

**Screening effects in plasma with charged Bose condensate**Alexander D. Dolgov,<sup>1,2,3,\*</sup> Angela Lepidi,<sup>1,2,†</sup> and Gabriella Piccinelli<sup>4,1,‡</sup><sup>1</sup>*Istituto Nazionale di Fisica Nucleare, Sezione di Ferrara, I-44100 Ferrara, Italy*<sup>2</sup>*Dipartimento di Fisica, Università degli Studi di Ferrara, I-44100 Ferrara, Italy*<sup>3</sup>*Institute of Theoretical and Experimental Physics, 113259 Moscow, Russia*<sup>4</sup>*Centro Tecnológico, FES Aragón, Universidad Nacional Autónoma de México, Avenida Rancho Seco S/N, Bosques de Aragón, Nezahualcóyotl, Estado de México 57130, México*

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The screening of a Coulomb field of test charge in plasma with a Bose condensate of an electrically charged scalar field is considered. It is found that the screened potential contains several different terms: one decreases as a power of distance (in contrast to the usual exponential Debye screening), and some others oscillate with an exponentially decreasing envelope. A similar phenomenon exists for fermions (Friedel oscillations), but fermionic and bosonic systems have quite different features. Several limiting cases and values of the parameters are considered and the resulting potentials are presented.

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**I. INTRODUCTION**

It is well known that an electric charge,  $Q$ , in plasma is screened according to the Debye law, so the long-ranged Coulomb field is transformed into the Yukawa type potential (see e.g. [1,2]):

$$U(r) = \frac{Q}{4\pi r} \rightarrow \frac{Q \exp(-m_D r)}{4\pi r}, \quad (1)$$

where the Debye screening mass,  $m_D$ , is expressed through the plasma temperature and chemical potentials of the charged particles; see below Eqs. (12) and (13). Physical interpretation of this result is evident: test charge polarizes plasma around, attracting opposite charge particles and thus the electrostatic field drops down exponentially faster than in vacuum. Formally the Debye screening appears from a pole at purely imaginary  $k$  in the photon propagator in plasma,  $(k^2 + \Pi_{00})^{-1}$ , where  $\Pi_{00}$  is the time-time component of the photon polarization operator.

By an evident reason the screening effects were studied historically first in fermionic i.e. in electron-proton and in electron-positron plasma. For degenerate fermionic plasma another and quite striking screening behavior was found. Namely, the screened potential drops down as a power of distance,  $1/r^3$  in the nonrelativistic case and  $1/r^4$  in the relativistic case multiplied by an oscillating function,  $\cos(k_F r)$  or  $\sin(k_F r)$ , where  $k_F$  is the Fermi momentum. This phenomenon is called Friedel oscillations [3,4]. Usually it is prescribed to a sharp (nonanalytic) cutoff of the Fermi distribution of degenerate electronic plasma at  $T = 0$ , but maybe it is better to say that the effect is related

to the logarithmic singularity of the photon polarization operator  $\Pi_{00}(\omega = 0, k)$ . This type of screening is discussed in Sec. III both in the nonrelativistic and relativistic cases for arbitrary, not necessarily zero, temperature.

Plasma with charged bosons attracted attention much later, both for pure scalar electrodynamics (for a review, see, e.g., Ref. [5]), or for quark-gluon plasma [2,6,7]. Surprisingly until last year the impact of a possible Bose condensate of charged fields on the photon polarization operator was not considered. Only recently an investigation of plasma with a Bose condensate of charged scalars was initiated [8–14]. It was found that in the presence of a Bose condensate the screened potential behaves similarly to that in the fermionic case, i.e., the potential oscillates, exponentially decreasing with distance [9,10]. This effect, however, in contrast to Friedel oscillations, does not come from the logarithmic branch point singularity in  $\Pi_{00}$  but from the pole in the photon propagator at a complex (not purely imaginary) value of  $k$ . It was shown that the polarization operator contains an infrared singular term  $\Pi_{00} \sim 1/k^2$  [10,11] that shifts the pole position from an imaginary axis (as in the Debye case) to a point with nonzero real and imaginary parts.

At nonzero temperature the polarization operator has another infrared singular term  $\sim 1/k$ . This term is odd with respect to the parity transformation,  $k \rightarrow -k$  and, as a result, the potential acquires the term which decreases as a power of distance but does not oscillate. Moreover, the polarization operator has logarithmic singularity as in the fermionic case and this singularity also generates an oscillating potential similar to the Friedel one. It is interesting that the screened potential is a nonanalytic function of the electric charge  $e$ . In particular, in a certain limit it may be inversely proportional to  $e$ , despite being calculated in the lowest order in  $e^2$ .

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Another oscillatory, exponentially damped, behavior of the potential between static charges has been reported in the literature: it was argued [15] that in nuclear matter at high densities and low temperatures, the Debye pole acquires a nonzero real part and so the screened potential oscillates (see also [16]). These *Yukawa oscillations* are short ranged oscillations and fade away with distance faster, as compared to the Friedel oscillations.

The screening of color charges in QCD in the presence of an uncharged pion condensate was considered recently in [17]. Since the condensate is uncharged its effect is quite different from those found in our work.

In this paper we further analyze the asymptotics of the screening effects arising in bosonic and fermionic plasma. In particular, we have taken into account all the contributions, including the ones from the logarithmic singularities in the photon polarization tensor and considered different limiting cases.

The content of the paper is the following. In Sec. II we reproduce our results for the photon polarization operator in plasma with a charged Bose condensate. Fermionic Friedel oscillations in nonrelativistic and relativistic cases both for  $T = 0$  and  $T \neq 0$  are considered in Sec. III. There we reproduce some already established results but use different techniques. We present this derivation because our method is new and more general than that presented in the literature. Moreover, the method is used in the bosonic case where such calculations have never been performed, and it is instructive to check the method in a simpler known case of fermions. On the other hand, our results for  $T \neq 0$  and relativistic electrons are new. In Sec. IV we calculate screening in bosonic plasma, taking into account the contributions from the poles in the complex  $k$  plane, from the integral along the imaginary axis, and from the logarithmic branch cuts. The last part has never been done before. We consider several conditions, in particular, bosons with or without condensate and eventually even in the absence of fermions. Finally, in Sec. V our conclusions are presented.

## II. POLARIZATION OPERATOR OF PHOTON IN MEDIUM

We confine ourselves to the lowest order in the electromagnetic coupling,  $e^2$ . The photon polarization operator,  $\Pi_{\mu\nu}(\omega, k)$ , in this approximation is well known; see, e.g., books [2,6]. For the calculation of the latter either imaginary or real time methods are used. However, the result can be obtained in a simpler way [10] just by including into the photon Green's function the effects of medium. This can be done by taking into account both the expectation value of the time ordered product of  $\langle A_\mu(x)A_\nu(y) \rangle$  over vacuum and the contribution of matter states, weighted by the particle distribution,  $f_j(q)$ , where  $j$  denotes the particle type and  $q$  is the particle momentum. The resulting expressions, found in many works—see, e.g., [10] and references therein—are the following:

$$\begin{aligned} \Pi_{\mu\nu}^B(k) = e^2 \int \frac{d^3q}{(2\pi)^3 E} [f_B(E, \mu) + \bar{f}_B(E, \bar{\mu})] \\ \times \left[ \frac{1}{2} \frac{(2q - k)_\mu (2q - k)_\nu}{(q - k)^2 - m_B^2} \right. \\ \left. + \frac{1}{2} \frac{(2q + k)_\mu (2q + k)_\nu}{(q + k)^2 - m_B^2} - g_{\mu\nu} \right], \end{aligned} \quad (2)$$

$$\begin{aligned} \Pi_{\mu\nu}^F(k) = 2e^2 \int \frac{d^3q}{(2\pi)^3 E} [f_F(E, \mu) + \bar{f}_F(E, \bar{\mu})] \\ \times \left[ \frac{q_\nu(k + q)_\mu - q^\rho k_\rho g_{\mu\nu} + q_\mu(k + q)_\nu}{(k + q)^2 - m_F^2} \right. \\ \left. + \frac{q_\nu(q - k)_\mu + q^\rho k_\rho g_{\mu\nu} + q_\mu(q - k)_\nu}{(k - q)^2 - m_F^2} \right], \end{aligned} \quad (3)$$

