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Semitransparency effects in the moving mirror model for Hawking radiation

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We discuss the particle production due to a semitransparent mirror accelerating on the trajectories which simulate the Hawking effect. We find in accordance with a previous result [3] that the number of emitted particles up to infinite times remains finite, but in contrast to the cited paper, we obtain that for large, but finite reflectivities of the mirror, the radiated spectrum is Bose-Einstein and not Fermi-Dirac. We compare the beta coefficients $\beta(\omega', \omega)$ for the perfectly reflecting and the semitransparency case and point out the differences in the sector of large frequencies ω'. For the perfect mirror, the source of the infinite number of particles are the frequencies $\omega' \to \infty$, while for the semitransparent one this contribution is eliminated due to the cutoff effects introduced by the finite barrier energy of the mirror.

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I. INTRODUCTION

It is well known that the quantum radiation due to a collapsing body can be reproduced with a great degree of accuracy by an accelerated mirror in Minkowski space [1]. Calculations exploiting this analogy are traditionally based on the two-dimensional perfect mirror model of Fulling and Davies [1,2]. Recently, an investigation [3] considered the implications for the effect if using instead a semitransparent mirror. The calculation in [3] relied on the mirror model originally proposed by Barton and Calogeracos [4], in which the mirror-field interaction is described by a repulsive delta-like potential localized on the mirror. The analysis revealed a significantly different picture from that based on the perfect reflector. Two main conclusions in [3] were that (1) the numbers of emitted particles up to infinite times remain finite, and (2) for sufficiently large, but finite reflectivities of the mirror, the radiated spectrum obeys a reversed statistics, i.e. it is Fermi-Dirac. We recall that in the standard calculation the particle numbers at infinite times diverge and that in this limit the radiated spectrum is precisely Bose-Einstein.

The intention of this paper is to reinvestigate the problem, and to point out that some of the results in [3] are incorrect. More exactly, we shall show that, while the first conclusion is correct, the second is not. We shall find that, for sufficiently large reflectivities of the mirror, the spectrum remains Bose-Einstein. Another significant disagreement with [3] concerns the dependence of the total number of emitted particles on the energy α, which characterizes the reflectivity of the mirror. The result in the cited paper implies that for α large the particle numbers diverge like α^2. Our calculation will show that the divergence is $\sim \ln\alpha$.

Note that our statement concerning the point (2) above simply means that the emitted spectrum continuously approaches that of the perfect mirror in the limit of infinite barrier energies $\alpha \to \infty$, which is indeed what one would naturally expect from a physical point of view. To our knowledge, the discontinuity reported by [3] seems to be unique in the literature on semitransparent moving mirrors. For example, a list of quantities for which one can explicitly see that semitransparency leads to the expected perfect reflectivity limit can include the scattering amplitudes for a uniformly accelerated mirror [5], the force acting on a mirror in arbitrary motion [6], the Casimir force for a cavity with trembling walls [7], or the effective action for a dynamically moving mirror [8].

Another question that we felt required a more thorough discussion is that of the relevance of the various sectors of the virtual frequencies ω'. Even for the perfect reflector, the existing results are somehow incomplete, in the sense that the beta coefficients $\beta(\omega', \omega)$ from which the thermal spectrum is usually derived [1,2,9–11] are systematically obtained in the limit of large frequencies ω', and assuming that the mirror accelerates for an infinite period of time.[1] It seems that there exists no precise picture concerning the contribution of the various frequencies ω' to the creation process, if one considers a finite acceleration time. We shall offer an intuitive answer to this question with the aid of a series of graphical representations.

The paper is organized as follows: We shall use the same mirror trajectories as in [3], in which the mirror is initially at rest and accelerates for a finite period of time. We describe these trajectories in Sec. II. In Sec. III, we focus on the case of the perfect mirror. We present a rigorous derivation for the thermal flux at infinite times and discuss the picture for finite acceleration times. In Sec. IV, we extend the analysis to the semitransparent mirror. Our main result is the formula for the total number of emitted particles in the limit of very large, but finite barrier energies of the mirror. The last section contains the conclusions and a number of connections with other works. In the Appendices we detail some of the calculations used in the body of the text. In order to make our paper self-

[1]More recent calculations also considered the case of finite acceleration times [12–14], but in these papers too the evolution of the beta coefficients in the ω' space is not fully clarified.

contained, we included a description of the semitransparent mirror model in Appendix A. It is essentially the same model of Barton and Calogeracos [4], but in a different treatment [15], which allows an exact construction of the quantum field.

II. THE MIRROR TRAJECTORIES

The mirror is assumed to be static up to the moment $t = 0$, after which it accelerates for a finite interval, and then continues to move inertially up to infinite times $t \to +\infty$. The accelerated part of the trajectory is that which provides the analogy with the Hawking effect, i.e. (for simplicity we choose the null coordinate of the horizon to be $v_H = 1/k$)

$$u(v) = -\frac{1}{k}\ln(1 - kv), \qquad k > 0. \tag{1}$$

We denote by t_A the maximum acceleration time, which means that trajectory (1) is restricted to the interval

$$0 \le t \le t_A. \tag{2}$$

Imposing the continuity of the mirror's position and velocity at $t = 0$ and $t = t_A$, the complete form of the trajectory is ($u_A = u(v_A)$)

$$u(v) = \begin{cases} v & \text{if } v \le 0 \\ -k^{-1}\ln(1 - kv) & \text{if } 0 \le v \le v_A < 1/k, \\ u_A + \varepsilon_A^{-1}(v - v_A), & \text{if } v \ge v_A, \end{cases} \tag{3}$$

where we denoted by u_A, v_A the null coordinates corresponding to the final acceleration time $t_A = (u_A + v_A)/2$ and

$$\varepsilon_A = 1 - kv_A, \qquad \varepsilon_A \in [0, 1). \tag{4}$$

This parameter will replace in many formulas the dependence on t_A. An approximation that will be useful is that for very large acceleration times the ε_A parameter is

$$\varepsilon_A \simeq e^{-2kt_A}, \qquad kt_A \gg 1. \tag{5}$$

We shall also need the trajectory function $u(\tau)$, with τ the proper time of the mirror. Using Eq. (3) a convenient form is ($d\tau = \sqrt{dudv}$)

$$u(\tau) = \begin{cases} \tau - \tau_0 & \text{if } \tau \le \tau_0 \\ -(2/k)\ln(\tau/\tau_0) & \text{if } \tau_0 \le \tau \le \tau_A < 0, \\ u_A + (\tau - \tau_A)\sqrt{\varepsilon_A}, & \text{if } \tau \ge \tau_A, \end{cases} \tag{6}$$

where

$$\tau_0 = -\frac{2}{k}, \qquad \tau_A = -\frac{2}{k}\sqrt{\varepsilon_A} \tag{7}$$

are the proper times at the extremities of the acceleration interval $\tau \in [\tau_0, \tau_A]$. It is important to note that the limit of infinite acceleration times $t \to \infty$ is equivalent to

$$v_A \to 1/k, \qquad \varepsilon_A \to 0, \quad \text{or} \quad \tau_A \to 0. \tag{8}$$

III. THE CASE OF THE PERFECT MIRROR

This section is primarily intended as a preliminary step for the more complex case of the semitransparent mirror. The quantity of main interest is the number of particles emitted in the ω mode as a function of the acceleration time t_A. We denote this number by $N_\omega(t_A)$. In terms of the beta coefficients, the particle numbers are

$$N_\omega(t_A) = \int_0^\infty d\omega|\beta(\omega', \omega)|^2. \tag{9}$$

An essential point in our calculation will be to distinguish in $\beta(\omega', \omega)$ between a contribution that corresponds to a transient phase associated to the initial inertial part of the trajectory, and a "significant part," which describes the late time creation process. The same procedure will be applied in the next section to the semitransparent mirror.

A. The coefficients $\beta(\omega', \omega)$

We are interested in the beta coefficients (we focus as usual on the field on the right side R of the mirror)

$$\beta(\omega', \omega) = (\varphi^{\mathrm{in}*}_{\omega',R}, \varphi^{\mathrm{out}}_{\omega,R}) \quad \text{with} \quad \omega, \omega' > 0. \tag{10}$$

The quantum modes are [1]

$$\varphi^{\mathrm{in}}_{\omega,R}(u, v) = \frac{1}{\sqrt{4\pi\omega}}(e^{-i\omega v} - e^{-i\omega g(u)}), \tag{11}$$

$$\varphi^{\mathrm{out}}_{\omega,R}(u, v) = \frac{1}{\sqrt{4\pi\omega}}(e^{-i\omega u} - e^{-i\omega f(v)}), \tag{12}$$

where the functions

$$v = g(u), \qquad u = f(v) \tag{13}$$

define the mirror trajectory in null coordinates. In order to evaluate the scalar products (10), we identify the integration hypersurface with the right past null infinity $\mathcal{J}_R^-$ (more precisely, with the null ray $u = t_0$, $v \in [t_0, +\infty)$ and let $t_0 \to -\infty$). This gives

$$\begin{aligned} \beta(\omega', \omega) &= i\int_{-\infty}^{+\infty} dv \varphi^{\mathrm{in}}_{\omega',R} \overleftrightarrow{\partial}_v \varphi^{\mathrm{out}}_{\omega,R} \\ &= -2i\int_{-\infty}^{+\infty} dv(\partial_v\varphi^{\mathrm{in}}_{\omega',R})\varphi^{\mathrm{out}}_{\omega,R} \\ &= \frac{1}{2\pi}\sqrt{\frac{\omega'}{\omega}}\int_{-\infty}^{+\infty} dv e^{-i\omega' v - i\omega f(v)}, \end{aligned} \tag{14}$$

where the second integral follows from an integration by parts and neglecting the boundary term from infinite distances $v \to \infty$. The boundary term from $v = t_0$ is absent because the modes vanish on the mirror. In the last integral we also ignored a delta-like quantity $\sim\delta(\omega')$, which is identically null because all frequencies are $\omega' > 0$.

We separate next in Eq. (14) the contributions from the uniform (U) and the accelerated (A) part of the trajectory. We write the integral as

$$\int_{-\infty}^{+\infty} = \int_{-\infty}^{0} + \int_{0}^{v_A} + \int_{v_A}^{+\infty}, \tag{15}$$

and define, in obvious notations,

$$\beta(\omega', \omega) = \beta_U(\omega', \omega)^- + \beta_A(\omega', \omega) + \beta_U(\omega', \omega)^+. \tag{16}$$

Using Eq. (3) a simple calculation gives (introducing the usual convergence factor to deal with the oscillatory functions at $v \to \pm\infty$):

$$\beta_U(\omega', \omega)^- = +\frac{i}{2\pi}\sqrt{\frac{\omega'}{\omega}}\frac{1}{\omega' + \omega}, \tag{17}$$

$$\beta_U(\omega', \omega)^+ = -\frac{i}{2\pi}\sqrt{\frac{\omega'}{\omega}}\frac{e^{-i\omega' v_A - i\omega u_A}}{\omega' + \omega/\varepsilon_A}, \tag{18}$$

$$\beta_A(\omega', \omega) = \frac{1}{2\pi k}\sqrt{\frac{\omega'}{\omega}}e^{-i\omega'/k}\int_{\varepsilon_A}^{1} dz e^{iz\omega'/k} z^{i\omega/k}, \tag{19}$$

where in the last relation we integrated with respect to $z = 1 - kv$.

