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No evidence for dark energy dynamics from a global analysis of cosmological data

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We use a variant of principal component analysis to investigate the possible temporal evolution of the dark energy equation of state, $w(z)$. We constrain $w(z)$ in multiple redshift bins, utilizing the most recent data from type Ia supernovae, the cosmic microwave background, baryon acoustic oscillations, the integrated Sachs-Wolfe effect, galaxy clustering, and weak lensing data. Unlike other recent analyses, we find no significant evidence for evolving dark energy; the data remain completely consistent with a cosmological constant. We also study the extent to which the time evolution of the equation of state would be constrained by a combination of current- and future-generation surveys, such as Planck and the Joint Dark Energy Mission.

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I. INTRODUCTION

One of the defining challenges for modern cosmology is understanding the physical mechanism responsible for the accelerating expansion of the Universe [1–3]. The origin of the cosmic acceleration can be due to a new source of stress energy, called "dark energy," a modified theory of gravity, or some mixture of both [4,5]. Careful measurement of the expansion history of the Universe as a function of cosmic epoch is required to elucidate the source of the acceleration. In particular, existing data already allow direct exploration of possible time variation of the dark energy equation of state (EOS).

While several recent papers have investigated the possibility of constraining the temporal evolution of dark energy (see, e.g., [6]), here we present an analysis improving and/or complementing existing work in two ways: First, we incorporate important recent data releases, including type Ia supernovae samples ("Constitution" and "Union" data sets) and baryon acoustic oscillation data (SDSS Data Release 7). These new data provide significant improvements in the dark energy constraints. Second, we utilize principal component analysis techniques to constrain the dark energy in a model independent manner, leading to more robust and unbiased constraints.

In the absence of a widely accepted model for dark energy, it is generally assumed that the dark energy EOS (the ratio of pressure to energy density) evolves with redshift with an arbitrary functional form [7]. Common parametrizations include a linear variation, $w(z) = w_0 + w_z z$ [8], or an evolution that asymptotes to a constant w at high redshift, $w(z) = w_0 + w_a z/(1+z)$ [9,10]. As we describe below, we extend the arbitrary description of the equation of state to include perturbations in the dark energy component when $w(z)$ is not -1. Given our complete ignorance of the underlying physical processes, we approach our analysis of dark energy with a minimum of assumptions. Fixing an *ad hoc* two parameter form could lead to bias in our inference of the dark energy properties.

In this paper we measure the evolution history of the dark energy using a flexible and almost completely model independent approach, based on a variant of the principal component analysis introduced in [11]. We determine the EOS parameter, $w(z)$, in five uncorrelated redshift bins, following the analysis presented in [12–15]. To be conservative, we begin by using data only from geometric probes of dark energy, namely, the cosmic microwave background radiation (CMB), type Ia supernovae (SNe), and baryon acoustic oscillation data (BAO). We perform a full likelihood analysis using the Markov chain Monte Carlo approach [16]. We then consider constraints on $w(z)$ from a larger combination of data sets, including probes of the growth of cosmological perturbations, such as large scale structure (LSS) data. An important consideration for such an analysis is to properly take into account dark energy perturbations, and we make use of the prescription introduced in [17]. We also generate mock data sets for future experiments, such as the Joint Dark Energy Mission (JDEM) and Planck, to see how much they improve the constraints.

II. ANALYSIS

The method we use to constrain the dark energy evolution is based on a modified version of the publicly available Markov chain Monte Carlo package COSMOMC [16], with a convergence diagnostics based on the Gelman-Rubin criterion [18]. We consider a flat cosmological model de-

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scribed by the following set of parameters:

$$\{w_i, \omega_b, \omega_c, \Theta_s, \tau, n_s, \log[10^{10}A_s]\}, \tag{1}$$

where ω_b ($\equiv \Omega_b h^2$) and ω_c ($\equiv \Omega_c h^2$) are the physical baryon and cold dark matter densities relative to the critical density, Θ_s is the ratio of the sound horizon to the angular diameter distance at decoupling, τ is the optical depth to reionization, and A_s and n_s are the amplitude of the primordial spectrum and the spectral index, respectively.

