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Gravitational wave bursts from cosmic superstrings with Y-junctions

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Cosmic superstring loops generically contain strings of different tensions that meet at Y-junctions. These loops evolve nonperiodically in time, and have cusps and kinks that interact with the junctions. We study the effect of junctions on the gravitational wave signal emanating from cosmic string cusps and kinks. We find that earlier results on the strength of individual bursts from cusps and kinks on strings without junctions remain largely unchanged, but junctions give rise to additional contributions to the gravitational wave signal coming from strings expanding at the speed of light at a junction and kinks passing through a junction.

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I. INTRODUCTION

In string theory models of brane inflation, fundamental (F) and D-strings are produced at the end of inflation when the branes collide and annihilate [1–4]. In certain scenarios the resulting superstring networks are stable and expand to cosmic size, with predicted string tensions in the range $10^{-11} \leq G\mu \leq 10^{-6}$. This raises the possibility that cosmic superstrings could provide an observational signature of string theory. Indeed it has been argued that the gravitational wave (GW) signals from cusps of oscillating loops on cosmic strings should be detectable by LIGO and LISA for string tensions as low as 10^{-13} [5–7].

However, the GW predictions of [6] may not be directly applicable to cosmic superstring networks, because F- and D-strings form an interconnected network in which they join and separate at Y-junctions. Each junction joins an F-string, a D-string and their bound state. This means that closed loops containing junctions in a cosmic superstring network do not evolve periodically in time. Furthermore, kinks interact with junctions, which leads to novel contributions to the GW signal as well as a more complicated network evolution. For these reasons we reexamine here the gravitational wave signal emanating from cusps and kinks on cosmic string loops taking in account the presence of junctions. We note that in a similar spirit, Damour and Vilenkin have calculated the effect on GW signatures of cosmic strings of a reduced reconnection probability of intersecting strings and of a reduced typical length of newly formed loops. It was found that earlier results obtained for field theory cosmic strings remain largely valid for a rather wide range of network parameters.

We first set up the formalism for calculating GW bursts emitted by cusps and kinks on nonperiodic cosmic string loops. The nonperiodic evolution of loops with junctions renders the GW calculation somewhat more involved because one can no longer factorize the Fourier transform of the GW amplitude. We show one can nevertheless integrate the stress energy over the string world sheet and obtain an analytic expression for the high frequency behavior of the various contributions to the GW bursts. We find that earlier results on the strength of individual bursts from cusps and kinks on strings without junctions remain largely unchanged, but junctions give rise to additional contributions coming from strings expanding at the speed of light at a junction and kinks passing through a junction. We analyze the latter contributions, which provide a possible observational discriminant between ordinary cosmic strings and cosmic superstrings.

We concentrate here on the calculation of individual gravitational bursts. This has the advantage of being a fully tractable exercise. By contrast, the observable signal of such bursts at the present time depends also on the details of the loop evolution as well as on the cosmological evolution of the network. In particular, junctions tend to enhance the number of kinks on loops. One might expect this amplifies the GW signal from cosmic superstring networks compared to the signal from cosmic string networks without junctions. A more detailed analysis of this effect will appear elsewhere [8].

II. GW EMISSION FROM COSMIC SUPERSTRINGS WITH JUNCTIONS

In this section we consider the GW emission from a cosmic superstring loop, and generalize the results of Damour and Vilenkin [5–7] on the emission of gravitational bursts from standard loops without junctions.

We consider closed loops of cosmic superstrings containing two Y-junctions, as shown in Fig. 1. (Generalization to loops with 4 or more junctions is straightforward in principle.) The loop consists of three local Nambu-Goto cosmic strings of tensions μ_q ($q = 1, 2, 3$), which meet at two Y-junctions where the junctions are

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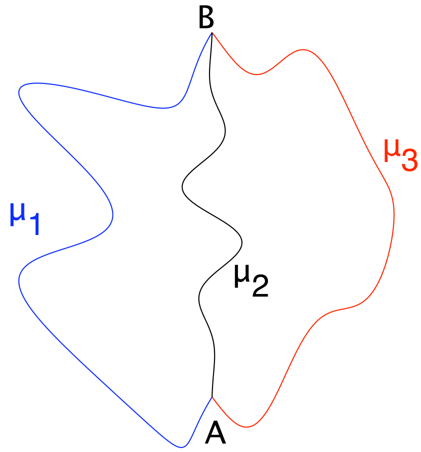

FIG. 1 (color online). Loop formed by three strings and two junctions.

labeled by A and B. The action describing the system has been set up and analyzed in [9], and is given by

$$S = -\sum_{q=1,2,3}\mu_q\int dt\int_{s_q^A(t)}^{s_q^B(t)} d\sigma_q\sqrt{\mathbf{x}_q'^2(1-\dot{\mathbf{x}}_q^2)} + \sum_{J=(A,B)}\sum_q\int dt\mathbf{f}_q^J\cdot[\mathbf{x}_q(t,s_q^J(t))-\mathbf{X}_J(t)]. \quad (1)$$

The first term is the Nambu-Goto action for each of the three strings in the loop, where we have assumed a flat space-time geometry with signature $(-+++)$ and used the standard conformal-temporal gauge to parametrize each string's world sheet. Thus each string is described by its spatial coordinates $\mathbf{x}_q(\sigma_q, t)$, where t coincides with background time. [As discussed in [10], cosmic superstrings should be described by the Dirac-Born-Infeld action, but the resulting equations of motion reduce to those derived from (1).] Since each string is bounded by the two junctions, the σ parameter runs between two time-dependent bounds

$$\sigma_q \in [s_{A,q}(t), s_{B,q}(t)]. \quad (2)$$

The conformal gauge constraints can be written (with $'$ and $\cdot$ standing for derivatives with respect to σ and t) as

$$\mathbf{x}_q'\cdot\dot{\mathbf{x}}_q = 0, \quad (3)$$

$$\mathbf{x}_q'^2 + \dot{\mathbf{x}}_q^2 = 1, \quad (4)$$

and as usual, away from the junctions, the wavelike equation of motion $\ddot{\mathbf{x}}_q - \mathbf{x}_q'' = \mathbf{0}$ yields

$$\mathbf{x}_q(\sigma, t) = \frac{1}{2}(\mathbf{a}_q(\sigma+t) + \mathbf{b}_q(\sigma-t)), \quad (5)$$

with $\mathbf{a}_q'^2 = \mathbf{b}_q'^2 = 1$ in order to satisfy the gauge constraints. This resembles the solution for standard closed loops. However, one must also take in account the second term of (1), which imposes, via the Lagrange multipliers $\mathbf{f}_q^J$, that the three strings meet at the positions of the junctions $\mathbf{X}_A(t)$ and $\mathbf{X}_B(t)$. Causality requires $|\dot{\mathbf{X}}_J(t)| \leq 1$, which together with (4) yields [9]

$$|\dot{s}_{A,q}| \leq 1, \qquad |\dot{s}_{B,q}| \leq 1. \quad (6)$$

The time evolution of $s_{A,q}(t)$ and $s_{B,q}(t)$ is given by the equations of motion at the junction, yielding [9]

$$\frac{\mu_1(1-\dot{s}_{B,1})}{\sum_q \mu_q} = \frac{M_1(1-c_{B,1})}{\sum_q M_q(1-c_{B,q})}, \quad (7)$$

where

$$c_{B,1} \equiv \mathbf{b}_2'(s_2-t)\cdot\mathbf{b}_3'(s_3-t), \qquad M_1 \equiv \mu_1^2 - (\mu_2-\mu_3)^2 \geq 0, \quad (8)$$

and similarly by circular permutation. As a result of the presence of the junctions, loops now evolve nonperiodically in time.

Like field theory cosmic strings, cosmic superstrings can contain cusps—points moving at the speed of light $|\dot{\mathbf{x}}_q| = 1$ (and hence $\mathbf{x}_q' = 0$) with $\mathbf{x}_q'' \neq 0$ [11,12]—and kinks, which correspond to a discontinuity in $\mathbf{x}_q'$. Here, we aim to study the gravitational wave emission by such localized sources. We first calculate the GW signal in the local wave zone of the source, at distances from the source that are larger than the wavelength of interest but smaller than the Hubble radius. In this regime we can take the space-time to be asymptotically flat: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, where $h_{\mu\nu} \ll 1$ is the metric perturbation generated by the source. The subsequent propagation of the gravity waves on cosmological scales in a Friedmann-Lemaître space-time is discussed at the end of this paper.