where  $k = [\omega, \mathbf{k}]$  and  $q = [E, \mathbf{q}]$  are four momenta of photon and charged particles living in plasma,  $E = (q^2 + m_{B,F}^2)^{1/2}$ , with  $m_{B,F}$  being either the mass of charged bosons or fermions.  $\Pi_{\mu\nu}^B$  and  $\Pi_{\mu\nu}^F$  are, respectively, the contribution to the polarization tensor from bosons and fermions, and  $\mu$  and  $\bar{\mu}$  are chemical potentials for particles and antiparticles. Chemical potentials for bosons and fermions are generally unequal; moreover, the chemical equilibrium is not necessarily maintained and  $\mu + \bar{\mu} \neq 0$ . Though in what follows we present all the results for  $\mu + \bar{\mu} = 0$ , it is straightforward to generalize them to arbitrary  $\mu$  and  $\bar{\mu}$ . The derivation of these intuitively clear expressions from the first principles in the case in which  $f_B$  contains a Bose condensate contribution can be found in our paper [10]. There, starting from the basics of the field theory, we solve the equation of motion for the photon propagator, averaging not only over vacuum fluctuations, but over the medium as well. In kinetic equilibrium the distribution functions take the form:

$$f_{B,F} = \frac{1}{\exp[(E - \mu)/T] \pm 1}, \quad (4)$$

where the signs  $+$  and  $-$  stay for fermions and bosons, respectively. In the case in which the boson chemical potential is equal to its maximum allowed value  $\mu = m_B$  (or  $\bar{\mu} = m_B$ ), the formation of the Bose condensate is possible and in equilibrium the Bose distribution function acquires an additional term describing accumulation of bosons in the zero momentum mode:

$$f_B = C\delta^{(3)}(\mathbf{q}) + \frac{1}{\exp[(E - m_B)/T] \pm 1}, \quad (5)$$

where  $C$  is a constant parameter describing the amplitude of the condensate. One can easily check that the distribution in Eq. (5) annihilates the collision integral and hence is the equilibrium distribution for  $\mu = m_B$ .

The screening of the test charge in the static case is determined by the zero frequency value of  $\Pi_{00}(0, k)$ . We assume that the plasma is homogeneous and isotropic, so the polarization tensor depends only upon the magnitude of vector  $\mathbf{k}$  but not on its direction. The corresponding expressions can be easily read from Eqs. (2) and (3):

$$\Pi_{00}^B(0, k) = \frac{e^2}{2\pi^2} \int_0^\infty \frac{dq q^2}{E_B} [f_B(E_B, \mu_B) + \bar{f}_B(E_B, \bar{\mu}_B)] \times \left[ 1 + \frac{E_B^2}{kq} \ln \left| \frac{2q+k}{2q-k} \right| \right], \quad (6)$$

$$\Pi_{00}^F(0, k) = \frac{e^2}{2\pi^2} \int_0^\infty \frac{dq q^2}{E_F} [f_F(E_F, \mu_F) + \bar{f}_F(E_F, \bar{\mu}_F)] \times \left[ 2 + \frac{(4E_F^2 - k^2)}{2kq} \ln \left| \frac{2q+k}{2q-k} \right| \right]. \quad (7)$$

In what follows we will omit the first argument in the polarization tensor, i.e., write  $\Pi_{00}(0, k) \equiv \Pi_{00}(k)$ .

Evidently the first (condensate) term in  $f_B$  gives rise to the quadratic infrared singularity  $\Pi_{00} \sim 1/k^2$ , as found in Refs. [10,11]. At nonzero temperature the pole singularity of the Bose distribution at  $q=0$  leads to an additional infrared pole  $\sim 1/k$  in the polarization tensor of photons [10]. Thus at low values of the photon momentum  $\Pi_{00}$  can be expanded as [10]

$$\Pi_{00}^B(0, k) = e^2 \left[ h(T) + \frac{m_B^2 T}{2k} + \frac{1}{(2\pi)^3} \frac{C}{m_B} \left( 1 + \frac{4m_B^2}{k^2} \right) \right], \quad (8)$$

where the function  $h(T)$  is independent of  $k$  and has the limiting values:

$$h(T) = \begin{cases} T^2/3 & (\text{high } T), \\ \zeta(3/2)(m_B T^3)^{1/2}/(2\pi)^{3/2} & (\text{low } T). \end{cases} \quad (9)$$

The low  $T$  limit of the function  $h(T)$  is however always subdominant with respect to the second term in Eq. (8) which comes from the logarithmic term in Eq. (6).

In the expression of the photon polarization tensor written above the singularities of  $\Pi_{00}$  due to pinching of the integration contour by the poles of  $f_B(E_B, m_B)$  and the logarithmic branch point in the integrand of Eq. (6) are not taken into account. It will be done in Sec. IV

The contribution of fermions into the polarization tensor is not infrared singular, so it is convenient to present the latter as

$$\Pi_{00}^F(k) = \Pi_{00}^F(0) + [\Pi_{00}^F(k) - \Pi_{00}^F(0)], \quad (10)$$

where

$$\Pi_{00}^F(0) = \frac{e^2}{\pi^2} \int \frac{dq}{E} (f + \bar{f})(q^2 + E^2). \quad (11)$$

In the case of relativistic fermions with nonzero chemical potential,  $\mu$ , the zero momentum limit of  $\Pi_{00}^F$  is [18]

$$\Pi_{00}^F(0) = e^2 \left( \frac{T^2}{3} + \frac{\mu^2}{\pi^2} \right). \quad (12)$$

This expression is valid in the limit  $m_F \ll \mu$  and  $m_F \ll T$ , while in the nonrelativistic case for positive  $(\mu - m)$  and small  $T$  we find

$$\Pi_{00}^F(0) = \frac{\sqrt{2}e^2 m_F^{3/2} (\mu - m_F)^{1/2}}{\pi^2} - \frac{e^2 T^2}{12\sqrt{2}} \left( \frac{m_F}{\mu - m_F} \right)^{3/2} + \dots \quad (13)$$

If  $\mu < m$ , the polarization tensor is exponentially suppressed,  $\Pi_{00} \sim \exp[-(m - \mu)/T]$ . For the Debye mass we find the well-known nonrelativistic result:

$$m_D^2 = \frac{e^2 n_F}{T}. \quad (14)$$

Here, as above in the bosonic case, the singularities of  $\Pi_{00}^F$  due to the logarithmic branch point in integral (7) are not included. For that, see the next section.

The potential of a test charge,  $Q$ , modified by the plasma screening effects is given by the Fourier transform of the photon propagator in plasma:

$$\begin{aligned} U(r) &= \frac{Q}{(2\pi)^3} \int \frac{d^3 k \exp(i\mathbf{k}\mathbf{r})}{k^2 + \Pi_{00}(k)} \\ &= \frac{Q}{2\pi^2} \int_0^\infty \frac{dk k^2}{k^2 + \Pi_{00}(k)} \frac{\sin(kr)}{kr} \\ &= \frac{Q}{2\pi^2 r} \text{Im} \int_0^\infty \frac{dk k e^{ikr}}{k^2 + \Pi_{00}(k)}. \end{aligned} \quad (15)$$

Usually the integrand in Eq. (15) is an even function on  $k$  and the integration along the line of positive real  $k$  can be transformed into the contour integral in the upper complex  $k$  plane. However, in the case of bosons with  $\mu_B = m_B$ , the polarization operator contains an odd term  $m_B^2 T/2k$ , Eq. (8), and the usual contour transformation is not applicable. So we express integral (15) through the integral along the imaginary upper  $k$  axis plus contribution of singularities in the upper  $k$  plane. If  $\Pi_{00}$  is an even function of  $k$  and  $(k^2 + \Pi_{00})^{-1}$  is regular on the imaginary  $k$  axis, the imaginary part of the integral along the imaginary axis vanishes. If the integrand has a pole at positive imaginary  $k = ik_D$ , i.e.,

$$-k_D^2 + \Pi_{00}(ik_D) = 0, \quad (16)$$

this poles contributes into the integral as  $i\pi\delta(k - ik_D)$  and gives rise to the usual exponential Debye screening. If  $\Pi_{00}$  contains an odd part, the integral along the imaginary  $k$  axis gives a contribution to the potential which decreases only as the power of distance [10].

