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Nonsingular ekpyrotic/cyclic model in loop quantum cosmology

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We study the role of nonperturbative quantum gravity effects in the ekpyrotic/cyclic model using the effective framework of loop quantum cosmology in the presence of anisotropies. We show that quantum geometric modifications to the dynamical equations near the Planck scale as understood in the quantization of Bianchi-I spacetime in loop quantum cosmology lead to the resolution of classical singularity and result in a nonsingular transition of the Universe from the contracting to the expanding branch. In the Planck regime, the Universe undergoes multiple small bounces and the anisotropic shear remains bounded throughout the evolution. A novel feature, which is absent for isotropic models, is a natural turn-around of the moduli field from the negative region of the potential leading to a cyclic phenomena as envisioned in the original paradigm. Our work suggests that incorporation of quantum gravitational effects in the ekpyrotic/cyclic model may lead to a viable scenario without any violation of the null energy condition.

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I. INTRODUCTION

One of the most intriguing issues in cosmology concerns the state of the Universe in the earliest epoch of its evolution. It is this phase where answers to some of the most difficult questions in conventional cosmology are hidden which includes understanding the initial conditions of our universe. It is difficult to understand the latter via cosmological models based on classical general relativity (GR) since it breaks down at very large spacetime curvatures and predicts an initial singularity. Therefore, any viable description of the history of our Universe must necessarily have inputs from a framework beyond GR. Not surprisingly, one expects that any such model must capture quantum aspects of gravity in order to be successful. Though a treatment based on a full theory of quantum gravity is beyond the scope at the present stage, nevertheless useful insights have been obtained on the nature of the Universe at the Planck scale in various frameworks. In one of the earliest works in this direction, Wheeler showed that the spacetime near the initial singularity resembles a quantum foam [1] (a picture shared with the analysis of quantization of conformal modes in gravity [2]). In recent years a different picture seems to emerge in some models based on nonperturbative canonical quantum gravity. In loop quantum cosmology (see Ref. [3] for an introductory review) a quantization of symmetry reduced spacetimes based on background independent loop quantum gravity, backward evolution of states for different stress-energy contents, show existence of a bounce near the Planck scale to a classical contracting branch [4–6]. The quantum bounce is nonsingular and allows a unitary evolution across the "big bang" *without any violation of null energy condition*. These results indicate that the Universe may have a "pre big bang" branch which plays an important role in the history and the fate of the Universe.

Interestingly, though loop quantum cosmology predicts the existence of a contracting universe preceding ours, the idea of a pre big bang branch of the Universe is not new. Various paradigms in theoretical cosmology have conjectured an existence of such a pre big bang branch and the existence of a nonsingular bounce is the most crucial element for them to be viable. As an example, a pre big bang branch is envisioned in the ekpyrotic/cyclic model paradigm [7–9] which is also considered as an alternative to the inflationary scenarios (see Ref. [10] for a review). It is based on the inputs from M-theory where it is envisioned that the Universe undergoes cycles of expansion and contraction governed by the interbrane dynamics of two boundary branes in a five dimensional bulk.[1] Our observable Universe is hypothesized to be confined on one of the boundary branes which interacts only gravitationally with the other brane. Attraction between the branes leads to their collision, an event which corresponds to a big bang or big crunch singularity for the observable Universe.

The transition from the contracting to the expanding branch when the branes collide and separate is a tricky issue in the ekpyrotic/cyclic models. Though insights on this problem from the bulk spacetime perspective have been gained (see Refs. [13–15]), its solution remains elusive in the 4-dimensional cosmological picture. In its absence a viable nonsingular cosmological model of the early

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[1] Based on string theory, idea of a bouncing Universe has also been extensively studied in pre big bang models [11] and more recently in string gas cosmologies [12].

Universe is difficult to construct. Insights on obtaining a nonsingular transition in the ekpyrotic/vyclic model is expected from understanding the role of nonperturbative quantum gravitational effects which are well understood in loop quantum gravity. In particular a lot of progress has been made in recent years on understanding the resolution of singularities in the framework of loop quantum cosmology [3,16–18]. This leads to a natural question (which is the focus of this work): Is it possible to obtain a nonsingular transition in the ekpyrotic/cyclic model using inputs from loop quantum cosmology?

Loop quantum cosmology is a canonical nonperturbative quantization of homogeneous spacetimes based on Dirac's method of constraint quantization. The gravitational part of the classical phase space is labeled by the Ashtekar variables: SU(2) connection A_a^i (which in a cosmological setting is proportional to rate of change of scale factor at the classical level) and conjugate triad E_i^a (proportional to the square of the scale factor). The elementary variables used for quantization in loop quantum cosmology are the holonomies of the connection components and triads. Quantization proceeds with expressing the classical Hamiltonian constraint, consisting of the field strength tensor of the Ashtekar connection, in terms of elementary variables. The resulting quantum constraint captures the underlying discreteness in quantum geometry (one of the predictions of loop quantum gravity) and turns out to be uniformly discrete in volume. This has been shown to be a property of isotropic models (for all values of curvature index k) [6,19–23] and Bianchi-I spacetimes [24] where one has rigorous analytical control and extensive numerical simulations have been performed.

Some of the novel features of loop quantization can be described as follows. Let us consider a isotropic universe filled with a massless scalar field. Take a state which is peaked at the classical trajectory at late times and evolve it with the loop quantum constraint toward the big bang. It turns out that the state remains peaked at the classical trajectory until spacetime curvature (R) is approximately 1% of the Planck value, however on further evolution there are significant departures from the classical theory. Instead of following the classical trajectory until the big bang as in the Wheeler-DeWitt theory, evolution leads to a bounce of the Universe when energy density of the Universe reaches a critical value, $\rho_{\rm crit} = 0.41\rho_{\rm Pl}$ [6]. After the bounce the state then evolves further to peak on the classical trajectory for a contracting universe. These turn out to be robust features of the theory. Using an exactly solvable model it can be proved that the bounce is a property *for all the states* in the physical Hilbert space and there exists a supremum for the energy density operator given by the above value [23]. Moreover, the fluctuations are bounded across the bounce and the state which is semiclassical at late times post bounce is also semiclassical at early times pre bounce [25]. It is important to note that existence of bounce does not require any fine tuning of initial conditions or a special choice of matter. In fact for the above case, matter satisfies the stiff equation of state throughout the evolution. Existence of bounce and nonsingular evolution has also been shown for the scalar field with a cosmological constant (for positive [26] as well as negative values [27]) and the inflationary potential [28]. Another important feature of loop quantum cosmology is that it turns out that there exists a unique consistent loop quantization for the isotropic spacetimes as well as the Bianchi-I spacetime [29,30].

Interestingly, it is possible to write an effective Hamiltonian in loop quantum cosmology with a resulting dynamics which approximates the underlying quantum dynamics to an excellent accuracy. The effective Hamiltonian can be derived using coherent state techniques for different matter sources [31–33]. Assuming that effective dynamics is valid for arbitrary matter (an assumption which turns out to be true for various cases), it is possible to prove that isotropic flat loop quantum cosmology is generically nonsingular and geodesically complete [34]. It resolves all known types of cosmological singularities. Similar results are expected for curved and anisotropic models. Results obtained in the full quantum theory, numerical simulations, and the effective theory strongly indicate that resolution of cosmological singularities is natural in loop quantum cosmology.

The availability of an effective description leads us to explore a viable nonsingular bouncing model in the effective description of loop quantum cosmology with the ekpyrotic/cyclic model potential. This serves as a first step to investigate the role of nonperturbative quantum gravity effects as derived in loop quantum gravity to the ekpyrotic/cyclic models. For the case when anisotropies are absent, such an analysis was performed earlier [35]. It was shown that a nonsingular bounce is generically possible for the flat isotropic ekpyrotic/cyclic model in loop quantum cosmology. Despite this a viable model was not possible due to lack of a turn-around of the moduli field in the epoch when the branes collide and separate, inhibiting a cyclic phenomena. It was shown that unless the ekpyrotic/cyclic model potential becomes positive when the singularity is approached it is not possible for the moduli field to turn around in the process of transition from contracting to the expanding branch (or vice versa) and lead to cycles.[2]

However in a realistic universe anisotropies are always present, even if their strength is very small. Hence it is important to analyze whether the limitations pointed out in the analysis of Ref. [35] were artifacts of the assumption of pure isotropy. It is pertinent to ask whether there exists a viable nonsingular ekpyrotic/cyclic model for effective

[2]Such a modification to the cyclic model potential has also been suggested in Refs. [36,37].

loop quantum dynamics incorporating anisotropic properties of spacetime. As we will show the answer turns out to be positive. We will see that the mere presence of the shear term in the cosmological dynamics leads to a turn-around of the moduli field from the negative region of the potential. Instead of a single bounce of the scale factor, the anisotropic effective dynamics obtained from the loop quantization of Bianchi-I model exhibits bounces for each of the directional scale factors. Numerical simulations show that the turn-around of the moduli occurs in the middle of the transition of the mean scale factor from the contracting to the expanding branch. Further, the anisotropic shear remains bounded throughout the evolution. Thus a nonsingular transition from the contracting to the expanding phase with a turn-around of the moduli as envisioned in the ekpyrotic/cyclic model paradigm is achieved in the effective loop quantum dynamics. This is the main result of our analysis.

