apply the definition of L_{DSR} using (25) with (11) to obtain the following expressions for energy-momentum tensor with respect to an arbitrary inertial frame in a DSR spacetime,

$$\begin{aligned} t^{00} &= L_{\mathrm{DSR}}(\tilde{t}^{00}) = F \circ L_{SR} \circ F^{-1}(\tilde{t}^{00}) \\ &= F \circ L_{\mathrm{SR}}\left(\frac{\tilde{T}^{00}}{(1+\bar{P}^0/\kappa)^4}\right) = F\left(\frac{\gamma^2(D+Pv^2)}{(1+\frac{\gamma}{\kappa}(\bar{P}^0 - v\bar{P}^1))^4}\right) \\ &= \frac{\gamma^2(\rho + pv^2)}{\bar{\alpha}^4}; \end{aligned} \tag{26}$$

$$t^{i0} = L_{\mathrm{DSR}}(\tilde{t}^{i0}) = \frac{\gamma^2(\rho+p)v^i}{\bar{\alpha}^4};$$

$$t^{ij} = L_{\mathrm{DSR}}(\tilde{t}^{ij}) = \frac{\gamma^2(\rho+p)v^i v^j}{\bar{\alpha}^4} + p\delta^{ij}.$$

It is very interesting to note that the above expressions can also be combined into a single form which is structurally very close to the fluid EMT in SR,

$$\begin{aligned} t^{\mu\nu} &= \frac{(1-\frac{\bar{p}_0}{\kappa})^2}{\bar{\alpha}^4}\left((p+\rho)u^\mu u^\nu + p\frac{\eta^{\mu\nu}}{(1-\frac{\bar{p}_0}{\kappa})^2}\right) \\ &= \frac{(1-\frac{\bar{p}_0}{\kappa})^2}{\bar{\alpha}^4}((p+\rho)u^\mu u^\nu + p\tilde{\eta}^{\mu\nu}). \end{aligned} \tag{27}$$

where we have defined the four-velocity u^μ in the DSR spacetime as:

$$\begin{aligned} u^0 &= dx^0/d\tau = \frac{\gamma}{(1-\bar{p}_0/\kappa)}, \\ u^i &= dx^i/d\tau = \frac{\gamma v^i}{(1-\bar{p}_0/\kappa)}. \end{aligned} \tag{28}$$

Note that the DSR four-velocity u^μ is actually the mapped form of the SR four-velocity U^μ since the parameter τ does not undergo any transformation. The other point to notice is that $\tilde{\eta}^{\mu\nu}$ of (9), (DSR analogue of the flat metric $\tilde{\eta}^{\mu\nu}$), appears in $t^{\mu\nu}$ making the final form of the DSR EMT transparent. Indeed $t^{\mu\nu}$ in (27) reduces smoothly to $T^{\mu\nu}$ of SR (12) in the large κ limit. Incidentally, again in analogy to the SR construction of many-body system for fluid [(13) and (14)], this form of $t^{\mu\nu}$ is consistent with the microscopic picture of DSR EMT for fluid that we have developed [(19)–(23)].

IV. EQUATION OF STATE

So far we have only provided the abstract form of DSR EMT, relevant for a fluid, from purely kinematical considerations. It is now time for application. Keeping an eye in our cosmological motivation, in the present paper we will take up the issue of equation of state for an ideal DSR fluid.

A. Equation of state in SR theory

In the standard SR version, one way of deriving [14] the equation of state is to return to the microscopic picture (13) and substitute the SR energy dispersion relation $P^0 = (\mathbf{P}^2 + m^2)^{1/2}$ into (14) to get the following expression for the equation of state,

$$P = \frac{1}{3}D - \frac{1}{3}\sum_i \frac{m^2}{P_i^0}\delta^3(X - X_i). \tag{29}$$

For a cool nonrelativistic gas we have $\mathbf{P} \ll m$; so the expression for the energy becomes: $E \simeq m + \frac{\mathbf{P}^2}{2m}$. Using (14) and (15) one gets the equation of state

$$D - mN = \frac{3}{2}P. \tag{30}$$

For a hot ultrarelativistic gas since $E \simeq \mathbf{P} \gg m$ using (14) the equation of state becomes

$$D = 3P. \tag{31}$$

B. Equation of state in DSR theory

Let us now we proceed to derive the ideal fluid equation of state in the DSR scheme. We start with the Magueijo-Smolin modified dispersion relation ([11]),

$$(\bar{p}^0)^2 - \bar{\mathbf{p}}^2 = m^2\left[1 - \frac{\bar{p}^0}{\kappa}\right]^2. \tag{32}$$