There may also be poles at complex  $k = k_p$ , when both real and imaginary parts of  $k_p$  are nonzero. Such poles have been found for plasma with a charged Bose condensate [9,10]. They produce an oscillating behavior superimposed

on the exponential decrease of the potential. It was argued [15] that complex poles also exist in the plasma of strongly interacting particles (pions and nucleons) and in QCD plasma.

There are also logarithmic singularities of  $\Pi_{00}(k)$  at some nonzero  $\text{Im}k$  and the integrals along the corresponding cuts also produce oscillations in the screened potential but the exponential cutoff is much weaker; it is proportional to temperature and for zero  $T$  it becomes a power law one. For fermions, this effect, called Friedel oscillations, is known for a long time [3,4], while for bosons a similar phenomenon has not been studied before.

### III. FRIEDEL OSCILLATIONS IN FERMIONIC PLASMA

We consider here the Friedel oscillations in fermionic plasma. The nonrelativistic case is discussed in Refs. [3,4,19], both at zero and nonzero temperatures. The relativistic case was studied in [19]. In what follows all four cases are presented, considered in somewhat different ways.

Singularities of  $\Pi_{00}(k)$  in the complex  $k$  plane appear when the singular points of the integrand in Eq. (7) in the complex  $q$  plane pinch the contour of integration or coincide with the integration limit at  $q = 0$ . The usual calculation is done at zero temperature when the fermion distribution tends to the  $\theta$  function and hence the integral over  $dq$  goes from zero to the Fermi momentum,  $q_f$ . The singularity in  $\Pi_{00}^F$  appears when the branch points of the logarithm at  $k = \pm 2q$  move to the integration limit at  $q = q_f$ . In a more general case of arbitrary temperature the integrand is a smooth function of  $q$  and integration goes up to infinity. The integrand has two kinds of singularities. First, there are poles in the distribution function  $f_F$  which are situated at

$$q_n^2 = [\mu \pm i\pi T(2n + 1)]^2 - m_F^2, \quad (17)$$

where  $n$  runs from 0 to infinity.

The second type of singularities are branch points of the logarithm at

$$q_b = \pm k/2. \quad (18)$$

The singularities of  $\Pi_{00}(k)$  are situated at such  $k_n$  for which  $q_n$  and  $q_b$  coincide,  $q_n = q_b$ , and the poles  $q_n$  and branch points  $q_b$  approach the integration contour in the  $q$  plane from the opposite sides. Since, according to the discussion in the previous section and Eq. (15), we consider  $k$  in the first quadrant of the complex  $k$  plane, only the singularities with  $\text{Re}k \geq 0$  and  $\text{Im}k \geq 0$  contribute to the asymptotics of the potential, i.e.,

$$k_n = 2q_n = [(\mu + i\pi T(2n + 1))^2 - m_F^2]^{1/2}. \quad (19)$$

Symbolically the integral on the right-hand side of Eq. (15) can be written as a sum of three contributions:

$$I_0 = \int_0^\infty [idk] + 2\pi i \sum [\text{Res}] + \sum_n \int_{k_n}^{k_n + i\infty} \Delta, \quad (20)$$

where the first integral goes along the positive imaginary axis in the  $k$  plane, the second one is the sum of the residues of the poles on the integrand (if the poles are on the imaginary axis, only one-half of the residue is to be taken), and the third term is the integral of the discontinuity over the branch line of the logarithmic singularity of  $\Pi_{00}(k)$ . The integration contour in the complex  $k$  plane is schematically depicted in Fig. 1, where only one pole and one branch cut are included.

Before calculating the singular part of  $\Pi_{00}$ , let us first note that we are interested only in singularities in the first quadrant in the  $k$  plane and thus only the contribution from  $-\ln|2q - k|$  should be taken. Since the absolute value of the argument can be written as the limit of  $\epsilon \rightarrow 0$  of  $|2q - k| = [(2q - k)^2 + \epsilon^2]^{1/2}$ , the logarithmic contribution into  $U(r)$  is given by

$$\begin{aligned} \ln \left| \frac{k + 2q}{k - 2q} \right| &\rightarrow -\ln|k - 2q| \\ &= -[\ln(k - 2q + i\epsilon) + \ln(k - 2q - i\epsilon)]/2 \\ &\rightarrow -\ln(k - 2q - i\epsilon)/2. \end{aligned} \quad (21)$$

The singular part  $\Pi_{00}^{(n)}$  near  $k_n$  can be determined as follows. The integral along the contour squeezed between  $q_n$  and  $q_b$  is equal to the residue of the integrand at the pole multiplied by  $2\pi i$  plus a regular part at  $k = k_n$ . The pole term near  $q = q_n + z$  is equal to

$$\frac{1}{\exp[(E_n - \mu)/T] + 1} = -\frac{E_n T}{z q_n}. \quad (22)$$

The residue in the pole gives the singular term in  $\Pi_{00}$  equal

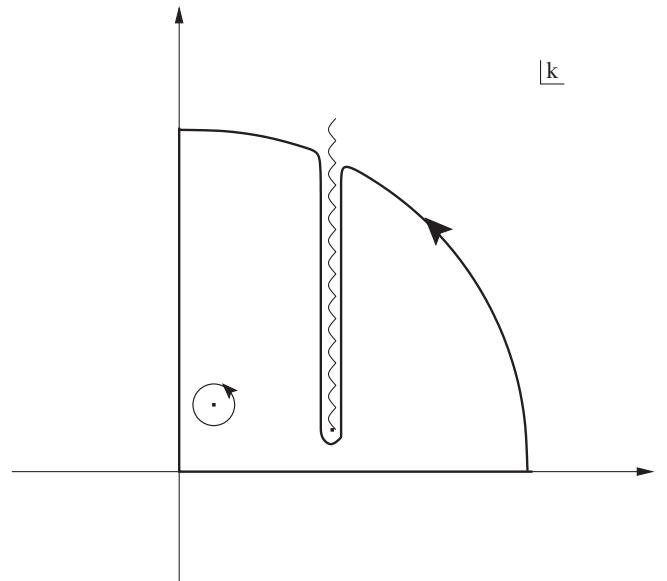


FIG. 1. Contour of integration in the complex  $k$  plane.

to

$$\Pi_{00}^{(n)}(k) = \frac{ie^2T}{4\pi k}(4E_n^2 - k^2)\ln(k - 2q_n - i\epsilon), \quad (23)$$

where  $q_n$  is the pole position given by Eq. (19) and  $E_n = \sqrt{q_n^2 + m^2}$ . We have neglected here the contributions of antiparticles assuming that the chemical potential is sufficiently large. The discontinuity of  $\Pi_{00}$  at the branch line  $k = 2q_n + iy$ , where  $y$  runs from zero to infinity, is equal to

$$\Delta\Pi_{00}^{(n)} = \Pi_{00}^{(n)+} - \Pi_{00}^{(n)-} = -\frac{e^2T(4E_n^2 - k^2)}{2k}, \quad (24)$$

where the upper index  $+$  or  $-$  indicates that the value of  $\Pi_{00}$  is taken on the right- or the left-hand side of the cut.