In a suitable gauge the GW are described by the transverse traceless $^{(TT)}$ part of the linear perturbation $h_{ij}^{(TT)}$ of the spatial metric. This satisfies the linearized Einstein equations,

$$\Box h_{ij}^{(TT)} = -16\pi G T_{ij}^{(TT)}. \quad (9)$$

where $T_{ij}^{(TT)}$ is the $^{(TT)}$ part of the stress-energy tensor of the source. In the local wave zone, for closed loops of characteristic size L localized around the origin, the solution is given by [6]

$$h_{ij}^{(TT)}(\mathbf{x}, \omega) = \frac{4G}{r}e^{i\omega r}T_{ij}^{(TT)}(\omega\mathbf{n}, \omega), \quad (10)$$

where $\mathbf{n} = \mathbf{x}/\parallel\mathbf{x}\parallel$ and $r = \parallel\mathbf{x}\parallel$. Note that $h_{ij}^{(TT)}$ is actually the time Fourier transform of $h_{ij}^{(TT)}$ and $T_{ij}^{(TT)}$ the space-time Fourier transform of $T_{ij}^{(TT)}$: we will use these abusive notations throughout the text. The wave-zone conditions are [13,14]

$$r \gg L, \qquad r \gg L^2\omega, \qquad r \gg 1/\omega.$$

Hence, to obtain the GW emission in this regime it suffices to evaluate $T_{ij}^{(TT)}$ at points $(\mathbf{k}, \omega)$ in the Fourier domain that satisfy the dispersion relation $\mathbf{k} = \omega\mathbf{n}$, where $\mathbf{n}$ is a unit vector pointing from the source toward the observer.

Let us put aside the $^{(TT)}$ projection. From now on we concentrate on the calculation of T_{ij} and merely indicate where the $^{(TT)}$ projection projects out a leading contribution. From the action (1) one gets [9]

$$T^{ij}(\mathbf{x_0}, t_0) = \sum_{\text{string } q} \mu_q \int dt \int_{s_{A,q}(t)}^{s_{B,q}(t)} (\dot{x}_q^i \dot{x}_q^j - x_q'^i x_q'^j)|_{(\sigma,t)} \times \delta^{(3)}(\mathbf{x_0} - \mathbf{x}_q(\sigma, t))\delta(t_0 - t) d\sigma, \tag{11}$$

namely a sum of contributions of all strings. Each term can be evaluated independently once the coupled dynamics of the system has been determined. From now on, we focus on one of these contributions and drop the q subscript indicating the string. Moving into Fourier space, we have

$$T_{ij}(\mathbf{k}, \omega) = \mu \int dt \int_{s_A(t)}^{s_B(t)} (\dot{x}_i \dot{x}_j - x_i' x_j')|_{(\sigma,t)} \times e^{i(\omega t - \mathbf{k}\cdot\mathbf{x}(\sigma,t))} d\sigma, \tag{12}$$

$$= \frac{\mu}{2} \int dt \int_{s_A(t)}^{s_B(t)} a'_{(i}(\sigma + t) b'_{j)}(\sigma - t) e^{i(\omega t - \mathbf{k}\cdot\mathbf{x}(\sigma,t))} d\sigma. \tag{13}$$

For the periodic loops with no junctions considered in [6], changing integration variables to

$$u = \sigma + t, \qquad v = \sigma - t, \tag{14}$$

and using the periodicity leads to a *factorized* expression of the form

$$T_{ij} \propto \left(\int a'_{(i}(u) e^{(i/2)(\omega u - \mathbf{k}\cdot\mathbf{a}(u))} du\right) \times \left(\int b'_{j)}(v) e^{-(i/2)(\omega v + \mathbf{k}\cdot\mathbf{b}(v))} dv\right). \tag{15}$$

With junctions, the absence of periodicity prevents one from writing the stress energy in a similar convenient form. Nevertheless, the same change of variables still proves useful to study T_{ij} (see Fig. 2). Indeed, after changing to (u, v) in (13) and evaluating at $\mathbf{k} = \omega\mathbf{n}$ we obtain

$$T_{ij}(\omega, \omega\mathbf{n}) = \frac{\mu}{4} \int_{-\infty}^{\infty} b'_j(v) e^{-(i\omega/2)(v + \mathbf{n}\cdot\mathbf{b}(v))} \times \left(\int_{u_A(v)}^{u_B(v)} a'_i(u) e^{(i\omega/2)(u - \mathbf{n}\cdot\mathbf{a}(u))} du\right) dv. \tag{16}$$

Notice that the u and v integrals are *no longer factorized* due to the v dependence of the bounds in the u integral. Indeed, the bounds $u_A(v)$ and $u_B(v)$ are defined as follows: Since $|\dot{s}_A|, |\dot{s}_B| < 1$, for each v there is a unique t_A and a unique t_B that satisfy $s_A(t_A) - t_A = v$, $s_B(t_B) - t_B = v$. Then, $u_A(v) = s_A(t_A) + t_A$ and $u_B(v) = s_B(t_B) + t_B$.

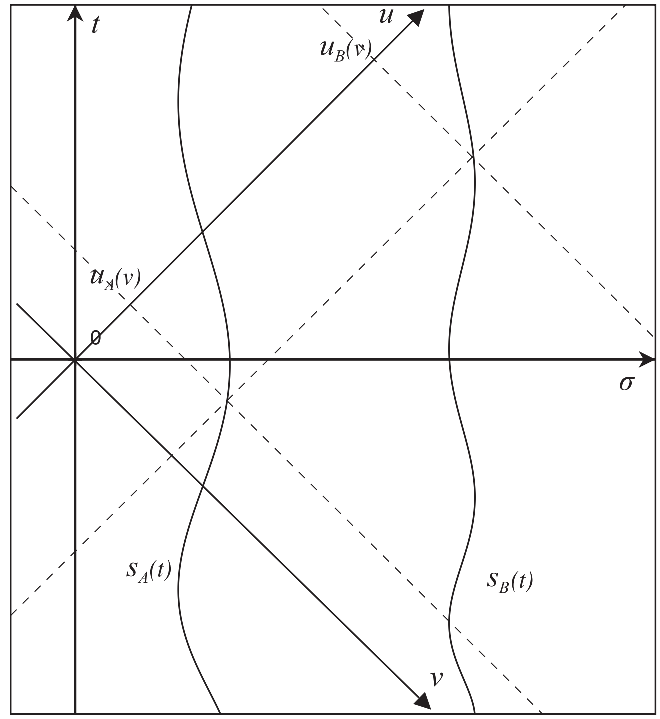

FIG. 2. World sheet of one of the strings.

We note for further use that, for $J = A$ or B,

$$\frac{du_J}{dv} = \frac{\dot{s}_J + 1}{\dot{s}_J - 1} \leq 0. \tag{17}$$

This means that for "typical" values of $|\dot{s}_J|$ not too close to 1, du_J/dv is negative and of order 1. However, one can also have the limiting values $du_J/dv = -\infty$ when $\dot{s}_J = 1$ and $du_J/dv = 0$ when $\dot{s}_J = -1$. Since such values correspond to a junction moving at the speed of light, a phenomenon which will turn out to lead to a gravitational burst, we will consider these more carefully below. Note, however, that the equations of motion forbid the strings from shrinking at the speed of light (either at junction A: $\dot{s}_A = 1$ and $du_A/dv = -\infty$, or at junction B: $\dot{s}_B = -1$ and $du_B/dv = 0$) [15]. Hence, one only needs to consider the case of a string expanding at the speed of light, again either at junction A with $\dot{s}_A = -1$ and $du_A/dv = 0$, or at junction B with $\dot{s}_B = 1$ and $du_B/dv = -\infty$).

We are interested in the production of gravitational bursts, that is in the high frequency ω regime. We can therefore restrict attention to the $\omega \to \infty$ limit of the integrals in (16), where standard techniques have been developed [6,16]. We note, however, that an analysis of this kind does not include the low frequency gravitational radiation from the slow motion of the string itself, and therefore the resulting stochastic background of gravitational waves arising from this.

III. HIGH FREQUENCY BEHAVIOR OF T_{ij}: GENERAL DISCUSSION

Let us consider integrals of the form

$$I(\omega) = \int_a^b f(t) e^{-i\omega\phi(t)} dt \tag{18}$$

in the $\omega \to \infty$ limit. We will use the following standard result (see, for example, [16]):

If

(i) $\forall k \geqslant 0$, $f^{(k)}(a) = f^{(k)}(b)$ and $\phi^{(k)}(a) = \phi^{(k)}(b)$

(ii) f and ϕ are C^∞ on $[a, b]$—that is, $f(t)$ and $\phi(t)$ are smooth in the interval; and

(iii) $\forall t \in [a, b]$, $\dot{\phi}(t) \neq 0$—that is, there is no stationary phase or saddle points, then $I(\omega) \to 0$ as $\omega \to \infty$, faster than any power of $1/\omega$.

(To see this, change the variable of integration to ϕ using (iii) and integrate by parts. (i) means the boundary term vanishes. Repeating this procedure N times shows that $I(\omega) = \mathcal{O}(\omega^{-N})$.) Thus, integrals of the form (18) are exponentially small for large values of ω if the conditions (i)–(iii) hold and the integrand has a rapidly varying phase, i.e. $\omega \gg |\dot{f}/(f\dot{\phi})|$.

