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Neutrino oscillations, Lorentz/*CPT* violation, and dark energy

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If dark energy (DE) couples to neutrinos, then there may be apparent violations of Lorentz/*CPT* invariance in neutrino oscillations. The DE-induced Lorentz/*CPT* violation takes a specific form that introduces neutrino oscillations that are energy independent, differ for particles and antiparticles, and can lead to novel effects for neutrinos propagating through matter. We show that ultra-high-energy neutrinos may provide one avenue to seek this type of Lorentz/*CPT* violation in ν_μ-ν_τ oscillations, improving the current sensitivity to such effects by 7 orders of magnitude. Lorentz/*CPT* violation in electron-neutrino oscillations may be probed with the zenith-angle dependence for high-energy atmospheric neutrinos. More compelling evidence for a DE-neutrino coupling would be provided by a dependence of neutrino oscillations on the direction of the neutrino momentum relative to our peculiar velocity with respect to the cosmic microwave background rest frame. While the amplitude of this directional dependence is expected to be small, it may nevertheless be worth seeking in current data and may be a target for future neutrino experiments.

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I. INTRODUCTION

The accelerated cosmic expansion [1] poses difficult questions for theoretical physics [2–4]. Is it simply due to a cosmological constant? Is some new negative-pressure dark energy (DE) required? Is general relativity modified at large distance scales? The major thrust of the empirical assault on these questions has been to determine whether the expansion history and growth of large-scale structure are consistent with a cosmological constant or require something more exotic [5].

However, it may be profitable to explore whether there are other experimental consequences of the new physics—which we collectively refer to as DE, although it may involve a modification of gravity rather than the introduction of some new substance—responsible for accelerated expansion. If cosmic acceleration is due to a cosmological constant (i.e., if general relativity is valid and the equation-of-state parameter is $w = -1$), then the vacuum is Lorentz invariant. If, however, something else is going on, then the "vacuum" has a preferred frame: the rest frame of the cosmic microwave background (CMB). If, moreover, dark energy couples somehow to standard-model particles, then there may be testable (apparent) violations of Lorentz invariance. For example, if DE is coupled to the pseudoscalar $F\tilde{F}$ of electromagnetism [6], there may be a "cosmological birefringence" that rotates the linear polarization of cosmological photons; CMB searches for such a rotation [7] constrain this rotation to be less than a few degrees [8].

Here, we explore DE-induced Lorentz/*CPT*-violating effects in the neutrino sector. We show that the form of a Lorentz-violating coupling between neutrinos and dark energy is highly restricted under fairly general assumptions.[1] The coupling engenders an additional source for neutrino mixing (e.g., Ref. [10]), resulting in neutrino oscillations with a different energy dependence than vacuum oscillations and different oscillation probabilities for neutrinos and antineutrinos. While similar Lorentz/*CPT*-violating oscillations have been considered before [11–13], we emphasize here that cosmic acceleration dictates a specific form for such effects.

Data from Super-Kamiokande and K2K [14] and AMANDA/IceCube [15] already tightly constrain *CPT*-violating parameters for ν_μ-ν_τ mixing, and those from solar-neutrino experiments and KamLAND [16] do so for ν_e-ν_μ mixing. However, the effects of DE-induced *CPT* violation become more significant at higher energies [17]. Here, we show that next-generation measurements of ultra-high-energy neutrinos produced by spallation of ultra-high-energy cosmic rays will increase the sensitivity to *CPT*-violating ν_μ-ν_τ oscillations by 7 orders of magnitude. We also show that these *CPT*-violating couplings may lead to novel effects in the zenith-angle dependence for atmospheric neutrinos in the $\sim$100 GeV range.

While such *CPT*-violating effects, if detected, could be attributed simply to intrinsic *CPT* violation in fundamental physics, not related to DE, a DE-neutrino coupling further predicts a directional effect: the neutrino-mixing parameters depend on the neutrino-propagation direction relative to our peculiar velocity with respect to the CMB rest frame.

[1]The coupling of neutrinos to dark energy has also been considered in the context of "mass-varying neutrinos" [9], but that implementation of the DE-neutrino coupling does not lead to the type of Lorentz/*CPT*-violating effects we discuss here.

While this signature will likely remain elusive even to next-generation experiments, it would, if detected, be strong support for DE beyond a cosmological constant. It is therefore worth considering as a long-range target for future neutrino experiments. It may also be worthwhile to search current data in case an implementation of DE-neutrino coupling different from that we consider here leads to a different energy dependence for these directional effects. We therefore work out explicitly the directional dependence to aid experimentalists who may wish to look for such correlations in current data.

Below, we first derive in Sec. II the form of the Lorentz/CPT violation allowed by a DE-neutrino coupling and discuss the resulting neutrino-oscillation physics. In Sec. III we apply the formalism to cosmogenic ultra-high-energy neutrinos, and obtain projected sensitivities of future detectors to these effects in ν_μ-ν_τ oscillations. In Sec. IV we discuss matter-induced effects for ν_e oscillations in high-energy atmospheric neutrinos in the presence of Lorentz-invariance–violating mixings. Concrete formulas for the directional dependence on oscillation probabilities are given in Sec. V. Finally, we discuss some theoretical implications in Sec. VI and summarize and conclude in Sec. VII.

