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Inverse problem: Reconstruction of the modified gravity action in the Palatini formalism by supernova type Ia data

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We introduce in $f(R)$ gravity-Palatini formalism the method of the inverse problem to extract the action from the expansion history of the Universe. First, we use an ansatz for the scale factor and apply the inverse method to derive an appropriate action for the gravity. In the second step we use the supernova type Ia data set from the Union sample and obtain a smoothed function for the Hubble parameter up to the redshift 1.7. We apply the smoothed Hubble parameter in the inverse approach and reconstruct the corresponding action in $f(R)$ gravity. In the next step we investigate the viability of reconstruction method, doing a Monte Carlo simulation we generate synthetic SNIa data with the quality of the Union sample and show that roughly more than 1500 SNIa data is essential to reconstruct correct action. Finally, with enough SNIa data, we propose two diagnosis in order to distinguish between the ΛCDM model and an alternative theory for the acceleration of the Universe.

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I. INTRODUCTION

A combination of SNIa + CMB data shows that the Universe is in the positive acceleration phase [1,2]. This result is in contradiction with our expectation from the behavior of the ordinary matter. The simplest solution is assuming a cosmological constant in the Einstein field equation as a constant of integration [3]. While this simple modification explains the observational data [4], however the cosmological constant suffers from the fine-tuning and coincidence problems [5]. One of the solutions is introducing a scalar field that provides a time-dependent negative equation of state [6].

The other possibility is the modification of the law of gravity in such a way that it behaves as standard general relativity in strong gravitational regimes and repulses particles in the low density cosmological scales [7]. The modified gravity models can be examined with three categories of observations: (a) cosmological dynamics [8], (b) local gravity [9], and (c) the evolution of large scale structure [10].

In this work we use SNIa data to reconstruct an appropriate $f(R)$ gravity model in Palatini formalism [11] with the inverse method. The method is the extension of the work introduced for the metric formalism by Rahvar and Sobouti in [12]. The inverse method also is introduced in the work by Capozziello *et al.* [13] in the metric formalism. In this method we need to know the dynamics of Hubble parameter from the observational data. Many methods for the extraction of the Hubble parameter have been introduced in the literature [14]. Here, we use the smoothing method suggested by Shafieloo *et al.* [15] and apply it to the SNIa Union sample [16]. We use $H(z)$ in the inverse method algorithm to reconstruct the corresponding action. We test the reliability of this method by doing a Monte Carlo simulation and generating the synthetic SNIa data according to an action and comparing the reconstructed action with the original one. Finally, we introduce two diagnoses for distinguishing the standard ΛCMD model from the alternative models.

The structure of this article is as follows: In Sec. II, we introduce $f(R)$ modified gravity in Palatini formalism, derive the equation of motion, and obtain the dependence of Hubble parameter to the Ricci scalar and scale factor. In Sec. III, we introduce the method of the inverse problem in Palatini $f(R)$ gravity. In Sec. IV, we use the method to the real data of SNIa, smoothing the supernova data we extract the Hubble parameter in terms of redshift and apply it to extract the action [15]. Also, in order to show the level of confidence of our results, we simulate 100 realization of SNIa data to extract the Hubble parameter and compare it with that obtained directly from fitting to the model. In the Sec. V, we examine the viability of the reconstruction method and dependence of the results to the number of SNIa data. In Sec. VI, we propose two diagnoses as a probe to distinguish between the ΛCDM and the alternative models. Section VII concludes the paper.

II. MODIFIED GRAVITY IN PALATINI FORMALISM

For $f(R)$ gravity, there are two main approaches to obtain the field equation. The first one is the so-called metric formalism, which is obtained by the variation of the action with respect to the metric. In this case the derived field equation is a fourth order nonlinear differential equation. In the second approach, which is called Palatini formalism, the connection and metric are considered as independent fields, and the variation of action with respect to these fields results in a set of second order differential equations. The Palatini formalism is a plausible candidate to be the effective classical theory of gravity from a more fundamental theory of loop quantum gravity [17].