Integrals of this type appear in the expression for the stress energy, Eq. (15) and (16). In the following, we will therefore be interested in situations where (at least) one of the above conditions does not hold, since these might lead to high frequency contributions to the stress energy proportional to small powers of $1/\omega$. But we first summarize the results obtained in [6] for the factorized stress-energy tensor (15) of cusps and kinks on periodic strings. In this case the above results can be applied independently to each integral, and since (i) is ensured by the periodicity, the main contributions to T_{ij} appear when

(i) there is a saddle point in each integral [(iii) violated in both integrals], leading to a contribution $T_{ij} \propto 1/\omega^{4/3}$ and corresponding to the physical situation of a cusp emitting around one specific direction;

(ii) there is a discontinuity of a'_i and a saddle point in the integral over v (or *vice versa*), leading to $T_{ij} \propto 1/\omega^{5/3}$. This is the case of a kink emitting in a one-dimensional fan-like set of directions throughout its propagation.

All other cases (discontinuities in higher derivatives of a' or b' for example) give smaller contributions to T_{ij}.

In the case of loops with junctions we consider here, the stress-energy tensor given in Eq. (16) cannot be factorized. It can only be written in the form

$$T_{ij}(\omega, \omega\mathbf{n}) = \frac{\mu}{4} \int_{-\infty}^{\infty} b'_j(v) e^{-(i\omega/2)(v+\mathbf{n}\cdot\mathbf{b}(v))} I_i(v) dv, \tag{19}$$

where

$$I_i(v) := \int_{u_A(v)}^{u_B(v)} a'_i(u) e^{-i\omega\phi(u)} du \tag{20}$$

with

$$\phi(u) = -\tfrac{1}{2}(u - \mathbf{n}\cdot\mathbf{a}(u)). \tag{21}$$

The previous results can only formally be applied to the integral in $I_i(v)$: indeed, its bound depends on v, so that the v integral in (19) receives contributions that could prevent it from being of the same type. However, as we now discuss, in most situations of interest these additional contributions have a simple dependence on v, allowing us to generalize the results of [6].

IV. HIGH FREQUENCY BEHAVIOR OF T_{ij}: FIRST INTEGRATION

Consider first the integral $I_i(v)$ given in (20). In order to be in the rapidly varying phase regime, we require $\omega \gg \| \mathbf{a}'' \|$. If the loop is not too wiggly, one can assume that $\| \mathbf{a}'' \| \sim 1/L$ (recall that $\| \mathbf{a}' \| = 1$), in which case the condition becomes $\omega \gg 1/L$ [note that this is an additional constraint on top of the wave-zone conditions (11)]. We now examine the high frequency contributions to $I_i(v)$ resulting from the violation of one of the conditions listed in the previous section.[1]

Part A. (i) is violated, i.e. $\mathbf{a}'(u_A(v)) \neq \mathbf{a}'(u_B(v))$.

This will generically be the case for a string bounded by junctions. Hence, this is a novel contribution with respect to [6]. After integrating by parts, the leading order contribution scales as ω^{-1} and is given by

$$I_i^{\text{boundary}}(v) \approx \frac{2}{i\omega}\left[\frac{a'_i(u)}{1-\mathbf{n}\cdot\mathbf{a}'(u)} e^{-i\omega\phi(u)}\right]_{u=u_A(v)}^{u=u_B(v)}. \tag{22}$$

In most cases of interest $|\frac{du_J}{dv}(v)| \sim 1$, so that $\frac{a'_i(u_J(v))}{1-\mathbf{n}\cdot\mathbf{a}'(u_J(v))}$ is a slowly varying function of v and $e^{-i\omega\phi(u_J(v))}$ is a rapidly varying phase term that must be taken in account when performing the integral over v in (19), though the latter remains of the same type as (18).[2]

[1] An analysis of situations in which several conditions are violated is beyond the scope of this work.

[2] As mentioned above, when the string expands at the speed of light *at junction B*, $\frac{du_B}{dv}$ diverges. One might expect this leads to a discontinuity-like contribution in the second integral. However, both the derivative with respect to v of the amplitude $\frac{a'_i(u_B(v))}{1-\mathbf{n}\cdot\mathbf{a}'(u_B(v))}$ and of the phase in the exponential diverge. The condition $\dot{\phi}\omega \gg \frac{\dot{f}}{f}$ still holds since $\frac{du_B}{dv}$ cancels on both sides. Therefore, no discontinuity-like contribution is introduced in the second integral.

Part B. (ii) is violated in the following way: There exists a $u_* \in]u_A(v), u_B(v)[$ where a_i' is discontinuous.

We could also consider non regularities in higher order derivatives of a_i but they would lead to higher order powers in $1/\omega$.)

To evaluate this term, we first rewrite

$$I_i(v) = \int_{u_A(v)}^{u_*} a_i'(u)e^{i\omega\phi}du + \int_{u_*}^{u_B(v)} a_i'(u)e^{i\omega\phi}du,$$

and then integrate once by parts as above to find

$$I_i^{\rm disc}(v) = -\frac{2}{i\omega}\left(\frac{a_i'(u_*^+)}{1-\mathbf{n}\cdot\mathbf{a}'(u_*^+)} - \frac{a_i'(u_*^-)}{1-\mathbf{n}\cdot\mathbf{a}'(u_*^-)}\right)e^{i\omega\phi(u_*)} \tag{23}$$

As above, we find an ω^{-1} falloff. The only dependence on v comes from the bound $u_A(v) < u_* < u_B(v)$. This means that when we will perform the integral over v in (19) in Sec. V below, we will need to restrict the v domain of integration appropriately.

Part C. (iii) is violated (saddle point)

There exists[3] a $u_s \in [u_A(v), u_B(v)]$ where the phase $\phi(u)$ has a vanishing derivative $\phi'(u_s) = -(1-\mathbf{n}\cdot\mathbf{a}'(u_s))/2 = 0$, or in other words $\mathbf{n} = \mathbf{a}'(u_s)$.

In this case the leading contribution to $I_i(v)$ comes from the vicinity of the saddle point and is obtained by Taylor expanding $\phi(u)$ and $a_i'(u)$ around u_s.

To do so, notice that the gauge conditions (on the world sheet) enforce $\phi''(u_s) = 0$. Indeed, $\phi''(u) = \frac{1}{2}\mathbf{n}\cdot\mathbf{a}''(u)$ and since $\mathbf{a}'(u)^2 = 1$, it follows that $\mathbf{a}'\cdot\mathbf{a}''(u) = 0$. Furthermore, $\phi'''(u_s) \leqslant 0$ because upon taking the derivative of $\mathbf{a}'\cdot\mathbf{a}''(u) = 0$, one gets $\mathbf{a}'\cdot\mathbf{a}'''(u) = -||\mathbf{a}''||^2$ so that $\phi'''(u) = -||\mathbf{a}''||^2/2$. Regarding the Taylor expansion of $a_i'(u)$, the first term is $a_i'(u_s) = n_i$ which is suppressed by the $^{(TT)}$ projection operator (it amounts to a gauge term [6]). To summarize, we therefore need to Taylor expand the phase to the third order and $a_i'(u)$ to the first order:

$$\begin{aligned}\phi(u) &\simeq \phi(u_s) + \frac{\phi'''(u_s)}{6}(u-u_s)^3\\ a_i'(u) &\simeq a_i'(u_s) + (u-u_s)a_i''(u_s),\end{aligned} \tag{24}$$

[3] Strictly speaking, u_s could lie outside $[u_A(v), u_B(v)]$, but close to u_A or u_B and still give rise to a non-negligible contribution (cf. the Appendix).