II. THE DARK-ENERGY–NEUTRINO COUPLING

A. General formalism

Following Ref. [13], the neutrino fields are denoted by Dirac spinors $\{\nu_e, \nu_\mu, \nu_\tau, \cdots\}$ and their charge conjugates by $\{\nu_{e^c}, \nu_{\mu^c}, \nu_{\tau^c}, \cdots\}$, where $\nu_{x^c} \equiv \nu_x^c \equiv C\bar{\nu}_x^T$ is the charge-conjugated spinor, and C is the charge-conjugation matrix. The $2N$ fields (where N is the number of flavors) and their conjugates are arranged in a single object ν_A, where A ranges over $e, \mu, \tau, \cdots, e^c, \mu^c, \tau^c, \cdots$.

With a canonical kinetic term in the neutrino Lagrangian, the most general Lorentz/CPT-violating Dirac equation is[2]

$$(i\gamma^\mu \partial_\mu - M_{AB})\nu_B = 0, \tag{1}$$

where

$$M_{AB} \equiv m_{AB} + im_{5AB}\gamma_5 + a^\mu_{AB}\gamma_\mu + b^\mu_{AB}\gamma_5\gamma_\mu + \tfrac{1}{2}H^{\mu\nu}_{AB}\sigma_{\mu\nu}. \tag{2}$$

The usual mass terms are $m + im_5\gamma_5 \equiv m_L P_L + m_R P_R$, where $m_R = (m_L)^\dagger = m + im_5$, $P_L = (1-\gamma_5)/2$, and $P_R = (1+\gamma_5)/2$. The $2N \times 2N$ mass matrix m_R is written in terms of $N \times N$ matrices L, R, and D, through

$$m_R = \begin{pmatrix} L & D \\ D^T & R \end{pmatrix}. \tag{3}$$

Here, R and L are the right- and left-handed Majorana neutrino masses ($L = 0$ is required if electroweak gauge invariance is preserved), and D is the Dirac-mass matrix. The R and L matrices are required to be symmetric, and R, L, and D can most generally be complex.

B. Dark-energy–induced Lorentz violation

Lorentz violation in Eq. (2) is parametrized by the four-vectors a^μ, b^μ, and the antisymmetric tensor $H^{\mu\nu}$. The parameters a^μ and b^μ are both CPT and Lorentz violating, while $H^{\mu\nu}$ is Lorentz violating but CPT conserving. While these parameters are nonzero for the most general Lorentz/CPT-violating Dirac equation [13], the allowable forms for a^μ, b^μ, and $H^{\mu\nu}$ are highly restricted if the Lorentz/CPT violation is induced by coupling to dark energy.

The smallness of the CMB quadrupole demands that the three-dimensional hypersurfaces of constant DE density must be closely aligned with those of constant CMB temperature [18]. The preferred frame associated with the cosmic expansion is then parametrized by a unit four-vector l^μ, which is orthogonal to surfaces of constant CMB temperature; i.e., in the CMB rest frame, it is $l^\mu = (1, 0, 0, 0)$. The symmetry of the problem thus dictates that $a^\mu \propto l^\mu$ and $b^\mu \propto l^\mu$. The tensor $H^{\mu\nu}$ is antisymmetric, and there is no way to construct an antisymmetric tensor $H^{\mu\nu}$ from a single four-vector; we thus expect $H^{\mu\nu} = 0$ for DE-neutrino coupling.

Furthermore, since neutrinos are produced and interact in weak eigenstates, it is only the combination $(a_L)^\mu_{ab} \equiv (a+b)^\mu_{ab}$ that is relevant for neutrino phenomenology. *Thus, the Lorentz/CPT violation induced in neutrino physics can be parametrized entirely by a single four-vector-valued $(a_L)^\mu_{ab} \propto l^\mu$ matrix in the flavor space.*

C. Neutrino oscillations

The propagation of the flavor eigenstates is then described by an effective Hamiltonian

$$(h_{\rm eff})_{ab} = \begin{pmatrix} p\delta_{ab} + (\tilde{m}^2)_{ab}/2p + (a_L)^\mu_{ab}p_\mu/p & 0 \\ 0 & p\delta_{ab} + (\tilde{m}^2)^*_{ab}/2p - (a_L)^{*\mu}_{ab}p_\mu/p \end{pmatrix}, \tag{4}$$

where the flavor indices a and b run over the flavor eigenstates e, μ, and τ and e^c, μ^c, and τ^c. Here, $p \equiv |\boldsymbol{p}|$, with $\boldsymbol{p}$ the neutrino momentum, and $\tilde{m}^2 \equiv m_l m_l^\dagger$ is the usual mass matrix, with $m_l = L - DR^{-1}D^T$.

Equation (4) has several implications: (i) Since the matrix is block-diagonal, there is no mixing between neutrinos and antineutrinos (as may arise in more general

[2]Additional possibilities arise with a noncanonical kinetic term; we comment briefly on possible consequences below.

Lorentz-violating scenarios [13]). (ii) Since a_L appears with opposite sign in the neutrino and antineutrino entries in the Hamiltonian, a nonzero a_L implies (apparent) *CPT* violation—i.e., the propagation of neutrinos and antineutrinos is not the same. Thus, for example, if the anomalous LSND results had stood, the *CPT*-violating explanations (e.g., Ref. [19]) for them [20] may have implied DE-neutrino coupling. (iii) The mixing induced by DE-neutrino coupling is energy independent (like in the Mikheev-Smirnov-Wolfenstein, or MSW, effect [21]), as opposed to vacuum mixing, which declines as E^{-1}. Thus, these effects will become increasingly visible at higher energies. The detailed form of *CPT* violation implied by this effect is also thus different than that obtained with different Δm^2 for neutrinos and antineutrinos. (iv) There may also be novel effects for neutrinos propagating through matter, an effect we discuss further in Sec. IV below.