We now concentrate on the integral (19). The integrand is analytical in the semiplane $\mathrm{Im}z > 0$, so that we can apply the Cauchy theorem. We choose for the integration contour the rectangle defined by (1) one edge identified with the real interval $z \in [\varepsilon_A, 1]$, two edges running parallel with the imaginary positive semi-axis passing through (2) $z = \varepsilon_A$ and (3) $z = 1$, and (4) the last edge closing the contour at $\mathrm{Im}z \to +\infty$. Because of the factor $e^{iz\omega'/k}$ with $\omega'/k > 0$, the contribution of the last edge is null. We add the contributions from the second/third edge to the components $\beta_U(\omega', \omega)^{+/-}$ and denote the sums by $\tilde{\beta}(\omega', \omega)^{+/-}$. The result is

$$\beta(\omega', \omega) = \tilde{\beta}(\omega', \omega)^- + \tilde{\beta}(\omega', \omega)^+, \tag{20}$$

where [$e^{-i\psi}$ is the phase factor in the long fraction in Eq. (18)]

$$\tilde{\beta}(\omega', \omega)^- = \frac{i}{2\pi}\sqrt{\frac{\omega'}{\omega}}\left\{\frac{1}{\omega' + \omega} - \frac{(\omega'/k)^{-i\omega/k}}{\omega'} \times \int_0^\infty dt e^{-t}(\omega'/k + it)^{i\omega/k}\right\}, \tag{21}$$

$$\tilde{\beta}(\omega', \omega)^+ = -\frac{ie^{-i\psi}}{2\pi}\sqrt{\frac{\omega'}{\omega}}\left\{\frac{1}{\omega' + \omega/\varepsilon_A} - \frac{(\omega'\varepsilon_A/k)^{-i\omega/k}}{\omega'} \times \int_0^\infty dt e^{-t}(\omega'\varepsilon_A/k + it)^{i\omega/k}\right\}. \tag{22}$$

The essential observation at this point is that only the plus component (22) is relevant for the particle numbers at $t_A \to \infty$, and hence for the late time flux. This can be already guessed from the fact that $\tilde{\beta}(\omega', \omega)^-$ is independent of ε_A, and thus of t_A. More precisely, the argument is that (i) for very large frequencies ω' the two components behave as[2]

$$\tilde{\beta}(\omega', \omega)^- \sim \frac{1}{\omega'^{3/2}}, \qquad \tilde{\beta}(\omega', \omega)^+ \sim \frac{1}{\omega'^{1/2}(\omega' e^{-2kt_A})}, \tag{23}$$

and (ii) for infinite times or $\varepsilon_A \to 0$ the plus component is $\sim\omega'^{-1/2}$, from which it is immediate that only $\tilde{\beta}(\omega', \omega)^+$ can produce an infinite quantity in Eq. (9) when $t_A \to \infty$. The conclusion is that for t_A sufficiently large the number of particles can be approximated by

$$N_\omega(t_A) \simeq \int_{\omega'_0}^{\infty} d\omega' |\tilde{\beta}^+(\omega', \omega)|^2, \qquad t_A \to \infty. \tag{24}$$

Note that we introduced in Eq. (24) an inferior nonzero integration limit $\omega'_0 > 0$, which is necessary in order to eliminate the infrared divergence implied by the $\sim\omega'^{-1/2}$ behavior in Eq. (22). [This is however only an artefact due to the integration in the complex plane; the divergence does not appear in the original expressions, see Eqs. (17)–(19).] We emphasize that the value of ω'_0 is of no relevance for the particle numbers at very large times. This follows from the fact that the "low" frequencies $\omega' \le \omega'_0$ contribute only with a finite quantity in $N_\omega(t_A)$, since the coefficients remain finite and bounded for all t_A and $\omega' > 0$.

In the limit $t_A \to \infty$, the integrand in Eq. (24) takes the form

$$\begin{aligned}\lim_{\varepsilon_A \to 0} |\tilde{\beta}^+(\omega', \omega)|^2 &= \frac{1}{4\pi^2 \omega\omega'}\left|\int_0^\infty dt e^{-t}(it)^{i\omega/k}\right|^2, \\ &= \frac{1}{4\pi^2}\frac{e^{-\pi\omega/k}}{\omega\omega'}|\Gamma(1 + i\omega/k)|^2 \\ &= \frac{1}{2\pi k\omega'}\left(\frac{1}{e^{2\pi\omega/k} - 1}\right).\end{aligned} \tag{25}$$

Formula (25) reproduces the known result for the squared beta coefficients in the limit of large frequencies ω', implying a Bose-Einstein flux at infinite times [1]. The fact that one can restrict to the contribution of ω' large in the late time flux is rigorously justified by the second relation in Eq. (23), which shows that as t_A increases the sector $\omega' \to \infty$ becomes increasingly relevant in the sum over frequencies (24). The exact picture concerning the contribution of "low" and "high" frequencies ω' will be discussed in Sec. C.

[2]This follows from Eqs. (21) and (22) writing e^{-t} as a derivative and integrating by parts. In the second relation we assumed $\omega' \gg k/\varepsilon_A$ and the large t_A approximation (5).

B. The particle flux at infinite times

We now extract the late time flux from Eq. (24). It is evident that it is sufficient to determine the divergent behavior of $N_\omega(t_A)$ with respect to $t_A \to \infty$. We present the calculation in detail,[3] as the same procedure will be applied in the next section. The main difference will be that for the semitransparent mirror we shall assume from the start $t_A \to \infty$ and consider the divergent behavior of $N_\omega(t_A \to \infty)$ with respect to the barrier energy of the mirror.

We observe that introducing

$$w = \omega' \varepsilon_A / k, \tag{26}$$

the relevant component (22) can be rewritten as

$$\tilde{\beta}^+(\omega', \omega) = \frac{1}{\sqrt{\omega'}} b_\omega(w), \tag{27}$$

where the function b_ω is (we ignore an irrelevant phase factor)

$$b_\omega(w) = \frac{1}{2\pi\sqrt{\omega}} \left\{ \frac{1}{1 + \omega/(kw)} - w^{-i\omega/k} \times \int_0^\infty dt e^{-t}(w + it)^{i\omega/k} \right\}. \tag{28}$$

Integrating with respect to w in Eq. (24) one finds

$$N_\omega(t_A) \simeq \int_{w_A}^\infty \frac{dw}{w} |b_\omega(w)|^2, \qquad w_A = \varepsilon_A \omega_0'/k, \tag{29}$$

where a key observation is that the dependence on t_A is now completely included in the inferior integration limit w_A. Since the limit of interest $\varepsilon_A \to 0$ is equivalent to $w_A \to 0$, it is clear from the behavior of the integrand $\sim 1/w$ for w small that the divergence can only come from the "infrared" contributions $w \to 0$. We isolate the divergence writing

$$\int_{w_A}^\infty = \int_{w_A}^\eta + \int_\eta^\infty, \tag{30}$$

where we introduced a fixed number $\eta > w_A$. Note that the second integral leads to a finite quantity independent of t_A, so that it is irrelevant for the divergent behavior. We now use the fact that we are interested in the limit $w_A \to 0$. This allows to consider $w_A \ll 1$, and thus $\eta \ll 1$. In these conditions we can assume in the first integral

$$b_\omega(w) \simeq b_\omega(w \to 0), \tag{31}$$

which makes the integral trivial. The result is

$$N_\omega(t_A) \simeq |b_\omega(w \to 0)|^2 \int_{w_A}^\eta \frac{dw}{w} = 2kt_A |b_\omega(w \to 0)|^2 + \dots, \tag{32}$$

where we used that $\ln w_A = -2kt_A + \dots$. An essential fact in Eq. (32) is that the neglected terms, which contain the dependence on η and ω_0', remain finite for $t_A \to \infty$, and thus give no contribution in the derivative that defines the limit flux below (at this point ω_0' disappears from the calculation).

The particle flux emitted by the mirror can be reasonably identified with

$$\mathcal{F}_\omega(t_A) = dN_\omega(t_A)/dt_A. \tag{33}$$

It is then immediate that [the limit $w \to 0$ can be read from Eq. (28)]

$$\mathcal{F}_\omega(t_A \to \infty) = 2k|b_\omega(w \to 0)|^2 = \frac{k}{2\pi^2 \omega} |\Gamma(1 + i\omega/k)|^2 = \frac{1}{\pi} \frac{1}{e^{2\pi\omega/k} - 1}. \tag{34}$$

The result (34) is practically the standard flux in the literature [1,2], with the only difference that the last quantity is smaller by a factor of 2. The discrepancy can be explained as follows: The observation is that Eq. (33) represents the *emitted* flux, which, for a source in motion, will differ from the flux detected by a static observer (which is the quantity evaluated in literature). If the source recedes from the observer with a velocity close to the speed of light, as is the case of the mirror for $t_A \to \infty$, it easy to show that the observed flux is smaller than the emitted flux precisely by the factor of 2 (see [13] for details).

C. Observations on the evolution at finite acceleration times

It is interesting to consider the form of the beta coefficients as a function of ω' for different times t_A. We recall that, intuitively, the quantum flux emitted by the mirror can be seen as resulting from the zero point oscillations, which come from the past null infinity $\mathcal{J}_R^-$ (the frequencies ω') and which due to the reflection on the mirror can absorb energy, and be transformed thus into real quanta (the frequencies ω). In a loose sense, the coefficient $\beta(\omega', \omega)$ can then be interpreted as an amplitude for this process.

We shall consider instead of $\beta(\omega', \omega)$ the new quantity

$$\mathcal{B}_\omega(\omega') = \omega' |\beta(\omega', \omega)|^2, \tag{35}$$

in terms of which

$$N_\omega(t_A) = \int_{-\infty}^\infty d(\ln\omega') \mathcal{B}_\omega(\omega'). \tag{36}$$

It is more convenient to refer to Eq. (35) because we shall deal with exponentially shifted frequencies $\omega' \sim e^{2kt_A}$, for which it is more suited to the logarithmic scale in Eq. (36).

A plot for $\mathcal{B}_\omega(\omega')$ as a function of $\ln(\omega'/k)$ for different times t_A is presented in Fig. 1. According to Eq. (36), the

[3] The derivation of the thermal flux that follows is a simplified version of our earlier calculation in [13].