Let us take a general form of the action in Palatini formalism as

$$S[f; g, \hat{\Gamma}, \Psi_m] = -\frac{1}{2\kappa}\int d^4x\sqrt{-g}f(R) + S_m[g_{\mu\nu}, \Psi_m], \quad (1)$$

where $\kappa = 8\pi G$ and $S_m[g_{\mu\nu}, \Psi_m]$ is the action of matter dependent on the metric $g_{\mu\nu}$ and the matter field Ψ_m. $R = R(g, \hat{\Gamma}) = g^{\mu\nu}R_{\mu\nu}(\hat{\Gamma})$ is the generalized Ricci scalar, and $R_{\mu\nu}$ is the Ricci tensor made of affine connection. Varying action with respect to the metric results in

$$f'(R)R_{\mu\nu}(\hat{\Gamma}) - \tfrac{1}{2}f(R)g_{\mu\nu} = \kappa T_{\mu\nu}, \quad (2)$$

where prime is the derivative with respect to the Ricci scalar, and $T_{\mu\nu}$ is the energy-momentum tensor

$$T_{\mu\nu} = \frac{-2}{\sqrt{-g}}\frac{\delta S_m}{\delta g^{\mu\nu}}. \quad (3)$$

On the other hand, varying the action with respect to the connection results in

$$\hat{\nabla}_\alpha[f'(R)\sqrt{-g}g^{\mu\nu}] = 0, \quad (4)$$

where $\hat{\nabla}$ is the covariant derivative defined from parallel transformation and is given by the affine connection. From Eq. (4), we define a new metric, $h_{\mu\nu} = f'(R)g_{\mu\nu}$ conformally related to the physical metric where the connection is the Christoffel symbol of this new metric.

We apply the flat Friedmann- Robertson-Walker (FRW) metric (namely, $K = 0$) for the Universe

$$ds^2 = -dt^2 + a(t)^2\delta_{ij}dx^i dx^j, \quad (5)$$

and assume that the Universe is filled with a perfect fluid with the energy-momentum tensor of $T^\nu_\mu = \mathrm{diag}(-\rho, p, p, p)$. Using the metric and energy-momentum tensor in Eq. (2), we obtain the generalized FRW equations. It should be noted that the conservation law of energy-momentum tensor, $T^{\mu\nu}{}_{;\mu} = 0$ is satisfied according to the covariant derivative with respect to the metric, and this definition guarantees the motion of particles on geodesic [18]. The combination of G_{00} and G^i_i results in [19]

$$\left(H + \frac{1}{2}\frac{\dot{f}'}{f'}\right)^2 = \frac{1}{6}\frac{\kappa(\rho + 3p)}{f'} + \frac{1}{6}\frac{f}{f'}. \quad (6)$$

Taking the trace of Eq. (2) results in

$$Rf'(R) - 2f(R) = \kappa T, \quad (7)$$

where $T = g^{\mu\nu}T_{\mu\nu} = -\rho + 3p$. The time derivative of this equation results in $\dot{R}$ in terms of the time derivative of density and pressure. Using the equation of state of cosmic fluid $p = p(\rho)$ and the continuity equation, the time derivative of Ricci is obtained as

$$\dot{R} = 3\kappa H\frac{(1 - 3dp/d\rho)(\rho + p)}{Rf'' - f'(R)}. \quad (8)$$

To obtain generalized first FRW equation we start with Eq. (7) and obtain the density of matter in terms of Ricci scalar as

$$\kappa\rho = \frac{2f - Rf'}{1 - 3\omega}, \quad (9)$$

where $w = p/\rho$. We substitute Eq. (9) in (6), and use Eq. (8) to change the time derivative to d/dR. We rewrite Eq. (6) as follows:

$$H^2 = \frac{1}{6(1 - 3\omega)f'}\frac{3(1 + \omega)f - (1 + 3\omega)Rf'}{[1 + \frac{3}{2}(1 + \omega)\frac{f''(2f - Rf')}{f'(Rf'' - f')}]^2}. \quad (10)$$

On the other hand, using Eq. (7) and the continuity equation, the scale factor can be obtained in terms of Ricci scalar

$$a = \left[\frac{1}{\kappa\rho_0(1 - 3\omega)}(2f - Rf')\right]^{-(1/(3(1+\omega)))}, \quad (11)$$

where ρ_0 is the energy density and a_0 is the scale factor (set to one, i.e. $a_0 = 1$) at the present time. Now for a generic modified action, omitting the Ricci scalar in favor of the scale factor between Eqs. (10) and (11) we can obtain the dynamics of the Universe (i.e. $H = H(a)$).