Thus, up to a gauge term proportional to n_i,

$$\begin{aligned}I_i &\simeq e^{-i\omega\phi(u_s)}\int_{u_A(v)}^{u_B(v)}(u-u_s)a_i''(u_s)e^{-(i\omega/6)\phi'''(u_s)(u-u_s)^3}du\\ &\simeq -e^{-i\omega\phi(u_s)}a_i''(u_s)\left(\frac{6}{\omega|\phi'''(u_s)|}\right)^{2/3}\int_{w_B}^{w_A}we^{-iw^3}dw\\ &\simeq -e^{-i\omega\phi(u_s)}a_i''(u_s)\left(\frac{6}{\omega|\phi'''(u_s)|}\right)^{2/3}\int_{-\infty}^{\infty}we^{-iw^3}dw\\ &\simeq \frac{1}{\omega^{2/3}}a_i''(u_s)e^{-i\omega\phi(u_s)}\left(\frac{6}{|\phi'''(u_s)|}\right)^{2/3}\left(\frac{i}{\sqrt{3}}\Gamma\left(\frac{2}{3}\right)\right),\end{aligned} \tag{25}$$

where we have changed variables according to $w = -(\frac{\omega|\phi'''(u_s)|}{6})^{1/3}(u-u_s)$ and have used $\int_{-\infty}^{\infty}we^{-iw^3}dw = -\frac{i}{\sqrt{3}}\Gamma(\frac{2}{3})$. We note that in going from the second to the third line, we have assumed that the saddle point lies *far* from the boundaries:

$$|u_s - u_J(v)||\omega\phi'''(u_s)|^{1/3} \gg 1, \tag{26}$$

thus allowing the domain of integration to be extended from $-\infty$ to $+\infty$. This will *not* always be the case, however, and the general result reads

$$\begin{aligned}I_i^{\rm saddle}(v) = \text{gauge term} + \Bigg[\frac{1}{\omega^{2/3}}a_i''(u_s)e^{(i\omega/2)(u_s-\mathbf{n}\cdot\mathbf{a}(u_s))}\\ \times\left(\frac{12}{|\mathbf{n}\cdot\mathbf{a}'''|}\right)^{2/3}\left(\frac{i}{\sqrt{3}}\Gamma\left(\frac{2}{3}\right)\right)C(w_A, w_B)\Bigg],\end{aligned} \tag{27}$$

with

$$\begin{aligned}C(w_A, w_B) &= \frac{\int_{w_B}^{w_A}we^{-iw^3}dw}{\int_{-\infty}^{\infty}we^{-iw^3}dw} = B(w_A) - B(w_B),\\ B(w_J) &= \frac{\int_{-\infty}^{w_J}we^{-iw^3}dw}{\int_{-\infty}^{\infty}we^{-iw^3}dw},\end{aligned} \tag{28}$$

and

$$\begin{aligned}w_A(v) &= \left(\frac{\omega|\mathbf{n}\cdot\mathbf{a}'''|}{12}\right)^{1/3}(u_s - u_A(v)),\\ w_B(v) &= -\left(\frac{\omega|\mathbf{n}\cdot\mathbf{a}'''|}{6}\right)^{1/3}(u_B(v) - u_s).\end{aligned}$$

The behavior of the function $C(w_A, w_B)$ is studied in detail in the Appendix. Its important features are the following:

(a) when the saddle point lies well in between the upper and lower bounds of the u integral $C(w_A(v), w_B(v))$ obviously reduces to 1, as in (25).
(b) when the saddle point is located far outside the domain of integration then $C(w_A(v), w_B(v))$ reduces to zero.
(c) when the saddle point is near $u_A(v)$ (resp $u_B(v)$), $w_A(v)$ (resp $w_B(v)$) is of order one. $C(w_A(v), w_B(v))$ then reduces to $B(w_A(v))$ (resp $B(w_B(v))$) given in Fig. 3. We note that $B(w_A) \to 1$ for large values of

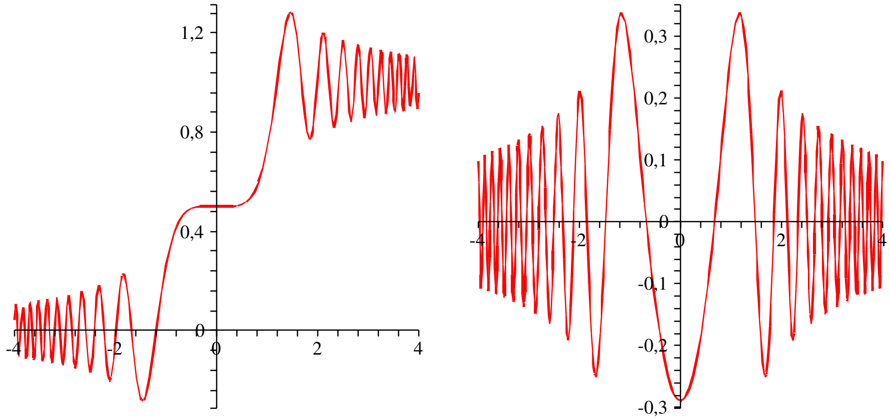

FIG. 3 (color online). Real and imaginary parts of $B(w_A)$ as a function of w_A.

w_A, with the envelope of its oscillations decreasing as $1/w$.

The entire dependence on v in (27) is contained in $C(v) = C(w_A(v), w_B(v))$, which is a smoothed version of the step function $\theta(u_s - u_A(v))\theta(u_B(v) - u_s)$ (cf. the Appendix). This enters the integral over v as follows: Let $v_{A,s}$ (resp. $v_{B,s}$) be the value of v for which $u_A(v) = u_s$ (resp. $u_B(v) = u_s$). Since u_A and u_B are decreasing functions of v, $C(w_A(v), w_B(v))$ is actually a smoothed version of the step function $\theta(v - v_{A,s})\theta(v_{B,s} - v)$. But we ought to verify whether C is rapidly varying (in the vicinity of $v_{A,s}$ and $v_{B,s}$) compared to the phase appearing in $e^{-\frac{i\omega}{2}(v+\mathbf{n}\cdot\mathbf{b}(v))}$. In typical cases where $|\frac{du_A}{dv}| \approx 1$, $|\frac{du_B}{dv}| \approx 1$, one has $\frac{dB}{dw}(w \approx w_A(v)) \sim \mathcal{O}(1)$ in the region of interest (see Fig. 3). Hence,

$$\frac{dC}{dv}(v \approx v_{A,s}) \approx \underbrace{\frac{dB}{dw}(w \approx w_A(v))}_{\approx 1}(\omega|\phi'''(u_s)|)^{1/3}\frac{du_A}{dv} \approx (\omega|\phi'''(u_s)|)^{1/3}. \tag{29}$$

For strings that are not too wiggly we expect $|\phi'''| \sim ||\mathbf{a}''||^2 \approx 1/L^2$ so that

$$\frac{dC}{dv}(v \approx v_{A,s}) \approx \left(\frac{\omega}{L^2}\right)^{1/3} \ll \omega, \tag{30}$$

since we assumed $\omega \gg 1/L$. Hence, $C(v)$ enters as a multiplicative, slowly varying amplitude in the integral over v.[4] In particular, it does not introduce a boundary term in the integral over v (unless of course du_B/dv diverges, see below), which implies junctions do not radiate spontaneously.

V. FULL HIGH FREQUENCY BEHAVIOR OF T_{ij}

We are now in a position to perform the remaining integral in (19) to obtain the high frequency behavior of the various contributions to the stress-energy source of GW bursts. The analysis of the previous section shows that, in all cases, the integral over v is of exactly the same type as the integral $I_i(v)$ over u. We can therefore apply the same methods. Generalizing the definition we used above in the saddle point case we denote by $v_A(u)$ the value of v for which $u_A(v) = u$ (and the dual definition for B). Since we are interested in the high frequency regime, we list the different contributions with increasing powers of $1/\omega$.

A. Contributions in $1/\omega^{4/3}$

(i) *saddle point in the u integral at u_s/ saddle point in the v integral at v_s (standard cusp)*
The contribution from the integral over u can be treated as a slowly varying amplitude. Hence, the integral over v is formally of the same type as the u integral, with the product $C(v)b_j'(v)$ now acting as the slowly varying amplitude. The derivative of this evaluated at v_s contains a term proportional to $b_j'(v_s)$, which is a gauge term for the gravitational waves, and a term proportional to $C(v_s)b_j''(v_s)$. Our final result reads

$$T_{ij} \approx \frac{\mu}{\omega^{4/3}} a_i''(u_s)b_j''(v_s)e^{(i\omega/2)[u_s - v_s - \mathbf{n}\cdot(\mathbf{a}(u_s)+\mathbf{b}(v_s))]} \times \left(\frac{12}{|\mathbf{n}\cdot\mathbf{a}'''(u_s)|}\right)^{2/3}\left(\frac{12}{\mathbf{n}\cdot\mathbf{b}'''(v_s)}\right)^{2/3} \times \frac{1}{12}\Gamma^2\left(\frac{2}{3}\right)C(w_A, w_B), \tag{31}$$

with

$$\mathbf{n} = \mathbf{a}'(u_s) = -\mathbf{b}'(v_s)$$

and

[4] Strictly speaking, we need to prove that $I(v)b_i'(v)$ is slowly varying in the sense that $\frac{d}{dv}(I(v)b_j'(v))\frac{1}{I(v)b_j'(v)} \ll \omega$. After applying the derivative one is left with two contributions: the first one is $\frac{b_j''(v)}{b_j'(v)}$ which, as before for the a' term, is of order $\frac{1}{L} \ll \omega$. The second one is $\frac{I'}{I} = \frac{C'}{C}$ which, according to what we just said, is $\ll \omega$.

$$w_A = \left(\frac{\omega|\mathbf{n}\cdot\mathbf{a}'''(u_s)|}{12}\right)^{1/3}(u_s - u_A(v_s)),$$
$$w_B = -\left(\frac{\omega|\mathbf{n}\cdot\mathbf{a}'''(u_s)|}{12}\right)^{1/3}(u_B(v_s) - u_s).$$

This is the case of a cusp that emits in the direction given by $\mathbf{a}'(u_s)$. If the cusp occurs away from the junction, C reduces to 1 and we recover the standard result of Damour and Vilenkin [6].