We solve this equation for $\bar{p}^0$ to $O(\kappa)$,

$$\bar{p}^{\,0} = (\bar{\mathbf{p}}^2 + m^2)^{1/2} - \frac{m^2}{\kappa}. \tag{33}$$

We substitute this expression in (19) and finally obtain,

$$p = \frac{1}{3}\rho - \frac{1}{3}\sum_i \frac{m^2}{\bar{p}^0}\delta^3(x - x_i) + \frac{2m^2 n}{\kappa}. \tag{34}$$

In the nonrelativistic regime $\bar{p}^0 \simeq m + \frac{\bar{\mathbf{p}}^2}{2m} - \frac{m^2}{\kappa}$, using (19) we have

$$\rho - mn = \frac{3}{2}p - \frac{m^2 n}{\kappa}. \tag{35}$$

However something interesting occurs in the extreme relativistic scenario due to the Planck energy upper bound $\bar{p}^0 \sim \kappa$. Referring once again to the Magueijo-Smolin dispersion relation (32), we find that for $\bar{p}^0 = \kappa$ the SR photon dispersion relation is recovered, $\bar{p}^0 = \mid \bar{\mathbf{p}} \mid = \kappa$ the rest mass m does not appear in the consideration. (In fact one can check that the energy ceiling κ can only be reached by a massless particle.) But this condition reduces the equation of state to,

$$\rho = 3p = n\kappa. \tag{36}$$

These equations of state might prove to be important signatures for quantum gravity effects if DSR happens to be the proper framework to address quantum gravity issues.

V. CONCLUSION AND FUTURE PROSPECTS

Doubly special relativity (DSR) is a generalization of special relativity (SR) that can be relevant in the context of quantum gravity since it possesses an observer invariant energy upper bound, naturally assumed to be the Planck energy. Also DSR is compatible with the κ-Minkowski form of noncommutative spacetime. DSR reduces to SR for low energy regime as indeed it should. In this paper, for the first time, we have tried to incorporate DSR effects in an ideal fluid since eventually we aim to consider a DSR based cosmology.

We generalize the conventional framework of deriving the covariant energy-momentum tensor by boosting its spherically symmetric form, where we exploit the DSR-Lorentz transformations (instead of the special theory transformations). We stress that effects of a noncommutative (in particular κ-Minkowski) spacetime enters through the DSR-Lorentz transformations. In the process we had to resort to some simplifying assumptions in describing the fluid as a many-body system (in the so called Lagrangian description of fluid). One might treat this problem as a more virulent form of the one we find even in SR if we try to treat a multiparticle system in a relativistic way. We have exploited the concept that DSR is a nonlinear realization of SR so that one can use a canonical phase space as a tool for obtaining DSR relations. We have demonstrated that, even in this simplified situation, there are effects of DSR that introduces the Planck scale in the equations of motion for an ideal fluid. Below we list some of the open problems that we plan to pursue in near future: (i) While boosting the comoving form of energy-momentum tensor in DSR, we had to utilize the average values of energy and momentum modulus for the whole system while (DSR) boosting. We require an improved way of applying the DSR boost keeping the dependence of DSR boost on individual particles of the fluid intact. (ii) Two of us are looking at the thermodynamics of ideal fluid for DSR explicitly from the partition function [18]. In this formulation DSR effects will appear from two sources, from the deformed mass-energy dispersion relation of particles and from the high energy cut off in the form of Planck energy. (iii) Generalization of Cosmology in DSR framework is the next program that we wish to take up.

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[1] G. Amelino-Camelia, Nature (London) **418**, 34 (2002); Phys. Lett. B **510**, 255 (2001); Int. J. Mod. Phys. A **11**, 35 (2002) arXiv:hep-th/0405273; For a review on DSR, see J. Kowalski-Glikman, Lect. Notes Phys. **669**, 131 (2005).

[2] J. Kowalski-Glikman and S. Nowak, Phys. Lett. B **539**, 126 (2002); Classical Quantum Gravity **20**, 4799 (2003).

[3] S. Ghosh and P. Pal, Phys. Rev. D **75**, 105021 (2007).