For the simple case of a matter dominant epoch $\omega = 0$, which is our concern, these equations reduce to

$$H^2 = \frac{1}{6f'}\frac{3f - Rf'}{[1 + \frac{3}{2}\frac{f''(2f - Rf')}{f'(Rf'' - f')}]^2}, \quad (12)$$

$$a = \left[\frac{1}{\kappa\rho_0}(2f - Rf')\right]^{-(1/3)}. \quad (13)$$

III. INVERSE METHOD IN PALATINI FORMALISM

In this section we introduce the inverse method to extract $f(R)$ action in Palatini formalism from the dynamics of the Universe. This method has been studied in the work by Rahvar and Sobuti in the metric formalism [12], and we extend it to the Palatini formalism.

Replacing f with the first derivatives of action from the Eq. (7),

$$f(R) = \tfrac{1}{2}[RF - \kappa(3p - \rho)], \quad (14)$$

Eq. (6) can be written as follows:

$$\left(H + \frac{1}{2}\frac{\dot{F}}{F}\right)^2 = \frac{R}{12} + \frac{\kappa}{4F}(\rho + p), \quad (15)$$

where F is defined as $F = df/dR$. It should be noted that the Ricci tensor in the Palatini formalism is given in terms of the conformal metric of $h_{\mu\nu} = F(R)g_{\mu\nu}$. Substituting the new metric in the definition of Ricci scalar results in

$$R_{\mu\nu} = R_{\mu\nu}(g) + \frac{3}{2}\frac{\nabla_\mu F \nabla_\nu F}{F^2} - \frac{\nabla_\mu \nabla_\nu F}{F} - \frac{1}{2}g_{\mu\nu}\frac{\nabla_\alpha \nabla^\alpha F}{F}, \tag{16}$$

where $R_{\mu\nu}(g)$ is the Ricci tensor defined in terms of the metric $g_{\mu\nu}$. By taking a trace from Eq. (16) we obtain the relation between the Ricci scalar in Palatini and metric as

$$R = R(g) + \frac{3}{2}\frac{\nabla_\alpha F \nabla^\alpha F}{F^2} - \frac{3\nabla_\alpha \nabla^\alpha F}{F}. \tag{17}$$

On the other hand, using the FRW metric, $R(g)$ as the Ricci scalar in the metric formalism is given by the Hubble parameter as

$$R(g) = 6\dot{H} + 12H^2, \tag{18}$$

where for simplicity in the calculation we rewrite this equation by changing the time derivative to the redshift derivative. In what follows we use prime for the derivative with respect to the redshift.

$$R(g) = -6HH'(1+z) + 12H^2. \tag{19}$$

For the Ricci scalar in Palatini formalism from Eq. (17), we obtain

$$R = -6HH'(1+z) + 12H^2 - \frac{3}{2}H^2(1+z)^2\left(\frac{F'}{F}\right)^2 + 3(1+z)^2HH'\frac{F'}{F} - 6H^2(1+z)\frac{F'}{F} + 3H^2(1+z)^2\frac{F''}{F}. \tag{20}$$

Substituting this equation in (15) results in a differential equation for the evolution of F as a function of redshift

$$F'' - \frac{3}{2}\frac{F'^2}{F} + F'\left(\frac{H'}{H} + \frac{2}{1+z}\right) - \frac{2H'}{H(1+z)}F + \frac{\kappa}{H^2(1+z)^2}(\rho + p) = 0. \tag{21}$$

In the matter dominant epoch we rewrite this equation by putting $p = 0$ and $\rho = \rho_0(1+z)^3$, where we can replace $\kappa\rho_0$ with $3H_0^2\Omega_m$. It should be noted that the definition of the Ω_m is different from that in the standard FRW equations.

For a given dynamics of the Universe (i.e. $a(t)$), we can extract the Hubble parameter in terms of redshift, and applying it in Eq. (21) will provide F in terms of redshift. On the other side, we can calculate the Ricci scalar from Eq. (20) in terms of redshift. Eliminate z in favor of Ricci scalar results in $F(R)$. Finally, by the numerical integration of this function we can obtain the modified gravity action $f(R)$.

