

Degeneracy between the dark components resulting from the fact that gravity only measures the total energy-momentum tensor

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(Received 27 February 2007; revised manuscript received 3 September 2009; published 2 December 2009)

Gravity probes only the total energy-momentum tensor, which leads to a perfect degeneracy for generalized dark energy models. Because of this degeneracy, Ω_m cannot be measured. We demonstrate this explicitly by showing that the combination of cosmic microwave background and supernova data is compatible with very large and very small values of Ω_m for a specific family of dark energy models. We also show that for the same reason interacting dark energy is always equivalent to a family of non-interacting models. We argue that it is better to face this degeneracy and to parametrize the actual observables.

DOI: 10.1103/PhysRevD.80.123001

PACS numbers: 95.36.+x, 95.35.+d, 98.80.Jk

I. INTRODUCTION

Over the last decades, cosmology has turned into an experimental science, with more and more high-quality data becoming available. The most surprising conclusion from this data is the existence of a dark contribution to the energy density in the Universe, which interacts through gravity with normal matter, and which seems to make up 95% of the total energy density today. To understand the nature of this dark sector is correspondingly considered one of the most important tasks in cosmology.

The line element of a perfectly homogeneous and isotropic space with zero curvature is

$$ds^2 = -dt^2 + a(t)^2 dx^2 \quad (1)$$

with only 1 degree of freedom, the scale factor $a(t)$. The energy-momentum tensor has to be compatible with perfect homogeneity and isotropy, which means that it has to have the form of a perfect fluid,

$$T_{\mu}^{\nu} = \text{diag}(-\rho(t), p(t), p(t), p(t)). \quad (2)$$

There are 2 degrees of freedom in the energy-momentum tensor (EMT), $\rho(t)$ and $p(t)$. The Friedmann equations can only fix the behavior of one of them, conventionally taken to be $\rho(t)$, while the other one is an *a-priori* free function of time, describing the physical properties of the perfect fluid. In cosmology one usually poses $p(t) = w(t)\rho(t)$ so that $w(t)$ is now a free function.

Photons and baryonic matter are detected through their nongravitational interactions, and their contribution to $T_{\mu\nu}$ can be measured directly. But the dark sector, by definition, is only constrained through gravity, which depends only on the *total* energy-momentum tensor. Gravity therefore only constrains the total $w(z)$. Any further freedom, like subdividing the dark EMT into dark matter and dark energy, or

introducing couplings between the dark constituents, cannot be directly measured and will introduce degeneracies.

II. BACKGROUND ON CONSTRAINTS ON Ω_m AND $w(z)$

One generally postulates that the “energy excess” in the Universe, the dark matter and dark energy, are two different components, with the dark matter being characterized by $w_m = 0$ and a relative energy density today of $\Omega_m = 8\pi G\rho_m(t_0)/(3H_0^2)$.

However, nothing stops us from adding the energy-momentum tensor of the dark energy and of the dark matter together into a combined “dark fluid” EMT. This provides also a solution of the Friedmann equations, but with a different equation of state. Worse, we can just as well split that dark fluid EMT *arbitrarily* into one part with $w = 0$ and another part with a time-varying equation of state [1–3]. To show this explicitly, we notice that for a flat universe composed of matter and dark energy with unknown $w(z)$, and given $H(z)$, we find from the Friedmann equations (see e.g. [3–5])

$$w(z) = \frac{H(z)^2 - \frac{2}{3}H(z)H'(z)(1+z)}{H_0^2\Omega_m(1+z)^3 - H(z)^2} \quad (3)$$

where we used the observationally more relevant redshift z as the time variable, and $H' = dH/dz$. This means that for *any* choice of Ω_m we find a $w(z)$ which reproduces the measured expansion history of the Universe. In other words, Ω_m cannot be measured if $w(z)$ is not known [6]. Although this has been noticed before, it seems to have been forgotten subsequently and it is worth demonstrating it explicitly before extending the result beyond this simplest case.

The fundamental point that dark constituents cannot be distinguished does not change if we add radiation and baryons. Their abundance can be measured in different ways thanks to their interactions. Curvature is also a dark

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component and exhibits a similar degeneracy [7,8], but it can at least in theory be distinguished from the other dark components due to its different geometric structure [9,10]. We will therefore limit this discussion to flat space.