B. Contributions in $1/\omega^{5/3}$

This is the first case in which we find novel contributions specific to strings with junctions.

(i) *discontinuity in a_i' at some u_*/saddle point in the v integral at v_s (standard kink)*
This is the standard case of a left-moving kink propagating on the string and emitting in the fan of directions $\mathbf{n} = -\mathbf{b}'(v)$ generated by the right-moving waves:

$$T_{ij} \approx \frac{\mu}{\omega^{5/3}}\left(\frac{a_i'(u_*^+)}{1-\mathbf{n}\cdot\mathbf{a}'(u_*^+)} - \frac{a_i'(u_*^-)}{1-\mathbf{n}\cdot\mathbf{a}'(u_*^-)}\right)b_j''(v_s)$$
$$\times\left(\frac{12}{\mathbf{n}\cdot\mathbf{b}'''(v_s)}\right)^{2/3}\frac{1}{2\sqrt{3}}\Gamma\left(\frac{2}{3}\right)\overline{C(w_A, w_B)}$$
$$\times e^{(i\omega/2)[u_* - v_s - \mathbf{n}\cdot(\mathbf{a}(u_*)+\mathbf{b}(v_s))]}, \tag{32}$$

with

$$\mathbf{n} = -\mathbf{b}'(v_s)$$

and

$$w_A = \left(\frac{\omega\mathbf{n}\cdot\mathbf{b}'''(v_s)}{12}\right)^{1/3}(v_s - v_A(u_*)),$$
$$w_B = -\left(\frac{\omega\mathbf{n}\cdot\mathbf{b}'''(v_s)}{12}\right)^{1/3}(v_B(u_*) - v_s).$$

In (32), $\bar{C}$ is the complex conjugate of C (induced from the fact that in the Taylor expansion of the phase of the v integral, we now have a *positive* $\phi'''(v)$, and hence this is equivalent to changing i to $-i$ in all integrals).
In order to illustrate how the presence of the function $C(w_A, w_B)$ modifies the case of a standard loop with no junction, let us consider a left-moving kink propagating between B and A and ask in which directions $\mathbf{n}$ (described by points on the 3D unit sphere) it emits. The set of points $\{-\mathbf{b}'(v)\}$ draws a curve $\mathcal{C}$ on the Kibble-Turok sphere, as illustrated in Fig. 4. Away from $\mathcal{C}$, $T_{ij}(\mathbf{n}) \simeq 0$, whereas on the curve the amplitude is given by (32). The interval between positions $v_A(u_*)$ and $v_B(u_*)$ on the curve contains the right-moving waves $-\mathbf{b}'(v)$ effectively seen by the kink while it propagates, so we expect to have an emission only in or around these directions. On this interval—and not too close to the endpoints—we have $C \simeq 1$. Far from this interval[5] (in blue on Fig. 4), $C \simeq 0$. Finally, C smoothly decreases from 1 to 0 in a short interval Δv of a few $(L^2/\omega)^{1/3}$ near $v_A(u_*)$ and $v_B(u_*)$. Hence, the overall probability that a GW burst comes with a value of C significantly different from one is of the order $(L\omega)^{-1/3} \ll 1$. We conclude therefore that up to small corrections the predictions of [6] apply to GW bursts from kinks on cosmic strings with junctions.

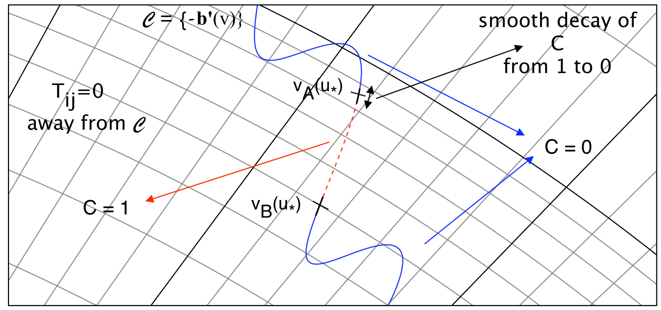

FIG. 4 (color online). Directions of emission of a propagating kink on the Kibble-Turok sphere.

(ii) *saddle point in the u integral at u_s/discontinuity of $\mathbf{b}_j'$ at some v_* (standard kink)*
This is a situation that mirrors the previous one, namely, a right-moving kink propagating on the string and emitting in a fan-like set of directions given by $\mathbf{n} = \mathbf{a}'$. The formulae are similar to those above.

(iii) *boundary term (A) in the integral over u/saddle point in the second integral at v_s*
Inserting (22) into (19), we obtain

$$T_{ij}(\omega, \omega\mathbf{n}) = \frac{i\mu}{2\omega}\int_{-\infty}^{\infty}\frac{b_j'(v)a_i'(u_A(v))}{1-\mathbf{n}\cdot\mathbf{a}'(u_A(v))}$$
$$\times e^{-(i\omega/2)[(v+\mathbf{n}\cdot\mathbf{b}(v))-(u_A(v)-\mathbf{n}\cdot\mathbf{a}(u_A(v))]}dv, \tag{33}$$

where $\frac{b_j'(v)a_i'(u_A(v))}{1-\mathbf{n}\cdot\mathbf{a}'(u_A(v))}$ is a slowly varying amplitude compared to the rapidly varying phase

$$\tilde{\phi}(v) \equiv \tfrac{1}{2}[(v + \mathbf{n}\cdot\mathbf{b}(v)) - (u_A(v) - \mathbf{n}\cdot\mathbf{a}(u_A(v))]. \tag{34}$$

The saddle point condition is that

[5] We note that although this is not taken into account by (32), T_{ij} smoothly vanishes when $\mathbf{n}$ moves away from $\mathcal{C}$.

$$\tilde{\phi}'(v_s) = \frac{1}{2}\Bigg[\underbrace{1 + \mathbf{n}\cdot\mathbf{b}'(v_s)}_{\geqslant 0} + \underbrace{\underbrace{\frac{du_A}{dv}(v_s)}_{\leqslant 0}\underbrace{(\mathbf{n}\cdot\mathbf{a}'(u_B(v_s)) - 1)}_{\leqslant 0}}_{\geqslant 0}\Bigg] = 0. \tag{35}$$

Since both terms in the sum are positive, they both need to vanish at v_s. Moreover, $\mathbf{n}\cdot\mathbf{a}'(u_A(v_s)) \neq -1$, otherwise we would have a cusp at the junction. Hence, we are led to the following conditions:

$$\begin{cases} \mathbf{n}\cdot\mathbf{b}'(v_s) = -1 \Rightarrow \mathbf{n} = -\mathbf{b}'(v_s) \\ \frac{du_A}{dv}(v_s) = 0. \end{cases} \tag{36}$$

The second condition means the string is expanding at the speed of light at junction A. The other case where the string expands at the speed of light at junction B would correspond to $\frac{du_B}{dv} = -\infty$, and thus to a discontinuity in the v integral. We will treat this separately below.

The amplitude of this contribution can be calculated using the saddle point treatment described previously (namely, a Taylor expansion of the amplitude and the phase). We have

$$\tilde{\phi}''(v_s) = \frac{1}{2}\Bigg[\underbrace{\mathbf{n}\cdot\mathbf{b}''(v_s)}_{=0} + \underbrace{\frac{d^2u_A}{dv^2}(v_s)(\mathbf{n}\cdot\mathbf{a}'(u_A(v_s)) - 1)}_{=0} + \underbrace{\mathbf{n}\cdot\mathbf{a}''(u_A(v))\left(\frac{du_A}{dv}(v_s)\right)^2}_{=0}\Bigg]. \tag{37}$$

The first term vanishes because of the gauge conditions on the world sheet, and the last term vanishes due to the second condition in (36). The second term also vanishes. Indeed, since $\frac{du_A}{dv}(v) \leqslant 0$ and $\frac{du_A}{dv}(v_s) = 0$, $\frac{du_A}{dv}(v)$ reaches its maximum at v_s. Hence, we need to Taylor expand the phase to third order:

$$\tilde{\phi}'''(v_s) = \frac{1}{2}\Bigg[\mathbf{n}\cdot\mathbf{b}'''(v_s) + \frac{d^3u_A}{dv^3}(v_s)(\mathbf{n}\cdot\mathbf{a}'(u_A(v_s)) - 1)\Bigg]. \tag{38}$$

On the other hand, Taylor expansion of the amplitude to first order gives

$$b'_j(v)\frac{a'_i(u_A(v))}{1-\mathbf{n}\cdot\mathbf{a}'(u_A(v))} \approx \text{gauge term} + (v - v_s) \times b''_j(v_s)\frac{a'_i(u_A(v_s))}{1-\mathbf{n}\cdot\mathbf{a}'(u_A(v_s))}, \tag{39}$$

where the gauge term is proportional to $b'_j(v_s) = -n_j$.