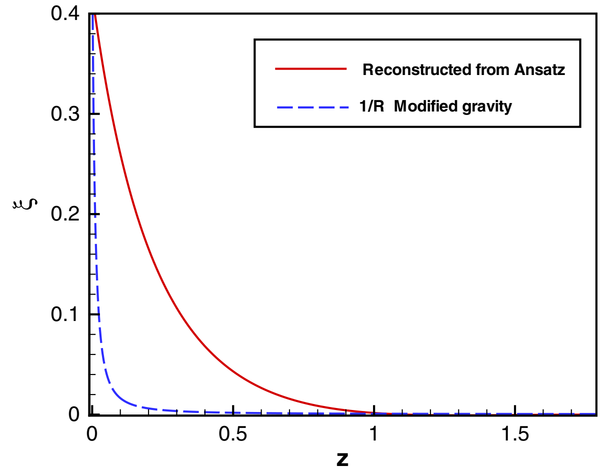

FIG. 1 (color online). Deviation parameter $\xi = F(R) - 1$ obtained from the ansatz scale factor (solid line) is compared with ξ parameter of $1/R$ modified gravity (dashed line) in terms of redshift. For the redshifts $z > 1$, the $f(R)$ gravity model converge to the Einstein-Hilbert action.

In the rest of this section to test this algorithm we use an ansatz for the scale factor and try to extract the corresponding action from a given dynamic. We apply the following ansatz for the scale factor proposed in [12]:

$$a(\tau) = \frac{1}{1+p}(\tau)^{(2/3)}[1 + p\tau^{(2\alpha/3)}] \tag{22}$$

in which $\tau = tH_0$ is a dimensionless time parameter defined in the interval of 0 to 1 and H_0 is the Hubble parameter at the present time. This proposed dynamic has two free parameters of α and p. We obtain the corresponding Hubble parameter from this scale factor and consequently the distance modulus and compare the model

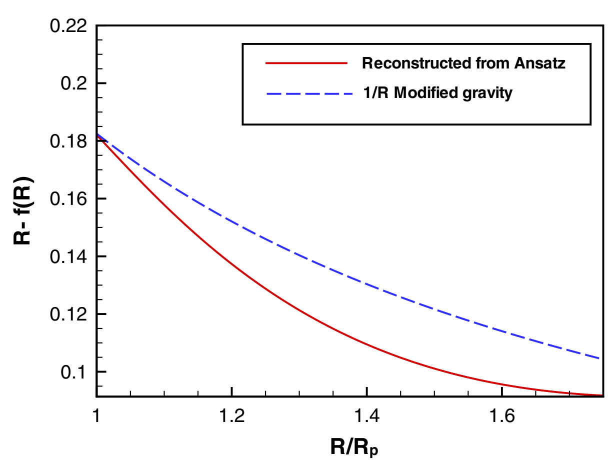

FIG. 2 (color online). The difference of modified gravity action from the Einstein-Hilbert action is plotted versus Ricci scalar (solid line) and compared with the action of $f(R) = R - \frac{\mu^4}{R}$ (dashed line). The Ricci scalar is normalized to its value at the present time, R_p.

with the observed SNIa data. The best value for the parameters of model has been obtained roughly as $p = \frac{1}{3}$ and $\alpha = 2$ in [12]. Substituting the scale factor in Eq. (21), we obtain the deviation parameter of $\xi \equiv F - 1$ from general relativity as a function of redshift, which is plotted in Fig. 1.

In order to reconstruct the action in terms of Ricci action, we obtain the Ricci scalar in terms of redshift and finally by eliminating redshift between F and the Ricci scalar, we obtain $F = F(R)$. Integrating this function provides the action in terms of Ricci scalar. We also plot the ξ parameter of $1/R$ modified gravity for comparison with the reconstructed modified gravity in Fig. 1 and $R - f(R)$ as a function of R in Fig. 2. Comparison of the extra term with the Einstein-Hilbert action roughly resembles the μ^4/R function with $\mu^2 \sim 10^{-1}R_p$, where R_p is the present value of Ricci scalar.

IV. RECONSTRUCTION OF THE DYNAMIC OF THE UNIVERSE BY SNIA DATA

In Sec. III, we showed that it is possible to reconstruct the modified gravity action by knowing the dynamics of the Universe. In this section, we use SNIa cosmological data to obtain the Hubble parameter and consequently reconstruct the action of modified gravity.