Finally, (v) the neutrino oscillations induced by DE-neutrino coupling are *frame dependent*. If the observer is in the rest frame of the CMB, then $(a_L)^\mu p_\mu \propto E$, and neutrino oscillations are independent of the neutrino direction. However, the Solar System moves with respect to the CMB rest frame with a velocity $v \simeq 370\ \mathrm{km\,s^{-1}}$. DE-induced neutrino oscillations will therefore depend on $(a_L)^\mu p_\mu \propto E(1 - \boldsymbol{v}\cdot\hat{\boldsymbol{p}})$, where $\hat{\boldsymbol{p}}$ is the neutrino-propagation direction, and $\boldsymbol{v}$ is our peculiar velocity with respect to the CMB rest frame. There will thus be an annual modulation in solar-neutrino oscillations, a diurnal modulation in laboratory neutrino-mixing experiments, and a direction dependence in oscillations of cosmogenic neutrinos.

Since neutrino mixing arises only as a consequence of the *traceless* part of the propagation Hamiltonian, the DE-neutrino coupling must (like the vacuum mass matrix) be flavor-violating if neutrino oscillations are to be affected.

D. Two-flavor oscillations

The evolution equation for DE-induced two-flavor mixing is of the form

$$i\frac{d}{dt}\begin{pmatrix}\nu_a\\ \nu_b\end{pmatrix} = \frac{1}{2}\begin{pmatrix} -\frac{\Delta m^2}{2E}\cos 2\theta_v - m_{\mathrm{eff}}(1-\boldsymbol{v}\cdot\hat{\boldsymbol{p}})\cos 2\theta_d & \frac{\Delta m^2}{2E}\sin 2\theta_v + m_{\mathrm{eff}}(1-\boldsymbol{v}\cdot\hat{\boldsymbol{p}})\sin 2\theta_d e^{i\eta} \\ \frac{\Delta m^2}{2E}\sin 2\theta_v + m_{\mathrm{eff}}(1-\boldsymbol{v}\cdot\hat{\boldsymbol{p}})\sin 2\theta_d e^{-i\eta} & \frac{\Delta m^2}{2E}\cos 2\theta_v + m_{\mathrm{eff}}(1-\boldsymbol{v}\cdot\hat{\boldsymbol{p}})\cos 2\theta_d \end{pmatrix}\begin{pmatrix}\nu_a\\ \nu_b\end{pmatrix}, \tag{5}$$

where m_{eff} is an effective mass parameter, and θ_d and η are a mixing angle and relative phase in the DE-neutrino coupling matrix, respectively. There is also the usual vacuum mass difference (squared) Δm^2 and the vacuum-mixing angle θ_v. The analogous propagation equations for antineutrinos are the same as Eq. (5) with the replacements $m_{\mathrm{eff}} \to -m_{\mathrm{eff}}$ and $\eta \to -\eta$, the changes in sign a manifestation of *CPT* violation.

Recall that if the propagation Hamiltonian is of the form

$$h = M\begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}, \tag{6}$$

then the probability for one species of neutrino to convert to a different neutrino after a distance L is

$$P(\nu_a \to \nu_b) = \sin^2 2\theta \sin^2(ML). \tag{7}$$

Here, we have neglected the *CP*-violating phase in Eq. (6) because it does not affect the oscillation probability. The propagation Hamiltonian in Eq. (5) can be written in the form of Eq. (6) with the following relations [22]:

$$M^2 = \left(\frac{\Delta m^2}{4E}\right)^2 + \frac{m_{\mathrm{eff}}^2(1-\boldsymbol{v}\cdot\hat{\boldsymbol{p}})^2}{4} + \frac{\Delta m^2}{4E} m_{\mathrm{eff}}(1-\boldsymbol{v}\cdot\hat{\boldsymbol{p}}) \times (\cos 2\theta_v \cos 2\theta_d + \sin 2\theta_v \sin 2\theta_d \cos\eta), \tag{8}$$

$$\sin^2 2\theta = \frac{1}{M^2}\left[\left(\frac{\Delta m^2}{4E}\right)^2 \sin^2 2\theta_v + \frac{m_{\mathrm{eff}}^2(1-\boldsymbol{v}\cdot\hat{\boldsymbol{p}})^2}{4}\sin^2 2\theta_d + \frac{\Delta m^2}{4E} m_{\mathrm{eff}}(1-\boldsymbol{v}\cdot\hat{\boldsymbol{p}})\sin 2\theta_v \sin 2\theta_d \cos\eta\right]. \tag{9}$$

Note that this time $\sin 2\theta$ does indeed depend on η, as it is not the overall phase, but the *relative* one, that cannot be rotated away by redefinition of wave functions. The oscillation length is then $L_{\mathrm{osc}} = \pi M^{-1}$. In the absence of DE-neutrino coupling, we recover the standard oscillation length $L_{\mathrm{osc}} = 4\pi E/\Delta m^2$ and mixing angle $\theta = \theta_v$. If $m_{\mathrm{eff}} \gg \Delta m^2/2E$, then the oscillation length is $L_{\mathrm{osc}} = 2\pi m_{\mathrm{eff}}{}^{-1}(1 - \boldsymbol{v}\cdot\hat{\boldsymbol{p}})^{-1}$.

In general, m_{eff} can be either positive or negative. However, from the symmetry of the Hamiltonian, the relevant parameter space can be limited to $m_{\mathrm{eff}} \geq 0$, $0 \leq \theta_d \leq \pi/4$, and $0 \leq \eta \leq \pi$ [14].