This paper is organized as follows. In the next section we revisit the classical theory of the Bianchi-I model in the Ashtekar variables. We describe the way the classical generalized Friedman equation for an anisotropic model can be derived in a Hamiltonian treatment. In Sec. III we consider the effective Hamiltonian of the Bianchi-I model for the loop quantization performed in Ref. [24]. This quantization for Bianchi-I model has recently been shown to be the only consistent choice which leads to a bounded shear and expansion factors [30]. Using Hamilton's equations we derive the dynamical equations for connections and triad components (the dynamical equations for matter such as the Klein-Gordon equation are not changed by quantum gravitational effects) and highlight some of the properties of the effective theory. In Sec IV, we use the effective dynamics to analyze the potential in the ekpyrotic/cyclic model. Using numerical techniques we study the evolution of the moduli field focusing, in particular, on the transition from the contracting branch to the expanding branch. We show that a nonsingular turn-around of the scale factor along with the same for the moduli field is possible in the effective 4-dimensional description without any extra inputs. We summarize the results and discuss open issues in the concluding section.

II. BIANCHI-I MODEL: CLASSICAL THEORY IN ASHTEKAR VARIABLES

The Bianchi-I spacetime is one of the simplest examples of spacetimes with anisotropies. It has vanishing intrinsic curvature and unlike the Bianchi-IX model, the classical dynamics do not exhibit Belinski-Khalatnikov-Lifshitz behavior as the singularity is approached. The isotropic limit of this spacetime is $k = 0$ Friedman-Robertson-Walker cosmological spacetime. We consider a homogeneous Bianchi-I anisotropic spacetime with a manifold $\Sigma \times \mathbb{R}$ where Σ is topologically flat ($\mathbb{R}^3$). The spatial manifold is noncompact and in order to define the symplectic structure and formulate a Hamiltonian theory, it is necessary to introduce a fiducial cell $\mathcal{V}$. The cell $\mathcal{V}$ has fiducial volume $V_o = l_1 l_2 l_3$ with respect to the fiducial metric $\mathring{q}_{ab}$ endowed on the spatial manifold. Here l_i refer to the coordinate lengths of the each side of the fiducial cell.[3]

Because of the homogeneity of the Bianchi-I spacetime, the Ashtekar variables take a simple form. The matrix valued connection A_a^i and triad E_i^a can be expressed as c_i and p_i respectively (where $i = 1, 2, 3$) [38]. The canonical conjugate phase space variables satisfy

$$\{c_i, p_j\} = 8\pi G\gamma\delta_{ij}, \tag{1}$$

where $\gamma = 0.2375$ is the Barbero-Immirzi parameter. The triad p_I are related to the three scale factors a_I of the Bianchi-I metric

$$ds^2 = -N^2dt^2 + a_1^2dx^2 + a_2^2dy^2 + a_3^2dz^2, \tag{2}$$

as

$$|p_1| = l_2l_3a_2a_3, \qquad |p_2| = l_1l_3a_1a_3, \qquad |p_3| = l_2l_3a_2a_3, \tag{3}$$

where the modulus sign arises because of orientation of the triads (and is suppressed in the following).

The only nontrivial constraint to be solved in this model is the Hamiltonian constraint which when expressed in terms of c_i and p_i takes the form:

$$\mathcal{H}_{\rm cl} = -\frac{N}{8\pi G\gamma^2 V}(c_1p_1c_2p_2 + c_3p_3c_1p_1 + c_2p_2c_3p_3) + \mathcal{H}_{\rm matt}, \tag{4}$$

where $\mathcal{H}_{\rm matt}$ is the matter Hamiltonian which may describe perfect fluid and/or scalar fields. Physical solutions are obtained by the vanishing of the Hamiltonian constraint: $\mathcal{H}_{\rm cl} \approx 0$. Equations of motions for the phase space variables are obtained by solving Hamilton's equations:

$$\dot{p}_i = \{p_i, \mathcal{H}_{\rm cl}\} = -8\pi G\gamma\frac{\partial\mathcal{H}}{\partial c_i}, \tag{5}$$

and

$$\dot{c}_i = \{c_i, \mathcal{H}_{\rm cl}\} = 8\pi G\gamma\frac{\partial\mathcal{H}}{\partial p_i}. \tag{6}$$

If we choose the lapse function $N = 1$, then using (4) and (6) we obtain

[3] As should be expected from any consistent treatment, physical predictions of this model are insensitive to the choice of l_i and one could choose these to be unity. We do not restrict to this choice in order to stress the independence of physics in the classical and especially the effective theory on the fiducial cell. The latter feature is not shared by alternative quantizations of Bianchi-I model in loop quantum cosmology.

$$c_i = \gamma l_i \dot{a}_i = \gamma l_i H_i a_i, \tag{7}$$

where $H_i \equiv \dot{a}_i/a_i$ is the Hubble rate in the i-th direction.

Similarly we can obtain the dynamical equations for the matter degrees of freedom. As an example, in case $\mathcal{H}_{\rm matt}$ corresponds to a minimally coupled scalar field ϕ with momentum p_ϕ (satisfying $\{\phi, p_\phi\} = 1$) and potential $V(\phi)$, the dynamical equations take the standard form

$$\dot{\phi} = \frac{\partial}{\partial p_\phi}\mathcal{H}_{\rm matt} \quad \text{and} \quad \dot{p}_\phi = -\frac{\partial}{\partial \phi}\mathcal{H}_{\rm matt}. \tag{8}$$

Taking the second derivative of ϕ and using $\dot{p}_\phi$, we obtain the standard Klein-Gordon equation for ϕ:

$$\ddot{\phi} + \sum_i H_i \phi = -\partial_\phi V(\phi). \tag{9}$$

Before we study the properties of dynamical equations it is useful to note the behavior of the classical Hamiltonian constraint under one of the underlying freedoms in the framework. This has to do with the change of the shape of the fiducial cell. Let us consider this change as $(l_1, l_2, l_3) \to (l'_1, l'_2, l'_3)$. Under this change $V \to l'_1 l'_2 l'_3 V$ and c_1, p_1 (and similarly other components) transform as

$$c_1 \to c'_1 = l'_1 c_1, \qquad p_1 \to p'_1 = l'_2 l'_3 p_1. \tag{10}$$

Thus, the gravitational part of the constraint transforms as $\mathcal{H}_{\rm grav} \to l'_1 l'_2 l'_3 \mathcal{H}_{\rm grav}$. It can be shown that the matter part of the constraint also transforms in the same way. Thus, $\mathcal{H}_{\rm class}/V$ is invariant under arbitrary change in shape of the fiducial cell.

Solving the classical constraint $\mathcal{H}_{\rm cl} \approx 0$, dividing by the total volume $V = V_o(a_1 a_2 a_3) = (p_1 p_2 p_3)^{1/2}$, and using (7) we obtain the following equation relating directional Hubble rates with energy density:

$$H_1 H_2 + H_2 H_3 + H_3 H_1 = 8\pi G \frac{\mathcal{H}_{\rm matt}}{V} = 8\pi G \rho, \tag{11}$$

where ρ is the energy density of the matter component: $\rho = \mathcal{H}_{\rm matt}/V$. As expected, using (10) one finds that Hubble rates and energy density are invariant under $(l_1, l_2, l_3) \to (l'_1, l'_2, l'_3)$.

An interesting property of equations of motion for p_i and c_i arises for matter with vanishing anisotropic stress. It can then be shown that $(p_i c_i - p_j c_j)$ is a constant of motion satisfying [38], i.e.

$$p_i c_i - p_j c_j = V(H_i - H_j) = \gamma V_o \alpha_{ij}, \tag{12}$$

where α_{ij} is a constant antisymmetric matrix.