The dynamics of the Hubble parameter, $H(z)$, can be obtained if the distance modulus of supernovas data as a function of redshift is known. In a FRW-flat universe, the Hubble parameter is related to the distance modulus of SNIa as follows:

$$H(z) = \left[\frac{d}{dz}\left(\frac{d_L(z)}{1+z}\right)\right]^{-1}. \tag{23}$$

The main challenging point in this procedure is the limited number of observed SNIa, which impose an uncertainty in calculating the continues function for $H(z)$. The overall number of supernovas which has been detected is in the order of 300–400. We use the latest supernova data of the Union sample to extract the Hubble parameter [16]. To make a continues Hubble parameter, we follow the procedure known as reconstruction method, proposed by Shafieloo *et al.* in [15]. In this algorithm, a nonparametric function is used for smoothing the distance modulus of supernova data over the redshift. Here, we choose a guess model resemble to the observed distance modulus of the supernova data. In the next step we subtract the distance modulus of the observed data from the guess model using a Gaussian function for smoothing the observed data as follows:

$$\mu^d(z) = N(z)\sum_i[\mu^g(z) - \mu^{\rm obs}(z_i)]\exp\frac{-(z-z_i)^2}{2\Delta^2}, \tag{24}$$

where

$$N(z)^{-1} = \sum_i \exp\left[\frac{-(z-z_i)^2}{2\Delta^2}\right], \tag{25}$$

where z_i represents the redshift of each SNIa in the Union sample. The sum term is considered for all 307 Union sample data, $\mu^{\rm obs}(z_i)$ is the observed distance modulus and $\mu^g(z)$ is a continues guess model for the distance modulus. $N(z)$ is a normalization factor, and Δ is a suitable redshift window function. $\mu^d(z)$ is a smoothed continues function for the residual of luminosity distance in terms of redshift. Now the corrected distance modulus is added to the guess model to generate the new smooth function for the distance modulus:

$$\mu^s(z) = \mu^g(z) + \mu^d(z). \tag{26}$$

We repeat this procedure using $\mu^s(z)$ as the new guess function. It can be shown that after a finite time of this irritation, χ^2 of the smoothed function with respect to observed data will converge to a fixed value. It means that we will find a continues distance modulus function with the best fit to the real data. It is shown in [15] that the result of the best continues function is independent of the choice of the first guess model. Having the smoothed luminosity distance we use Eq. (23) to obtain the Hubble parameter $H(z)$. Another point is that the results will clearly depend upon the value of Δ in Eq. (24). A large value of Δ produces a smooth result, but the accuracy of reconstruction worsens, while small value Δ gives a more accurate, but noisy result. Considering the frequency of SNIa data observed in the Union sample, we choose $\Delta = \sqrt{M}(1+z)\Delta_0$ for the window function, where M is the number of irritation need to converge χ^2 and $\Delta_0 = N^{-1/3}$ where N is the number of SNIa [15].

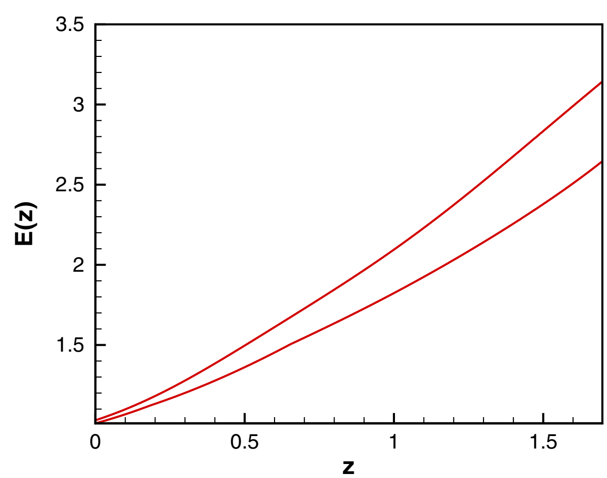

FIG. 3 (color online). The uncertainty in $E(z) = \frac{H}{H_0}$ from the smoothing method, resulting from the Monte Carlo simulation. We generate 100 realizations for the distance modulus and obtain the Hubble parameter and all these continues functions reside inside the boundaries of the figure.