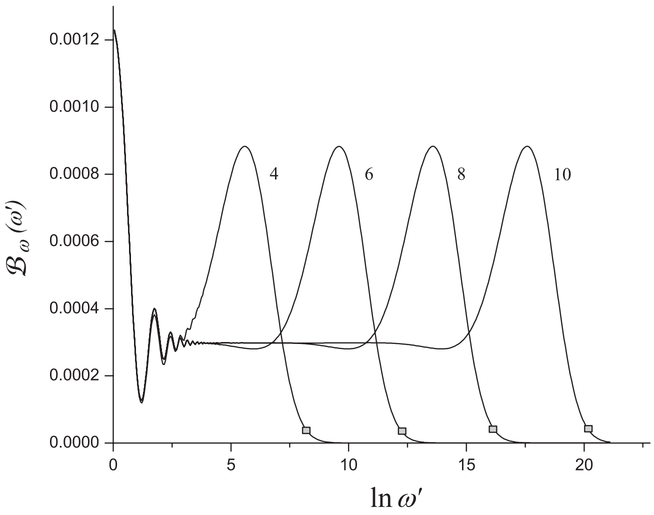

FIG. 1. The coefficients $\mathcal{B}_\omega(\omega') = \omega'|\beta(\omega', \omega)|^2$ for the perfect mirror represented as a function of $\ln(\omega'/k)$ for $\omega/k = 1$. The time t_A is shown near the curves. The square on the curves indicate the theoretical cutoff frequencies (39). In this and all other diagrams the unit scale for the dimension full quantities is fixed by $k = 1$.

areas below the curves are practically[4] the numbers $N_\omega(t_A)$. One sees that the areas increase with t_A, and that this happens because of the contribution of increasingly large frequencies ω'. In particular, it becomes evident that the flux at infinite times is determined only by the infinite frequencies $\omega' \to \infty$.

A notable property of the curves is that for each time t_A there exists a cutoff frequency $\omega'_{\rm max}$ beyond which the coefficients are negligibly small. In other words, frequencies larger than $\omega'_{\rm max}$ do not essentially contribute to the creation process. A simple evaluation for $\omega'_{\rm max}$ can be made using the ray tracing method [1]. We recall that this states that, adopting the geometric optics approximation, the frequencies ω' which contribute to the creation of a particle of frequency ω at $t \to +\infty$ are obtained by propagating the associated emergent wave backwards in time to $t \to -\infty$. In our case, the relation between the two frequencies is simply given by the Doppler shift due to the reflection on the mirror. For a mirror trajectory defined by $u(v)$, the frequency shift is $\omega'/\omega = (du/dv)_{\rm ref}$, where the derivative is evaluated at the reflection point $v = v_{\rm ref}$. Using Eq. (1) one finds (it is evidently sufficient to consider only the reflection for the accelerated part of the trajectory)

$$\frac{\omega'}{\omega} = \frac{1}{1 - kv_{\rm ref}}, \qquad v_{\rm ref} \in [0, v_A]. \tag{37}$$

It is immediate then that the maximum frequencies ω' are

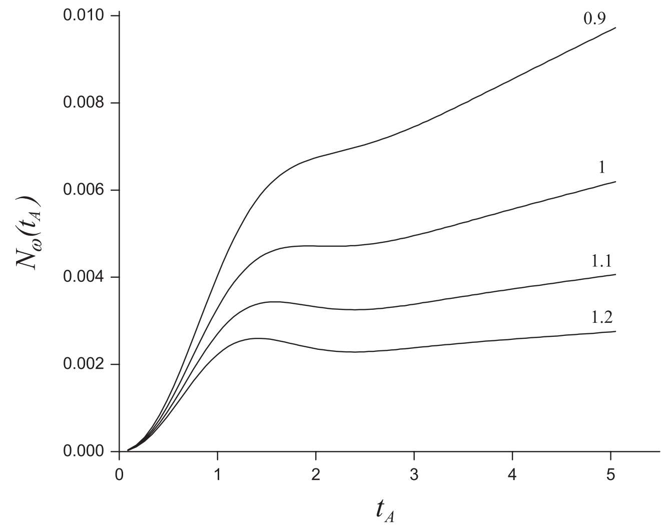

FIG. 2. The particle numbers for the perfect mirror $N_\omega(t_A)$ represented as a function of t_A. The frequency ω/k is indicated near the curves.

obtained for the reflection at the end of the accelerated trajectory (where the velocity, and thus the Doppler shift is maximum)

$$v_{\rm ref} = v_A, \tag{38}$$

from which

$$\frac{\omega'_{\rm max}}{\omega} = \frac{1}{\varepsilon_A}, \quad \text{or} \quad \ln(\omega'_{\rm max}/k) \simeq 2kt_A + \ln(\omega/k)$$

for $kt_A \gg 1$. (39)

This is indeed in good agreement with the cutoff frequen-

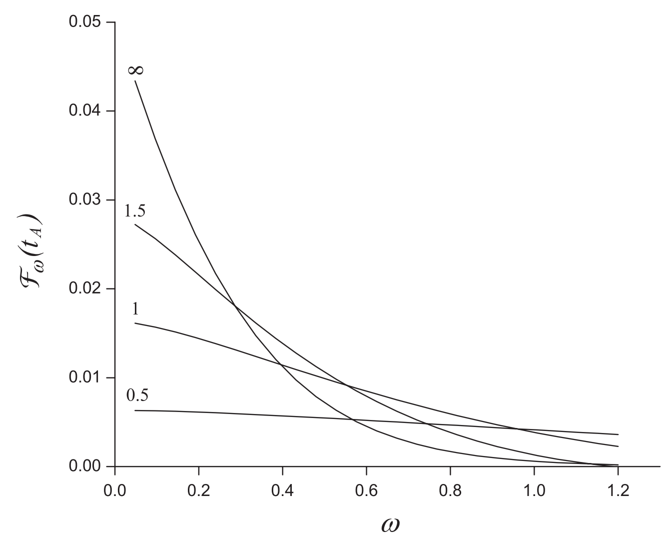

FIG. 3. The particle flux emitted by the perfect mirror $\mathcal{F}_\omega(t_A)$ represented as a function of ω. The time t_A is indicated near the curves. The upper curve reproduces the Bose-Einstein spectrum (34).

[4]One should also add the invisible contribution due to the low frequency sector $\ln(\omega'/k) < 0$, but the curves clearly indicate that this contribution is the same for all t_A, so that it is irrelevant for the flux.

cies determined numerically (see Fig. 1). Note the linear dependence $\ln\omega'_{\rm max} \sim t_A$, which basically explains the linear divergence $N_\omega(t_A) \sim t_A$.

A representation of $N_\omega(t_A)$ as a function of t_A for different frequencies ω is shown in Fig. 2. One sees that, as expected, the evolution becomes linear for t_A sufficiently large. Numerical calculations show that $N_\omega(t_A)$ on the linear piece of the curves is well approximated by Eq. (24) with a lower integration limit $\omega'_0 \simeq k$.

In Fig. 3 we represented the emitted flux (33) as a function of ω for different times t_A. The curves illustrate the progressive evolution of the flux toward the final spectrum (34). [In passing, it is interesting to remark that the stationary flux at $t_A \to \infty$ is obtained with a great degree of accuracy already for t_A of the order of a few units $1/k$. This might well be considered a short time at a "macroscopic" scale, if one has in mind that the characteristic period of the thermal quanta in the final flux is also $\sim 1/k$.]

IV. THE CASE OF THE SEMITRANSPARENT MIRROR

We first present some preliminary facts. The mirror model is discussed in Appendix A. The basic input is the barrier energy α, which defines the reflectivity of the mirror.

A. The modes and beta coefficients for an arbitrary mirror trajectory

It is clear that the modes for the semitransparent mirror extend both in the R and L region, and that they can be grouped into two classes, corresponding to the incident/emergent unperturbed waves $e^{-i\omega u}$ and $e^{-i\omega v}$. We denote the two sets of modes by U_ω and V_ω. The *in* modes have the following form:

$$U^{\rm in}_{\omega,L}(u,v) = \frac{1}{\sqrt{4\pi\omega}}\{e^{-i\omega u} - \mathcal{R}^{\rm in}_{\omega,L}(v)e^{-i\omega f(v)}\}, \quad (40)$$

$$U^{\rm in}_{\omega,R}(u,v) = \frac{1}{\sqrt{4\pi\omega}}\mathcal{T}^{\rm in}_{\omega,R}(u)e^{-i\omega u}, \quad (41)$$

and

$$V^{\rm in}_{\omega,R}(u,v) = \frac{1}{\sqrt{4\pi\omega}}\{e^{-i\omega v} - \mathcal{R}^{\rm in}_{\omega,R}(u)e^{-i\omega g(u)}\}, \quad (42)$$

$$V^{\rm in}_{\omega,L}(u,v) = \frac{1}{\sqrt{4\pi\omega}}\mathcal{T}^{\rm in}_{\omega,L}(v)e^{-i\omega v}. \quad (43)$$

It is also clear that the *out* modes, which have to be considered in our problem of interest, are only those describing the right-moving particles, corresponding to the emergent waves $e^{-i\omega u}$. Their form is

$$U^{\rm out}_{\omega,R}(u,v) = \frac{1}{\sqrt{4\pi\omega}}\{e^{-i\omega u} - \mathcal{R}^{\rm out}_{\omega,R}(v)e^{-i\omega f(v)}\}, \quad (44)$$

$$U^{\rm out}_{\omega,L}(u,v) = \frac{1}{\sqrt{4\pi\omega}}\mathcal{T}^{\rm out}_{\omega,L}(u)e^{-i\omega u}. \quad (45)$$

The expression of the reflection ($\mathcal{R}$) and transmission ($\mathcal{T}$) coefficients as a function of the mirror's trajectory is given in Appendix A. (We emphasize that no approximations are involved.) In the limit of infinite barrier energies, the coefficients assume the expected values

$$\lim_{\alpha\to\infty}\mathcal{R} = 1, \qquad \lim_{\alpha\to\infty}\mathcal{T} = 0. \quad (46)$$

In this case the wave functions in the R region containing the reflected component $V^{\rm in}_{\omega,R}$, $U^{\rm out}_{\omega,R}$ reduce to the perfect mirror modes $\varphi^{\rm in}_{\omega,R}$, $\varphi^{\rm out}_{\omega,R}$, and the transmitted wave functions $U^{\rm in}_{\omega,R}$ are identically null.

The two types of *in* modes determine two sets of beta coefficients:

$$\beta^{(V)}(\omega',\omega) = (U^{\rm out*}_\omega, V^{\rm in}_{\omega'}),$$
$$\beta^{(U)}(\omega',\omega) = (U^{\rm out*}_\omega, U^{\rm in}_{\omega'}). \quad (47)$$

We shall call them for obvious reasons "reflected" (V) and "transmitted" (U) coefficients [see Eqs. (48) and (49)], and similarly for the particle numbers in Eq. (51).