The directional Hubble rates can be considered to be the diagonal elements of an expansion matrix H_{ij}. The trace of this matrix is related to the expansion rate of geodesics in this spacetime as

$$\theta = \frac{1}{3}(H_1 + H_2 + H_3) = \frac{1}{a}\frac{da}{dt}, \tag{13}$$

where a is identified as the mean scale factor for our anisotropic model:

$$a = (a_1 a_2 a_3)^{1/3}. \tag{14}$$

The trace-free part of the expansion matrix leads to the shear term σ_{ij}:

$$\sigma_{ij} = H_{ij} - \theta\delta_{ij}, \tag{15}$$

and defines the shear scalar $\sigma^2 \equiv \sigma_{\mu\nu}\sigma^{\mu\nu}$ given by

$$\begin{aligned}\sigma^2 &= \sum_{i=1}^{3}(H_i - \theta)^2 \\ &= \frac{1}{3}((H_1 - H_2)^2 + (H_2 - H_3)^2 + (H_3 - H_1)^2) \\ &= \frac{1}{3a^6}(\alpha_{12}^2 + \alpha_{23}^2 + \alpha_{31}^2),\end{aligned} \tag{16}$$

where to obtain the last expression we have used Eq. (12).

The generalized Friedman equation for the mean scale factor can be obtained by considering the mean Hubble rate

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\sum_{i=1}^{3} H_i, \tag{17}$$

which yields

$$\begin{aligned}H^2 = {}& \tfrac{1}{3}(H_1 H_2 + H_2 H_3 + H_3 H_1) + \tfrac{1}{18}((H_1 - H_2)^2 \\ & + (H_2 - H_3)^2 + (H_3 - H_1)^2).\end{aligned}$$

On using Eqs. (11) and (16) we obtain

$$H^2 = \frac{8\pi G}{3}\rho + \frac{\Sigma^2}{a^6}, \tag{18}$$

with

$$\Sigma^2 \equiv \tfrac{1}{6}\sigma^2 a^6. \tag{19}$$

From (16) it follows that the shear scalar Σ is a constant of motion in the classical theory. As we will show later, this feature does not hold in the loop quantization where only at the classical scales Σ approaches a constant value.

Analysis of the generalized Friedman equation (18) immediately shows that the singularity is inevitable in the classical theory as the scale factor approaches zero. It is important to note that in the classical theory the anisotropic term dominates on the approach to singularity unless ρ corresponds to matter with the equation of state $w > 1$, i.e. with an equation of state of an ultrastiff fluid. In the case when matter has the equation of state $w < 1$, the energy density grows slower than the anisotropic term as $a(t) \to 0$. We will discuss later that in the ekpyrotic/cyclic model the equation of state $w \gg 1$ during the ekpyrosis phase (when the moduli is in the steep negative region of the potential). This causes the anisotropic term to be subdued in the subsequent evolution. Thus, as the Universe approaches classical big bang/crunch singularities in this model one expects anisotropies to become very small [39].

III. EFFECTIVE DYNAMICS

In the loop quantization the elementary variables are the holonomies of the connection and the flux of the triad (related by a constant for the present case). Elements of the holonomies are of the form $\exp(i\mu c_i)$ where μ labels the edge length along which a holonomy is computed. The classical Hamiltonian constraint is expressed in terms of holonomies and triads and then quantized. The resulting quantum constraint is generically nonsingular and the picture at the Planck scale in Bianchi-I model turns out to be similar to the one in the isotropic model [24]. One can also derive the effective Hamiltonian using similar techniques as used for the isotropic model [31] and for $N = 1$ is given by[4]

$$\mathcal{H}_{\rm eff} = -\frac{1}{8\pi G\gamma^2 V}\left(\frac{\sin(\bar{\mu}_1 c_1)}{\bar{\mu}_1}\frac{\sin(\bar{\mu}_2 c_2)}{\bar{\mu}_2} p_1 p_2 + \text{cyclic terms}\right) + \mathcal{H}_{\rm matt}, \tag{20}$$

where

$$\bar{\mu}_1 = \lambda\sqrt{\frac{p_1}{p_2 p_3}}, \qquad \bar{\mu}_2 = \lambda\sqrt{\frac{p_2}{p_1 p_3}}, \quad \text{and} \quad \bar{\mu}_3 = \lambda\sqrt{\frac{p_3}{p_1 p_2}}. \tag{21}$$

Here Δ arises due to regularization of the field strength of the connection by the underlying quantum geometry. Its value is given by [40]

$$\lambda^2 = 4\sqrt{3}\pi\gamma\ell_{\rm Pl}^2. \tag{22}$$

It is to be noted that the effective dynamics resulting from the above Hamiltonian is invariant under the choice of the fiducial cell. In particular this is the only known loop quantization of Bianchi-I spacetime which is independent of the shape of the fiducial cell [30]. Comparing (20) and (4) we notice that the change from classical to effective theory only comes in the form of replacement of connection components c_I with $\sin(\bar{\mu}_I c_I)/\bar{\mu}_I$. Under the transformation: $(l_1, l_2, l_3) \to (l_1', l_2', l_3')$, $\bar{\mu}_I \to \bar{\mu}_I' = (1/l_I')\bar{\mu}_I$. Hence $\sin(\bar{\mu}_I c_I)/\bar{\mu}_I$ transforms in the same way as c_I. Thus, transformation properties remain same as in the classical theory and the resultant dynamics and physical predictions are unaffected by the freedom of the choice of the fiducial cell.[5]

Substituting (21) in (20) and solving for the Hamiltonian constraint $\mathcal{H}_{\rm eff} \approx 0$ we obtain

$$\rho = \frac{1}{8\pi G\gamma^2\lambda^2}(\sin(\bar{\mu}_1 c_1)\sin(\bar{\mu}_2 c_2) + \text{cyclic terms}). \tag{23}$$

Since $\sin(\bar{\mu}_i c_i)$ are bounded functions, the above equation implies that the energy density can never diverge in the effective loop quantum cosmology. The maximum value of the terms in the parenthesis determines the upper bound for the energy density:

$$\rho \leq \rho_{\rm max}, \qquad \rho_{\rm max} = \frac{3}{8\pi G\gamma^2\lambda^2}. \tag{24}$$

The upper bound for the energy density in the Bianchi-I anisotropic model coincides with the value in the isotropic model [6].

It should be noted that the matter Hamiltonian $\mathcal{H}_{\rm matt}$ in (20) is unmodified from its classical expression, due to which the dynamical equations for matter remain the same as Eqs. (8) and (9). Modified dynamical equations can be obtained from the Hamilton equations using (20). As an example, for the triad component p_1 we get

$$\dot{p}_1 = -8\pi G\frac{\partial\mathcal{H}_{\rm eff}}{\partial c_1} = \frac{p_1}{\gamma\lambda}\cos(\bar{\mu}_1 c_1)(\sin(\bar{\mu}_2 c_2) + \sin(\bar{\mu}_2 c_2)), \tag{25}$$

(and similarly for p_2 and p_3).

In order to find the equation for directional Hubble rates we first note that from (3) we get

$$a_1 = \frac{1}{l_1}\left(\frac{p_2 p_3}{p_1}\right)^{1/2}, \quad a_2 = \frac{1}{l_2}\left(\frac{p_3 p_1}{p_2}\right)^{1/2}, \quad \text{and} \quad a_3 = \frac{1}{l_3}\left(\frac{p_1 p_2}{p_3}\right)^{1/2}. \tag{26}$$

Taking their derivatives and using Eq. (25) and corresponding equations for p_2 and p_3 we obtain

$$H_1 = \frac{\dot{a}_1}{a_1} = \frac{1}{2\gamma\lambda}(\sin(\bar{\mu}_1 c_1 - \bar{\mu}_2 c_2) + \sin(\bar{\mu}_1 c_1 - \bar{\mu}_3 c_3) + \sin(\bar{\mu}_2 c_2 + \bar{\mu}_3 c_3)). \tag{27}$$

Similar equations can be derived for the directional Hubble rates H_2 and H_3. It is clear from the above equation, unlike in the classical theory the directional Hubble rates in effective loop quantum dynamics are always bounded.

The Hamilton equation for connection components can be derived in a similar way. As an example, for c_1 one obtains

[4]In order to compare with earlier works (for e.g. equations in Refs. [30,38]), in our convention $\bar{\mu}_i$ correspond to often used $\bar{\mu}_i'$.