Thus far, no deviations from standard three-flavor neutrino oscillations have been discovered in experimental data (except LSND [20]), and this yields constraints on *CPT*-violating parameters, especially for m_{eff}. By analyzing solar-neutrino and KamLAND data, Ref. [16] obtained an upper limit of $m_{\mathrm{eff}} < 3.1 \times 10^{-20}$ GeV for ν_e-ν_μ mixing. Atmospheric and accelerator data provide an upper limit for ν_μ-ν_τ mixing of $m_{\mathrm{eff}} < 5 \times 10^{-23}$ GeV [14].

III. ULTRA-HIGH-ENERGY NEUTRINOS

A. Prediction

Given that DE-induced neutrino mixing becomes increasingly important, relative to vacuum mixing, at high energies, the DE-neutrino coupling can be probed with ultra-high-energy cosmogenic neutrinos. These neutrinos are produced by the interaction of ultra-high-energy cosmic-ray protons with CMB photons [23]:

$$p\gamma \to n\pi^+ \to n\mu^+\nu_\mu \to ne^+\nu_e\nu_\mu\bar{\nu}_\mu. \quad (10)$$

The fact that the Greisen-Zatsepin-Kuzmin cutoff [24] has now been observed by the HiRes [25] and Auger [26] Collaborations implies that this interaction must be occurring. And if so, there must be a population of cosmogenic neutrinos with energies 10^{17}–10^{20} eV [23,27–29].

The characteristic distance between the source of these neutrinos and the Earth is the Hubble distance cH_0^{-1}, which is much longer than the oscillation length—i.e., $cH_0^{-1} \gg M^{-1}$—as long as $m_{\rm eff} \gg H_0 = 10^{-42}$ GeV, as is always the case here. Therefore, any oscillatory features in neutrino mixing will be washed out; the probability for conversion of a cosmogenic neutrino from its production flavor to another flavor en route from the source is then simply $\sin^2 2\theta/2$. Cosmogenic neutrinos mostly originate from pion decays, with the characteristic flavor ratio $\nu_e{:}\nu_\mu{:}\nu_\tau = 1{:}2{:}0$. The result of standard vacuum mixing would be a flavor ratio at the Earth of $\nu_e{:}\nu_\mu{:}\nu_\tau = 1{:}1{:}1$. While possible corrections to this flavor ratio can be induced by small three-flavour oscillation effects or other new physics, here we concentrate on exploring the consequences of the DE-induced mixing.

For the sake of simplicity, we focus on ν_μ-ν_τ mixing (and their antiparticles). In the absence of a DE-neutrino interaction, these two flavors are maximally mixed; i.e., $\theta_v = \pi/4$, and thus even if only ν_μ are produced at the source, an equal number of ν_μ and ν_τ is generated by mixing. However, this can be altered if there is a DE-neutrino interaction. The flux of ν_μ and ν_τ at the detector is related to the ν_μ flux at the source through,

$$\phi_{\nu_\mu} = (1 - \tfrac{1}{2}\sin^2 2\theta)\phi^0_{\nu_\mu}, \quad (11)$$

$$\phi_{\nu_\tau} = \tfrac{1}{2}\sin^2 2\theta\phi^0_{\nu_\mu}, \quad (12)$$

and θ will in general differ from θ_v if $m_{\rm eff} \neq 0$.

B. Proposed measurement

Here, we investigate the possibility of measuring θ using current or future ultra-high-energy-neutrino experiments such as Auger [30] and ANITA [31]. It is in principle possible to discriminate ν_τ from ν_μ by separately measuring the Earth-skimming events (ν_τ) and almost horizontal events originating in air (ν_μ).

We assume that a given experiment detects $N_\nu^{\rm tot}$ neutrino events. This quantity is for the *total* neutrino and antineutrino flux; i.e., $N_\nu^{\rm tot} = N_{\nu_\mu} + N_{\nu_\tau}$ (here ν represents both neutrinos and antineutrinos). If the flavor democracy expected from vacuum mixing is realized, then one expects $N_{\nu_\mu} = N_{\nu_\tau} = N_\nu^{\rm tot}/2$. The number of neutrino events at the detector is related to the flux through

$$N_\nu = \int_{E_{\rm min}}^{E_{\rm max}} dE\phi_\nu(E)\Xi(E), \quad (13)$$

where $\Xi(E)$ is the detector exposure to neutrinos in units of cm^2 s sr, and it generally depends on neutrino energy. Here, we assume $\phi_\nu^{\rm tot} = \phi^0_{\nu_\mu} = KE^{-2}$ with a normalization constant K, $E_{\rm min} = 2\times 10^{17}$ eV, and $E_{\rm max} = 2\times 10^{19}$ eV. This provides a good approximation for the spectrum of cosmogenic neutrinos (e.g., Ref. [29]). For simplicity we further assume that the detector exposure is independent of energy; the Auger exposure indeed depends on neutrino energy only weakly [30]. Therefore, the total number of neutrino events is given by

$$N_\nu^{\rm tot} = \frac{K\Xi}{E_{\rm min}}, \quad (14)$$

and the number of ν_τ events is given by

$$\begin{aligned} N_{\nu_\tau} &= \int_{E_{\rm min}}^{E_{\rm max}} dE\frac{1}{2}\sin^2 2\theta\phi^0_{\nu_\mu}(E)\Xi \\ &= \frac{K\Xi}{2}\int_{E_{\rm min}}^{E_{\rm max}} dEE^{-2}\sin^2 2\theta, \\ &= \frac{N_\nu^{\rm tot}E_{\rm min}}{2}\int_{E_{\rm min}}^{E_{\rm max}} dEE^{-2}\sin^2 2\theta, \end{aligned} \quad (15)$$

where we used Eq. (14) in the last equality.