The coefficients (47) as a function of $\mathcal{R}$, $\mathcal{T}$ for an arbitrary mirror trajectory are determined in Appendix B. The result is

$$\beta^{(V)}(\omega',\omega) = \int_{-\infty}^{+\infty} dv\,\mathcal{R}^{\rm out}_{\omega,R}(v)e^{-i\omega' v - i\omega f(v)}, \quad (48)$$

$$\beta^{(U)}(\omega',\omega) = \int_{-\infty}^{+\infty} du\,\mathcal{T}^{\rm out}_{\omega,R}(u)e^{-i\omega' u - i\omega u}. \quad (49)$$

In the perfect reflectivity limit (46), the reflected coefficients reduce, as it should, to the quantities for the perfect reflector (14) and the transmitted coefficients are null,[5]

$$\lim_{\alpha\to\infty}\beta^{(V)}(\omega',\omega) = \beta(\omega',\omega), \qquad \lim_{\alpha\to\infty}\beta^{(U)}(\omega',\omega) = 0. \quad (50)$$

We define corresponding to the two sets of coefficients

$$N^{(V)}_\omega(t_A) = \int_0^\infty d\omega' |\beta^{(V)}(\omega',\omega)|^2,$$
$$N^{(U)}_\omega(t_A) = \int_0^\infty d\omega' |\beta^{(U)}(\omega',\omega)|^2. \quad (51)$$

The physically measurable number of particles in the ω mode is

$$N_\omega(t_A) = N^{(V)}_\omega(t_A) + N^{(U)}_\omega(t_A). \quad (52)$$

Our interest will lie in the evaluation of $N_\omega(t_A \to \infty)$ in the limit of large, but finite energies α. We first make a couple of observations on the two terms in Eq. (52).

[5] In fact this is not generally true, as the case under consideration shows; see the comments concerning Eq. (54).

An immediate consequence from the first limit in Eq. (50) and the fact that for the perfect mirror $N_\omega(t_A \to \infty) = \infty$ is that the reflected numbers diverge with α,

$$\lim_{\alpha\to\infty} N_\omega^{(V)}(t_A \to \infty) = \infty. \tag{53}$$

As concerns the transmitted numbers, one would naturally expect from the second limit in Eq. (50) that they vanish for $\alpha \to \infty$. Quite surprisingly, an explicit calculation shows that

$$\lim_{\alpha\to\infty} \lim_{t_A\to\infty} \beta^{(U)}(\omega', \omega) \neq 0, \tag{54}$$

and

$$\lim_{\alpha\to\infty} N_\omega^{(U)}(t_A \to \infty) = \text{finite}. \tag{55}$$

The nonzero result (54) is not in contradiction with $\lim_{\alpha\to\infty} \mathcal{T} = 0$, because the coefficient $\mathcal{T}$ in Eq. (49) is under an integral that extends over a noncompact domain, in which case the limit and the integral do not necessarily commute. For finite acceleration times $t_A < 0$, one can show that the integration domain in (49) can be reduced to a finite interval, and the vanishing property in Eq. (50) will always follow. The nonzero coefficients have to be viewed thus as strictly related to the case $t_A \to \infty$.

We shall however not insist on the quantity in Eq. (55), since it is of no relevance for the final result. We are content to mention that one can prove that $N_\omega^{(U)}(t_A \to \infty)$ admits a fixed *finite* upper bound for all barrier energies $\alpha > 0$. It is immediate then from Eqs. (53) and (55) that for α large enough one can approximate the particle numbers with the reflected component, i.e.

$$N_\omega(t_A \to \infty) \simeq N_\omega^{(V)}(t_A \to \infty), \qquad \alpha \text{ large}. \tag{56}$$

(The same conclusion was reached in [3].) It will suffice thus for discussing the divergent behavior of $N_\omega(t_A \to \infty)$ to focus only on the reflected coefficients $\beta^{(V)}(\omega', \omega)$.

B. The reflection coefficients $\mathcal{R}^{\text{out}}_{\omega,R}$

We need as a first step the coefficients $\mathcal{R}^{\text{out}}_{\omega,R}(v)$, which appear in Eq. (48). They are defined by the integral formula (A13) as a function of the mirror's proper time τ in terms of the trajectory function $u(\tau)$ [see Eq. (6)]. It is clear that the analytical form of $\mathcal{R}^{\text{out}}_{\omega,R}(\tau)$ will depend on the interval to which τ belongs. A simple calculation leads to the following expressions:

(1) $\tau \geq \tau_A$: The integral (A13) can be exactly performed and yields (note that the coefficients are independent of τ):

$$\mathcal{R}^{\text{out}}_{\omega,R} = \frac{\alpha\sqrt{\varepsilon_A}}{2i\omega + \alpha\sqrt{\varepsilon_A}}. \tag{57}$$

(2) $\tau_0 \leq \tau \leq \tau_A$: The integration variable can belong in this case both to the uniform part of the trajectory $\tau' \geq \tau_A$ and to the accelerated part $\tau' \in [\tau, \tau_A]$. We separate the integral as

$$\int_\tau^\infty = \int_\tau^{\tau_A} + \int_{\tau_A}^\infty, \tag{58}$$

and define, corresponding to the two terms,

$$\mathcal{R}^{\text{out}}_{\omega,R}(\tau) = \mathcal{R}_U(\tau) + \mathcal{R}_A(\tau). \tag{59}$$

The two components are

$$\mathcal{R}_U(\tau) = \left(\frac{\tau_A}{\tau}\right)^{2i\omega/k} e^{-\alpha(\tau_A-\tau)/2} \times \frac{\alpha\sqrt{\varepsilon_A}}{2i\omega + \alpha\sqrt{\varepsilon_A}}, \tag{60}$$

$$\mathcal{R}_A(\tau) = \int_0^{\alpha(\tau_A-\tau)/2} ds e^{-s}\left(1 + \frac{2s}{\alpha\tau}\right)^{2i\omega/k}. \tag{61}$$

The coefficients as a function of coordinate v follow from substituting in Eqs. (60) and (61) [the function $\tau(v)$ uniquely follows from Eqs. (3) and (6)]

$$\tau \to \tau(v) = -\frac{2}{k}\sqrt{1 - kv}. \tag{62}$$

(3) $\tau \leq \tau_0$: A decomposition like that in Eq. (58) would lead in this case to a complicated expression containing three terms. We use at this point the fact that we are interested in the case of α large. More precisely, we shall assume that

$$\alpha/k \gg 1. \tag{63}$$

We show in Appendix C that in these conditions the coefficients can be approximated with the coefficients for the mirror at rest, i.e. [compare with Eq. (57)]

$$\mathcal{R}^{\text{out}}_{\omega,R} = \frac{\alpha}{2i\omega + \alpha}. \tag{64}$$

For a more intuitive picture, we presented in Fig. 4 a plot for $|\mathcal{R}^{\text{out}}_{\omega,R}(\tau)|$ as a function of τ for different energies α. We considered the case of infinite acceleration times $\tau_A \to 0$ and restricted to the accelerated part of the trajectory $k\tau \in [-2, 0)$. The decreasing behavior with τ can be understood as follows: The essential observation is that, according to Eq. (44), one can interpret $\mathcal{R}^{\text{out}}_{\omega,R}$ as the coefficients that describe the reflection on the mirror of the emergent wave $e^{-i\omega u}$ which propagates backwards in time (with the mirror also moving backwards in time). Assuming a sharply localized packet, the argument τ in $\mathcal{R}^{\text{in}}_{\omega,R}(\tau)$ can be interpreted as the proper time at which the wave collides with the mirror. In the time reversed picture, the wave and the mirror are in "head-on" collision,[6] so that the ω frequencies appear blue shifted in the mirror's proper

[6]The reversed velocities of the mirror and wave packet are positive and, respectively, negative.

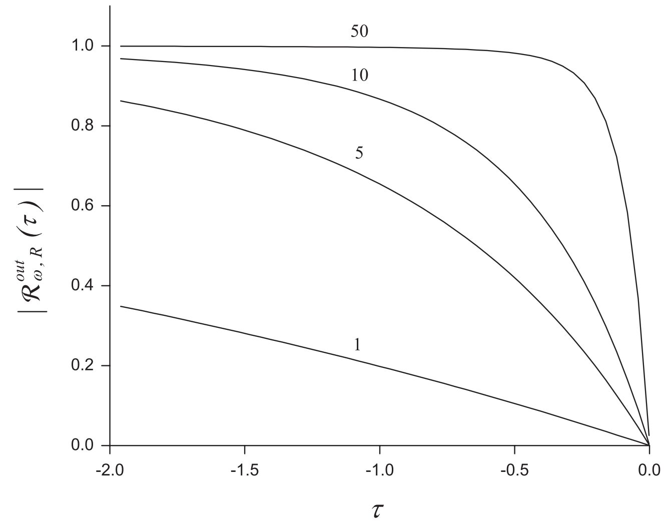

FIG. 4. The modulus of the coefficient $\mathcal{R}^{\rm out}_{\omega,R}(\tau)$ for $\omega/k = 1$ represented as function of τ on the accelerated part of the trajectory $\tau_A \to 0$. The numbers near the curves represent the energy α/k. Note the nearly constant values around $k\tau_0 = -2$ for α large, making the junction with the time independent coefficients (64) on the initial part of the trajectory $\tau \leq \tau_0$.

frame. This is equivalent to a diminishing of the reflectivity of the mirror, and thus of the $\mathcal{R}$ coefficients. The decreasing behavior with τ reflects the fact that the velocity of the mirror increases with τ. Note also that the coefficients completely vanish at the end of the acceleration interval, i.e.

$$\lim_{\tau_A\to 0}\mathcal{R}^{\rm out}_{\omega,R}(\tau \geq \tau_A) = 0. \tag{65}$$

This is a consequence of the fact that for $\tau_A \to 0$ the final velocity of the mirror approaches the speed of light, which means an infinite blue shift in the mirror's proper frame, and thus a null reflectivity.