Hence, the leading term of the energy-momentum tensor is given by

$$T_{ij}(\omega, \omega\mathbf{n}) \approx \frac{\mu}{\omega^{5/3}} b''_j(v_s)\frac{a'_i(u_A(v_s))}{1-\mathbf{n}\cdot\mathbf{a}'(u_A(v_s))} \times (\tilde{\phi}'''(v_s))^{2/3}\frac{6^{2/3}}{2\sqrt{3}}\Gamma\left(\frac{2}{3}\right) \times e^{-(i\omega/2)[(v_s+\mathbf{n}\cdot\mathbf{b}(v_s))-(u_A(v_s)-\mathbf{n}\cdot\mathbf{a}(u_A(v_s)))]}. \tag{40}$$

This contribution corresponds to the situation in which the string expands at the speed of light at junction A and emits in a direction corresponding to $\mathbf{n} = -\mathbf{b}'(v_s)$, with an amplitude proportional to $1/\omega^{5/3}$. We now turn to the case where the string expands at the speed of light at junction B.

(iv) *saddle point in the u integral at u_s/ string expanding at the speed of light at junction B*

In this case, du_B/dv diverges, which gives rise to a discontinuity-like contribution in the integral over v. From (19) and (27) we have

$$T_{ij}(\omega, \omega\mathbf{n}) \approx \frac{\mu}{4}\frac{1}{\omega^{2/3}} a''_i(u_s)e^{(i\omega/2)(u_s-\mathbf{n}\cdot\mathbf{a}(u_s))} \times \left(\frac{12}{|\mathbf{n}\cdot\mathbf{a}'''(u_s)|}\right)^{2/3}\frac{i}{\sqrt{3}}\Gamma\left(\frac{2}{3}\right) \times \int_{-\infty}^{\infty} C(v)b'_j(v)e^{-(i\omega/2)(v+\mathbf{n}\cdot\mathbf{b}(v))}dv, \tag{41}$$

where $C(v) = C(w_A(v), w_B(v))$ was defined in (28). We are interested in cases in which a discontinuity in C (or at least a variation faster than that of the phase in the exponential, $|dC/dv| \gg \omega$) is induced by the divergence of du_B/dv or equivalently dw_B/dv at some v_*. Yet, to achieve such a discontinuity in C, we need v_* to satisfy

$$|w_B(v_*)| \lesssim 1. \tag{42}$$

Indeed, in this region we have $\partial C/\partial w_B \approx 1$, whereas elsewhere C does not vary with w_B at leading order so that a strong variation of $w_B(v)$ would have no effect on $C(v)$. Physically, this requirement means that the saddle point condition is satisfied near the junction at a moment when $\dot{s}_B$ reaches 1.

From now on we assume that (42) is satisfied. For $|dC/dv| \gg \omega$, we can treat C as being discontinuous at v_*, where dC/dv diverges. In this case, the leading contribution is the same as that of a discontinuous C with a gap $\Delta C(v*)$ equal to the variation of the true function C over the interval where $|dC/dv| \gg \omega$:

$$T_{ij}(\omega, \omega\mathbf{n}) \approx \frac{\mu}{\omega^{5/3}} a_i''(u_s) \frac{b'(v_*)}{1 + \mathbf{n}\cdot\mathbf{b}'(v_*)} \times \left(\frac{12}{|\mathbf{n}\cdot\mathbf{a}'''(u_s)|}\right)^{2/3} \times \Delta C(v_*) \times \left(-\frac{1}{2\sqrt{3}}\Gamma\right)\left(\frac{2}{3}\right) \times e^{(i\omega/2)(u_s - v_* - \mathbf{n}\cdot(\mathbf{a}(u_s)+\mathbf{b}(v_*)))}. \tag{43}$$

We therefore need to evaluate the order of magnitude of $\Delta C(v_*)$. Let us start by determining the interval around v_* over which $|dC/dv| \gg \omega$. As was already explained in (29) and (30), $|dC/dv| \approx |du_B/dv|(\omega/L^2)^{1/3}$ so the condition $|dC/dv| \gg \omega$ translates into $|du_B/dv| \gg (\omega L)^{2/3}$. Since $|du_B/dv| = (1 + \dot{s}_B)/(1 - \dot{s}_B) \approx 2/(1 - \dot{s}_B)$ (because $\dot{s}_B \approx 1$ in the region of interest), this finally yields the condition

$$1 - \dot{s}_B \ll \frac{1}{(\omega L)^{2/3}}. \tag{44}$$

We therefore need to determine how $\dot{s}_B(t)$ behaves around a point t_* where $\dot{s}_B(t_*) = 1$. To this end we investigate the dynamics of a junction using the equations recalled in Sec. II (we momentarily drop the B index in Eq. (7), and assume the above discussion applies to string 1, namely $\dot{s}_1(t_*) = 1$, while strings 2 and 3 are assumed *not* to be expanding at the speed of light.) For convenience define $v_{2*} = s_2(t_*) - t_*$ and $v_{3*} = s_3(t_*) - t_*$. Then, from (7), $c_1(t_*) = 1$ so that $\mathbf{b}_2'(v_{2*}) = \mathbf{b}_3'(v_{3*}) \equiv \mathbf{B}$. Expansion of $\mathbf{b}_2'(v_2)$ and $\mathbf{b}_3'(v_3)$ around v_{2*} and v_{3*}, respectively, then yields

$$\mathbf{b}_2'(v_2) \approx \mathbf{B} + \mathbf{b}_2''(v_{2*})(v_2 - v_{2*}) + \mathbf{b}_2'''(v_{2*}) \times \frac{(v_2 - v_{2*})^2}{2}$$

$$\mathbf{b}_3'(v_3) \approx \mathbf{B} + \mathbf{b}_3''(v_{3*})(v_3 - v_{3*}) + \mathbf{b}_3'''(v_{3*}) \times \frac{(v_3 - v_{3*})^2}{2},$$

where we have used the fact that $\mathbf{b}_2''(v_{2*})\cdot\mathbf{B} = \mathbf{b}_3''(v_{3*})\cdot\mathbf{B} = 0$ because of the gauge constraints on the world sheets. Thus,

$$c_1(t) \approx 1 + \frac{(s_3(t) - t - v_{3*})^2}{2}\mathbf{b}_3'''(v_{3*})\cdot\mathbf{B} + \frac{(s_2(t) - t - v_{2*})^2}{2}\mathbf{b}_2'''(v_{2*})\cdot\mathbf{B} + (s_2(t) - t - v_{2*})(s_3(t) - t - v_{3*}) \times \mathbf{b}_2''(v_{2*})\cdot\mathbf{b}_3''(v_{3*}). \tag{45}$$

We can also expand $s_2(t) - t \approx v_{2*} + (\dot{s}_2(t_*) - 1)\times(t - t_*)$ and $s_3(t) - t \approx v_{3*} + (\dot{s}_3(t_*) - 1)(t - t_*)$, where generically $(\dot{s}_2(t_*) - 1)$ and $(\dot{s}_3(t_*) - 1)$ are of order 1. Then, using the fact that the three scalar products involved in (45) are of order $1/L^2$, we can write, around t_*

$$c_1(t) - 1 \approx -\frac{1}{L^2}(t - t_*)^2, \tag{46}$$

where the minus sign ensures that $c_1(t) \leq 1$. Plugging this into Eq. (7), we finally get the expansion around t_*

$$1 - \dot{s}_1(t) \approx \frac{1}{L^2}(t - t_*)^2. \tag{47}$$

Therefore, (44) holds over an interval of time around t_* of size Δt of order $\Delta t \approx L/(\omega L)^{1/3} = L^{2/3}\omega^{-1/3}$. Since $u_B = s_B(t) + t$, the variation of u_B during this interval Δt is of the same order as Δt, i.e. $\Delta u_B \approx L^{2/3}\omega^{-1/3}$ and so the variation of w_B is $\Delta w_B \approx (\omega L^{-2})^{1/3}\Delta u_B \approx (\omega L^{-2})^{1/3} L^{2/3}\omega^{-1/3} \approx 1$.