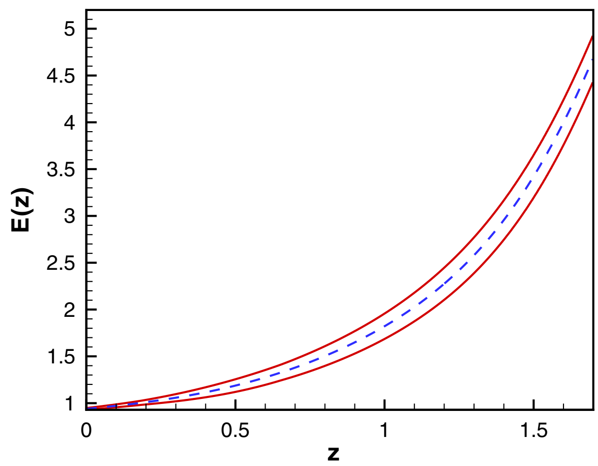

FIG. 4 (color online). The dashed line is the reconstructed Hubble parameter from the SNIa Union sample data. The bold lines are the confidence level of the Hubble parameter.

In what follows we find the uncertainty in $H(z)$ from this method to use it for reconstructing the appropriate action in the Palatini formalism. For this purpose we do a Monte Carlo simulation, generating 100 realization of SNIa data and using the same distribution of SNIa in terms of redshift reported by Kowalski *et al.* [16]. Also, we use the same error bars of distance modulus in the observed data. In order to simulate the synthetic distance modulus of SNIa, we assume a dark energy model for the Universe with a constant equation of state of $\omega = -0.75$. Choosing ΛCDM as the guess model for these data, we obtain the Hubble parameter $E(z) = H(z)/H_0$ for 100 realization of supernova data. Figure 3 shows the boundaries for $E(z)$, resulted from 100 realization of supernova data.

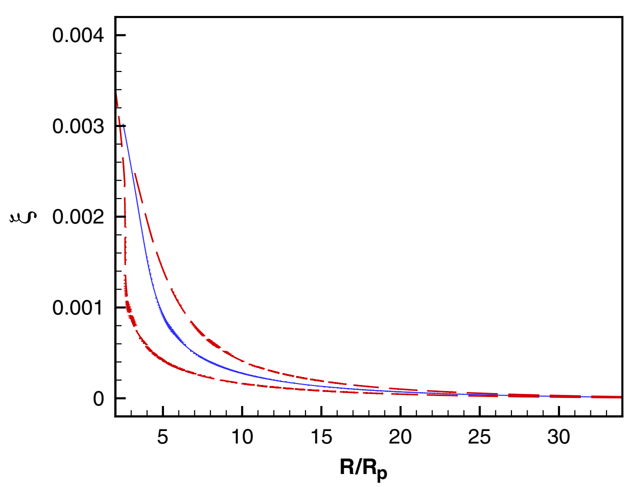

FIG. 5 (color online). The bold line shows the reconstructed modified gravity $\xi \equiv F - 1$ parameter versus the Ricci scalar normalized to its present value. The dashed lines show that the confidence level of the ξ parameter corresponds to the uncertainty of the Hubble parameter resulting from the smoothing procedure.

Now we can apply this uncertainty to $H(z)$, resulting from the smoothing procedure on the real Union sample [20]. We plot the Hubble parameter from the Union sample as shown in Fig. 4 with a margin represents the uncertainty that is obtained form the Monte-Carlo simulation. Using the method described in Sec. III, we extract the corresponding modified gravity in Palatini formalism. The parameter of $\xi = df(R)/dR - 1$ in terms of Ricci scaler shows a small deviation from the Einstein-Hilbert action as shown in Fig. 5 with the uncertainty of this parameter. Here, the deviation from the Einstein-Hilbert action is in the order of $\xi \sim 10^{-3}$. We ask the reliability of this result in terms of the number of SNIa data. In the next section we will discuss this issue.

V. VIABILITY OF SMOOTHING METHOD AND THE NUMBER OF SNIA DATA

In this section we examine the viability of smoothing of the Hubble parameter. The results will be applied to the modified gravity models in the next section.