C. The coefficients $\beta^{(V)}(\omega', \omega)$

We follow the same steps as in Sec. III. With a similar separation of the U and A components in Eq. (48) we find

$$\beta^{(V)}_U(\omega', \omega)^- = +\frac{i}{2\pi}\sqrt{\frac{\omega'}{\omega}}\frac{1}{\omega'+\omega}\mathcal{R}^{\rm out}_{\omega,R}(0), \tag{66}$$

$$\beta^{(V)}_U(\omega', \omega)^+ = -\frac{i}{2\pi}\sqrt{\frac{\omega'}{\omega}}\frac{e^{-i\omega' v_A - i\omega u_A}}{\omega'+\omega/\varepsilon_A}\mathcal{R}^{\rm out}_{\omega,R}(v_A), \tag{67}$$

where the $\mathcal{R}$ coefficients are given by Eqs. (57) and (64). The A component is

$$\beta^{(V)}_A(\omega', \omega) = \frac{1}{2\pi k}\sqrt{\frac{\omega'}{\omega}}e^{-i\omega'/k}\int_{\varepsilon_A}^1 dz\bar{\mathcal{R}}^{\rm out}_{\omega,R}(z)e^{iz\omega'/k}z^{i\omega/k}, \tag{68}$$

where $\bar{\mathcal{R}}^{\rm out}_{\omega,R}(z)$ is the coefficient (59) seen as a function of $z = 1 - kv(\tau)$, which amounts to make in Eqs. (60) and (61)

$$\tau \to \tau(z) = -\frac{2}{k}\sqrt{z}. \tag{69}$$

We transform Eq. (68) with the rectangular integration contour in the complex plane and make an identical regrouping of terms. The complex integration requires $\bar{\mathcal{R}}^{\rm out}_{\omega,R}(z)$ to be an analytical function of z. We discuss this property in Appendix D. The result is

$$\beta^{(V)}(\omega', \omega) = \tilde{\beta}^{(V)}(\omega', \omega)^- + \tilde{\beta}^{(V)}(\omega', \omega)^+, \tag{70}$$

where [compare with Eqs. (21) and (22)]

$$\begin{aligned}\tilde{\beta}^{(V)}(\omega', \omega)^+ = {}& -\frac{ie^{-i\psi}}{2\pi}\sqrt{\frac{\omega'}{\omega}}\bigg\{\frac{1}{\omega'+\omega/\varepsilon_A}\frac{\alpha\sqrt{\varepsilon_A}}{2i\omega+\alpha\sqrt{\varepsilon_A}} \\ & -\frac{1}{\omega'}\int_0^\infty dte^{-t}\bar{\mathcal{R}}^{\rm out}_{\omega,R}(\varepsilon_A + itk/\omega') \\ & \times(\varepsilon_A + itk/\omega')^{i\omega/k}\bigg\},\end{aligned} \tag{71}$$

$$\begin{aligned}\tilde{\beta}^{(V)}(\omega', \omega)^- = {}& -\text{identical to Eq.}(71) \\ & \text{with} \quad \varepsilon_A = 1, \qquad \psi = 0.\end{aligned} \tag{72}$$

Note that the minus component (72) is again independent of t_A. The plots in Sec. E are based on the expressions (71) and (72).

D. The numbers $N_\omega(t_A \to \infty)$ for large energies α

We begin by observing that, since for $\alpha \to \infty$ the reflection coefficients are $\mathcal{R} \to 1$, for α sufficiently large the same distinction between the plus and minus components as for perfect mirror will be valid: i.e., the minus component will produce only a finite quantity in $N^{(V)}_\omega(t_A \to \infty)$, so that for t_A very large the particle numbers can be approximated with [compare with Eq. (24)]

$$N_\omega(t_A) \simeq \int_k^\infty d\omega'|\tilde{\beta}^{(V)}(\omega', \omega)^+|^2. \tag{73}$$

For simplicity, we have chosen in the integral $\omega'_0 = k$, which as mentioned provides a good approximation for the perfect mirror.

We now concentrate on Eq. (73) for $t_A \to \infty$. In this limit the plus component (71) reduces to (we neglect an irrelevant phase factor)

$$\begin{aligned}\lim_{t_A\to\infty}\tilde{\beta}^{(V)}(\omega', \omega)^+ = {}& \frac{1}{2\pi\sqrt{\omega\omega'}}\int_0^\infty dte^{-t}\bar{R}_\omega(itk/\omega') \\ & \times(itk/\omega')^{i\omega/k},\end{aligned} \tag{74}$$

where we defined [see Eqs. (60), (61), and (69)]

$$\bar{R}_{\omega}(z) \equiv \lim_{\varepsilon_A \to 0} \bar{\mathcal{R}}^{\text{out}}_{\omega,R}(z) = \int_0^{\alpha\sqrt{z}/k} ds e^{-s}\left(1 - \frac{ks}{\alpha\sqrt{z}}\right)^{2i\omega/k}. \tag{75}$$

For infinite barrier energies $\alpha \to \infty$ the above quantity is $\bar{R}_{\omega}(z) \to 1$ and Eq. (74) reproduces, as it should, the perfect mirror coefficients (25).

One of the main results in [3] was that if one keeps α finite, the particle numbers $N_{\omega}(t_A \to \infty)$ remain finite. We present rapid proof of this fact in Appendix E, although the property will be immediate from the calculation below. The reader will also find there a connection between Eq. (74) and the beta coefficients obtained in [3], as well as a discussion of the source of error behind the affirmation that the radiated spectrum is Fermi-Dirac.

We now evaluate $N_{\omega}(t_A \to \infty)$. We introduce

$$u = \frac{k\omega'}{\alpha^2}, \tag{76}$$

and observe that Eq. (74) can be rewritten as

$$\lim_{t_A \to \infty} \tilde{\beta}^{(V)}(\omega', \omega)^{+} = \frac{1}{\sqrt{\omega'}} b^{(V)}_{\omega}(u), \tag{77}$$

where the function $b^{(V)}_{\omega}$ is (up to an irrelevant phase factor)

$$b^{(V)}_{\omega}(u) = \frac{1}{2\pi\sqrt{\omega}} \int_0^{\infty} dt e^{-t}(it)^{i\omega/k} \times \left\{ \int_0^{\sqrt{it/u}} ds e^{-s}\left(1 - \frac{s}{\sqrt{it/u}}\right)^{2i\omega/k} \right\}. \tag{78}$$

Integrating with respect to u in Eq. (73) one finds

$$N_{\omega}(t_A \to \infty) \simeq \int_{u_\alpha}^{\infty} \frac{du}{u} |b^{(V)}_{\omega}(u)|^2, \qquad u_{\alpha} = k^2/\alpha^2, \tag{79}$$

with the essential observation that the dependence on α is completely included in the integration limit u_α. The calculation from now on becomes identical with that for the particle numbers for the perfect mirror for t_A large [compare Eqs (29) and (79)]. We separate as in Eq. (30) the contributions from $u_\alpha < u < \eta$ with η fixed and use the fact that, since we are interested in the case of large energies $k/\alpha \ll 1$, we can choose $\eta \ll 1$. This allows to approximate

$$b^{(V)}_{\omega}(u) \simeq b^{(V)}_{\omega}(u \to 0). \tag{80}$$

Performing the integral one finds

$$N_{\omega}(t_A \to \infty) \simeq 2|b^{(V)}_{\omega}(u \to 0)|^2 \times \ln(\alpha/k) + \ldots, \tag{81}$$

where dots stand for a quantity that does not depend on α. In order to determine the squared modulus in Eq. (81), we observe that for $u \to 0$ the integral in the brackets in Eq. (78) is

$$\lim_{u\to 0}\left\{ \int_0^{\sqrt{it/u}} ds \ldots \right\} = 1. \tag{82}$$

It is then immediate comparing with Eqs. (28) and (34) that

$$2b^{(V)}_{\omega}(u \to 0) = \mathcal{F}_{\omega}(t_A \to \infty)/k, \tag{83}$$

and thus Eq. (81) can be rewritten as

$$\frac{N_{\omega}(t_A \to \infty)}{T_A} \simeq \mathcal{F}_{\omega}(t_A \to \infty), \qquad T_A = (1/k)\ln(\alpha/k). \tag{84}$$

We emphasize that Eq. (84) becomes a strict identity for $\alpha \to \infty$.

The above formula represents our main result. It implies that, for a sufficiently large barrier energy of the mirror, (1) the total particle numbers are proportional to the Bose-Einstein flux emitted by the perfect reflector (34) and (2) the numbers diverge as $\sim \ln\alpha$. The physical significance of the parameter T_A will be clarified in the next subsection. Numerical calculations show that Eq. (84) becomes a good approximation for energies larger than $\alpha/k \simeq 10^3$ [with $N_{\omega}(t_A \to \infty)$ including the neglected transmitted component $N^{(U)}_{\omega}(t_A \to \infty)$].

An interesting point in our calculation is that in arriving to Eq. (84) we have *not* used the explicit form of $\mathcal{R}^{\text{out}}_{\omega,R}(v)$. Note that these coefficients enter the final result only via the $u \to 0$ limit (82). If one observes that the integral under the limit is the (analytically extended) coefficient (75) with $\alpha\sqrt{z}/k \equiv \sqrt{it/u}$, one sees that the unit limit is nothing but the generally expected relation $\lim_{\alpha\to\infty} \bar{\mathcal{R}}(z, \alpha) = 1$. This invites to conjecture that Eq. (84) could be valid in a wider class of situations. It is thus plausible that the same formula will apply for *any* semitransparent mirror, with an appropriate interpretation of α as an "effective" barrier energy of the mirror. In the last section we shall mention an independent result that supports this idea.

E. Observations on the evolution for finite acceleration times

We first focus on the beta coefficients $\beta^{(V)}(\omega', \omega)$. In analogy with Eq. (35), we introduce

$$\mathcal{B}^{(V)}_{\omega}(\omega') = \omega' |\beta^{(V)}(\omega', \omega)|^2, \tag{85}$$

in terms of which

$$N^{(V)}_{\omega}(t_A) = \int_{-\infty}^{\infty} d(\ln\omega')\mathcal{B}^{(V)}_{\omega}(\omega'). \tag{86}$$

A plot for $\mathcal{B}^{(V)}_{\omega}(\omega')$ as a function of $\ln(\omega'/k)$ for different times t_A is shown in Fig. 5. As for the perfect mirror, the areas below the curves are practically the numbers $N^{(V)}_{\omega}(t_A)$. A comparison with Fig. 1 shows the evolution of curves for t_A large is significantly different now. The essential fact is that the extension into the region of large frequencies ω' stops beyond a certain t_A. (On our graphic the curves $kt_A = 10$ and $t_A \to \infty$ are indistinguishable.) It

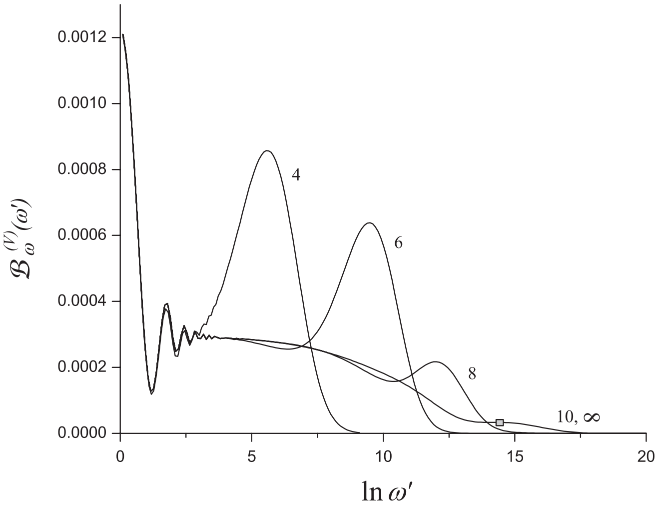

FIG. 5. The coefficients $\mathcal{B}_\omega^{(V)}(\omega') = \omega'|\beta^{(V)}(\omega', \omega)|^2$ for the semitransparent mirror represented as a function of $\ln(\omega'/k)$ for $\omega/k = 1$ and the barrier energy $\alpha/k = 10^3$. The time t_A is shown near the curves. The square on the $t_A \to \infty$ curve marks the theoretical cutoff frequency (89).

becomes clear thus the mechanism that makes $N_\omega(t_A \to \infty)$ finite.