Finally, since we assumed that $w_B(v_*)$ is in the region where $\partial C/\partial w_B \approx 1$, this yields

$$\Delta C(v_*) \approx 1 \tag{48}$$

To conclude, if $\mathbf{n} = \mathbf{a}'(u_s)$ and du_B/dv diverges at some v_* such that $|w_B(v_*)| = (\frac{\omega|\mathbf{n}\cdot\mathbf{a}'''(u_s)|}{12})^{1/3}\times |u_B(v_*) - u_s| \lesssim 1$, then the integral over v receives a discontinuity-like contribution leading to

$$T_{ij}(\omega, \omega\mathbf{n}) \approx \frac{\mu}{\omega^{5/3}} a_i''(u_s) \frac{b_j'(v_*)}{1 + \mathbf{n}\cdot\mathbf{b}'(v_*)} \times \left(\frac{12}{|\mathbf{n}\cdot\mathbf{a}'''(u_s)|}\right)^{2/3}\left(-\frac{1}{2\sqrt{3}}\Gamma\right)\left(\frac{2}{3}\right) \times \Delta C(v_*) e^{(i\omega/2)(u_s - v_* - \mathbf{n}\cdot(\mathbf{a}(u_s)+\mathbf{b}(v_*)))}, \tag{49}$$

where with respect to (43) we now write $\Delta C(v_*)$ on the third line to indicate that this is a numerical factor of order 1. Physically, this emission in a given direction arises when the string expands at the speed of light at junction B.

C. Contributions in $1/\omega^2$

(i) *Discontinuity in a_i' at some u_*/discontinuity in b_j' at some $v_* \in [v_A(u_*), v_B(u_*)]$*

$$T_{ij} \approx \frac{\mu}{\omega^2}\left(\frac{a_i'(u_*^+)}{1-\mathbf{n}\cdot\mathbf{a}'(u_*^+)} - \frac{a_i'(u_*^-)}{1-\mathbf{n}\cdot\mathbf{a}'(u_*^-)}\right) \times \left(\frac{b_j'(v_*^+)}{1+\mathbf{n}\cdot\mathbf{b}'(v_*^+)} - \frac{b_j'(v_*^-)}{1+\mathbf{n}\cdot\mathbf{b}'(v_*^-)}\right) \times e^{(i\omega/2)[u_*-v_*-\mathbf{n}\cdot(\mathbf{a}(u_*)+\mathbf{b}(v_*))]}. \tag{50}$$

This is the situation where a left-moving and a right-moving kink meet during their propagation on the string and emit in all directions.

(ii) *Discontinuity in a_i' at some u_*/boundary term in the integral over v*

$$T_{ij} \approx -\frac{\mu}{\omega^2}\left(\frac{a_i'(u_*^+)}{1-\mathbf{n}\cdot\mathbf{a}'(u_*^+)} - \frac{a_i'(u_*^-)}{1-\mathbf{n}\cdot\mathbf{a}'(u_*^-)}\right) \times \left(\frac{b_j'(v_B(u_*))}{1+\mathbf{n}\cdot\mathbf{b}'(v_B(u_*))}\right) \times e^{(i\omega/2)[u_*-v_B(u_*)-\mathbf{n}\cdot(\mathbf{a}(u_*)+\mathbf{b}(v_B(u_*)))]}. \tag{51}$$

This corresponds to a left-moving kink passing through a junction and emitting in all directions in space.

(iii) *boundary term* (A *or* B) *in the integral over u/discontinuity* of b_j' at some v_*

$$T_{ij} \approx -\frac{\mu}{\omega^2}\left(\frac{b_j'(v_*^+)}{1+\mathbf{n}\cdot\mathbf{b}'(v_*^+)} - \frac{b_j'(v_*^-)}{1+\mathbf{n}\cdot\mathbf{b}'(v_*^-)}\right) \times \left(\frac{a_i'(u_A(v_*))}{1-\mathbf{n}\cdot\mathbf{a}'(u_A(v_*))}\right) \times e^{(i\omega/2)[u_A(v_*)-v_*-\mathbf{n}\cdot(\mathbf{a}(u_A(v_*))+\mathbf{b}(v_*)]}. \tag{52}$$

This describes a right-moving kink passing through a junction (A here) and emitting in all directions in space. As one expects, the symmetry with respect to the case of a left-moving kink passing through B is recovered despite the explicit breaking of symmetry made by choosing to integrate on u first.

V. CONCLUSION

The transverse traceless projections of the various contributions to the stress-energy tensor found in Sec. V fully determine the GW emission from cusps and kinks in the local wave zone of the source through Eq. (10). The observed signal is obtained by parallel propagation of the gravity waves in a cosmological background, along the null geodesic followed by the GW. This gives rise to the usual redshifting of time intervals between emission and reception, which in the Fourier domain corresponds to $f_{\rm em} = (1+z)f_{\rm rec}$. One obtains the following order-of-magnitude estimates for the logarithmic Fourier transform of the observed amplitude of individual GW bursts emanating from the various high frequency sources discussed here [6,7,17],

$$h(f_{\rm rec}) \sim \frac{G\mu L}{((1+z)Lf_{\rm rec})^\alpha}\frac{1+z}{t_0 z}, \tag{53}$$

where t_0 denotes the present age of the Universe. Most importantly, the exponent α in this expression is determined by the high frequency behavior $\sim\omega^{-(1+\alpha)}$ of the stress-energy sources calculated above.

In the time domain (53) corresponds to a signal of the form $\sim|t-t_c|^\alpha$, where t_c is the arrival time of the center of the burst. The total GW signal of a superstring loop consists of the sum of the various bursts and a slowly varying component due to the low frequency modes of the strings. Despite its vanishing at $t=t_c$, bursts are distinguishable from the slowly varying component because the curvature associated with (53) diverges as $\sim|t-t_c|^{\alpha-2}$, exhibiting clearly the spiky nature of GW bursts.[6]

We are now in a position to summarize the modifications of the GW signal from bursts induced by the presence of junctions, with respect to the case of standard loops. As expected, *away from the junctions* we recover the same GW signal as Damour and Vilenkin [6,7], both for a kink propagating on a string—for which $\alpha = 2/3$ and the emission occurs in a fan-like set of directions—as well as for a cusp, where $\alpha = 1/3$ and the emission occurs in a small cone around a particular direction.

However for bursts from sources near junctions we find the predicted GW amplitude is modified by a smooth correction factor C. This means that for GW bursts from cusps and kinks near junctions one effectively measures a combination of the tension μ and C. Since these are rather improbable events, however, the overall effect of this on the (statistical) predictions of bursts from cosmic superstrings is likely to be small. The smoothness of the correction factor also implies that despite their spiky nature, junctions do not radiate spontaneously (at least for strings without structure, as considered here).

Furthermore, the presence of junctions gives rise to several novel sources of GW bursts, most notably from

(i) a string expanding at the speed of light at the level of a junction, which emits in a specific direction with an amplitude (53) with $\alpha = 2/3$.

(ii) a kink passing through a junction, which emits in all directions with an amplitude (53) with $\alpha = 1$.

Finally, the situation where a left-moving and a right-moving kink pass through each other during their propagation remains unchanged and has $\alpha = 1$.

To translate our results into observable waveforms one ought to sum the individual contributions from cusps and kinks in a cosmological network of string loops. However,

[6] In practice, the observer will never lie exactly in the direction of emission. However, the signal is detectable inside a small cone around this direction. This introduces a high frequency cutoff on the waveform or, equivalently, a smoothing of the signal around t_c in the time domain.

the rate of occurrence of events of different types strongly depends on the dynamics of the loops and on the evolution of the network, which is currently poorly understood. This gives rise to significant uncertainties in the resulting GW signal. Indeed, an increase in the number of events of a given type lowers the threshold redshift for observation and hence reduces the expected dilution of the signal. The individual contribution with the smallest value of α therefore need not necessarily provide the dominant contribution to the observed signal.

In particular, concerning the case at hand, one might expect junctions to enhance the number density of kinks, whereas it appears unlikely that the rate of occurrence of cusps is significantly affected by the presence of junctions [8]. This would mean that even though individual GW bursts from cusps are stronger, the increased number density of kinks on loops with junctions might compensate for the difference in strength. Since we have identified GW bursts from events involving kinks that are specific to strings with junctions, this would provide a promising route to observationally distinguish between gauge theory cosmic strings with no junctions and superstrings.

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APPENDIX A: DETAILED STUDY OF THE FUNCTION $C(v) \equiv C(w_A, w_B)$

In (27), we gave the general result for the integral (20) due to a saddle point:

$$I_i^{\text{saddle}}(v) = \text{gauge term} + \left[\frac{a_i''(u_s)}{\omega^{2/3}} e^{(i\omega/2)(u_s - \mathbf{n}\cdot\mathbf{a}(u_s))} \times \left(\frac{12}{|\mathbf{n}\cdot\mathbf{a}'''|}\right)^{2/3}\left(\frac{i}{\sqrt{3}}\Gamma\left(\frac{2}{3}\right)\right) C(w_A, w_B)\right], \quad \text{(A1)}$$

where

$$C(w_A, w_B) = \frac{\int_{w_B}^{w_A} we^{-iw^3}dw}{\int_{-\infty}^{\infty} we^{-iw^3}dw} = B(w_A) - B(w_B),$$
$$B(w_J) = \frac{\int_{-\infty}^{w_J} we^{-iw^3}dw}{\int_{-\infty}^{\infty} we^{-iw^3}dw}, \quad \text{(A2)}$$

and

$$w_A(v) = \left(\frac{\omega|\mathbf{n}\cdot\mathbf{a}'''|}{12}\right)^{1/3}(u_s - u_A(v)),$$
$$w_B(v) = -\left(\frac{\omega|\mathbf{n}\cdot\mathbf{a}'''|}{6}\right)^{1/3}(u_B(v) - u_s). \quad \text{(A3)}$$

The function C depends on v because the position of the saddle point relative to the bounds $u_A(v)$ and $u_B(v)$ of the integral does. This Appendix contains a detailed analysis of $C(v)$, whose behavior needs to be understood in order to determine the saddle point contributions in the three cases listed under (28).