The contribution of this discontinuity into the asymptotic behavior of  $U(r)$ , Eq. (15), is equal to

$$U_n(r) = \frac{Q}{2\pi^2 r} \text{Im} \int_0^\infty idyk \exp(-yr + 2iq_n r) \frac{(-\Delta\Pi_{00})}{[k^2 + \Pi_{00}^{(n)+}(k)][k^2 + \Pi_{00}^{(n)-}(k)]}. \quad (25)$$

Here  $k = 2q_n + iy$ . For the fermionic plasma we can neglect  $y$  in comparison to  $q_n$ , because in the limit of large distances  $y \sim 1/r$ . However, in the bosonic case a non-vanishing contribution comes from subdominant in  $y$  terms; see below.

Below we consider separately, first, relativistic and, second, nonrelativistic cases. In the relativistic limit  $E_n = q_n$  and the factor in front of the logarithm, Eq. (23), and discontinuity (24) vanish at the branch point and the discontinuity becomes purely imaginary in the leading order,  $\Delta\Pi_{00}^{(n)} = ie^2Ty$ . This leads to a faster decrease of the screened potential in comparison with the nonrelativistic case,  $1/r^4$  instead of  $1/r^3$ , and to the change of phase,  $\sin(2\mu r)$  instead of  $\cos(2\mu r)$ .

In the relativistic case, when  $m \ll T$  but  $\mu$  may be large, the poles are situated at

$$E_n = q_n = \mu \pm i\pi T(2n + 1). \quad (26)$$

Since  $|k|^2 > 4|q_n|^2 > 4(\mu^2 - m_F^2)$ , then for sufficiently large  $\mu$ ,  $\mu > m_F$ , and low  $T$  we can neglect  $\Pi_{00} \sim e^2\mu^2$  in the denominator in comparison with  $4q_n^2$  and obtain

$$\begin{aligned} U_n(r) &= \frac{Qe^2T}{16\pi^2 q_n^3 r^3} \text{Im} e^{2iq_n r} \\ &= \frac{Qe^2T}{16\pi^2 q_n^3 r^3} \sin(2\mu r) e^{-2\pi(2n+1)Tr}. \end{aligned} \quad (27)$$

For non-negligible  $T$ , the dominant term is that with  $n = 0$  and though it decreases exponentially, the power of the exponent may be much smaller than the standard one, Eq. (1) with  $m_D = e\mu/\pi$ , as follows from Eq. (12).

At small  $T$  the result is proportional to the temperature and thus formally vanishes at  $T = 0$ . However, at small  $T$

the total contributions of the branch points diverge as  $1/T$ , so summing up all  $U_n$  we find

$$U_{\text{cut}} = \sum_{n=0}^{\infty} U_n = \frac{e^2QT}{16\pi^2 r^3 \mu^3} \frac{\sin(2\mu r) \exp(-2\pi r T)}{1 - \exp(-4\pi r T)}. \quad (28)$$

For  $T \rightarrow 0$  and large  $r$  we can take  $q_n = \mu$  because the effective  $n$ 's are of the order of  $n_{\text{eff}} \sim 1/(4\pi r T)$  and  $nT \sim 1/r \ll \mu$ .

For very small  $T$  such that  $rT \ll 1$  we obtain

$$U_{\text{cut}} = \frac{e^2Q}{64\pi^3} \frac{\sin(2\mu r)}{r^4 \mu^3}, \quad (29)$$

in agreement with Ref. [19]. However, if  $rT \geq 1$ , then, as we mentioned above, the screened potential decays exponentially, similar to normal Debye screening with the important difference that the screening mass does not contain the electromagnetic coupling,  $e$ . On the other hand, the magnitude of the screened potential is proportional to  $e^2$ . So formally for  $e = 0$  the oscillating potential vanishes, while the Debye one tends to the vacuum Coulomb expression.

The ratio of the main term in the potential at  $T \neq 0$  to that at  $T = 0$  is equal to

$$\frac{U(r, T)}{U(r, T = 0)} = \frac{4\pi r T e^{-2\pi r T}}{1 - e^{-4\pi r T}}. \quad (30)$$

It is always smaller than unity, i.e., the screening is weakest at  $T = 0$ .

Let us turn now to the nonrelativistic limit, when  $m_F \gg T$ ,  $\mu - m_F \ll m_F$ , and for simplicity  $\tilde{\mu} = \mu - m_F \gg T$ . The calculations go along the same lines with evident modifications. The poles of the distribution function  $f$  are located at

$$\begin{aligned} q_n &= [(\mu^2 - m_F^2) + 2i\pi\mu T(1 + 2n)]^{1/2} \\ &\approx \sqrt{2m_F\tilde{\mu}} \left[ 1 + \frac{i\pi\mu T(1 + 2n)}{\mu^2 - m_F^2} \right]. \end{aligned} \quad (31)$$

The logarithmic singular part of  $\Pi_{00}$  corresponding to this pole is given by the same Eq. (23) and the discontinuity on the cut is given by Eq. (24). An essential difference now is that the discontinuity does not vanish near the branch point,  $(4E_n^2 - 4q_n^2) = 4m_F^2 \neq 0$ :

$$\Delta\Pi_{00} \approx e^2Tm_F^2/k. \quad (32)$$

Thus the contribution of the  $n$ th pole into the screened potential is equal to

$$U_n(r) = \frac{e^2QTm_F^2}{\pi^2 r} \text{Im} \int_0^\infty \frac{idye^{2iq_n r - yr}}{[k^2 + \Pi_{00}^{(n)+}(k)][k^2 + \Pi_{00}^{(n)-}(k)]}. \quad (33)$$

Here, as in the relativistic case above,  $k = 2q_n + iy$ . Neglecting  $k^2$  in comparison with  $\Pi_{00}$ , see Eq. (12) and

discussion below Eq. (26), we obtain

$$\begin{aligned}
 U_n(r) &= \frac{Qe^2 T m_F^2}{16\pi^2 q_n^4 r^2} \text{Im}[ie^{2iq_n r}] \\
 &= \frac{Qe^2 T}{64\pi^2 r^2 \tilde{\mu}^2} \cos(2\sqrt{2m_F \tilde{\mu}} r) \\
 &\quad \times \exp\left[-2\pi(2n+1)\frac{rT\mu}{\sqrt{2m_F \tilde{\mu}}}\right]. \quad (34)
 \end{aligned}$$

If temperature is not extremely small, the term with  $n = 0$  gives the slowest decreasing part of the potential, but for  $T \rightarrow 0$  we need to take into account the whole sum  $U_{\text{cut}}(r) = \sum U_n(r)$ :

$$\begin{aligned}
 U_{\text{cut}}(r) &= \frac{Qe^2 T m_F^2}{64\pi^2 r^2 \tilde{\mu}^2} \cos(2\sqrt{2m_F \tilde{\mu}} r) \\
 &\quad \times \frac{\exp(-\pi r T \sqrt{2m_F/\tilde{\mu}})}{1 - \exp(-2\pi r T \sqrt{2m_F/\tilde{\mu}})}. \quad (35)
 \end{aligned}$$

Asymptotically for large  $r$  but  $2\pi r T \sqrt{2m/\tilde{\mu}} < 1$  the potential tends to

$$U_{\text{cut}}(r) = \frac{Qe^2 m_F \cos(2q_F r)}{64\pi^3 r^3 q_F^3}, \quad (36)$$

where  $q_F = \sqrt{2\tilde{\mu}m_F}$ . The result agrees with that presented in Ref. [19]. The potential in Eq. (35) is plotted in Fig. 2 as a function of distance  $r$  and temperature  $T$  for  $m_F = 0.5$  MeV and  $\mu_F = 0.55$  MeV. Temperatures vary from  $10^{-4}$  and  $10^{-2}$  MeV, which corresponds to  $(1.16 \times 10^6 - 1.16 \times 10^8)$  K. Distances vary from 1 to 100  $\text{MeV}^{-1}$ , corresponding to  $(2 \times 10^{-11} - 2 \times 10^{-9})$  cm. The main features for the plot in the relativistic case are similar to the nonrelativistic one.