It is interesting to determine the cutoff frequency $\omega'_{\max}$ for the limit curve $t_A \to \infty$. We similarly apply the ray tracing method. The main observation is that for the semitransparent mirror the amplitude of the back propagated wave will be reduced by the coefficient $\mathcal{R}^{\text{out}}_{\omega,R}(\tau_{\text{ref}})$, where τ_{ref} is the proper time corresponding to the reflection point. Recall now that in the limit of interest $\tau_A \to 0$ the coefficients at the end of the acceleration interval are null, i.e. $\mathcal{R}(\tau_{\text{ref}} = \tau_A) \to 0$ [see Eq. (65)], which means that the back scattered frequencies ω' determined by Eq. (39) will be relevant only as long as τ_{ref} is not very close to $\tau_A = 0$. This implies an upper limit for the Doppler shifts that define $\omega'_{\max}$, which basically explains why $\omega'_{\max}$ remains finite for $t_A \to \infty$.

Let us suppose that (as on the graphic) the barrier energy is $\alpha/k \gg 1$ and that ω is comparable to k, so that the initial coefficients (64) are $\mathcal{R} \simeq 1$. As a simple approximation, let us admit that the back scattered waves are relevant only as long as

$$\mathcal{R}^{\text{out}}_{\omega,R}(\tau_A) \simeq 1. \tag{87}$$

Using Eq. (57) this implies for the ε_A parameter the inferior limit

$$\sqrt{\varepsilon_A} \simeq \frac{\omega}{\alpha}, \tag{88}$$

which gives in combination with the first relation in Eq. (39)

$$\omega'_{\max} \simeq \frac{\alpha^2}{\omega} \quad \text{or} \quad \ln(\omega'_{\max}/k) \simeq 2\ln(\alpha/k) + \ln(k/\omega). \tag{89}$$

The maximum frequencies (89) are in good agreement with the cutoff frequencies determined by numerical calculations (see Fig. 5).

A plot for the particle numbers $N_\omega(t_A)$ as a function of t_A for different energies α is shown in Fig. 6. (The transmitted component $N_\omega^{(U)}(t_A)$ is also included; however, because of the large energies $\alpha/k \gg 1$ the curves practically represent $N_\omega^{(V)}(t_A)$.) One sees that for t_A sufficiently large $N_\omega(t_A)$ becomes constant. Note also that, as intuitively expected, for t_A fixed the numbers increase with α.

A question that can be naturally asked considering the curves in Fig. 6 is for which time $t_A \equiv T_A$ the number $N_\omega(t_A)$ becomes close to $N_\omega(t_A \to \infty)$ (in other words, the emission process can be considered to stop around T_A). The answer follows from the same argument that led to Eq. (89). Assuming as before a large barrier energy $\alpha/k \gg 1$, one can admit that the interval within which the majority of particles is emitted is characterized by a not very small value of $\mathcal{R}^{\text{out}}_{\omega,R}$, which is equivalent to condition (87). The time T_A is then simply determined by the inferior limit of the ε_A parameter (88). It is clear that a large α implies a large emission time T_A, so that we can assume the large t_A approximation (5). One finds

$$T_A \simeq (1/k)\ln(\alpha/\omega). \tag{90}$$

This is in acceptable agreement with the evolution in Fig. 6. Note also that condition (87) can be equivalently interpreted by saying that T_A is the interval $\Delta t \simeq T_A$ within

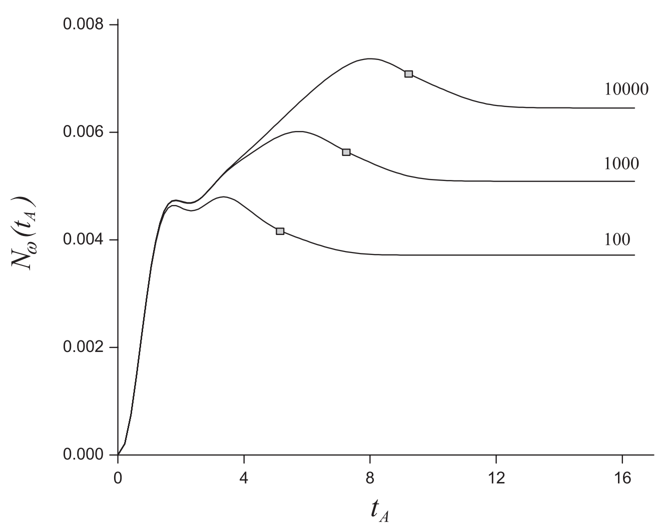

FIG. 6. The particle numbers for the semitransparent mirror $N_\omega(t_A)$ represented as a function of t_A for $\omega/k = 1$. The energy α/k is indicated by the numbers near the curves. The squares are placed at the time (90), which approximates the end of the emission phase.

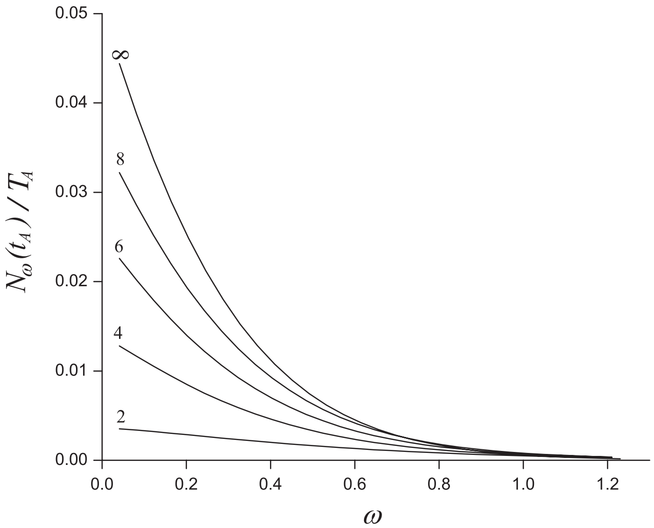

FIG. 7. The plot for $N_\omega(t_A)/T_A$ as a function of ω for the barrier energy $\alpha/k = 10^4$. The time t_A is indicated near the curves. The upper curve is identical with the Einstein-Bose spectrum (34).

which the mirror radiates in an ''almost perfect reflector'' regime. With this interpretation, the physical significance of Eq. (84) becomes obvious.

[The definition of T_A in Eq. (90) is different from that in Eq. (84) via $\omega \to k$, but this is inessential in the limit of α large: the two times differ only by a finite quantity independent of α, which is irrelevant in Eq. (84), where both the numerator and the denominator diverge with α.]

We represented the ratio $N_\omega(t_A)/T_A$ as a function of ω for different times t_A in Fig. 7 [for T_A we used Eq. (90)]. We have chosen a large barrier energy, in order to assure a great degree of accuracy in Eq. (84). In accordance with our result, the curve $t_A \to \infty$ practically reproduces the Bose-Einstein spectrum (34).

It is interesting that the curves have a very similar shape with those for the perfect mirror flux in Fig. 3. However, a closer look reveals a significative difference in the high frequency sector: i.e., the curves in Fig. 7 approach more rapidly the horizontal axis. In other words, semitransparency suppresses the emission at high frequencies. This is of course not unexpected, and can be naturally understood as a consequence of the fact that semitransparency tends to eliminate the high frequencies from the theory.

Finally, we included in Appendix F a few more comments related to the high frequency spectrum and the fact that we use a trajectory with discontinuous derivatives.

V. CONCLUSIONS

We investigated in this paper the particle production phenomenon due to a semitransparent mirror, which accelerates on the trajectories which provide the analogy with the Hawking effect. We confirmed a previous conclusion [3] that, in contrast to the case of the perfect reflector, the numbers of particles emitted up to infinite times remain finite. In disagreement to [3], however, we showed that for sufficiently large, but finite barrier energies of the mirror, the radiated spectrum remains Bose-Einstein. More exactly, in this limit the final numbers of particles in a certain ω mode can be approximated with the thermal flux emitted by the perfect mirror at infinite times, multiplied by a time $\sim \ln\alpha$. We conjectured that a similar result might apply for any semitransparent mirror.

We also discussed the relevance of the various large frequencies ω' to the creation process, considering trajectories with a finite acceleration time t_A. For the perfect mirror, we showed that the beta coefficients become negligible beyond a maximum frequency $\omega'_{\max}$, which behaves as $\ln\omega'_{\max} \sim t_A$. This frequency is well approximated by the emergent frequency propagated backwards in time, considering that the reflection on the mirror occurs at the end of the accelerated trajectory, where the frequency shift is maximum. The infinite numbers of particles for $t_A \to \infty$ emitted by the perfect reflector can be seen as a consequence of the indefinite increase of $\omega'_{\max}$ with t_A.

By contrast, for the semitransparent mirror, the frequencies $\omega'_{\max}$ remain finite for $t_A \to \infty$, and thus the particle numbers remain finite. The finiteness of $\omega'_{\max}$ in this case is as a consequence of the fact that the *out* reflection coefficients, on which the beta coefficients depend, vanish at the end of the accelerated part of the trajectory $t_A \to \infty$. More exactly, in the time reversed picture the coefficients $\mathcal{R}^{\text{out}}_\omega$ reduce the amplitude of the back scattered waves, and thus the vanishing behavior $\mathcal{R}^{\text{out}}_\omega \to 0$ introduces a cutoff for the shifted frequencies ω'.

An important feature which automatically follows from the finiteness of the particle numbers at $t_A \to \infty$ is that the particle flux vanishes at infinite times. As remarked in [3], this is consistent with a previous result [16], which states that the energy flux radiated by the mirror vanishes in the infinite future. (The calculation in [16] refers to the same mirror model and the same trajectories; the energy flux means more exactly the renormalized energy-momentum tensor).

Let us make a few comments in connection to the Hawking effect. First of all, one has to admit that it remains an open question whether the semitransparent mirror model has any relevance at all to the gravitational problem. If one has in mind that the effect of semitransparency is practically to eliminate the high frequencies ω' from the theory, one possible view is that it might offer a model for the Hawking flux in the presence of a frequency cutoff. From this perspective, the vanishing of the flux is of no surprise, as it confirms the well-known fact that the frequencies $\omega' \to \infty$ are essential for the existence of the Hawking flux at infinite times (see e.g. [17]).

In the same context, it is worth mentioning a recent investigation [18] that explicitly displayed the vanishing of the Hawking flux in the absence of the high frequencies

ω', and which arrived to a result very similar to our formula (84). The calculation considered the usual collapse scenario,[7] assuming a cutoff frequency $\omega' < \Lambda$. The relation obtained in [18] between the particle numbers at infinite times and the late time flux[8] is the same with Eq. (84), with the only difference that the barrier energy α is replaced by Λ. This supports the conjecture that Eq. (84) might apply to a wider class of situations.

Let us draw attention to the tempting idea that the vanishing flux of the mirror might somehow be an indication for the vanishing of the Hawking flux at infinite times, due to the complete evaporation of the black hole. We think, however, that this analogy is purely superficial. Most probably the flux of the semitransparent mirror can not be a model for the flux of an evaporating black hole. A simple argument is that in the evaporation process the temperature of the flux increases in time, while no such behavior can be seen in our case. The increased temperature should have left an imprint in the ω dependence of $N_\omega(t_A \to \infty)$, in contradiction with the spectrum (84) with the fixed temperature $T = k/2\pi$.