[5]This is in contrast to other proposals for quantization of Bianchi-I spacetimes such as in Ref. [41] and those motivated by lattice refinement considerations [42]. As emphasized in Ref. [30], since the effective dynamics in these models is not invariant under the change in shape of the fiducial cell, they lack predictive power. It turns out that in the effective dynamics of these proposals the expansion factor and shear are also unbounded, even if one fixes the fiducial cell.

$$\dot{c}_1 = 8\pi G\gamma \frac{\partial H_{\rm eff}}{\partial c_1}$$
$$= \frac{1}{2p_1\gamma\lambda}[c_2p_2\cos(\bar{\mu}_2c_2)(\sin(\bar{\mu}_1c_1) + \sin(\bar{\mu}_3c_3)) + c_3p_3\cos(\bar{\mu}_3c_3)(\sin(\bar{\mu}_1c_1) + \sin(\bar{\mu}_2c_2))$$
$$- c_1p_1\cos(\bar{\mu}_1c_1)(\sin(\bar{\mu}_2c_2) + \sin(\bar{\mu}_3c_3)) - \bar{\mu}_1p_2p_3[\sin(\bar{\mu}_2c_2)\sin(\bar{\mu}_3c_3) + \sin(\bar{\mu}_1c_1)\sin(\bar{\mu}_2c_2)$$
$$+ \sin(\bar{\mu}_3c_3)\sin(\bar{\mu}_1c_1)]] + 8\pi G\gamma\sqrt{\frac{p_2p_3}{p_1}}\left(\frac{\rho}{2} + p_1\frac{\partial\rho}{\partial p_1}\right). \quad (28)$$

Using (25) with (28) and similar equations for other connection and triad components, it can be shown that matter with vanishing stress satisfies

$$\frac{d}{dt}(p_ic_i - p_jc_j) = 0. \quad (29)$$

Thus, as in the classical theory $(p_ic_i - p_jc_j)$ is a constant of motion in the effective loop quantum dynamics. However, due to Eq. (27), unlike in the classical theory the relation (7) is no longer satisfied. A consequence is that in the effective dynamics

$$p_ic_i - p_jc_j \neq V(H_i - H_j). \quad (30)$$

In the classical approximation $\bar{\mu}_ic_i \ll 1$, Eq. (27) leads to $c_i \approx \gamma l_i\dot{a}_i$ which implies $p_ic_i - p_jc_j \approx V(H_i - H_j)$ as we approach the classical scales.

We can now evaluate the shear scalar σ^2 in the effective description of loop quantum cosmology. Since $(H_i - H_j)$ are no longer constant except at the classical scales, we note from Eq. (16) that σ^2 is not a constant in the effective loop quantum cosmology . Using the Hamilton equations for p_i, a straightforward calculation leads to

$$\sigma^2 = \frac{1}{3\gamma^2\lambda^2}[(\cos(\bar{\mu}_3c_3)(\sin(\bar{\mu}_1c_1) + \sin(\bar{\mu}_2c_2)) - \cos(\bar{\mu}_2c_2)(\sin(\bar{\mu}_1c_1) + \sin(\bar{\mu}_3c_3)))^2 + (\cos(\bar{\mu}_3c_3)(\sin(\bar{\mu}_1c_1)$$
$$+ \sin(\bar{\mu}_2c_2)) - \cos(\bar{\mu}_1c_1)(\sin(\bar{\mu}_2c_2) + \sin(\bar{\mu}_3c_3)))^2 + (\cos(\bar{\mu}_2c_2)(\sin(\bar{\mu}_1c_1) + \sin(\bar{\mu}_3c_3))$$
$$- \cos(\bar{\mu}_1c_1)(\sin(\bar{\mu}_2c_2) + \sin(\bar{\mu}_3c_3)))^2]. \quad (31)$$

This implies that Σ^2 defined via Eq. (19) is not a constant of motion in the effective loop quantum dynamics. Nevertheless, it is clear that σ^2 is bounded above by a fundamental value in loop quantum cosmology. A detailed analysis of the behavior of anisotropy and energy density will be done else where. It is worth pointing out some of the notable features of the effective dynamics. These include:

(i) Unlike the bounce in the isotropic model, due to the presence of anisotropies the energy density at the bounce may be less than $\rho_{\rm max}$. In fact it is possible for the bounce to occur with $\rho = 0$. In this case the bounce occurs purely because of interplay of quantum geometric effects with anisotropy.

(ii) The maximum value of the anisotropic shear at the bounce is

$$\sigma^2_{\rm max} = \frac{4}{3\gamma^2\lambda^2}. \quad (32)$$

(iii) It is also possible for the effective dynamics to saturate Eq. (24) at the bounce. However, in that case anisotropy vanishes at the bounce.

These featuresthe exhibit richness of the effective dynamics in the Bianchi-I model. They result due to incorporation of quantum geometry effects in the effective Bianchi-I spacetime. When the components of the spacetime curvature become large, there are significant departures from the classical theory. When these components are small compared to the Planck scale then dynamical equations approximate their classical counterparts and one recovers the classical description. It is important to note that the upper bounds on the energy density, shear, and Hubble rate are direct consequences of the underlying quantum geometry. In the classical limit $\lambda \rightarrow 0$, $\rho_{\rm max}$, $H_{\rm max}$, and $\sigma_{\rm max}$ diverge and the evolution is singular. The boundedness of these quantities plays the crucial role to construct nonsingular ekpyrotic/cyclic model as we discuss in the following section.

IV. EVOLUTION WITH THE CYCLIC POTENTIAL

In the previous section we analyzed the effective dynamics of the Bianchi-I model in loop quantum cosmology for matter with vanishing anisotropic stress. An important feature of quantum geometry modified dynamics turns out to be singularity resolution. We showed that the energy density, Hubble rates, and shear are always bounded in loop quantum cosmology independent of the equation of state of the matter content. This result is very encouraging for the primary question posed in this work: Is there a viable nonsingular evolution with the ekpryotic/cyclic potential in the effective loop quantum cosmology? We now explore the answer to this question.

Let us first briefly recall some of the salient features of the ekpyrotic/cyclic model which is motivated by the M-theory. It is hypothesized that the observable Universe is constrained on a visible brane which interacts gravitationally with a shadow brane. The collision between these branes constitutes the big bang/big crunch singularities for the observable Universe in the 4-dimensional effective description. The interbrane separation is determined by a moduli field (ϕ) and the interaction potential is given by [8]

$$V = V_o(1 - e^{-\sigma_1\phi})\exp(-e^{-\sigma_2\phi}), \tag{33}$$

where V_o, σ_1, and σ_2 are the parameters of the potential.

An attractive feature of the ekpryotic/cyclic models is the different roles which the moduli plays in various epochs. When the branes approach each other, the moduli field moves slowly in an almost flat and positive part of the potential which in the effective 4-dimensional description leads to a period of dark energy domination. Eventually when the potential energy of the moduli balances the Hubble expansion rate, the moduli field turns around and the interbrane separation starts decreasing leading to a contracting phase in the Universe. When the branes approach each other, the moduli potential is steep and negative leading to an ultrastiff equation of state, i.e. $w > 1$. It turns out that this feature leads to a decrease in the anisotropy of the Universe at small scale factors [39]. The ultrastiff equation of state is also responsible for producing a scale invariant spectrum of fluctuations with a significant nongaussian contribution [43]. The latter serves as a distinct prediction to the inflationary scenarios, and is available for test in the near future. The branes collide at $\phi = -\infty$ which follows their separation and running of the moduli field towards the positive value of potential, leading to a cyclic phenomena. Thus, the 4-dimensional effective dynamics description of the ekpyrotic/cyclic model is very rich and interesting.

A primary issue in the ekpyrotic/cyclic model is to understand the transition from the contracting to the expanding branch. This transition must be nonsingular and is vital to propagate the perturbations generated prior to the collision of the branes to the regime following the collision. In the 5 dimensional picture, the bulk spacetime near the collision of the branes appears as a compactified Milne space and it has been argued that the resulting singularity is milder than the conventional big bang/crunch singularity [44]. Further, in this picture it may be possible to match the perturbations across the transition [13]. This however results in the mixing of the modes with a strong sensitivity of predictions to the process of transition which ignores quantum gravitational effects. Other ideas to understand this transition have been proposed including perturbative string theoretic treatment [45] and AdS/CFT correspondence [14,15]. Though promising directions to explore, these reveal little about the transition from the contracting to the expanding branch in the 4-dimensional effective theory. If one assumes that the 4-dimensional effective theory has no inputs from a theory beyond GR, this difficulty is not surprising. The reason is that in GR such a nonsingular turn-around is forbidden unless matter violates the null energy condition. Though it is not possible to have this violation in the ekpyrotic/cyclic models, a variant of these has been proposed in form of a new ekpyrotic model [46,47] which includes a ghost condensate. The latter is responsible for the violation of a null energy condition and can result in a bounce. Whether such a nonsingular bounce is a generic feature of this model and is not affected by the instabilities is not an open issue. Further, it is not clear whether the 4-dimensional effective picture with a ghost condensate as conjectured in this model is valid at high energies of interest [48].