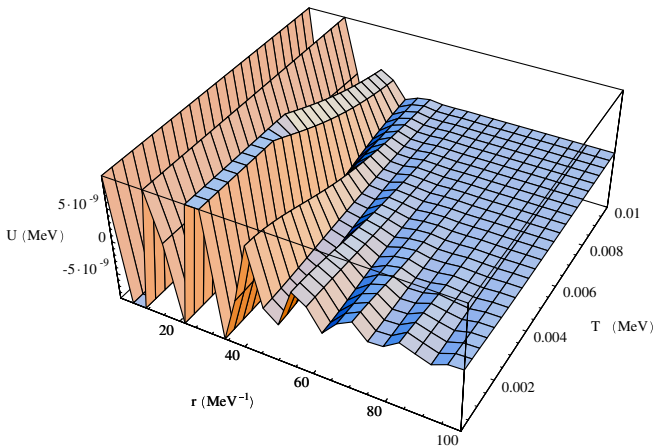


FIG. 2 (color online). Friedel oscillations for massive fermions—see Eq. (35)—with  $m_F = 0.5$  MeV, and  $\mu_F = 0.55$  MeV. Temperatures are in MeV and distances in  $\text{MeV}^{-1}$ . The exponential damping at large distance and/or temperature, as well as oscillations as a function of the distance  $r$ , can be seen.

Note in conclusion that above we have neglected  $\Pi_{00}$  in comparison with  $4q_n^2$ . It is justified for sufficiently small  $e^2$ . Otherwise one has to calculate the integral more accurately taking into account the mild logarithmic singularity in  $\Pi_{00}$ , which goes to infinity at the branch point for nonrelativistic fermions and goes to zero for relativistic ones.

#### IV. SCREENING IN BOSONIC PLASMA

As we have already mentioned the photon polarization tensor in the presence of the Bose condensate is infrared singular, having at small  $k$  form (8). The terms  $\sim 1/k^2$  have been found in Refs. [10,11], while the  $1/k$  term, which vanishes at  $T = 0$ , has been found in Ref. [10]. Because of the  $1/k^2$  term the pole of the photon Green's function shifts from an imaginary axis in contrast to the usual Debye case when the pole is purely imaginary. Because of its real part the screened potential acquires an oscillating factor superimposed on the exponential decrease [9,10]. The positions of poles in integral (15) are given by the equation  $k^2 + \Pi_{00}(k) = 0$ , which is convenient to write as

$$k^2 + e^2 \left( m_0^2 + \frac{m_1^3}{k} + \frac{m_2^4}{k^2} \right) = 0, \quad (37)$$

where

$$m_0^2 = \frac{C}{(2\pi^3)m_B} + h(T) + m_D^{(F)2}(T, \mu_F), \quad (38)$$

$$m_1^3 = \frac{m_B^2 T}{2}, \quad (39)$$

$$m_2^4 = \frac{4m_B C}{(2\pi)^3}, \quad (40)$$

where  $h(T)$  is defined in Eq. (9) and  $m_D^{(F)}$  is the fermionic Debye mass. For relativistic fermions it is given by Eq. (12) and for nonrelativistic ones by Eq. (14). If plasma is electrically neutral because of the mutual compensation of bosons and fermions, the chemical potential of fermions is expressed through the amplitude of the Bose condensate and  $\mu_B = m_B$ . However, one can imagine the case when there are two types of charged bosons and neutrality is achieved by the opposite charge densities of these bosons. In such plasma the fermionic Debye mass is zero.

In what follows we analyze different contributions to the electrostatic potential  $U(r)$  for different limiting values of the parameters. In Sec. IVA we investigate further the contribution from the poles in integral (15). In Sec. IVB we present the contribution from the imaginary axis which arises when the integrand in Eq. (15) is not an even function of  $k$ . Finally in Sec. IVC we calculate the contributions from integration along the branch cuts of the logarithmic terms in  $\Pi_{00}$ ; see Eq. (6). The integration contour is similar to that for fermions, Fig. 1, but the

positions of the poles are evidently shifted; see the following section.

### A. Contribution from poles

At low temperatures the four roots of Eq. (37) are given by

$$k_{1,2,3,4} = \pm \frac{i}{\sqrt{2}} [e^2 m_0^2 \pm \sqrt{e^4 m_0^4 - 4e^2 m_2^4}]^{1/2}. \quad (41)$$

As is mentioned above, we are interested only in the poles in the first quadrant in the complex  $k$  plane. If  $e^4 m_0^4 > 4e^2 m_2^4$ , all the poles are purely imaginary and the Coulomb potential is screened exponentially, similar to the usual Debye situation. The poles on the positive imaginary axis are situated at

$$k_{1,2} = \frac{iem_0}{\sqrt{2}} (1 \pm \sqrt{1 - 4m_2^4/e^2 m_0^4})^{1/2}. \quad (42)$$

The contribution of these poles into the potential is

$$U(r) = \frac{Q}{4\pi r} \frac{k_1^2 e^{ik_1 r} - k_2^2 e^{ik_2 r}}{k_1^2 - k_2^2}. \quad (43)$$

In the limit of the small ratio  $m_2^2/em_0^2$  the potential becomes

$$U(r)_{\text{pole}} \approx \frac{Q}{4\pi r} \left[ \exp\left(-em_0 r \left(1 - \frac{m_2^4}{2e^2 m_0^4}\right)\right) - \frac{m_2^4}{e^2 m_0^4} \exp(-m_2^2 r/m_0) \right]. \quad (44)$$

Thus for a small  $m_2$ , the screening, though exponential, can be much weaker than the usual Debye one.

In the opposite case,  $e^4 m_0^4 < 4e^2 m_2^4$ , the poles acquire a real part and now only one pole is situated in the first quadrant. The potential oscillates around the exponentially decreasing envelope [9,10]. The result is especially simple in the limit of large  $m_2$ :

$$U(r)_{\text{pole}} = \frac{Q}{4\pi r} \exp(-\sqrt{e/2} m_2 r) \cos(\sqrt{e/2} m_2 r). \quad (45)$$

A more interesting situation is realized at larger temperatures, when the term  $m_1^3/k$  in the polarization operator Eq. (37) is non-negligible. The contribution of the poles into the asymptotics of the screened potential is similar to the above considered case of low  $T$  if  $m_2$  dominates in  $\Pi_{00}$ , but for a small  $m_2$ , e.g., if  $C = 0$ , the poles are situated at  $k = e^{2/3}(-1)^{1/3}(m_B^2 T/2)^{1/3}$ . The potential exponentially decreases at large distances but the power of the exponent is proportional to temperature and at small  $T$  the decrease of  $U(r)$  may be rather weak.

### B. Contribution from the integral along the imaginary axis

Because of the odd term,  $m_1^3/k$ , in the polarization operator the imaginary part of integral (15) along the imaginary axis in the complex  $k$  plane is nonzero, and the screened potential drops as a power of  $r$ :

$$U(r) = -\frac{Qe^2 m_1^3}{2\pi^2 r^2} \int_0^\infty \frac{dz \exp(-z)}{[-(z/r)^2 + e^2(m_0^2 - m_2^4 r^2/z^2)]^2 + e^4 m_1^6 r^2/z^2}. \quad (46)$$

The previous expression has been obtained by substituting  $k = iy$  and then  $z = yr$ . If  $m_2 \neq 0$  the dominant term at large  $r$  behaves as

$$U(r) = -\frac{12Qm_1^3}{\pi^2 e^2 r^6 m_2^8}. \quad (47)$$

However, if the temperature is not zero and the bosonic chemical potential reaches its upper limit,  $\mu = m_B$ , but the condensate is not yet formed, the term proportional to  $m_1$  dominates and the asymptotic decrease of the potential becomes much slower:

$$U(r) = -\frac{Q}{\pi^2 e^2 r^4 m_1^3} = -\frac{2Q}{\pi^2 e^2 r^4 m_B^2 T}. \quad (48)$$

So the formation of the condensate manifests itself by a strong asymptotic increase of screening. This effect may be a signal of formation of the Bose condensate.

It is interesting that the screened potential is inversely proportional to the fine structure constant  $\alpha = e^2/4\pi$ .