To illustrate the real nature of this fundamental degeneracy we first notice that a flat universe composed of dark matter with density parameter $\hat{\Omega}_m$ and a fluid with equation of state parameter \hat{w} given by

$$\hat{w}(z) = -\frac{1 - \Omega_m}{(1 - \Omega_m) + (\Omega_m - \hat{\Omega}_m)(1 + z)^3} \quad (4)$$

leads to exactly the same expansion history $H(z)$ as a flat Λ CDM model with a dark matter density parameter of Ω_m . Therefore, knowing that Λ CDM provides a good fit to current data for $\Omega_m \approx 0.25$, we expect that the above model gives a fit that is just as good for any $\hat{\Omega}_m$ as long as we adjust \hat{w} accordingly. Inspired from this result we set $w(z) = -1/(1 - \lambda(1 + z)^3)$ and try to measure simultaneously λ and Ω_m from the Supernova Legacy Survey (SNLS) data [11] and the $R_{0.35}$ constraint from the baryon acoustic oscillations (BAO) measured by Sloan Digital Sky Survey Luminous Red Galaxy (SDSS LRG) data [12]. As expected, we find a strong degeneracy and no limit on Ω_m , see Fig. 1. This does not change if we add further background data, as this is a fundamental degeneracy that is present in *all* probes of the expansion history.

For our choice of $H(z)$ we find, not surprisingly, that $\rho_{\text{DE}}(z) \propto 1 - \lambda(1 + z)^3$. In the range $\lambda < 0$ the dark energy absorbs part of the dark matter, and becomes more similar to it. For $\lambda > 0$ the dark energy has to evolve in the different direction, becoming a kind of “anti-dark matter.”

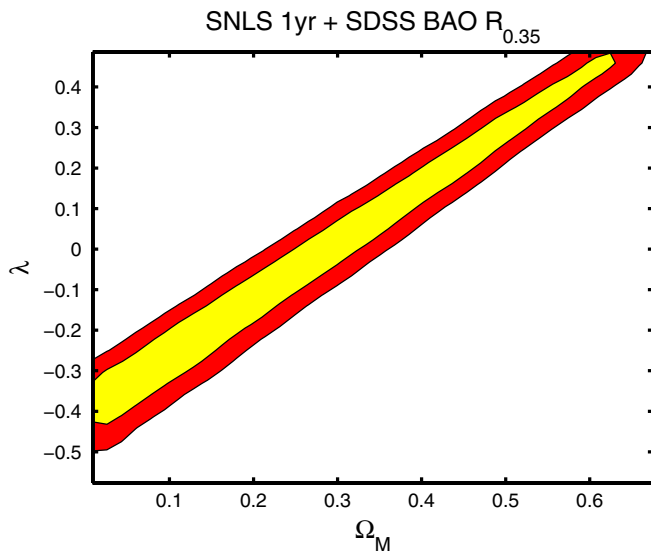


FIG. 1 (color online). We try to determine Ω_m with the SNLS 1 yr data and the $R_{0.35}$ constraint from the baryon acoustic oscillations measured by SDSS LRG, but using an equation of state that exhibits a degeneracy between dark matter and dark energy. We find that the background data cannot determine Ω_m .

In order to achieve this, ρ_{DE} becomes negative. This may appear strange, but is very similar to the behavior of a scalar field in a potential which is negative for some field values. Apparently negative energy-densities also appear in some theories of modified gravity with a nonstandard Friedmann equation [13,14]. Since we have derived the form of $w(z)$ from a well-behaved $H(z)$ we know that even an apparently strange equation of state leads to a well-behaved expansion history of the Universe.

Measuring only the expansion history of the Universe therefore does not allow us to make separate statements about the dark matter and the dark energy. Rather, Ω_m becomes a parameter which enumerates a family of dark energy models according to Eq. (4). All members of this family lead to exactly the same expansion history. From an experimental point of view, they should be regarded as forming an equivalence class of models. It is possible that all the matter is baryonic, and that the dark energy is characterized by $w(z) = -1/(1 + 0.3(1 + z)^3)$. Analyzing this scenario with the usual parametrizations of the equation of state, we would be led to conclude wrongly that $\Omega_m \approx 0.3$ and that the dark energy has a fine-tuning problem.