Given that nonperturbative quantum geometric modifications and their role in singularity resolution is well understood in the homogeneous models of loop quantum cosmology, we now explore the effective 4-dimensional loop quantum dynamics of the moduli field to understand the transition from the contracting to the expanding branch. An underlying assumption of such an analysis is the treatment of the potential (33) as an effective one for the field ϕ in four spacetime dimensions. This enables us to use effective equations derived in the previous section with ϕ as the source of matter. An investigation on these lines was carried out earlier with an additional assumption that the spacetime be isotropic [35]. Its main conclusions were:

(i) The big bang/big crunch singularity is generically avoided irrespective of the choice of the parameters of the potential and initial conditions for the field ϕ.

(ii) However, it is not possible in the isotropic effective loop quantum description for the field ϕ to turn around from $\phi = -\infty$. Thus a cyclic evolution as envisioned in the ekpyrotic/cyclic model paradigm was not possible in the isotropic loop quantum cosmology.[6]

It should be noted that the absence of the turn-around of ϕ from large negative values is not a limitation of the loop quantum dynamics, but is a feature shared also by the classical theory. This can be easily seen from the classical Friedmann equation for the isotropic model. For simplicity

[6]If the interbrane potential is modified such that it is positive for large negative values of ϕ, then the turn-around of the field and a cyclic description is possible [35]. A modification to the cyclic potential as proposed in the bicyclic scenario [36] would hence result in a desired nonsingular evolution.

let us assume that the only contribution to the matter energy density originates from the moduli field

$$H^2 = \frac{8\pi G}{3}\rho_\phi, \tag{34}$$

where

$$\rho_\phi = \tfrac{1}{2}\dot{\phi}^2 + V(\phi). \tag{35}$$

For the moduli field to turn around it is essential that $\dot{\phi}$ must vanish. Using (35) in (34) we get

$$\dot{\phi}^2 = 2\left(\frac{3}{8\pi G}H^2 - V(\phi)\right). \tag{36}$$

It is clear that a turn-around of the field is not possible when the potential is negative. Since in the ekpyrotic/cyclic model the potential is negative (and asymptotes to zero) when the branes are supposed to collide, $\dot{\phi}$ can not vanish and the moduli can not turn around. The presence of additional energy density components such as radiation is unlikely to affect this conclusion. The reason being that in the vicinity of brane collision ($a \to 0$), the dominant contribution to the energy density comes from the kinetic energy of the moduli.

Interestingly, the above situation changes if the spacetime has a nonvanishing anisotropic shear. In this case, the classical generalized Friedmann equation yields

$$\dot{\phi}^2 = 2\left(\frac{3}{8\pi G}\left(H^2 - \frac{\Sigma^2}{a^6}\right) - V(\phi)\right). \tag{37}$$

The shear term can be seen as an additional component of energy density which scales the same way as ρ_ϕ. Thus depending on the strength of the shear term, it is possible that ϕ can turn around even when $V(\phi) < 0$. However, the classical theory is singular and the above equation breaks down when spacetime curvature becomes very large. Hence, though we get some insights on the possible role of anisotropic shear to alleviate the above problem, it is not possible to obtain a nonsingular cyclic evolution without any additional inputs from quantum gravity or going beyond the scope of the classical theory.

Given the nontrivial role the shear term can play in the dynamics of the moduli field in this model, it becomes important to analyze the effective loop quantum equations with anisotropies. For simplicity we assume that the only contribution to the matter density originates from the field ϕ. By specifying the $\mathcal{H}_{\rm matt}$ corresponding to the cyclic potential [8] we can derive the dynamics from the Hamilton equations for c_i, p_i, the moduli ϕ, and its momentum p_ϕ as obtained in Sec. III. These equations can be numerically integrated to obtain the behavior of directional scale factors, the mean scale factor, Hubble rates, energy density, and the shear [which is calculated from its definition (16)].

We performed various numerical simulations with different initial data for the moduli field, expansion rates, and anisotropy. The initial conditions to solve the dynamics were provided for a contracting universe with small initial anisotropy and the value of the field ϕ in the positive part of the cyclic model potential. Since the primary aim of our analysis is to investigate the resolution of a singularity, we choose without the loss of generality, parameters and initial conditions independent of the consideration from the ones constrained by observations. We now illustrate some numerical results obtained from the effective dynamics.

The first set of the evolution is depicted in Figs. 1–4. Parameters in the potential (33) were chosen as $V_o = 0.02$, $\sigma_1 = 0.3\sqrt{8\pi}$, and $\sigma_2 = 0.09\sqrt{8\pi}$. (For simplicity we choose $G = \hbar = c = 1$.) Initial conditions for the moduli field were $\phi = 0.43$ and $\dot{\phi} = -0.038$. Initial conditions (provided at time $t = 0$) for the triad and connection components were $p_1 = 64$, $p_2 = 72$, $p_3 = 68$, $c_1 = -0.8$, $c_2 = -0.7$ with the initial anisotropy $\Sigma_i^2 = 5.804\,07$. (The connection component c_3 was determined by solving the Hamiltonian constraint $\mathcal{H}_{\rm eff} \approx 0$.) These initial conditions correspond to a contraction of all the scale factors, i.e. $\dot{a}_1 < 0$, $\dot{a}_2 < 0$, and $\dot{a}_3 < 0$. Figure 1 shows the evolution of the mean scale factor $a = (a_1 a_2 a_3)^{1/3}$. We find that the mean scale factor undergoes multiple bounces and recollapses in the Planck regime and there is a nonsingular evolution from the contracting to the expanding branch. This is a sharp contrast to the classical theory where such a transition is forbidden and the contracting and expanding branches are disjointed in the 4-dimensional description. Interestingl, as depicted in the Fig. 2 we also obtain a turn-around of the moduli field. The moduli starts from the positive part of the potential

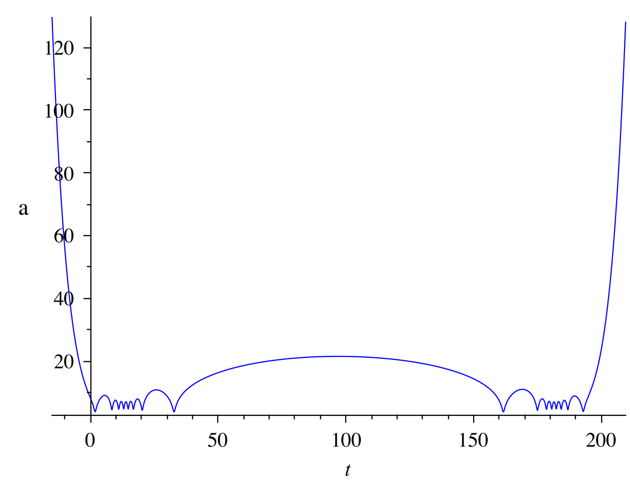

FIG. 1 (color online). The plot of the mean scale factor (in Planck units), $a = (a_1 a_2 a_3)^{1/3}$ is shown for the initial conditions (all values in Planck units) $\phi = 0.43$, $\dot{\phi} = -0.038$, $p_1 = 64$, $p_2 = 72$, $p_3 = 68$, $c_1 = -0.8$, $c_2 = -0.7$, and $\Sigma^2 = 5.804\,07$. The mean scale factor experiences multiple small bounces as a result of bounces of the individual scale factors (a_i). Unlike the classical theory, there is a nonsingular transition from the contracting to the expanding branch.