### C. Contribution from the logarithmic branch cuts

Let us estimate now the effects of the logarithmic singularities of  $\Pi_{00}$  on the asymptotics of the screened potential (analog of the Friedel oscillations). Technically the calculations are similar to those made in Sec. III but the results are noticeably different. We assume here that the chemical potential of bosons reaches its maximum value,  $\mu = m_B$ . For smaller  $\mu$  there is not much difference between bosons and nondegenerate fermions, while for  $\mu = m_B$  new phenomena arise, which are absent for fermions.

The poles in the integrand of Eq. (6), which lead to the singularities of  $\Pi_{00}(k)$  in the first quadrant of the complex  $k$  plane, are situated at

$$q_n = (4i\pi n T m_B)^{1/2} (1 + i\pi n T/m_B)^{1/2}. \quad (49)$$

Here  $n$  runs from 1 to infinity, because there is no pole at  $q = 0$  since the numerator of the integrand is proportional to  $q^2$ .

The singularities in  $\Pi_{00}(k)$  are situated at such  $k$  where the singularities of the integrand in Eq. (6) pinch the

integration contour, i.e. as above, at  $k_n = 2q_n$ . The singular part of  $\Pi_{00}$  is calculated in the same way as it has been done for fermions and is equal to the residue of the integrand:

$$\Pi_{00}^{(n)B} = -\frac{ie^2TE_n^2}{2\pi k} \ln\left(\frac{k-2q_n-i\epsilon}{k+2q_n+i\epsilon}\right), \quad (50)$$

where  $E_n = \sqrt{q_n^2 + m_B^2}$ .

The discontinuity of this term across the logarithmic cut is  $\Delta\Pi_{00}^{(n)B} = e^2TE_n^2/k$ . Correspondingly the contribution of this singularity into the asymptotics of  $U(r)$  is given by

$$U_n^B(r) = -\frac{Qe^2T}{2\pi^2r} \operatorname{Re} \int_0^\infty \frac{dyE_n^2e^{2iq_nr}e^{-ry}}{[k^2 + \Pi_{00}^{(+)}][k^2 + \Pi_{00}^{(-)}]}, \quad (51)$$

where  $k = 2q_n + iy$  and  $E_n^2 = q_n^2 + m_B^2$ , and  $\Pi_{00}^\pm$  are the values of the polarization tensor on right and left banks of the cut. Note that at  $r \rightarrow \infty$  the effective  $y$  is small,  $y \sim 1/r$ .

An important difference between bosonic and fermionic cases is that the position of the pole for fermions, Eqs. (26) and (31), does not move to zero when  $T \rightarrow 0$ , while for bosons  $q_n^2 \sim T$ . Correspondingly one can neglect  $\Pi_{00}^F$  in comparison with  $k_n^2$ , while it may be an invalid approximation for bosons.

Let us first consider the case of low temperatures when  $\Pi_{00}$  is dominated by the constant fermionic contribution,  $\Pi_{00}^F \approx m_D^2$ , where  $m_D^2$  is given either by Eq. (12) or (13). At large  $r$  and nonzero  $T$  the logarithmic contribution into the screened potential is essentially given by the first term with  $n = 1$ :

$$U_1(r) = -\frac{Q\pi^2}{2e^2} \frac{Tm_B^2}{r^2\mu_F^4} \exp(-2\sqrt{2\pi m_B T}r) \times \cos(2\sqrt{2\pi m_B T}r). \quad (52)$$

Here we took the relativistic limit for  $\Pi_{00}^F$ . The result is easy to rewrite in the nonrelativistic case. The potential in Eq. (52) is plotted in Fig. 3. The bosonic chemical potential is taken to be equal to its limiting value,  $\mu_B = m_B$ , and the boson mass is assumed to be the same as the fermion mass in Fig. 2,  $m_B = m_F = 0.5$  MeV. Such a low mass of bosons is chosen simply for illustration. In the realistic case charged bosons are much heavier than the charged fermions, though it is not excluded that there exists an unknown gauge symmetry with charged bosons lighter than fermions.

The temperature in Fig. 3 varies from  $10^{-4}$  to 0.1 MeV, corresponding to  $(1.16 \times 10^8 - 1.16 \times 10^9)$  K, while distances vary from 1 to 100  $\text{MeV}^{-1}$ , corresponding to  $(2 \times 10^{-11} - 2 \times 10^{-9})$  cm.

Figure 4 shows the same potential but with higher mass for bosons,  $m_B = 100$  MeV, that is of the order of the pion mass. The fermion mass and chemical potential are taken the same as above. The temperature varies in the range

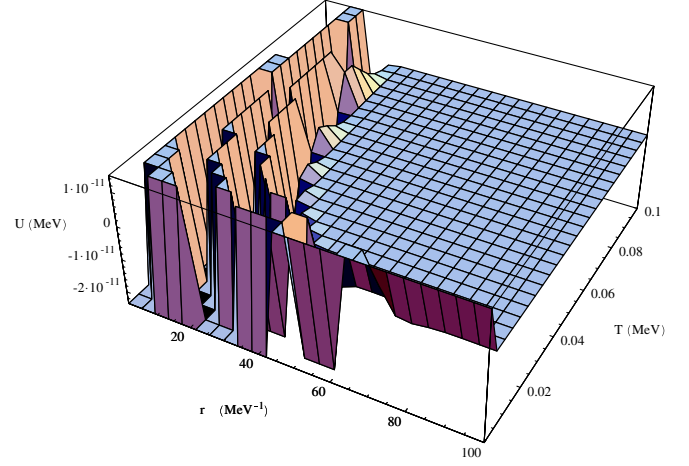


FIG. 3 (color online). Oscillation of the electrostatic potential in the presence of bosonic plasma; see Eq. (52). The boson mass is equal to the fermion one in Fig. 2,  $m_B = 0.5$  MeV and the chemical potential is  $\mu_B = m_B$ . Temperatures are in MeV and distances in  $\text{MeV}^{-1}$ .

$10^{-6} - 5 \times 10^{-2}$  MeV or  $1.16 \times 10^4 - 5.8 \times 10^8$  K and the distance in  $10^{-2} < r(\text{MeV}^{-1}) < 10$  corresponding to  $2 \times 10^{-13} < r(\text{cm}) < 2 \times 10^{-10}$ . We can see from these figures that if we increase the boson mass, the bosonic potential fades away faster.

In the limit of  $T \rightarrow 0$  (analogous to the discussed above Friedel case) we should take the sum  $\sum_{n=1}^\infty U_n$ , because all the terms are of the same order of magnitude and  $n_{\text{eff}} \sim 1/(4\pi m_B T r^2)$ . So we could expect that the sum is inversely proportional to  $T$  and the potential is nonvanishing at  $T = 0$ , the same as in the fermionic case. However, the summation is not so simple as previously thought, because we do

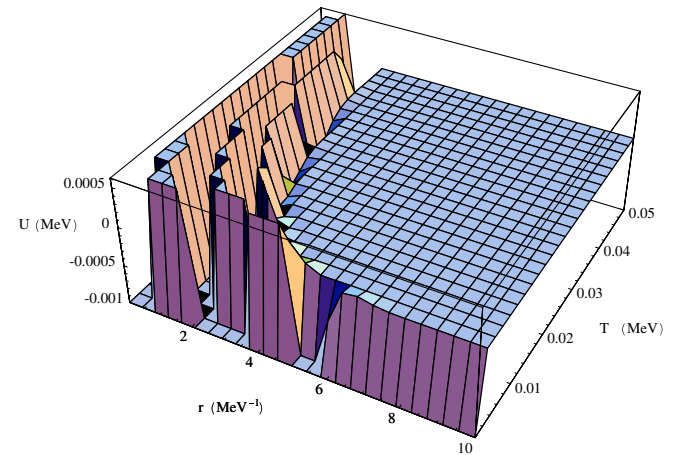


FIG. 4 (color online). Oscillation of the electrostatic potential in the presence of bosonic plasma; see Eq. (52). The boson chemical potential is equal to its mass,  $\mu_B = m_B = 100$  MeV. Temperatures are in MeV and distances in  $\text{MeV}^{-1}$ . In the picture, the oscillations due to both the temperature  $T$  and the distance  $r$ , as well as the exponential damping in both the directions, are evident.