To investigate the sensitivity of a given experiment, we assume a null detection of new physics; i.e., the result of N_{ν_τ} is consistent with the standard expectation $N_\nu^{\rm tot}/2$ within statistical errors (we do not take systematic uncertainties into account). This will reject a certain range of parameter space for ($m_{\rm eff}$, $\sin^2 2\theta_d$). More specifically, to obtain 95% C.L. (2σ) limits for these parameters, we solve

$$N_{\nu_\tau} > \frac{N_\nu^{\rm tot}}{2} - 2\sqrt{\frac{N_\nu^{\rm tot}}{2}}, \quad (16)$$

for $m_{\rm eff}$ and θ, using Eq. (15) for the left-hand side. In Fig. 1, we show the sensitivity of detectors that are expected to collect 12 and 100 neutrino events[3] (total) and that also have a ν_τ-identification capability. If the true values of $m_{\rm eff}$ and $\sin^2 2\theta_d$ are above these curves, then we will see an anomalously suppressed ν_τ flux compared with the standard mixing scenario. We also show the current upper limit on $m_{\rm eff}$ obtained from the combined analysis of Super-K and K2K data performed in Ref. [14]. One can see from this Figure that by detecting cosmogenic neutrinos and by studying their flavor content, one can largely improve the current sensitivity to $m_{\rm eff}$ and θ_d, quantifying further the suggestion of Ref. [17]. We also note that a weaker sensitivity, albeit still much better than

[3]The current Auger exposure is $\Xi \sim 10^{16}$ cm^2 s sr [30], and an optimistic estimate for the flux of cosmogenic neutrinos is close to the Waxman-Bahcall bound [28], $E^2\phi_\nu(E) \sim 10^{-8}$ GeV cm^{-2} s^{-1} sr^{-1} [29]. Therefore, from Eq. (14), we expect $N_\nu^{\rm tot} \lesssim 1$, which is still consistent with nondetection by Auger.

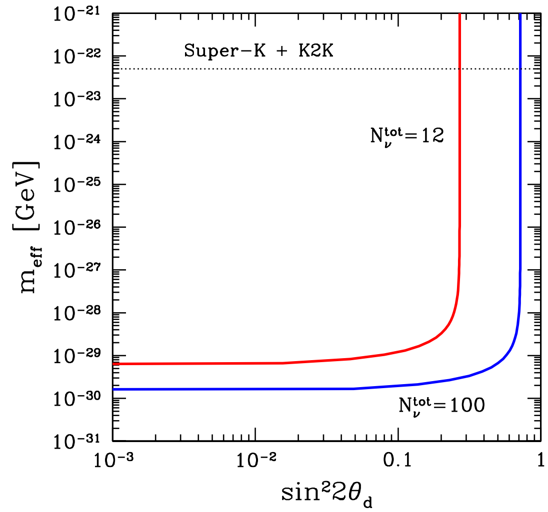

FIG. 1 (color online). Sensitivity on (m_{eff}, $\sin^2 2\theta_d$) plane of future experiments that would yield $N_\nu^{\mathrm{tot}} = 12$ and 100 total neutrino events.

the current sensitivity, may be achieved with neutrinos of slightly lower energies [32].

IV. MATTER EFFECTS IN ATMOSPHERIC NEUTRINO OSCILLATIONS IN THE PRESENCE OF A DARK-ENERGY COUPLING

We now turn our attention to Lorentz/*CPT*-violating effects in electron-neutrino oscillations, showing here that novel effects may arise with DE-neutrino coupling as neutrinos propagate through the Earth. These effects may allow us to access with atmospheric neutrinos regions of the DE-neutrino-coupling parameter space significantly below those currently probed. In this section, we consider two-flavor and three-flavor oscillations.

As neutrinos travel through matter, there is an additional contribution to oscillations from the matter potential $\sqrt{2}G_F N_e$ (where G_F and N_e are, respectively, the Fermi constant and electron density) relevant if electron neutrinos are involved. Recalling that the matter potential is $\gtrsim 10^{-22}$ GeV, the vacuum-mixing term $\Delta m^2/2E$ is small for neutrino energies $\gtrsim 10$ GeV. The mixing matrix Eq. (5) then becomes for ν_e-ν_μ mixing (neglecting the overall factor of 1/2, the directional dependence, and the phase η),

$$\begin{pmatrix} -m_{\mathrm{eff}}\cos 2\theta_d + \sqrt{2}G_F N_e & m_{\mathrm{eff}}\sin 2\theta_d \\ m_{\mathrm{eff}}\sin 2\theta_d & m_{\mathrm{eff}}\cos 2\theta_d - \sqrt{2}G_F N_e \end{pmatrix}. \tag{17}$$

Note that here, both the DE term and the matter potential change sign for antineutrinos, unlike the usual MSW effect, in which the vacuum term does not change sign. Unlike MSW mixing, there is essentially no energy dependence, at sufficiently high energies, in this mixing matrix.