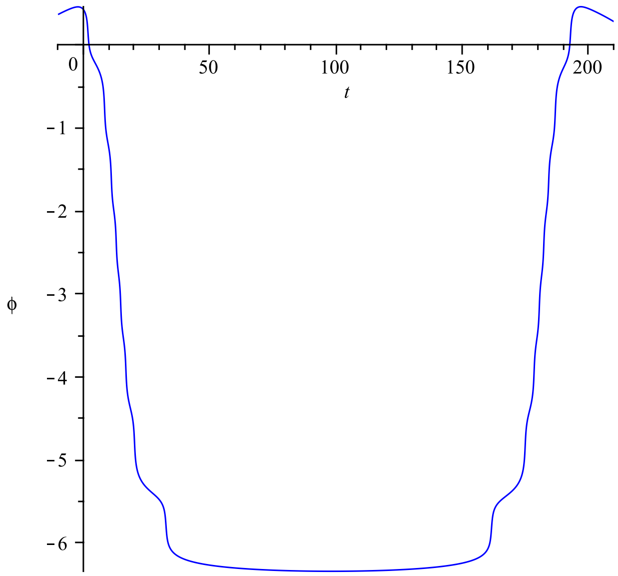

FIG. 2 (color online). The evolution of the moduli field in Planck units is shown for the initial conditions in Fig. 1. The moduli field starts from a positive region of the potential, rolls to the negative part, and turns around in the Planck regime. After the turn-around the moduli again reaches the positive part of the potential, ready for another cycle.

when the Universe is contracting, rolls down the negative region of potential and when the spacetime curvature reaches the Planck value, it turns around and goes back to the positive part. In the subsequent evolution the classical friction term stops the moduli field and it turns around causing a contraction of the Universe. Thus leading to a cyclic phenomena. In the above numerical simulation, the turn-around of the moduli field occurs at approximately $t \sim 100$ (in Planck units) coinciding with the midpoint of the transition from contraction to expansion of the scale factor.

Figure 3 shows the evolution of the shear term obtained by the numerical integration. We find that the shear term varies significantly from its classical value in the region where the transition occurs. However, it is everywhere bounded and does not affect the occurrence of bounces. As discussed earlier, due to the property of evolution of the shear in loop quantum cosmology, the value of the shear scalar in the low curvature regime before and after the transition turns out to be the same. Also for this numerical run, the shear scalar is very small when the moduli field turns around. We also find that the behavior of Σ^2 consists of spikes in the Planck regime where the mean scale factor of the Universe undergoes multiple bounces. Using the expression of the shear factor [obtained from (16) and (19) for loop quantum evolution], the cause of these spikes turns out to be due to rapid variation in the Hubble rates in different directions occurring during multiple bounces. This is illustrated in Fig. 4 where we have shown the behavior of the directional Hubble rate in the x direction (The directional Hubble rates in other directions have similar behavior.) As can be seen the spikes in the shear scalar coincide with the spikes in the directional Hubble rates which occur due to multiple bounces and recollapses of anisotropic scale factors in the Planck regime.

Another example of results obtained from numerical integration are shown in Figs. 5–8. The parameters of the potential (33) were chosen as $V_o = 0.01$, $\sigma_1 = 0.3\sqrt{8\pi}$, and $\sigma_2 = 0.09\sqrt{8\pi}$. The initial conditions specified at time $t = 0$ were $\phi = 0.4$, $\dot{\phi} = -0.03$, $p_1 = 64$, $p_2 = 72$, $p_3 = 68$, $c_1 = -0.6$, $c_2 = -0.5$, and $\Sigma^2 = 9.2365$. These initial conditions correspond to two of the anisotropic directions contracting ($\dot{a}_1 < 0$, $\dot{a}_2 < 0$) and one expanding ($\dot{a}_3 > 0$). To understand the nontrivial role played by the anisotropic

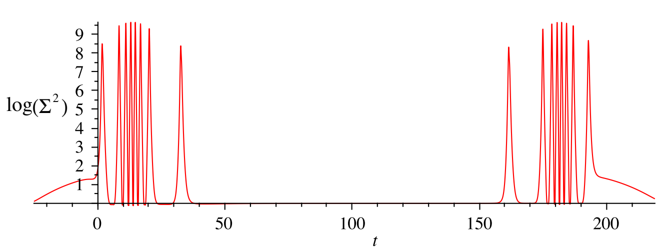

FIG. 3 (color online). The plot shows the variation of the shear Σ^2 in the transition region from the contracting to the expanding branch for the initial conditions in Fig. 1. Considerable variation for a short period before and after the bounce is evident from the spikes in value of $\log(\Sigma^2)$. We also see that in the classical regimes at both small and large values of t, the shear approaches similar values.

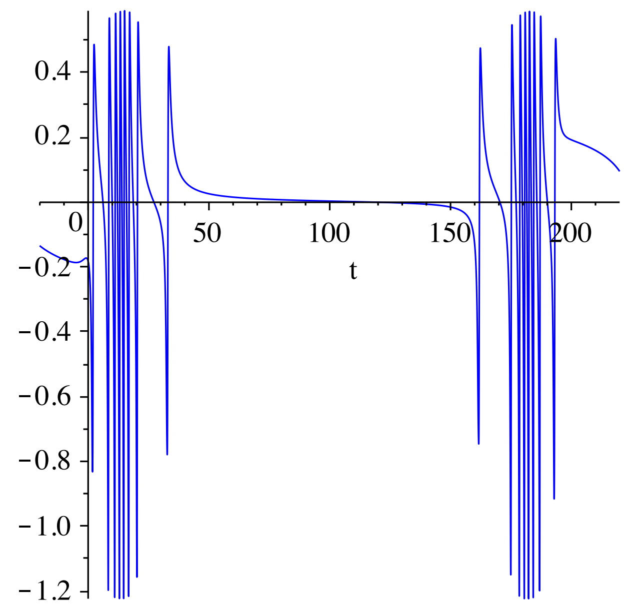

FIG. 4 (color online). The variation in the directional Hubble rate H_1 is plotted for the initial data in Fig. 1. The spikes reflect the bounces and recollapses of individual scale factors in the Planck regime during the transition from the contracting to the expanding branch.

term we also performed an analysis of the evolution in the isotropic loop quantum model for the same initial conditions for the moduli field and the isotropized initial conditions for triad and connection. In Fig. 5 we have shown the evolution of the scale factor in the present model (solid curve) and the isotropic model (dashed curve). We find that the bounce of the scale factor is present both in the presence and absence of anisotropy. The loop quantum evolution with the cyclic potential is thus nonsingular with or without anisotropy. Figure 6 shows the evolution of the moduli field and we find that as for the previous case (Fig. 2) there is a turn-around of the moduli field from the negative values of the potential to the positive part resulting in a cyclic phenomena. As before, the turn-around of the field occurs at the midpoint of the transition of the mean scale factor from the contracting to the expanding branch. Figure 6 also shows that for the same initial conditions the turn-around of the moduli is absent for the isotropic model. Thus, anisotropies play a very important role to obtain a nonsingular viable cyclic description.

The evolution of the shear and energy density are depicted in Figs. 7 and 8. We see that the shear term undergoes a significant variation in the region of nonsingular transition but remains bounded. The spikes in the variation correspond to the spikes in the directional Hubble rates which occur due to multiple bounces and recollapses of the scale factors (a_1, a_2, a_3). Further, its value before and after the transition is preserved. Unlike the previous case in Fig. 2, the shear scalar is not small at the turn-around of the moduli field but shows a spike. The behavior of the energy density is shown in Fig. 7 which demonstrates that at the bounces of the directional scale factors (associated with the occurrence of spikes) the energy density does not saturate to its maximum value, a feature due to the non-vanishing anisotropy at the bounces. In contrast, the energy density at the bounce in the isotropic model always reaches its maximum value, $\rho_{\rm crit} = 0.41\rho_{\rm Pl}$ at the bounce.