According to the methodology of the smoothing procedure, the Hubble parameter depends on the quantity and quality of the SNIa data. Similar to the simulation in the previous section we generate 100 realization of 307 SNIa data, using the redshift and the uncertainty of the distance modulus in the Union sample within the framework of the ΛCDM model. We generate continues Hubble parameter with the margin of uncertainty which is shown in Fig. 6. On the other hand, we obtain the Hubble parameter in the ΛCDM model through fitting the observed distance modulus of the data to the model as shown in Fig. 6. The margin represents 1σ level of confidence for the Hubble parameter. Comparing the uncertainties from these two methods

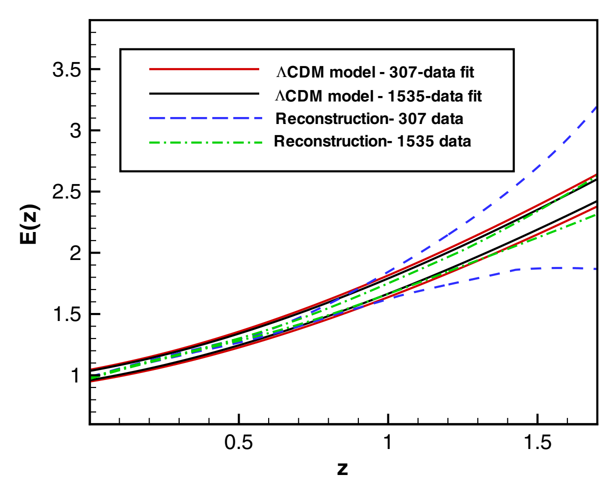

FIG. 6 (color online). The dashed lines and dotted-dashed line are the reconstructed Hubble parameters from 307 and 1535 SNIa data, respectively. The solid lines are the 1-σ confidence level of the Hubble parameter from data fitting to 307 and 1535 data in the ΛCDM model.

shows that the smoothing method is not reliable with 307 number of supernova data.

We increase the number of SNIa data with the quality of Union sample to have the same uncertainty in the Hubble parameter from the two methods. For $N > 1500$ as shown in Fig. 6 we can rely on the smoothing method. We can repeat this simulation with precise data than the Union sample to achieve this goal with a smaller number of supernova data.

VI. DISTINGUISHING BETWEEN THE MODIFIED GRAVITY MODELS AND ΛCDM

In this section we present two diagnosis in order to distinguish between ΛCDM from an alternative model. The first method is (a) comparison of the Hubble parameters and the second method is (b) the Ξ function. Both diagnoses are applicable by using the inverse method.

A. Comparison of the Hubble parameters

In the previous section we have seen the smoothing method generate a continues Hubble parameter from the observed data. If the dynamics of the Universe follow rather than the ΛCDM model, can we distinguish the real model of the Universe, knowing the Hubble parameter from the data?

We can compare the reconstructed Hubble parameter directly from the observed data with that obtained from fitting data to the ΛCDM model. We subtract the two Hubble parameters and compare it with 1σ deviation of the smoothed Hubble parameter. As we simulated in the previous section using $N > 1500$ is sufficient to reliable on the uncertainty of the Hubble parameter.

We use the $f(R) = R - \frac{\mu^4}{R}$ action in the Palatini formalism and generate 1535 supernova data with the quality of the Union sample. Using the smoothing method we obtain the Hubble parameter. On the other hand we fit the generated data with the ΛCDM model. Figure 7 compares the two Hubble parameters and shows that for the redshifts with $z > 1$, the two models are distinguishable.

B. The method of the Ξ function

We define a new criterion to distinguish the ΛCDM from an alternative model as follows:

$$\Xi \equiv \frac{2E(z)E'(z)}{3(1+z)^2}, \tag{27}$$

where prime is differential with respect to the redshift. For the case of $F = 1$ in a ΛCDM universe, from Eq. (21), Ξ reduces to $\Xi = \Omega_m^0$. If we have the smoothed normalized Hubble parameter from the observed data, $E(z)$, then Ξ can be obtained. Any deviation from a constant value for this function is a diagnosis for the ΛCDM universe. This method is applicable both for the dark energy and modified gravity models. Let us take $\omega(z)$ as the equation of state of dark energy, we can use FRW equations to obtain Ξ as follows:

$$\Xi = \Omega_m^0 + (1 - \Omega_m^0)\frac{1+\omega(z)}{(1+z)^3}, \tag{28}$$

where for the constant value of the equation of state, $\omega(z) = -1$, this equation reduces to $\Xi = \Omega_m^0$.