To see when DE-induced mixing may be significant, recall that the value of the matter potential is $\sqrt{2}G_F N_e = 7.6 \times 10^{-14} Y_e(\rho/\mathrm{g\,cm^{-3}})$ eV. The Earth core has average density $\rho_{\mathrm{core}} = 11.83\ \mathrm{g\,cm^{-3}}$ and electron fraction $Y_e^{\mathrm{core}} = 0.466$, while the mantle has average density $\rho_{\mathrm{mantle}} = 4.66\ \mathrm{g\,cm^{-3}}$ and $Y_e^{\mathrm{mantle}} = 0.494$, with the surface layer of the Earth having density as low as $2.6\ \mathrm{g\,cm^{-3}}$. The matter potential is thus about 10^{-13} eV, so the effects of DE-induced mixing may be manifest for m_{eff} around 10^{-22} GeV, well below current upper limits. In the absence of matter, as discussed in Ref. [11], it is possible to obtain a resonance when all mixing angles involved are maximal

$$\frac{\Delta m^2}{2E}\cos 2\theta_v + m_{\mathrm{eff}}\cos 2\theta_d = 0. \tag{18}$$

Here, in the presence of matter and at high energies, a resonance can occur for a small mixing angle θ_d when

$$m_{\mathrm{eff}}\cos 2\theta_d = \sqrt{2}G_F N_e. \tag{19}$$

The presence of a resonance is thus entirely determined by the densities encountered along the path and the DE coupling parameters, with no (or very weak) energy dependence at high energies.

To illustrate the possibilities, we integrate the neutrino-propagation equation (including the small vacuum-mixing term) to calculate the ν_μ-to-ν_e transition probability as a function of (cosine of) the zenith angle for atmospheric neutrinos propagating through the Earth. We use the density profile of the Earth as given by the preliminary reference earth model [33]. Figure 2 shows the results for two-flavor oscillations for different values of m_{eff} for $\theta_d = \pi/4$. When $m_{\mathrm{eff}} \gg \sqrt{2}G_F N_e$, the oscillation probability is determined almost entirely by the DE term; there are regular large-amplitude variations of the oscillation probability as a function of zenith angle. As m_{eff} decreases to values comparable to $\sqrt{2}G_F N_e$, the oscillation probability

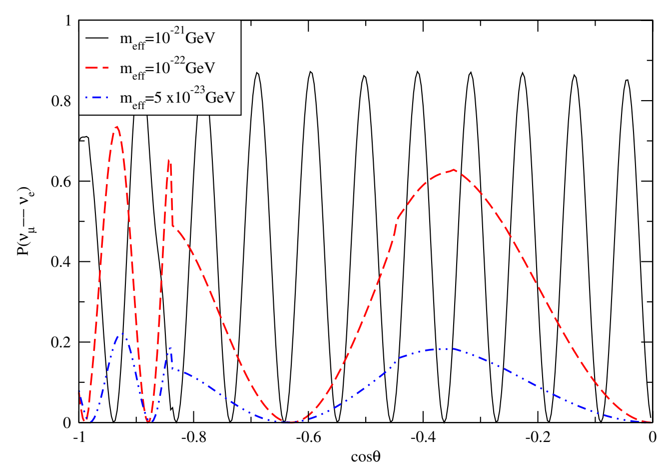

FIG. 2 (color online). Oscillation probability as a function of zenith angle for atmospheric neutrinos of $E = 50$ GeV, obtained with $\theta_d = \pi/4$.

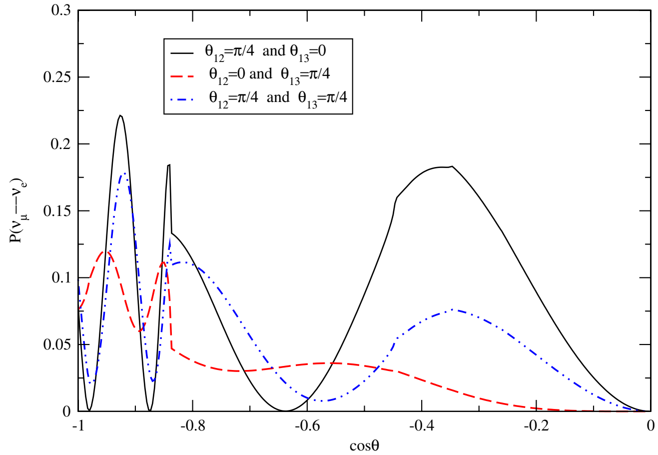

FIG. 3 (color online). Oscillation probability as a function of zenith angle for atmospheric neutrinos of $E = 50$ GeV, obtained for three-flavor oscillations with various values of θ_{12}^d and θ_{13}^d and $m_{\rm eff} = 5 \times 10^{-23}$ GeV.

decreases, and the oscillation length is seen to differ for trajectories that do ($\cos\theta \lesssim -0.8$) and do not ($\cos\theta \gtrsim -0.8$) pass through the core.

In Fig. 3 we show the oscillation probabilities for $m_{\rm eff} = 5 \times 10^{-23}$ GeV for three-flavor mixing in the Lorentz-violating sector. The case where $\theta_{13}^d = \pi/4$ and $\theta_{12}^d = 0$ also corresponds to an effective two-flavor scenario, just like the previous results. It leads, however, to a very different behavior due to the different contribution of the standard neutrino oscillations. The case where $\theta_{13}^d = \theta_{12}^d = \pi/4$ corresponds to a full three-flavor oscillation scenario. We have also studied the effects for other values of the mixing angles and the same features remain present. A nonzero value of θ_{13}^ν for standard neutrino oscillations leads to similar features in the zenith-angle distribution. However, the effects are extremely small at the high energies considered here, orders of magnitude below those coming from the Lorentz-invariance–violating terms.