If we assume that there is a period of matter domination at high redshift so that $H(z) \propto (1 + z)^{3/2}$, then the numerator of Eq. (3) is zero and so the behavior of the dark energy approaches that of matter at high redshift. This can only be avoided if the denominator vanishes as well, which singles out one specific value of Ω_m for which w does not go to zero at high redshift. It is this value which is conventionally considered to be the “true” one, but we have to be aware that this is a philosophical choice which cannot be supported by experimental evidence. Furthermore, this is the choice which, by construction, creates a fine-tuning problem for the dark energy, as its relative density will decrease with z .

Given this degeneracy, we can still compare models and exclude those which agree significantly worse with data than others. However, we cannot actually measure quantities like Ω_m and $w(z)$ with background data alone.

It is also worrying that this degeneracy seems to have escaped notice of the numerous analyses trying to constrain $w(z)$ with supernova data and other distance measures. This illustrates once more that parametrizations impose strong priors on the kind of dark energy models probed, as e.g. argued in [15].

III. INTERACTING DARK ENERGY MODELS

The above effect has also implications for other models, for example, those where the dark energy and the dark matter are interacting. In this case their total energy-momentum tensor has to be conserved for consistency with the Bianchi identities of the Einstein tensor (e.g. [16]),

$$(T_{\mu\nu}^{(m)} + T_{\mu\nu}^{(\text{DE})})_{;\mu} = 0. \quad (5)$$

But this means that the total EMT is just of the same class as the one discussed in the previous section. We can either keep it as a single unified dark fluid model, we can divide it into a coupled dark matter—dark energy system, or we can also divide it into uncoupled dark matter and dark energy. The only quantity that we measure is again $H(z)$ and this only fixes the *total* EMT and does not tell us anything about how it should be split. Correspondingly the interaction between dark matter and dark energy is also perfectly degenerate with the dark energy $w(z)$. In other words, we cannot measure it unless we fix *both* Ω_m and $w(z)$.

What happens is the following: The interaction modifies the conservation equations of the two dark components,

$$\dot{\rho}_m + 3H\rho_m = C(t) \quad (6)$$

$$\dot{\rho}_{\text{DE}} + 3H(1+w)\rho_{\text{DE}} = -C(t). \quad (7)$$

The sum of the two equations has to be zero in order to satisfy Eq. (5). These two equations, together with one of the Friedmann equations, determine $H(z)$. From $H(z)$ we can then derive a family of uncoupled models, using Eq. (3), or also families of models with other interactions.

Let us look in more detail at an especially simple case that is often used, e.g. [17], where $C = \gamma H\rho_m$ and γ is constant, and we also take w to be constant. This makes it easy to integrate the above equations, giving

$$\rho_m = \rho_m^{(0)}(1+z)^{3-\gamma} \quad (8)$$

$$\rho_{\text{DE}} = \left(\rho_{\text{DE}}^{(0)} + \rho_m^{(0)} \frac{\gamma}{\gamma + 3w} \right) (1+z)^{3(1+w)} - \rho_m^{(0)} \frac{\gamma}{\gamma + 3w} (1+z)^{3-\gamma} \quad (9)$$

and the Hubble parameter is (assuming flatness again)

$$H^2 = H_0^2 \left[\Omega_m \left(1 - \frac{\gamma}{\gamma + 3w} \right) (1+z)^{3-\gamma} + \left(1 - \frac{3\Omega_m w}{\gamma + 3w} \right) (1+z)^{3(1+w)} \right]. \quad (10)$$

In the case where our data is actually due to a decaying cosmological constant ($w = -1$) with $\Omega_m = 0.3$ and a constant γ , we find that we can just as well fit it with uncoupled dark matter and dark energy with an equation of state

$$\hat{w}(z) = \frac{-0.3\gamma + (\gamma - 2.1)(1+z)^{-3+\gamma}}{0.9 + (1+z)^\gamma(\hat{\Omega}_m(\gamma - 3) - (\gamma - 2.1)/(1+z)^3)} \quad (11)$$

We are again free to choose an apparent $\hat{\Omega}_m$ different from 0.3. Conversely, given a noninteracting dark matter–dark energy model with a certain w , we can pretend that we are actually dealing with a coupled cosmological constant by solving Eq. (11) for γ . (Although one would have to

generalize the above discussion to time-varying couplings in order to do that.) As has been noticed before [18,19] this could be used to replace phantom dark energy (with $w < -1$) with a nonphantom interacting model.