It might be also useful to point out that the vanishing of the flux in our investigation has nothing in common with the moving mirror model for the evaporation process proposed by Carlitz and Wiley [19]. Their analysis is completely based on the perfect reflector model, and the vanishing of the flux, as well as the increase of temperature, are direct consequences of a suitably devised mirror trajectory. (The acceleration parameter k is considered a time dependent quantity which is chosen to vanish at large times in the future.)

As a final question, let us discuss a more delicate aspect concerning the frequency cutoff. In recent years, it became increasingly clear from the study of acoustic black holes (see e.g. [20]) that the existence of the Hawking effect is actually not incompatible with a cutoff. It is important to recall that in these models the cutoff acts not relative to the static Minkowski frame, but with respect to the comoving frame of the fluid. (In the gravitational problem, this would translate into a cutoff relative to a freely falling observer.) It should be clear that this is not the case here, where the frequencies are defined with respect to the fixed Minkowski frame.

However, the attentive reader might raise the following question. The observation is that the finite barrier energy introduces a well defined cutoff with respect to the *rest* frame of the mirror, while we are dealing here with a mirror in motion. We recall that the trajectories of interest are such that the mirror recedes from the R region with a velocity that steadily increases in time, and which for the trajectories $t_A \to \infty$ becomes infinitely close to the speed of light at $t \to +\infty$. The point is that, from the perspective of a static observer, this implies for the ω' frequencies incident from the R zone (i.e. the virtual frequencies relevant for the emergent flux) a blue shifted cutoff that indefinitely increases in time, from which the natural conclusion would be that the cutoff will be ultimately eliminated (i.e. become infinite) at $t \to +\infty$. This might appear to be in contradiction with the finite cutoff in the beta coefficients.

Although the argument seems correct, the conclusion regarding the elimination of the cutoff is not. In a rigorous way, the question can be reformulated by asking if the coefficients that describe the reflection on the mirror of the waves thatcome from the R region still posses the usual vanishing behavior with the incident frequency at infinite acceleration times (i.e. for $\tau \to 0$ when $\tau_A \to 0$). As can be seen from Eq. (42), these coefficients are $\mathcal{R}^{\text{in}}_{\omega,R}$ (the role of ω' is played now by ω). An explicit calculation using Eq. (A9) shows that in the limits of interest the coefficients for large frequencies behave as

$$\lim_{\tau\to 0}\mathcal{R}^{\text{in}}_{\omega,R}(\tau) \sim \alpha/\sqrt{\omega k}, \qquad \tau_A \to 0. \tag{91}$$

This confirms that a cutoff exists also with respect to the Minkowksi frame, and not only in the mirror's proper frame. (Note that Eq. (91) implies that the cutoff is $\sim\alpha^2/k$. This is similar, but not identical with the cutoff in the beta coefficients (89). One could not have expected a complete identity between the two quantities, since the last one contains the extra dependence on the *out* frequency ω.)

From a physical point of view, the finite cutoff implied by Eq. (91) can be understood as follows: The idea is that the nonlocality of the wave/quantum particle makes it impossible to localize the reflection point strictly at $\tau \to 0$, i.e. when the velocity of the mirror becomes infinitely close to the speed of light. This means that the reflection process has to be viewed as taking place also at subluminal velocities, in which conditions the finite cutoff from the mirror's proper frame can survive.

APPENDIX A

We present here the construction of the quantum modes[9] in the semitransparent mirror model [15]. The interaction between the mirror and the field is described by an external delta-like potential V, which for the static mirror located in $z = 0$ has the form

[7] In two dimensions the problem is completely equivalent to the accelerated mirror model with an infinite acceleration time.

[8] The relation is actually not explicitly written in [18]. It follows from combining Eqs. (15) and (19)–(22) in the cited paper. Notably, the result appears now as a strict identity.

[9] We include these facts since [15] contains only the derivation of the *in* modes, while we also need here the *out* modes. Note, however, that our calculation of the beta coefficients does not use the explicit form of the *in* coefficients (A9). On the other hand, the calculation in [3] uses the *in* coefficients and does not use the *out* coefficients (A13). The difference originates in the fact that in [3] the scalar products (47) are evaluated with the integration hypersurface Σ identified with the future null infinity, while in our case Σ is the past null infinity; see Appendix B.

$$\hat{V}(z) = \alpha\delta(z), \qquad \alpha > 0. \tag{A1}$$

The equation imposed on the quantum field is

$$(\Box + V)\varphi = 0, \tag{A2}$$

where $V(t, z)$ is the generalization of $\hat{V}(z)$ to an arbitrary trajectory. The explicit form of Eq. (A2) is however not necessary for determining the modes. Let $\varphi_{L/R}$ denote the field at the left/right of the mirror. It is clear that within the L and R regions

$$\Box\varphi_L = \Box\varphi_R = 0. \tag{A3}$$

Equation (A3) has to be supplemented with the junction conditions on the mirror (M). In obvious notations, the continuity condition is

$$\varphi_L = \varphi_R|_M. \tag{A4}$$

The equation for the derivatives of φ can be most easily established considering first the mirror at rest and then covariantly generalizing the result. One finds (the overdot represents derivation with respect to the mirror's proper time)

$$n^\mu\partial_\mu\varphi_L - n^\mu\partial_\mu\varphi_R + \alpha\varphi = 0|_M, \qquad n^\mu = (\dot{z}, \dot{t}). \tag{A5}$$

For the fixed mirror $\hat{n}^\mu = (0, 1)$ and Eq. (A5) reduces as it should to the equation for the static case [which follows as usual by integrating Eq. (A2) with respect to z in the vicinity of the mirror].

1. The in modes

The wave functions (42) are an evident generalization of the modes for the perfect reflector (11). Note that the dependence on u, v in $\mathcal{R}$, $\mathcal{T}$ in all cases is chosen such that the free field Eq. (A3) is automatically satisfied. The coefficients $\mathcal{R}$, $\mathcal{T}$ can be determined as follows. The continuity condition implies (we refer for economy only to the V_ω^{in} modes)

$$\mathcal{R}^{\text{in}}_{\omega,R} + \mathcal{T}^{\text{in}}_{\omega,L} = 1. \tag{A6}$$

The essential point is that Eq. (A5) can be transformed into an evolution equation for $\mathcal{R}$. To this end, we consider the coefficients as functions of the proper time of the mirror τ via

$$\mathcal{R}^{\text{in}}_{\omega,R}(\tau) \equiv \mathcal{R}^{\text{in}}_{\omega,R}(u(\tau)), \tag{A7}$$

and eliminate $\mathcal{T}$ in favor of $\mathcal{R}$. After expressing all derivatives in terms of $d/d\tau$, the equation translates into

$$\frac{d}{d\tau}\mathcal{R}^{\text{in}}_{\omega,R} - \left(i\omega\frac{dv}{d\tau} - \frac{\alpha}{2}\right)\mathcal{R}^{\text{in}}_{\omega,R} = \frac{\alpha}{2}. \tag{A8}$$

The solution of Eq. (A8) will depend on the initial condition at some time τ_0. If one chooses $\tau_0 \to -\infty$, one finds that the initial condition becomes irrelevant and the solution is

$$\mathcal{R}^{\text{in}}_{\omega,R}(\tau) = \frac{\alpha}{2}\int_{-\infty}^{\tau} d\tau' e^{i\omega(v(\tau)-v(\tau'))-\alpha(\tau-\tau')/2}. \tag{A9}$$

For the corresponding formulas for the U_ω^{in} modes it is sufficient to make everywhere in the expressions above $L \leftrightarrow R$ and $u \leftrightarrow v$ (and the same for the *out* modes below). This completes the derivation of the *in* modes.

2. The out modes

A similar construction applies to the coefficients in the *out* modes. The continuity condition implies

$$\mathcal{R}^{\text{out}}_{\omega,R} + \mathcal{T}^{\text{out}}_{\omega,L} = 1. \tag{A10}$$

Eliminating $\mathcal{T}$ and considering $\mathcal{R}$ as a function of the proper time τ via

$$\mathcal{R}^{\text{out}}_{\omega,R}(\tau) \equiv \mathcal{R}^{\text{out}}_{\omega,R}(v(\tau)), \tag{A11}$$

Eq. (A4) becomes

$$\frac{d}{d\tau}\mathcal{R}^{\text{out}}_{\omega,R} - \left(i\omega\frac{du}{d\tau} + \frac{\alpha}{2}\right)\mathcal{R}^{\text{out}}_{\omega,R} = -\frac{\alpha}{2}. \tag{A12}$$

Choosing this time $\tau_0 \to +\infty$ the solution is

$$\mathcal{R}^{\text{out}}_{\omega,R}(\tau) = \frac{\alpha}{2}\int_{\tau}^{+\infty} d\tau' e^{i\omega(u(\tau)-u(\tau'))+\alpha(\tau-\tau')/2}. \tag{A13}$$

Note that, as it should, the *in* coefficients depend on the past trajectory, while the *out* coefficients on the future trajectory of the mirror.

APPENDIX B

Here, we determine the form of the beta coefficients for an arbitrary trajectory of the mirror. We refer only to the reflected coefficients (48), since the calculation for the transmitted coefficients (49) is basically the same. It is clear that in the semitransparency case the scalar products (47) have to be evaluated using a Cauchy hypersurface Σ for the entire Minkowski space. In obvious notations, we have

$$\Sigma = \Sigma_L \cup \Sigma_R \Rightarrow \int_\Sigma = \int_{\Sigma_L} + \int_{\Sigma_R}. \tag{B1}$$

This allows to decompose the beta coefficients as

$$\beta^{(V)}(\omega', \omega) = \beta^{(V)}(\omega', \omega)_L + \beta^{(V)}(\omega', \omega)_R. \tag{B2}$$

We identify Σ_R as for the perfect mirror with the right past null infinity $\mathcal{J}_R^-$ [see above Eq. (14)] and Σ_L with the left past null infinity $\mathcal{J}_L^-$ (more exactly the ray $v = t_0$, $u \in [t_0, \infty)$ with $t_0 \to -\infty$). One finds for the R component

$$\beta^{(V)}(\omega', \omega)_R = i\int_{-\infty}^{+\infty} dv V^{\rm in}_{\omega',R}\overleftrightarrow{\partial}_v U^{\rm out}_{\omega,R} = -2i\int_{-\infty}^{+\infty} dv(\partial_v V^{\rm in}_{\omega',R})U^{\rm out}_{\omega,R} - iV^{\rm in}_{\omega',R}U^{\rm out}_{\omega,R}|_M, \quad \text{(B3)}$$

where the second expression follows from an integration by parts and neglecting the boundary term from infinite distances $v \to \infty$. The M term is the contribution due to the extremity on the mirror at $v = t_0$.[10] A similar calculation gives for the L component

$$\beta^{(V)}(\omega', \omega)_L = i\int_{-\infty}^{+\infty} du V^{\rm in}_{\omega',L}\overleftrightarrow{\partial}_u U^{\rm out}_{\omega,L} = +2i\int_{-\infty}^{+\infty} du V^{\rm in}_{\omega',L}(\partial_u U^{\rm out}_{\omega,L}) + iV^{\rm in}_{\omega',L}U^{\rm out}_{\omega,L}|_M. \quad \text{(B4)}$$