V. DIRECTIONAL DEPENDENCE

While detection of CPT/Lorentz-violating effects would be spectacular—it would imply new physics regardless of whether it is DE-related or not—more compelling evidence for a DE effect would be the directional dependence, $\propto (1 - \boldsymbol{v} \cdot \hat{\boldsymbol{p}})$, of neutrino-oscillation parameters. Given that our peculiar velocity with respect to the CMB rest frame is 10^{-3} times the speed of light, the magnitude of this effect is going to be suppressed relative to the other effects, discussed above, of a DE-neutrino interaction. Statistics well beyond the reach of current and forthcoming neutrino experiments will be required to detect this effect. Still, it is worth keeping in mind for future generations of experiments.

It may also be worth searching for such a directional dependence in current data, just in case there is a DE-neutrino coupling that is manifest in ways different than we have foreseen here. For example, if DE somehow produces Lorentz violation through a modification of the kinetic term in the Dirac equation, the energy dependence of the mixing induced by Lorentz/CPT violation could be different [13]. We therefore work out in this section expressions for the factor $\boldsymbol{v} \cdot \hat{\boldsymbol{p}}$ to aid experimentalists who may wish to look for direction-dependent effects in their neutrino (or other) data.

To proceed, we first set our coordinate system. We set the origin at the center of Earth and align the z axis along the rotational axis of Earth, so that the North Pole has positive z coordinate. We set the x axis along the direction to the Sun at vernal equinox. Since the Sun moves eastbound, its position at summer solstice aligns with the y axis. We can thus represent the seasonal shift by an azimuthal angle φ, where $\varphi = 0$, $\pi/2$, π, and $3\pi/4$ for vernal equinox, summer solstice, autumn equinox, and winter solstice, respectively. Note also that the orbital plane of the Sun is inclined from the x-y plane by $\theta_{\rm inc} = 23.5°$ [34].

The Sun is moving with respect to the CMB rest frame with a speed of $v_\odot = 369\ {\rm km\,s^{-1}}$ towards the direction $\alpha = 168°$, $\delta = -7.22°$ [35], where α is right ascension and δ is declination of the celestial coordinates [34]. In our coordinates, the velocity of the Sun is $\boldsymbol{v}_\odot = v_\odot(\cos\delta\cos\alpha, \cos\delta\sin\alpha, \sin\delta) = (-358, 76.1, -46.4)\ {\rm km\,s^{-1}}$. The Earth is moving around the Sun with average orbital speed of $V = 29.8\ {\rm km\,s^{-1}}$. Thus, the velocity of the Earth with respect to the CMB rest frame is

$$\boldsymbol{v}_\oplus = \boldsymbol{v}_\odot + V\begin{pmatrix} \sin\varphi \\ -\cos\varphi\cos\theta_{\rm inc} \\ -\cos\varphi\sin\theta_{\rm inc} \end{pmatrix} = \begin{pmatrix} -358 & +29.8\sin\varphi \\ 76.1 & -27.3\cos\varphi \\ -46.4 & -11.9\cos\varphi \end{pmatrix} {\rm km\,s^{-1}}. \tag{20}$$

We neglect the contribution from the rotation of Earth ($\lesssim 0.5\ {\rm km\,sec^{-1}}$) to our velocity with respect to the CMB rest frame.

Now we evaluate the direction of the neutrino beam $\hat{\boldsymbol{p}}$. We suppose that the beam runs from some point A on the Earth's surface to another point B on its surface (or *vice versa*). It should be straightforward to generalize the arguments below so that extraterrestrial neutrino production can be taken into account. We set the origin of time coordinate T to "noon" (i.e., when the Sun reaches highest) at the point A. Therefore, the positions of A and B in our coordinate are

$$\boldsymbol{x}_{\rm A} = R_\oplus \begin{pmatrix} \cos\phi_{\rm A}\cos(\omega T_{\rm A} + \varphi) \\ \cos\phi_{\rm A}\sin(\omega T_{\rm A} + \varphi) \\ \sin\phi_{\rm A} \end{pmatrix}, \tag{21}$$

$$\boldsymbol{x}_{\rm B} = R_\oplus \begin{pmatrix} \cos\phi_{\rm B}\cos(\omega T_{\rm A} + \Delta\lambda + \varphi) \\ \cos\phi_{\rm B}\sin(\omega T_{\rm A} + \Delta\lambda + \varphi) \\ \sin\phi_{\rm B} \end{pmatrix}, \quad (22)$$

where $R_\oplus$ is the radius of the Earth, ω is the rotational frequency (2π/day), $T_{\rm A}$ is the time at the position A relative to noon, $\phi_{\rm A,B}$ is the geometric latitude of the points A and B, and $\Delta\lambda = \lambda_{\rm A} - \lambda_{\rm B}$ is the difference of the geometric longitude. The quantity $\Delta\lambda$ appears in $\boldsymbol{x}_{\rm B}$ because we measure the time (for both A and B) with respect to noon of the point A, so the time difference is given by the longitude difference (note also that the longitude increases to the west). The direction of the neutrino beam $\hat{\boldsymbol{p}}$ is then proportional to $\boldsymbol{x}_{\rm B} - \boldsymbol{x}_{\rm A}$ with proper normalization as

$$\hat{\boldsymbol{p}} = \frac{1}{\sqrt{2(1 - \cos\phi_{\rm A}\cos\phi_{\rm B}\cos\Delta\lambda - \sin\phi_{\rm A}\sin\phi_{\rm B})}} \begin{pmatrix} \cos\phi_{\rm B}\cos(\omega T_{\rm A} + \Delta\lambda + \varphi) - \cos\phi_{\rm A}\cos(\omega T_{\rm A} + \varphi) \\ \cos\phi_{\rm B}\sin(\omega T_{\rm A} + \Delta\lambda + \varphi) - \cos\phi_{\rm A}\sin(\omega T_{\rm A} + \varphi) \\ \sin\phi_{\rm B} - \sin\phi_{\rm A} \end{pmatrix}. \quad (23)$$

Therefore, by combining Eqs. (20) and (23), we obtain the directional factor $1 - \boldsymbol{v}_\oplus \cdot \hat{\boldsymbol{p}}$. Since it is a scalar quantity, the final result does not depend on the choice of the coordinate system.