It is only possible to constrain the couplings by imposing a prior on the space of possible models. It is important that we are aware of this limitation, as we do not know the nature of the dark energy. We can always trade off a specific form of the interaction against a different $w(z)$ and a change in Ω_m . Finally, coupling dark matter and dark energy does not lead to any new phenomena in the dark sector, beyond those which can be achieved by general uncoupled dark energy.

IV. BEYOND THE BACKGROUND

Is this degeneracy just a problem at the background level, and can it be broken when we take into account that the Universe is not homogeneous and isotropic? In general, it cannot: the fundamental reason of this “dark degeneracy” stems directly from the structure of the full Einstein equations,

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}. \quad (12)$$

The Einstein tensor $G_{\mu\nu}$ on the left-hand side describes the geometric aspects of general relativity, while the energy-momentum tensor on the right-hand side defines the energy and pressure content. Although the equations are highly nonlinear in $g_{\mu\nu}$, they are linear in the components of $T_{\mu\nu}$. If the only information on a part of $T_{\mu\nu}$ comes from gravitational probes, as is true by definition for “dark stuff,” then we can decompose this part in any way we want—we cannot learn anything about the subparts, only about the whole.

As an illustration, in first-order perturbation theory the dark fluids influence the “bright side” through the gravitational potentials ϕ and ψ , which describe the scalar metric perturbations. The physical properties of fluids are described by two additional quantities, for example, the pressure perturbations δp and the anisotropic stresses Π . They can be different for each fluid, but as discussed e.g. in [20,21] it is always possible to combine several fluids into a single one with an effective δp and Π . This single fluid then contributes in exactly the same way to ϕ and ψ as its constituent fluids. Conversely, any fluid can be split in a basically arbitrary way into subfluids.

We can escape the degeneracy by considering specific models, for example, scalar field dark energy for which we know that δp is given by a rest-frame sound speed $c_s^2 = 1$, and $\Pi = 0$. In this way we basically define the dark energy to be the dark part which does not cluster. This may be a reasonable way to break the degeneracy, but we should not forget that it may well be that there is only one dark fluid that clusters partially, rather than one strongly clustering and one nonclustering part. Also modifications of gravity like DGP can act effectively like a clustering form of dark

energy [22]. We show in Fig. 2 that indeed nonclustering dark energy leads to a well-defined Ω_m when using the Wilkinson Microwave Anisotropy Probe 3-yr cosmic microwave background (CMB) data [23] together with the SNLS 1 yr supernova data (open contours) and using the equation of state parameter (4).

In 2004 Sandvik *et al.* [24] concluded that many unified models are ruled out due to the excessive fine-tuning required, based on an analysis of Chaplygin gas models. Since unified models lie also on the dark degeneracy, in the limit $\Omega_m \rightarrow 0$, is it possible to break the degeneracy with the same argument? Indeed, δp changes in a precise way along the degeneracy, and the expression can be quite complicated in any specific example. For the Λ CDM mimicry model, the dark matter has $w = c_s^2 = 0$ and the cosmological constant does not carry perturbations so that we find $\delta p = 0$ in the Newtonian gauge for the “degeneracy fluid.” Enforcing this condition precisely would require an unnatural looking choice of sound speed since $\partial_t \hat{w} \neq 0$ in Eq. (4). On the other hand, the required sound speed is small, and a vanishing rest-frame sound speed, $c_s^2 = 0$, would not look unnatural or fine-tuned. Indeed, this choice restores the degeneracy (filled contours of Fig. 2). Does dark energy cluster? We do not know. Figure 2 shows that clustering dark energy with $c_s^2 = 0$ is perfectly compatible with CMB and supernova data. This may change as the data improves, in which case fine-tuning arguments may point the way to the true model. But for the fundamental reasons given here, it will still not be possible to provide experimental proof from cosmological measurements alone.