In order to show the deviation from the Einstein-Hilbert action, we can calculate $\Xi - \Omega_m^{(0)}$ as a function of redshift. Here, we know $\Omega_m^{(0)}$ from a direct cosmological observations such as gravitational weak lensing. To show how

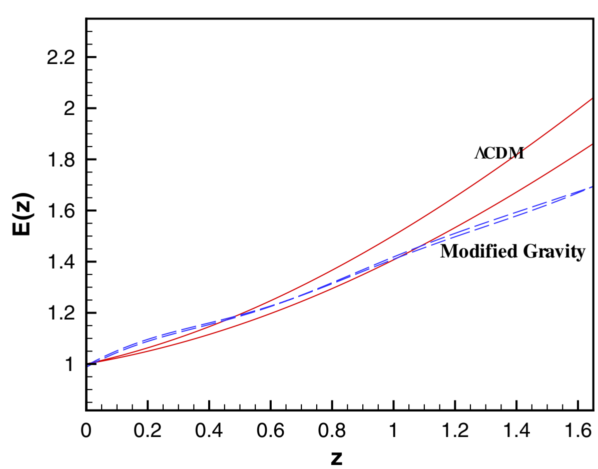

FIG. 7 (color online). The solid line is the Hubble parameter with 1-σ error bar from fitting the simulated data to the ΛCDM model. The dashed line is the confidence level of the Hubble parameter extracted from the smoothing method of the Hubble parameter. The figure shows the deviation of the two curves for $z > 1$.

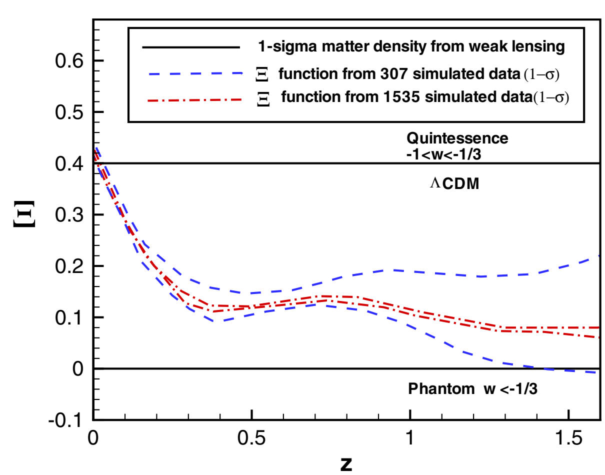

FIG. 8 (color online). Ξ function versus redshift is plotted for two series of SNIa data, with 1σ level of confidence. The dashed lines and dashed-dotted lines indicate 307 and 1535 data, respectively. The solid lines indicate the matter density of the Universe derived from gravitational weak lensing.

this method works, we generate SNIa data from $f(R) = R - \mu^4/R$ action resemble to the Union sample and after smoothing the Hubble parameter, we plot Ξ as a function of redshift. We generate Ξ function for two cases with 307 and 1535 supernova data as represented in Fig. 8. To compare this function with a direct measurement of $\Omega_m^{(0)}$, we use the weak lensing data, which puts a limit on the matter content of the Universe in the range of $\Omega_m^{(0)} = 0.2\pm^{0.2}_{0.2}$ [21]. As shown in Fig. 8, increasing SNIa data will pin down the Ξ function more precisely where today's SNIa data is not sufficient. On the other hand, direct observations of matter density of the Universe are precise enough to claim any model with distinguishing results.

VII. CONCLUSION

One of the most puzzling questions in the cosmology is the physical mechanism for the acceleration of the Universe. Is it driven by a cosmological constant or is the Universe filled with an exotic dark energy or should the Einstein gravity be modified? In this work we proposed an inverse method to extract the action of a modified gravity in the Palatini formalism from the expansion history of the Universe.

We used the smoothing method to obtain a continues Hubble parameter from the supernova type Ia Union sample data. We showed that more than 1500 supernova data with the quality of the Union sample is essential to have a reliable Hubble parameter. Finally, we proposed two cosmological diagnoses in order to distinguish between ΛCDM and alternative models. The first one compares the smoothed Hubble parameter from the SNIa data with the standard ΛCDM model. In the second approach we define a new parameter where it is constant, equal to the Ω_m for the ΛCDM universe and varies with the redshift for any alternative model. A precise measurement of the matter content of the Universe from one hand and enough number of supernova data from the other hand will enable us to identify either our Universe follows ΛCDM universe or some modification is necessary.

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