In both of the previous simulations, though the initial anisotropic shear is small it is nevertheless of the same order. An interesting question which arises is whether the nonsingular transition of the scale factor and the turn-around of the moduli field occurs if the shear is decreased. It is to be noted that as shown in Ref. [30] the loop quantization of Bianchi-I model leads to an upper bound on the expansion rate and anisotropic shear. These bounds are generic and independent of initial conditions for the matter content. The upper bound on the expansion rate implies that the mean scale factor would always bounce for the Cyclic model potential as the classical singularity is

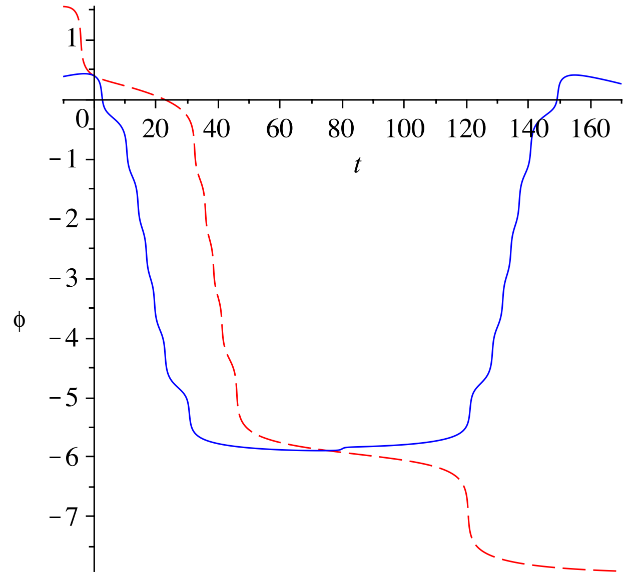

FIG. 6 (color online). The evolution of the moduli field is compared for the initial conditions in Fig. 5 for the anisotropic (blue solid curve) and isotropic (red dashed curve) model. As in Fig. 2 the moduli field turns around in the presence of anisotropies in the Planck regime and leads to a nonsingular cyclic model. It fails to turn around, and approaches $\phi = -\infty$, in the absence of anisotropies in confirmation with results of Ref. [35].

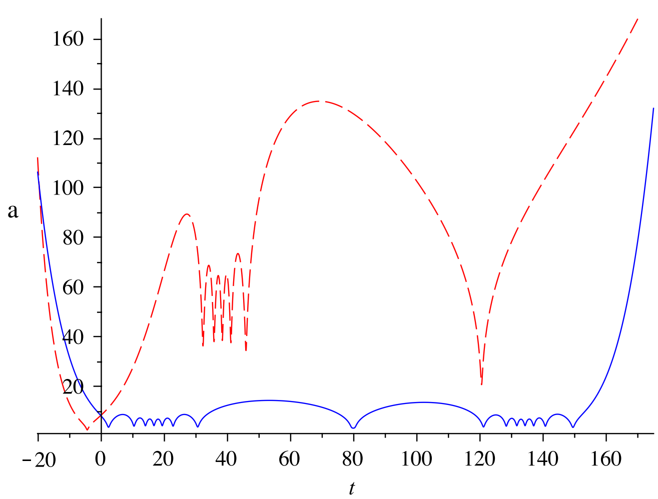

FIG. 5 (color online). The plot shows the evolution of the scale factor in the presence (solid curve) and absence (dashed curve) of anisotropy for initial conditions $\phi = 0.4$, $\dot{\phi} = -0.03$, $p_1 = 64$, $p_2 = 72$, $p_3 = 68$, $c_1 = -0.6$, $c_2 = -0.5$, and $\Sigma^2 = 9.2365$. The classical singularity is avoided in both the cases.

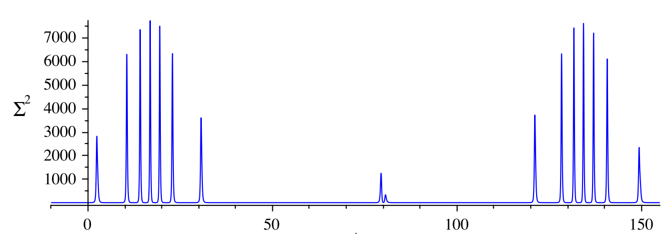

FIG. 7 (color online). The shear scalar Σ is plotted for the numerical run in Fig. 5. We see that its value is preserved before and after the transition period. The shear term is bounded across the evolution with spikes corresponding to multiple bounces and recollapses of the individual scale factors.

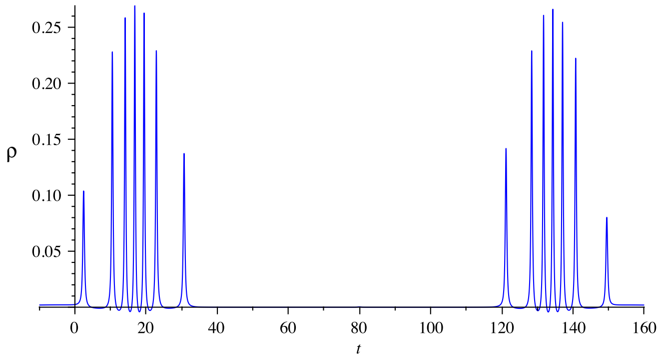

FIG. 8 (color online). The plot of the energy density of the moduli field in the transition regime is shown. Bounces of the individual scale factors result in spikes in its behavior. Because of the presence of nonvanishing anisotropy, the energy density at these bounces does not need to saturate to its maximum allowed value.

approached. Further, from the discussion of the classical theory we recall that the turn-around of moduli field would require appropriate choice of initial conditions of the scalar field and anisotropy. In various simulations which we performed, we found that there exists a large range of initial data for which the turn-around of moduli field can occur even for very small anisotropies. Results from one such example are depicted in Figs. 9 and 10. We chose the same values of parameters as in previous simulations. The initial values are $p_1 = 64$, $p_2 = 72$, $p_3 = 68$, $c_1 = -0.6$, $c_2 = -0.532$, and $\Sigma^2 = 0.8517$. As can be seen from these plots a significant decrease in the initial anisotropy does not affect the bounce of the scale factor or the turn-around of the moduli field.

It is interesting to note that in these simulations the behavior of the mean scale factor and the scalar field seems symmetric across the middle of the transition from the contracting to the expanding branch. To understand this we first note that the weak symmetry of the variation of the scalar field in the transition regime stems from a lack of interaction with any other form of matter. We expect that the presence of additional matter of degrees of freedom would lead to a pronounced asymmetry. Further, we note that in the plots we depict the mean scale factor which actually suppresses the asymmetry of the individual scale factors across the bounce. This becomes clear if we plot the anisotropic scale factors as shown in Fig. 10 for the numerical simulation for Fig. 9. In Fig. 11 we show results from another numerical simulation where the asymmetry in the mean scale factor is not suppressed. The parameters are chosen as the same in previous cases and the initial conditions are $p_1 = 64$, $p_2 = 72$, $p_3 = 68$, $c_1 = -0.6$, $c_2 = -0.1$, and $\Sigma^2 = 1449.4278$. As we can see the be-

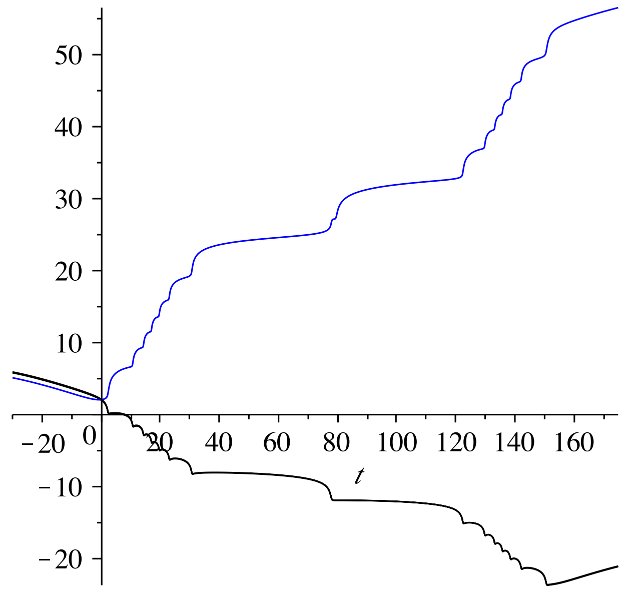

FIG. 10 (color online). This plot shows the evolution of different scale factors for the simulation corresponding to Fig. 9. We have plotted the logarithm of directional scale factors. The darker curve depicts a_1 and lighter curve a_3. The behavior of a_2 (not shown in the figure) is very similar to that of a_1.