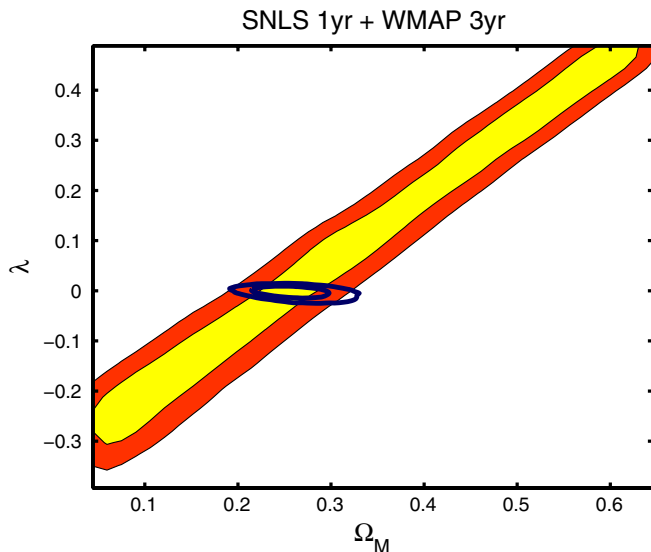


FIG. 2 (color online). This figure shows that supernova and CMB data together cannot measure Ω_m for generalized dark energy models. The filled contours show 1 and 2σ limits for a model with $c_s^2 = 0$, while the open contours show the limits for an effective scalar field model. The lower limit on Ω_m is due to the baryons which we know to exist.

These results, derived using a modified version of CAMB [25] also illustrate the dangers of using standard parametrizations of experimental results for studying non-standard dark energy models. The shift parameter R of the CMB as well as A parameter of BAO data contain Ω_m explicitly. They are therefore only valid for very specific models, and would wrongly rule out the clustering dark energy model shown in Fig. 2 for most values of Ω_m .

As argued above using the full Einstein equation, this game can be played to all orders. For example, galaxy rotation curves fix the amount of clustered dark stuff. So we can use this to determine the amount of dark matter only if we (arbitrarily) impose that dark energy does not cluster. Stars feel gravity, not the dark matter itself.

V. CONCLUSIONS

Gravity only responds to the total energy-momentum tensor $T_{\mu\nu}$. We have been able to measure the physical properties of some of its constituents, like baryons and photons, in laboratory experiments, and their contribution to $T_{\mu\nu}$ can be separated out. However, by definition the dark parts do not show up in laboratory experiments and interact only with gravity, so that we can only probe the total dark EMT. With probes of the background evolution, we can measure for example $H(z)$, corresponding to the overall dark equation of state parameter $w_{\text{tot}}(z)$. At the level of first-order perturbation theory, the observables are, for example, the gravitational potentials ϕ and ψ , or the overall anisotropic stress Π and the pressure perturbations δp of the dark sector.

Conventionally, the dark sector is subdivided into dark matter and dark energy. Here we have shown that, being in a state of total ignorance about the nature of a single one of the dark components, we can also not completely measure the others. In this situation the separation into dark matter and dark energy becomes merely a convenient parametrization without experimental reality. Indeed, we need to impose a specific condition to make this split well-defined: for example, that the dark energy has to vanish at high redshift, or that the dark energy constitutes the nonclustering part of the dark EMT. Such a split is useful for *testing* specific models of the dark constituents, but gravity alone cannot *measure* their physical properties independently.

We have also demonstrated the dangers of using fitting formulas for experimental results, like the peak locations in the CMB or the baryonic oscillations. Great care has to be taken when analyzing nonstandard dark energy models with such formulas, and it is generally preferable to err on the side of caution and to recalculate the actually measured experimental quantities *ab-initio*.

We now need either a theoretical breakthrough which produces a well-motivated model of all things dark and of their properties, which agrees with the data, or else a direct (nongravitational) detection of the dark matter, for example, with the LHC at CERN or with scattering experi-

ments. Fixing the abundance and evolution of the dark matter (and putting strong constraints on its couplings to other constituents) allows us to remove the word “dark” from its name and subsequently to probe the remaining part of $T_{\mu\nu}$ which describes the dark energy—assuming that such a split is realized in nature, which we do not know yet. Dark matter experiments are therefore of the highest importance for dark energy studies as well. Without their results, we can only deduce the *overall* properties of the dark side from cosmological data and state that they are compatible (or not) with a given model (e.g. Λ CDM). At best cosmological measurements can break the dark de-

generacy with the help of fine-tuning arguments. This seems rather unsatisfactory for such a fundamental question, and current data is fully compatible with a simple model where the dark energy has a vanishing sound speed.

ACKNOWLEDGMENTS

It is a pleasure to thank Luca Amendola, Ruth Durrer, Andrew Liddle, David Parkinson and Domenico Sapone for numerous interesting discussions. M. K. acknowledges funding by the Swiss NSF and STFC (United Kingdom).

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