not deal now with the geometric progression,  $\exp(-an)$  but with a more complicated function,  $\exp(-b\sqrt{n})$ . Since the effective values of  $n$  are big, we can express the sum as an integral and obtain, in the leading approximation  $\Pi_{00} = m_D^2$ , that the potential is proportional to the temperature  $T$  and hence vanishes:

$$U(r)^B = -\frac{QT\pi^2}{2e^2r^2\mu_F^4} \operatorname{Re} \sum_{n=1}^{\infty} E_n^2 e^{2iq_n r} \\ \approx -\frac{QT\pi^2}{2e^2r^2\mu_F^4} \operatorname{Re} \int_1^{\infty} dn E_n^2 e^{2iq_n r} \sim T. \quad (53)$$

The real part of the integral  $\int_1^{\infty} dn e^{2iq_n r}$  written above is equal to

$$\frac{\exp(-2\sqrt{2\pi Tmr})}{4r} \left[ \sqrt{\frac{2}{\pi m T}} (\cos(2\sqrt{2\pi Tmr}) - \sin(2\sqrt{2\pi Tmr})) - \frac{1}{2\pi m r T} \sin(2\sqrt{2\pi Tmr}) \right],$$

that goes to the constant value  $-1$  in the  $T \rightarrow 0$  limit. So the whole expression in Eq. (53) is proportional to  $T$ .

It is important to stress that the previous result is valid in the limit  $Tm_B r^2 \ll 1$ , which means that it is applicable at small distances  $r_B \ll 1/\sqrt{m_B T}$ . On the other hand, at large distances  $r$  and nonvanishing  $T$  one should consider the expression in Eq. (52) which is similar to the fermionic Friedel term in Eq. (35) but has a different dependence on the coupling constant  $e$  since it goes like  $e^{-2}$ , while the Friedel oscillations go like  $e^2$ . Hence we have nonanalytic dependence on the coupling constant  $e$  in the presence of bosons. A similar dependence on  $e^{-2}$  was found in Sec. IV B.

There are also differences arising from the fact that in the limit  $T \rightarrow 0$  the poles of the boson distribution function go to zero, see Eq. (49), while the poles of the fermion distribution function tend to the nonvanishing value  $q_F$ , see Eq. (17). Hence Friedel oscillations for fermions start from their maximum amplitude at  $T = 0$  and then exponentially decrease with temperature, while for bosons the effect vanishes at  $T = 0$ , then linearly increases with  $T$  and finally exponentially decreases. Another consequence is that the argument of the oscillating cosine function depends on  $T$  for bosons but not for fermions. Hence the boson potential does not oscillate at small temperatures.

At high fermionic chemical potential  $\mu_F$  and small temperature  $T$ , the boson oscillations typically go to 0 at smaller distances than the fermionic ones, which are observable at distances  $r \leq T$ . On the other hand, lowering the boson mass  $m_B$  the exponential damping is weaker but at the same time oscillations fade away.

If the condensate is formed,  $\Pi_{00}$  would be dominated by the singular term  $e^2 m_2^4/k^2$  and according to Eq. (51) the contribution of the  $n$ th branch point into the screened potential becomes

$$U_n^B(r) = -\frac{QTm_B^2}{2\pi^2 e^2 m_2^8 r^2} \operatorname{Re}[k_n^4 e^{ik_n r}]. \quad (54)$$

Again, at large  $r$  and nonzero  $T$  the  $n = 1$  term is dominant. It oscillates and exponentially decreases according to Eq. (52). However, the sum  $\operatorname{Re} \sum_n U_n^B$  vanishes as above, Eq. (53). Probably the vanishing of  $U^B(r)$  at small  $T$  in the leading order is a more general feature. At least the sub-leading (at small  $T$ ) terms in  $k_n$  and in  $\Pi_{00}$  vanish as well. If we take into account the imaginary part of  $\Pi_{00}$  due to the logarithmic cut, the result still remains proportional to a power of temperature after summation. On the other hand, as we see below, in the absence of a condensate the potential not only survives at  $T \rightarrow 0$  but rises as an inverse power of  $T$ .

Let us turn now to a more interesting though probably less realistic case when fermions are absent in the plasma, the chemical potential of bosons is maximally allowed,  $\mu_B = m_B$  but the condensate is not formed. In the standard model a neutral system has necessarily a fermionic component because fermions are lighter than bosons. Anyway we can imagine systems where the electric charge is compensated by other heavier bosons which do not condense or models with an extra  $U(1)$  sector and different particle content. In this situation fermions may be absent. Under these conditions  $\Pi_{00}$  vanishes when  $T \rightarrow 0$ . The position of the branch points of the logarithm  $k_n = 2q_n$  also tends to zero and the screening due to logarithmic discontinuity may be nonvanishing at  $T = 0$ . Indeed, let us turn again to Eq. (51). The integral goes along the contour  $k = k_n + iy$  and  $y \sim 1/r$  is very small. We assume that  $r > 1/\sqrt{Tm_B}$ . Thus  $k^2 \approx k_n^2 = 16i\pi n T m_B$ . Let us now estimate  $\Pi_{00}$  at  $k = k_n$ . At small temperatures, when  $z^2 \equiv (E_B - m_B)/T \approx q^2/(2m_B T)$ ,  $\Pi_{00}$  can be presented as

$$\Pi_{00}(k) = \frac{e^2 m_B^2 T}{\pi^2 k} \int \frac{dz z}{\exp(z^2) - 1} \ln \left| \frac{\sqrt{8m_B T} z + k}{\sqrt{8m_B T} z - k} \right|. \quad (55)$$

Notice in passing that if  $k < \sqrt{8m_B T}$ , then  $\Pi_{00}$  behaves as  $m_B^3/k$  in agreement with Eqs. (37) and (39), while at large  $k$ ,  $k > \sqrt{8m_B T}$ , it has the following asymptotic behavior:

$$\Pi_{00}(k) \approx \frac{\sqrt{2} e^2 m_B^{5/2} T^{3/2} \zeta(3/2)}{\pi^{3/2} k^2}, \quad (56)$$

where  $\zeta(3/2) \approx 2.6$ . The singular part of  $\Pi_{00}$ , Eq. (50), at  $k = k_n + iy$  is equal to

$$\Pi_{00}^{(+)}(k_n + iy) = -\frac{i^{1/2} e^2 T^{1/2} m_B^{3/2}}{8\pi^{3/2} n^{1/2}} \\ \times [\ln(y/8\sqrt{\pi n m_B T}) + i\pi/2]. \quad (57)$$

For  $\Pi_{00}^{(-)}$  the last factor is changed to  $(\ln y/8\sqrt{\pi n m_B T} - 3i\pi/2)$ . The factor in the denominator of the logarithm comes from  $|k + 2q_n| = 4|q_n|$  in Eq. (50).

The screened potential (51) at large distances, i.e. for  $8\pi T m_B r^2 > 1$ , is dominated by  $n = 1$ . One can check that  $|\Pi_{00}(k_1)| > |k_1^2|$ , so the latter can be neglected in the denominator of Eq. (51). Keeping in mind that we will use the result below for arbitrary  $n$  for which  $|\Pi_{00}(k_1)| > |k_1^2|$ , we write

$$U_n(r) \approx \frac{32\pi Q n}{e^2 m_B r^2} \operatorname{Re} \left[ i e^{2iq_n r} \int_0^\infty \frac{dx e^{-x}}{\ln^2(x/8\sqrt{m_B \pi n T r}) - i\pi \ln(x/8\sqrt{m_B \pi n T r}) + 3\pi^2/4} \right], \quad (58)$$

where  $x = yr$ . For a large logarithm the leading part of the integral can be approximately evaluated leading to the result:

$$U_1(r) = -\frac{32\pi Q}{e^2 m_B r^2} \frac{e^{-2\sqrt{2\pi T m_B} r}}{\ln^2(8\sqrt{\pi m_B T r})} \sin(2\sqrt{2\pi T m_B} r). \quad (59)$$

Note that  $U_1(r)$  is inversely proportional to the electric charge and formally vanishes at  $T \rightarrow 0$ , but remains finite if  $\sqrt{T m_B} r$  is not zero.