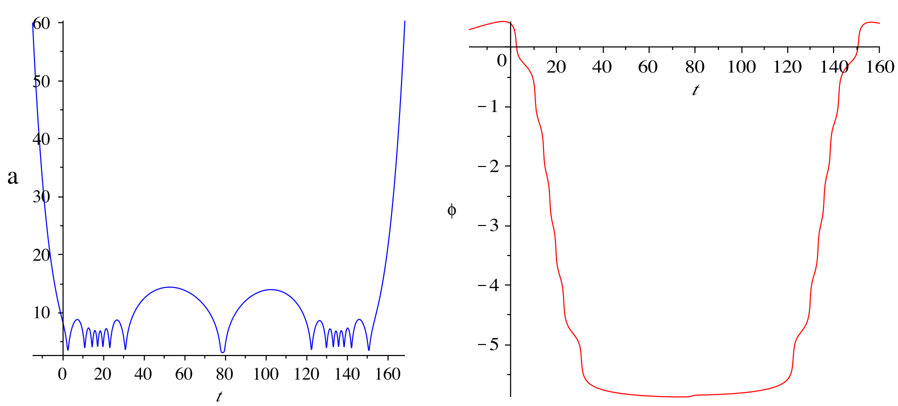

FIG. 9 (color online). The plots of the mean scale factor a and the scalar field ϕ are shown for the initial conditions with very small anisotropies. These are $\phi = 0.4$, $\dot{\phi} = -0.03$, $p_1 = 64$, $p_2 = 72$, $p_3 = 68$, $c_1 = -0.6$, $c_2 = -0.532$, and $\Sigma^2 = 0.8517$.

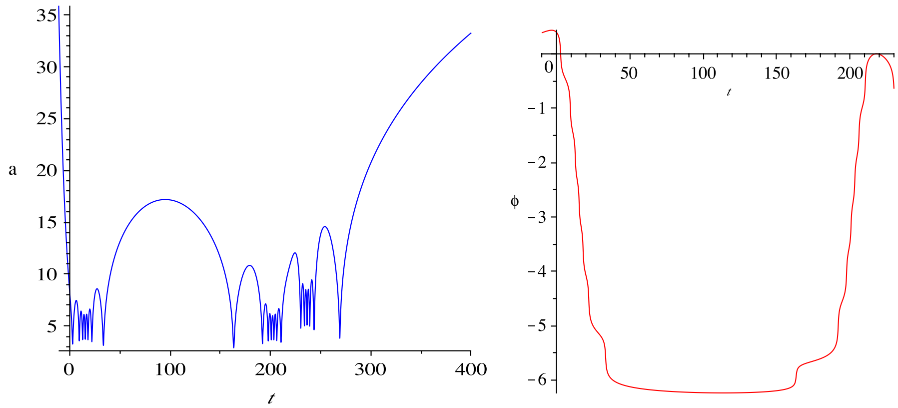

FIG. 11 (color online). These plots depict the evolution of the scale factor and scalar field for the initial conditions: $\phi = 0.4$, $\dot{\phi} = -0.03$, $p_1 = 64$, $p_2 = 72$, $p_3 = 68$, $c_1 = -0.6$, $c_2 = -0.1$, and $\Sigma^2 = 1449.4278$. Unlike the previous cases, we find that the asymmetry in the mean scale factor and the moduli field around the bounce point is enhanced. Note that the moduli field reaches the positive value for $t \sim 220$ before starting a new cycle.

havior of the scale factor and the scalar field across the bounce is not symmetric.

We summarize the main results of the numerical analysis as follows:

(1) Irrespective of the choice of initial conditions, the classical singularity at $a = 0$ is generically avoided in the effective dynamics of loop quantum cosmology for the cyclic model potential.

(2) Starting from arbitrary anisotropic conditions, scale factors in different directions bounce when the spacetime curvature becomes close to the Planck value. This causes a bounce of the mean scale factor in the Planck regime. The shear term remains bounded during the evolution and it approaches the constant classical value when spacetime curvature becomes small.

(3) Interestingly, it is not difficult to choose the initial conditions such that avoidance of the singularity is accompanied by a turn-around of the moduli field in the negative regime of the potential. The moduli then rushes towards the positive part of the potential, stops at certain positive value of the potential, and rolls back toward the negative value of the potential. This leads to a cyclic model of the Universe.

V. SUMMARY AND OPEN ISSUES

The ekpyrotic/cyclic model is a very interesting paradigm for the early Universe which is considered as an alternative to the inflationary scenarios. A key issue in this model is to understand the transition from the contracting to the expanding branch in the 4-dimensional spacetime picture. In the 5-dimensional picture this transition corresponds to the collision between boundary branes in the bulk. Though novel insights have been gained in the latter phenomenon [14,15,44,45], obtaining a nonsingular transition in the 4-dimensional picture has remained an open issue. Understanding which is important for various reasons including the way cosmological perturbations propagate from the contracting to the expanding branch. Given the Penrose-Hawking singularity theorems, a nonsingular transition is not possible in the framework of classical theory. There are hopes to alleviate this problem in the classical framework by the introduction of a ghost condensate, however, the approach has its own limitations [46,48].

Our analysis is based on the widely accepted notion that the resolution of singularities involves going beyond the classical description of gravity. In particular, inputs from the nonperturbative quantization of gravity may be necessary to avoid singularities. In our approach we have used the effective spacetime description of loop quantum cosmology to analyze the dynamics of the ekpyrotic/cyclic model. It is a nonperturbative quantization of homogeneous spacetimes based on loop quantum gravity and it has successfully addressed the resolution of cosmological singularities in various settings [4,6,19,23,24] with a general picture of the replacement of big bang with big bounce at the Planck scale. In this work we have focussed on the resolution of the big bang/crunch singularity in the ekpyrotic/cyclic model. Such an investigation has been performed earlier [35][7] using the assumption that the spacetime be purely isotropic. It was found that though the 4-dimensional scale factor generically bounces in the Planck regime a viable cyclic model is not possible due to lack of a turn-around of the moduli field from the negative region of the cyclic model potential.

To probe whether the conclusions reached in the previous work [35] were artifacts of ignoring the anisotropic shear, we include them in the analysis and investigate the dynamics with the effective description of loop quantiza-

[7]See also Ref. [49] for an earlier work which ignored modifications to the gravitational part of the Hamiltonian.

tion of the Bianchi-I spacetime. Our analysis assumes vanishing intrinsic curvature (whose consequences are discussed below) and the moduli field as the only source of the matter energy density. The effective dynamical equations are complicated to solve analytically due to which various numerical simulations were performed with different choices of the parameter of the cyclic model potential and initial conditions. We find that the Universe undergoes a nonsingular transition from the contracting to the expanding branch accompanied by small multiple bounces for individual scale factors in the Planck regime. In contrast to the previous results, we find that it is possible to easily choose initial conditions such that the moduli field turns around from the negative part of the potential in the Planck regime. Thus leading to a potentially viable nonsingular ekpyrotic/cyclic model without introduction of any exotic matter. This is the novel result of our analysis.

It should be noted that the anisotropies play a very nontrivial role to obtain a nonsingular cyclic model in our analysis. At first sight it seems perplexing because in the ekpryosis phase, which occurs when the moduli field is in the steep negative region of the potential, anisotropies become small and the Universe evolves towards an isotropic phase. However as we demonstrated, even though anisotropies may become very small during the evolution, their nonzero value is important to turn around the moduli field in the negative part of the potential. Recall that such a turn-around is not possible even for the classical theory in the absence of anisotropies. Hence the loop quantum evolution successfully leads to a turn-around of the scale factor and the moduli without affecting the nice features of the ekpyrotic/cyclic model.

There remain several open questions which require further investigations. One of them deals with generalizing our model to include the intrinsic curvature of the spacetime which will require studying loop quantization of spacetimes such as Bianchi-IX. It will also help in obtaining insights on the Belinski-Khalatnikov-Lifshitz behavior in loop quantum cosmology and whether it affects the conclusion reached in this work. A second issue is to gain more analytical control on the effective dynamics of the anisotropic loop quantum cosmology. Because of its complexity it is difficult to obtain an analog of generalized Friedmann equations with the shear term in loop quantum cosmology, however, various insights have been obtained on general features of the effective dynamics. These include, showing that the expansion parameter and the shear term are universally bounded [30]. Further analytical studies in this direction on the ekpyrotic/cyclic model will be reported elsewhere. It will also be useful to understand the full loop quantum dynamics with the cyclic potential and compare them with the effective dynamics treatment as done presently. Also in order to have a more realistic ekpyrotic/cyclic model, it is important to include radiation, and study its interaction with the moduli field and its influence on the nonsingular transition obtained in this work. Finally, it is an open problem to introduce inhomogeneities in our framework and understand the role of quantum gravitational effects on their propagation through the bounce. Recent results on the study of Fock quantized inhomogeneous modes on the loop quantum spacetime are encouraging in this respect [50]. It is quite possible that incorporation of these inhomogeneities may reveal subtle imprints of quantum gravity on the predictions of the ekpyrotic/cyclic model.

ACKNOWLEDGMENTS

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