VI. THEORETICAL IMPLICATIONS

Before closing, we discuss, for illustration, the implications of a measurement of a particular value of the Lorentz-invariance–violating effective mass parameter $m_{\rm eff}$ in terms of a specific model of DE-neutrino coupling.

Perhaps the simplest interaction of this kind has the form

$$\mathcal{L}_{\rm int} = -\lambda_{\alpha\beta}\frac{\partial_\mu\phi}{M_*}\bar{\nu}_\alpha\gamma^\mu(1-\gamma_5)\nu_\beta, \quad (24)$$

where ϕ is a quintessence field, $\lambda_{\alpha\beta}$ is a coupling-constant matrix, and M_* is some mass scale. Thus, $a_L^\mu \sim \lambda\dot{\phi}(t)l^\mu/M_*$, and $m_{\rm eff} \sim \Delta\lambda\dot{\phi}(t)/M_*$, where $\Delta\lambda$ is the difference between eigenvalues of the λ matrix. For quintessence, one expects $\dot{\phi} \sim M_{\rm Pl}H_0(1+w)^{1/2}$ (e.g., Ref. [3]), where $M_{\rm Pl}$ is the Planck energy scale. In this case, the mass scale M_* corresponding to a given $m_{\rm eff}$ is

$$M_* \simeq 10^6(\Delta\lambda)\left(\frac{1+w}{0.01}\right)^{1/2}\left(\frac{m_{\rm eff}}{10^{-30}\ {\rm GeV}}\right)\ {\rm GeV}, \quad (25)$$

to the mass scale that controls the DE-neutrino interaction. The ultra-high-energy ν_μ-ν_τ oscillation effects we have discussed thus probe up to mass scales $M_* \sim 10^6$ GeV. The ν_e-ν_μ oscillations induced by the matter effects we discussed probe up to mass scales $M_* \sim 100$ MeV.

VII. CONCLUSIONS

We studied the implications of an interaction between dark energy and neutrinos for neutrino oscillations. The most general Lorentz/*CPT*-violating term induced by DE takes the form $(a_L)^\mu\bar{\nu}\gamma_\mu(1-\gamma_5)\nu$, where $(a_L)^\mu$ is a four-vector normal to the CMB rest frame. This introduces a new source for neutrino oscillations that are energy independent and different for neutrinos and antineutrinos. Furthermore, the motion of the Earth with respect to the cosmic rest frame induces a directional dependence in the oscillation probabilities.

The current best limits to the DE-neutrino coupling we considered are obtained from atmospheric- and accelerator-neutrino experiments for ν_μ-ν_τ mixing, and from solar and reactor experiments for ν_e-ν_μ mixing. However, the higher the neutrino energy, the more prominent the effect of the DE-neutrino interaction. We therefore considered in this paper cosmogenic ultra-high-energy (energies of 10^{17}–10^{19} eV) neutrinos produced by the interaction of ultra-high-energy cosmic rays with CMB photons. We showed that future experiments targeting these neutrinos will improve the sensitivity to a DE-neutrino interaction by 7 orders of magnitude, down to $m_{\rm eff} \sim 10^{-30}$ GeV compared with the current upper bound $m_{\rm eff} \lesssim 5\times10^{-23}$ GeV (Fig. 1). This corresponds to a sensitivity to an energy scale as large as $\sim10^6$ GeV for the DE-neutrino interaction. We then showed that the interplay of DE- and matter-induced neutrino mixing could induce a novel zenith-angle dependence for ν_e oscillations in atmospheric neutrinos. This effect may extend the sensitivity to Lorentz/*CPT*-violating parameters in the ν_e by roughly 3 orders of magnitude.

More compelling evidence of a DE-neutrino interaction (as opposed to some other origin for Lorentz/*CPT* violation) would be a directional dependence of the oscillation probabilities. The notion that Lorentz violation may give rise to a directional dependence is not new (e.g., Ref. [36]) and searches for directional dependence in neutrino experiments have already been carried out (e.g., Ref. [37]), but prior work has considered Lorentz-violating parameters introduced in an *ad hoc* manner and/or tested for direction-dependent effects in a Sun-centered inertial frame. We emphasize here that cosmic acceleration suggests that we seek a specific form of Lorentz violation, that where the preferred frame is aligned with the CMB rest frame. Even though such a signal is expected to be small, it is still worth seeking in existing and future experimental data.

We have not discussed specific models for a DE-neutrino interaction, beyond an illustrative toy model, but it may be interesting to do so (see also Ref. [38]). The theoretical motivation to expect such a coupling may admittedly be slim. However, we are at square one in our understanding of DE, and such a coupling is no less likely to be expected, perhaps, than any of the many other manifestations of new cosmic-acceleration physics that have been considered. Discovery of Lorentz/CPT-violating effects would be extremely important, even if not attributable directly to dark energy. A directional dependence, if discovered, would be absolutely remarkable, as it would provide moreover clear evidence that there is more to cosmic acceleration than simply a cosmological constant.

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