1. Envelope of oscillations of B

When the saddle point is far from w_B, we can write $C(w_A, w_B) \sim B(w_A)$, where $B(w_A)$ is displayed in Fig. 3. We first show that the envelope of the oscillations of B scales as $1/w_A$.

Consider, for example, the real part $\Re(B(w_A)) = \alpha \int_{-\infty}^{w_A} w\sin(w^3)dw$, where $\alpha = -i/(\int_{-\infty}^{\infty} we^{-iw^3}dw)$, which oscillates around its asymptotic value of 1 with decreasing amplitude. Let $w_k = (\pi k)^{1/3}$ be the values where its derivative ($\alpha w\sin(w^3)$) vanishes. These points correspond to the relative maxima (for odd values of k) and minima (for even values of k) of $\Re(B(w_A))$. Then, because $w_{k+1} - w_k$ is small, the amplitude of the oscillations around a certain w_k (say k even, for example) is given (for large values of k) by

$$\frac{B(w_{k+1}) - B(w_k)}{2} = \frac{\alpha}{2}\int_{w_k}^{w_{k+1}} w\sin(w^3)dw \sim \frac{\alpha}{2}\left[\frac{-\cos(w^3)}{3w}\right]_{w_k}^{w_{k+1}} \sim \frac{\alpha}{3w_k}. \quad \text{(A4)}$$

We infer that $B(w)$ is of order $1/w$ in the limit $w \to -\infty$. In particular, if w_B is negative and large in absolute value, we have $C(w_A, w_B) \approx B(w_A)$.

2. Contributions to (27) due to a saddle point inside $[u_A(v), u_B(v)]$

Following the discussion in Sec. III, we investigate the contribution due to a saddle point lying between $u_A(v)$ and $u_B(v)$. This corresponds to studying $C(v)$ between the points $v_{A,s}$ and $v_{B,s}$ defined at the end of Sec. IV. Since $\phi'''(u_s) \leqslant 0$, $u_s \in [u_A(v), u_B(v)]$ translates into $w_A(v) \geqslant 0$ and $w_B(v) \leqslant 0$. We need to distinguish between two regimes:

(a) *Saddle point far from both bounds*: $|u_s - u_J(v)||\omega\phi'''(u_s)|^{1/3} \gg 1$
Then we have $w_A(v) \gg 1$ and $|w_B(v)| \gg 1$ so according to the discussion above, $C(v) \approx 1$ up to corrections of the order of $1/w_A$, $1/w_B \ll 1$

(b) *Saddle point near one of the boundaries (e.g. A)*: $|u_s - u_A(v)||\omega\phi'''(u_s)|^{1/3} \lesssim 1$
First, note that the saddle point cannot be close to both bounds at the same time: $(\omega\phi''')^{1/3}$ is of the order of $\omega L^{-2} \gg 1/L$ so $|u_s - u_A(v)| \ll L$; since $|u_A(v) - u_B(v)|$ is of the order of L, $|u_s - u_B(v)|$ has to be of the order of L too so we still have $|w_B(v)| \gg 1$.

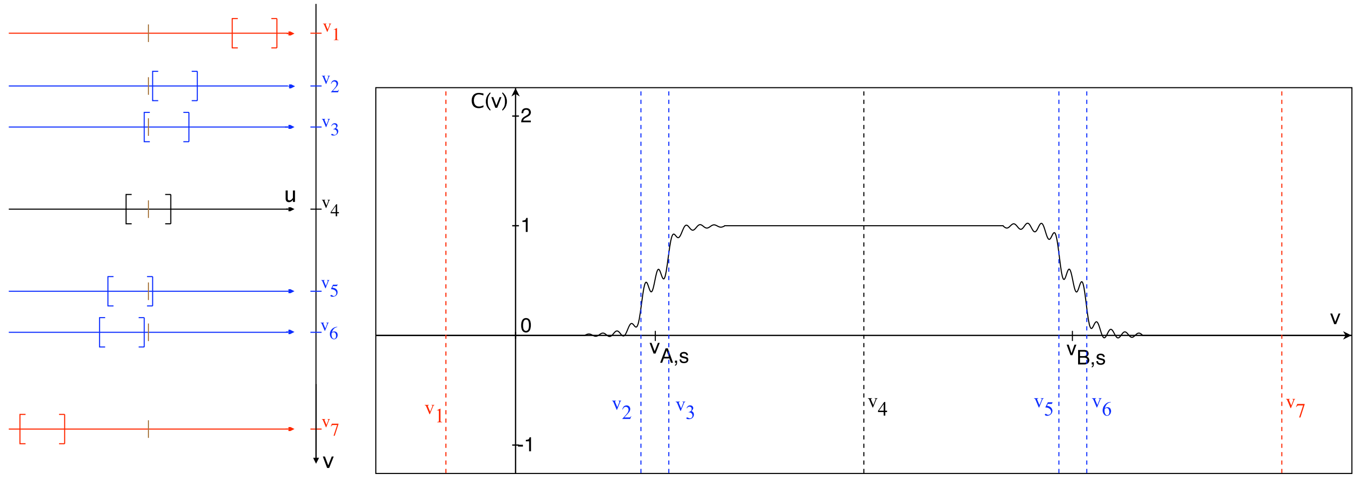

FIG. 5 (color online). Typical behavior of the function $C(v)$. The different cases are represented on the left panel. The left and right brackets, respectively, indicate the position of $u_A(v)$ and $u_B(v)$ on the u axis, while the brown tick is the position of u_s.

Therefore, up to a negligible correction of the order of $1/w_B$, $C(w_A(v), w_B(v)) \approx B(w_A(v))$. This time w_A is of order 1 (and still positive). The behavior of $B(w_A)$ is plotted in Fig. 3. The contribution in this case is of the same order as the one far from the bounds but the result is multiplied by the complex factor $B(w_A)$ of order 1.

3. Contributions to (27) due to a saddle point *outside* $[u_A(v), u_B(v)]$

Up to now we calculated the leading contribution from a saddle point present anywhere *inside* the interval $[u_A(v), u_B(v)]$. Does this mean that for the values of v for which the saddle point lies outside this interval, $I(v)$ discontinuously drops to zero?

We argued in Sec. III that, if $I_i(v)$ obeys all conditions (i)–(iii), then it is negligible because it vanishes exponentially in the infinite ω limit. However, since ω is actually large but finite, there might also exist non-negligible contributions if $I(v)$ is only *close to* satisfying these conditions. In particular, if some saddle point lies just outside the interval of integration, the derivative at the nearest bound is very close to vanishing so we can expect to have some contribution. Another way to say this is that the contribution comes from an interval around the saddle point of size $\approx (L^2/\omega)^{1/3}$, which can intersect $[u_A(v), u_B(v)]$ even if the saddle point is outside it. Obviously, in the case where the saddle point is very far from the interval of integration, the contribution needs to be negligible.

(a) *Saddle point outside* $[u_A(v), u_B(v)]$ *and near one of the boundaries (e.g. A)*:
$|u_s - u_A(v)||\omega\phi'''(u_s)|^{1/3} \lesssim 1$. Here, $w_B \leqslant 0$. For the same reasons as before, we have $|w_B| \gg 1$ and $C(w_A(v), w_B(v)) \approx B(w_A(v))$. The only difference is that now $w_A \leqslant 0$. This is why in Fig. 3 we also plotted B for negative values of its argument.

(b) *Saddle point far from* $[u_A(v), u_B(v)]$
In this case, both w_A and w_B have the same sign and their absolute values are $\gg 1$. According to the discussion in the first section of this Appendix, we have at leading order $C(v) \approx 0$ up to corrections in $1/w_A$, $1/w_B \ll 1$.

4. Summary

It should now be clear that $C(v)$ is a smoothed version of $\theta(v - v_{A,s})\theta(v_{B,s} - v)$. In Fig. 5 we sum up the different cases and show the typical behavior of the function $C(v)$ provided that the derivatives $\frac{du_I}{dv}$ are of order 1, which is the generic case.

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