[4] For a very recent review on DSR related Quantum Gravity signatures and related issues, see G. Amelino-Camelia, arXiv:0806.0339; Richard J. Szabo, arXiv:0906.2913.

[5] Recent interest in Non-Commutative geometry and Quantum Field Theory was generated mainly by N. Seiberg and E. Witten, J. High Energy Phys. 09 (1999) 032; For reviews on Non-Commutative geometry inspired quantum physics, see M. R. Douglas and N. A. Nekrasov, Rev. Mod. Phys. **73**, 977 (2001); R. J. Szabo, Phys. Rep. **378**, 207 (2003).

[6] C. Rovelli and L. Smolin, Nucl. Phys. **B442**, 593 (1995); **B456**, 734(E) (1995); G. Amelino-Camelia, Mod. Phys. Lett. A **17**, 899 (2002).

[7] D. Amati, M. Ciafaloni, and G. Veneziano, Phys. Lett. B **216**, 41 (1989); M. Maggiore, Phys. Lett. B **304**, 65 (1993); Phys. Rev. D **49**, 5182 (1994); Phys. Lett. B **319**, 83 (1993); L. J. Garay, Int. J. Mod. Phys. A **10**, 145 (1995); S. Hossenfelder, M. Bleicher, S. Hofmann, J. Ruppert, S. Scherer, and H. Stoecker, Phys. Lett. B **575**, 85 (2003); S. Das and E. C. Vagenas, Phys. Rev. Lett. **101**, 221301 (2008).

[8] S. Doplicher, K. Fredenhagen, and J. E. Roberts, Phys. Lett. B**331**, 39 (1994).

[9] J. Lukierski, A. Nowicki, H. Ruegg, and V. N. Tolstoy, Phys. Lett. B **264**, 331 (1991); S. Majid and H. Ruegg, Phys. Lett. B **334**, 348 (1994); J. Lukierski, H. Ruegg, and W. J. Zakrzewski, Ann. Phys. (N.Y.) **243**, 90 (1995).

[10] J. Lukierski, H. Ruegg, and W. J. Zakrzewski, Ann. Phys. (N.Y.) **243**, 90 (1995); J. Lukierski and A. Nowicki, Int. J. Mod. Phys. A **18**, 7 (2003).

[11] J. Magueijo and L. Smolin, Phys. Rev. Lett. **88**, 190403 (2002); Phys. Rev. D **67**, 044017 (2003).

[12] Alex Granik, arXiv:hep-th/0207113; arXiv:physics/0108050; arXiv:physics/0108031; Phys. Lett. A **316**, 173 (2003); Phys. Lett. B **672**, 186 (2009); S. Ghosh and P. Pal, Phys. Lett. B **618**, 243 (2005); S. Ghosh, Phys. Lett. B **623**, 251 (2005).

[13] Alessandra Agostini, Giovanni Amelino-Camelia, Michele Arzano, Antonino Marcian, and Ruggero Altair Tacchi, Mod. Phys. Lett. A **22**, 1779 (2007); M. Daszkiewicz, J. Lukierski, and M. Woronowicz, Phys. Rev. D **77**, 105007 (2008); F. Girelli, T. Konopka, J. Kowalski-Glikman, and E. R. Livine, Phys. Rev. D **73**, 045009 (2006).

[14] S. Weinberg, *Gravitation and Cosmology: Principles and Applications of General Theory of Relativity* (John Wiley & Sons, U. K., 1972).

[15] S. Judes and M. Visser, Phys. Rev. D **68**, 045001 (2003).

[16] N. R. Bruno, G. Amelino-Camelia, and J. Kowalski-Glikman, Phys. Lett. B **522**, 133 (2001).

[17] Our methodology requires some elaboration. The well-known ‘‘soccer ball problem’’ manifests itself when one wants to construct multiparticle states in DSR. Quite clearly (if energy is added linearly) the total energy of a multiparticle state, or a soccer ball, can exceed the energy upper bound (Planck energy). To cure this problem it has been suggested before, by J. Magueijo, Phys. Rev. D **73**, 124020 (2006), albeit in an ad-hoc way, that one should use the average energy of a bound state in the DSR transformations; On the other hand, S. Hossenfelder, Phys. Rev. D **75**, 105005 (2007), has proposed that in a field theoretic framework, it is more natural to consider energy or momentum densities which would solve the ‘‘soccer ball problem’’.

[18] S. Das and D. Roychowdhury (work in progress).