For smaller distances, or such small temperatures that  $8\pi T m_B r^2 \ll 1$ , all  $n$  up to  $n_{\max} \sim 1/(8\pi T m_B r^2)$  make comparable contributions. Thus we have to sum over  $n$ . If  $n_{\max} \gg 1$  the sum can be evaluated as an integral over  $n$ . Now, for large  $n$ ,  $k_n^2 \sim n$  and may be comparable to  $\Pi_{00}(k_n)$  which, according to Eq. (57), drops as  $1/\sqrt{n}$ .  $\Pi_{00}(k_n)$  would be smaller by magnitude than  $k_n^2$  for

$$n > n_0 \approx 10^{-3} (m_B/T)^{1/3} \ln^{2/3}(\sqrt{8m_B T r^2}). \quad (60)$$

This condition makes sense if  $n_0 < n_{\max}$  or  $r \ln^{1/3}(\sqrt{8m_B T r^2}) < 5/(T m_B^2)^{1/3}$ . For larger  $r$  we return to domination of  $\Pi_{00}$ . We should check that the condition

$$r \ln^{1/3}(\sqrt{8m_B T r^2}) > 5/(T m_B^2)^{1/3} \quad (61)$$

does not contradict the condition of large  $n_{\max}$ . The latter reads

$$r < 1/\sqrt{8\pi T m_B}. \quad (62)$$

If we neglect the logarithmic factor, both conditions would be compatible for  $T/m_B < 4 \times 10^{-9}$ . Thus both cases of dominant  $\Pi_{00}(k_n)$  or  $k_n^2$  can be realized depending upon the relation between  $r$ ,  $T$ , and  $m_B$ .

Let us consider smaller temperatures when  $|\Pi_{00}(k_n)| > |k_n^2|$ . The potential in the limit of small  $\pi T m_B r^2$  is equal to

$$U(r) = \frac{32\pi Q}{e^2 m_B r^2} \operatorname{Im} \left[ \sum_n n e^{2iq_n r} \frac{\ln^2(\sqrt{8m_B T r}) + i\pi \ln(\sqrt{8m_B T r}) + 3\pi^2/4}{(\ln^2(\sqrt{8m_B T r}) + 3\pi^2/4)^2 + \pi^2 \ln^2(\sqrt{8m_B T r})} \right]. \quad (63)$$

Since the sum

$$\sum_n n e^{2iq_n r} \approx 2 \int d\eta \eta^3 e^{4i\sqrt{\pi T m_B} r \eta} \approx -\frac{12}{256\pi^2 T^2 m_B^2 r^4}, \quad (64)$$

where  $\eta = \sqrt{n}$ , is real in leading order in  $1/(16\pi T m_B r^2)$ , a nonvanishing contribution comes from the imaginary part of the numerator of the integrand and we obtain for the analog of Friedel oscillations in the purely bosonic case:

$$U(r) \approx -\frac{3Q}{2e^2 T^2 m_B^3 r^6 \ln^3(\sqrt{8m_B T r})}. \quad (65)$$

The result has some unusual features. First, the potential decreases monotonically without any oscillations. Second, it is inversely proportional to the temperature, so the smaller  $T$  is, the larger the potential is. However, the effect exists for sufficiently small  $r$ ,  $r \ll 1/\sqrt{16\pi T m_B}$ , i.e. if  $T = 0.1$  K and  $m_B = 1$  GeV the distance should be bounded from above as  $r \ll 3 \times 10^{-8}$  cm. Another obstacle to realization of such screening behavior is that with fixed charge asymmetry the Bose particle should condense and the dominant term in  $\Pi_{00}$  becomes  $4m_B C/(2\pi)^3$ . In

this condition we arrive to potential (54) which vanishes at  $T = 0$ .

## V. CONCLUSION

We have calculated the electrostatic potential between two test charges in plasma with electrically charged bosons and fermions. The new part of our consideration is an inclusion of the effects of the Bose condensate into the screening phenomena in plasma. To this end the chemical potential of bosons is taken equal to the maximally allowed value, that is to the boson mass,  $\mu = m_B$ . In this case the bosonic contribution to the time-time component of the photon polarization operator in plasma,  $\Pi_{00}(k)$ , acquires an infrared singular contribution proportional to  $T/k$  even before formation of the condensate and  $1/k^2$  after formation of the condensate. Such terms drastically change the form of the screened potential  $U(r)$ .

All the calculations have been done in the lowest order in the electromagnetic coupling constant,  $e$ . We have imposed the condition of electric neutrality of the plasma, assuming that bosons and fermions compensate each other's charge. We have noticed, however, that the

screened potential demonstrates a very interesting and unusual behavior as a function of temperature if fermions are absent. Such a situation cannot be realized in realistic equilibrium plasma because the lightest charged fermions (electron/positrons) are lighter than charged bosons. However, one can imagine a hypothetical case of a new gauge  $U(1)$  symmetry with lighter charged bosons.

We started with a purely fermionic plasma and reconstructed the known Friedel oscillations of  $U(r)$  both in relativistic and nonrelativistic limits using a somewhat different technique. We obtained an explicit expression for  $U(r)$  at nonzero temperature which, to the best of our knowledge, is absent in the literature.

The main part of our work is dedicated to the new phenomena created by the singularities of  $\Pi_{00}(k)$  in the complex  $k$  plane. We reproduced the previously obtained results, where due to the  $1/k^2$  term the pole of the static photon propagator acquires a nonzero real part, and because of that the screened potential oscillates with an exponentially decreasing envelope.

At nonzero temperature and  $\mu = m_B$  the polarization operator obtains an odd contribution with respect to the transformation  $k \rightarrow -k$  and because of that the integral along the imaginary axis in the complex  $k$  plane, which determines the asymptotic behavior of  $U(r)$ , becomes nonvanishing. It leads to monotonic power law screening  $U \sim 1/(e^2 r^4 T)$ , Eq. (48), if the condensate has not yet been formed. After the condensate formation the screened potential behaves as  $U(r) \sim T/(e^2 r^6 C^2)$ , Eq. (47). Such a change in the screening may be a signal of the condensate formation.

We have also considered an analog of the Friedel oscillations in the bosonic case. The origin of the phenomenon is the same as in the fermionic case but the resulting

potential is quite different. The Friedel oscillations can be understood as a result of pinching the integration contour in the complex  $k$  plane by the logarithmic branch point of  $\Pi_{00}$  and the poles of the bosonic (or fermionic) distribution functions. However, the poles of bosonic distribution move to zero when temperature tends to zero, while the fermionic ones keep a finite value. This leads to a completely different behavior of the potential as a function of temperature. The potential vanishes when  $T$  goes to zero for mixed bosonic and fermionic plasma. In the case in which it is dominated by the first pole, for large  $r$  and nonzero  $T$ , it goes as in Eq. (52) and at small  $T$  the exponential screening is quite mild. For purely bosonic plasma the ‘‘Friedel’’ part of the screening is given by Eqs. (58) and (65). If  $Tm_B r^2$  is not small the potential oscillates and exponentially decreases, while for smaller  $T$  it does not oscillate and is proportional to  $1/(e^2 T^2)$ . The  $1/e^2$  behavior looks puzzling but one should remember that it is an asymptotic result for large distances. However, if we take the formal limit  $e \rightarrow 0$  the screening would disappear together with  $e$ . Similar reasoning is applicable to  $1/T^2$  behavior: this is true only for large but simultaneously sufficiently small distances  $r < 1/\sqrt{16\pi m_B T}$ , when the  $k^2$  part of the photon Green’s function is subdominant.

We see that the screening is quite different in different limits and it would be very interesting to study this rich behavior experimentally.

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