We now observe that, because of the continuity of the modes on the mirror, the two M terms are equal (and of opposite signs), so that they cancel in Eq. (B2). Introducing Eqs. (42) and (44) in the integral term in $\beta^{(V)}(\omega', \omega)_R$ one obtains formula (48). It remains to justify the vanishing of the integral term in $\beta^{(V)}(\omega', \omega)_L$. Using Eqs. (43) and (45) one finds that the integral is [the integrand is of the form $F(v)\partial_u G(u)$, which makes the result immediate]

$$\int_{t_0}^{+\infty} du \ldots \sim \mathcal{T}^{\rm in}_{\omega',L}(t_0)\mathcal{T}^{\rm out}_{\omega,L}(t_0)e^{-i(\omega'+\omega)t_0}, \qquad t_0 \to -\infty. \quad \text{(B5)}$$

If one considers wave packets, this is indeed a vanishing quantity because distributionally $\lim_{t_0\to-\infty} e^{-i(\omega'+\omega)t_0} = 0$, since $\omega' + \omega > 0$. The $\mathcal{T}$ factors do not interfere with the vanishing property, because $u = v = t_0 \to -\infty$ corresponds to the initial static part of the trajectory at $t \to -\infty$, for which $\mathcal{T} = 1 - \mathcal{R}$ reduces to the finite, time independent solution for the mirror at rest.

APPENDIX C

We justify here that for large barrier energies $\alpha/k \gg 1$ the coefficients $\mathcal{R}^{\rm out}_{\omega,R}(\tau \le \tau_0)$ can be approximated with the coefficients for the static mirror (64). The simplifying idea is that it is sufficient to show that the approximation is valid for $\tau = \tau_0$. The conclusion then follows from the facts that (1) the solution $\mathcal{R}^{\rm out}(\tau \le \tau_0)$ of the differential Eq. (A12) is uniquely determined by the initial condition at τ_0 and the velocity of the mirror for $\tau \le \tau_0$, and (2) the velocity of the mirror on this interval is exactly null. To prove the first statement, we consider Eq. (59) for $\tau = \tau_0 = -2/k$. For economy, we only refer to the case of interest $\varepsilon_A, \tau_A \to 0$. Note from Eq. (60) that in this limit the U term is identically null,

$$\lim_{\varepsilon_A\to 0}\mathcal{R}_U(\tau) = 0. \quad \text{(C1)}$$

In the A term (61), we observe that condition $k/\alpha \ll 1$ together with the factor e^{-s} under the integral allows to make the approximation (we use $\ln(1-z) \simeq -z$ for z small)

$$\lim_{\tau_A\to 0}\mathcal{R}_A(\tau_0) = \int_0^{\alpha/k} ds e^{-s}\exp\left\{\frac{2i\omega}{k}\ln\left(1-\frac{sk}{\alpha}\right)\right\} \simeq \int_0^{\infty} ds e^{-s-2i\omega s/\alpha} = \frac{\alpha}{2i\omega+\alpha}, \quad \text{(C2)}$$

which reproduces Eq. (64).

APPENDIX D

We discuss here the analytical extension of the coefficient (59) as a function of the complex z variable in Eq. (69). The analyticity of the nonintegral term (60) is evident from the analyticity with respect to $\tau \equiv -2\sqrt{z}/k$. In the integral term (61), the situation is a bit more complicated, because τ also appears in one of the integration limits, which requires Eq. (61) to be seen as an integral in the complex s plane, and thus one has to prescribe an integration contour for s. A simple solution is to rewrite first the integral using the real integration variable σ defined by $s = \alpha(\tau_A - \tau)\sigma/2$, after which to perform the analytical extension. The result is

$$\bar{\mathcal{R}}_A(z) = \frac{\alpha(\sqrt{z}-\sqrt{\varepsilon_A})}{k}\int_0^1 d\sigma e^{-\alpha(\sqrt{z}-\sqrt{\varepsilon_A})\sigma/k} \times\left(1+\frac{\sqrt{\varepsilon_A}-\sqrt{z}}{\sqrt{z}}\sigma\right)^{2i\omega/k}. \quad \text{(D1)}$$

In this way the analytical extension is unambiguously fixed.

It is clear that the only source of nonanalyticity in Eq. (D1) can be the power function $(1 + f(z, \sigma))^{2i\omega/k}$. Let us consider that the cut in the complex z plane is along the negative real semi-axis. Recall now that the edges of the rectangular contour parallel with the imaginary axis are located at $z = \varepsilon_A$ and $z = 1$, so that everywhere within the contour $\mathrm{Re}z > \varepsilon_A$. In these conditions one can easily check that $|f(z, \sigma)| < 1$, and thus the argument in $(\ldots)^{2i\omega/k}$ never crosses the cut. This proves the analyticity of $\bar{\mathcal{R}}^{\rm out}_{\omega,R}(z)$ within the integration contour.

In the discussion of the coefficients for $\varepsilon_A \to 0$ and $\omega' \to \infty$ in Appendix E, we shall need the small z approximation of Eq. (75). Note that in the first limit only the A contribution (D1) survives [see Eq. (C1)], i.e.

[10] This term is not null in the semitransparency case, because the modes do not vanish on the mirror.

$$\bar{R}_{\omega}(z) \equiv \lim_{\varepsilon_A \to 0} \bar{\mathcal{R}}_A(z). \tag{D2}$$

The observation is that for z very small in Eq. (D1), one can admit that in the exponential under the integral $\alpha\sqrt{z}/k \simeq 0$ and thus $e^{-\alpha(\ldots)\sigma/k} \simeq 1$. More precisely, the approximation is allowed when[11]

$$\alpha^2/(k\omega') \ll 1. \tag{D3}$$

A simple calculation then gives

$$\lim_{\varepsilon_A \to 0} \bar{\mathcal{R}}_A(z) \simeq \frac{\alpha\sqrt{z}}{k}\int_0^1 d\sigma(1-\sigma)^{2i\omega/k} = \frac{\alpha\sqrt{z}}{k}\frac{1}{2i\omega/k+1}. \tag{D4}$$

APPENDIX E

We have to check that Eq. (73) remains finite for $t_A \to \infty$. It is clear that it is sufficient to consider the integrand for ω' large. We note that for $\omega' \to \infty$ the argument of the function $\bar{R}_{\omega}$ in Eq. (74) is

$$z \equiv ikt/\omega' \to 0, \tag{E1}$$

so that we can use the small z approximation (D2) and (D4). Inserting Eq. (D4) in the limit coefficients (74), a simple integration leads to

$$\lim_{t_A \to \infty} \tilde{\beta}^{(V)}(\omega', \omega)^+ \simeq \frac{\alpha}{4\pi k\sqrt{\omega\omega'}}\left(\frac{ik}{\omega'}\right)^{i\omega/k+1/2} \times \Gamma(1/2 + i\omega/k), \qquad \omega' \text{ large.} \tag{E2}$$

This shows that the integrand in Eq. (73) is $\sim 1/\omega'^2$ and thus the integral converges.

Notably, Eq. (E2) reproduces (*modulo* a not so relevant numerical factor) the beta coefficients obtained in [3] [see Eq. 35 therein]. The expression suggests indeed Fermi-Dirac statistics with respect to the energy ω (the $1/2$ term in the argument of the Gamma function) and a number of particles $\sim\alpha^2$. Our observation is that, however, the coefficients (E2) cannot be used to establish the number of particles, since they correctly apply only in the limit of ω' large.

To be more precise, let us estimate the error one makes in $N_{\omega}(t_A \to \infty)$ if one uses Eq. (E2). Consistency requires of course to restrict to the frequencies for which the result is valid, i.e. condition (D3), or equivalently $\omega' \gg \alpha^2/k$. We appeal at this point to our analysis in Sec. E, where we conclude that for ω' large compared with the cutoff frequency $\omega'_{\max} \simeq \alpha^2/\omega$, the beta coefficients are negligibly small. Consider now, for the sake of the argument, a frequency $\omega \sim k$ (i.e. the typical frequency in the thermal flux of the perfect reflector) or larger. It is then immediate that for such frequencies Eq. (E2) will apply only within the irrelevant sector $\omega' \gg \omega'_{\max}$. Most probably, with a more accurate analysis one can show that the same property applies for all frequencies $\omega > 0$. The conclusion is thus that the Fermi-Dirac form of the coefficients (E2) is valid precisely within that part of the spectrum of the frequencies ω' that gives a practically null contribution in $N_{\omega}(t_A \to \infty)$. An illustrative picture is provided by Fig. 5, which shows indeed that frequencies much larger than $\omega'_{\max}$ contribute in the area below the curves with a negligible quantity.

APPENDIX F

We make here a few observations related to the high frequency behavior of the spectrum and the fact that we use a piecewise defined trajectory. Recall that the acceleration of the mirror is discontinuous at $t = 0$ and $t = t_A$. Since the beta coefficients (14) are basically the Fourier transform of a function depending on the trajectory $u = f(v)$, one would naturally expect that the discontinuities will make the particle numbers slowly decreasing functions of ω. This might seem to be in contradiction with the perfect mirror flux at infinite times (34), which exponentially decreases as $\sim e^{-2\pi\omega/k}$. A similar remark can be made for the particle numbers for the semitransparent mirror in Eq. (84).

The basic observation is that the beta contain indeed a slowly decreasing part, but in the limit of large acceleration times this contribution becomes negligible in the flux or the particle numbers. For simplicity, let us focus on the perfect mirror flux. One sees from Eqs. (17) and (18) that the U terms in the beta coefficients have the slowly decreasing behavior $\sim 1/\omega^{3/2}$. The idea is that, as t_A increases, these quantities become less and less relevant in $\mathcal{F}_{\omega}(t_A) = dN_{\omega}(t_A)/dt_A$. For the U plus term (18), this is immediate from the fact that for infinite times $\varepsilon_A \to 0$ the term is identically null. For the U minus term (17), the property follows from the fact that, as part of the minus component (21), it introduces only a finite quantity in $N_{\omega}(t_A \to \infty)$, and thus it gives a vanishing contribution in the limit flux. The bottom line is that for $t_A \to \infty$ the flux will be determined only by the infinitely differentiable trajectory $t \in [0, t_A]$, in agreement with the exponential form of $\mathcal{F}_{\omega}(t_A \to \infty)$. The progressive disappearance of the slowly decreasing behavior with ω in the flux can be nicely observed from the tails of the curves in Fig. 3.

[11] The exponent can take in fact arbitrary large values for any given ω' since $\sqrt{z} \sim \sqrt{t/\omega'} \in [0, \infty)$, but condition (D3) still makes sense because the e^{-t} factor in Eq. (74) practically restricts t below a quantity around unity.

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