As discussed above, we bin the dark energy EOS in five redshift bins, $w_i(z)$ $(i = 1, 2, \ldots 5)$, representing the value at five redshifts, $z_i \in [0.0, 0.25, 0.50, 0.75, 1.0]$. We have explicitly verified that the use of more than five bins does not significantly improve the dark energy constraints. We need $w(z)$ to be a smooth, continuous function, since we evaluate $w'(z)$ in calculating the dark energy perturbations (and their evolution with redshift). We thus utilize a cubic spline interpolation to determine values of $w(z)$ at redshifts in between the values z_i.

For $z > 1$ we fix the EOS parameter at its $z = 1$ value, since we find that current data place only weak constraints on $w(z)$ for $z > 1$. To summarize, our parametrization is given by

$$w(z) = \begin{cases} w(z = 1), & z > 1 \\ w_i, & z \leq z_{\max}, z \in \{z_i\} \\ \text{spline}, & z \leq z_{\max}, z \notin \{z_i\}. \end{cases} \tag{2}$$

When fitting to the temporal evolution of the dark energy EOS using cosmological measurements that are sensitive to density perturbations, such as LSS or weak lensing, one must take into account the presence of dark energy perturbations. To this end, we make use of a modified version of the publicly available code CAMB [19], with perturbations calculated following the prescription introduced by [17]. This method implements a parametrized post-Friedmann prescription for the dark energy perturbations following [20,21].

Moreover, the dark energy EOS parameters $\mathbf{w} = w_i$ are correlated; we follow [12,13] to determine uncorrelated estimates of the dark energy parameters. Our first analysis considers constraints from "geometric" data: CMB, type Ia SN luminosity distances, and BAO data. We subsequently include data sets that probe the growth of cosmic structures, incorporating weak lensing, as well as integrated Sachs-Wolfe measurements through cross correlations between CMB and galaxy survey data. We include the latter data sets separately, since our understanding of the cosmic clustering in dark energy models still suffers from several limitations. These LSS uncertainties are mainly related to our poor understanding of both the bias between galaxies and matter fluctuations (with a possible scale dependence of the bias itself, see [22–24]) and nonlinearities at small redshifts (see [25,26]). For the CMB, we use data and the likelihood code from the WMAP team's 5-year release [25] (both temperature TT and polarization TE; we will refer to this analysis as WMAP5). In this respect, our approach is more extensive than that in [6] and other recent studies, since we fully consider the CMB data set instead of simply using the constraint on the θ parameter from the analysis of [25]. This constraint is model dependent (see, e.g., [27]), and changes with dark energy parametrizations.

Supernova data come from the Union data set (UNION) produced by the Supernova Cosmology Project [28]; however, to check the consistency of our results, we also used the recently released Constitution data set (Constitution) [29] which, with 397 type Ia supernovae, is the largest sample to date. We also used the latest SDSS release (DR7) BAO distance scale [30,31]: at $z = 0.275$ we have $r_s(z_d)/D_V(0.275) = 0.1390 \pm 0.0037$ [where $r_s(z_d)$ is the comoving sound horizon at the baryon drag epoch, $D_V \equiv [(1+z)^2 D_A^2 cz/H(z)]^{1/3}$, $D_A(z)$ is the angular diameter distance, and $H(z)$ is the Hubble parameter] and the ratio of distances $D_V(0.35)/D_V(0.20) = 1.736 \pm 0.065$. Weak lensing (WL) data are taken from CFHTLS [32] and we use the weak lensing module provided in [33,34], with some modifications to assess the likelihood in terms of the variance of the aperture mass (Eq. 5 of [32]) with the full covariance matrix [35]. The cross correlation between CMB and galaxy survey data is employed using the public code at [36]. We modify it to take into account the temporal evolution of the dark energy EOS, since the code only considers *w*CDM cosmologies. We refer to [37,38] for a description of both the methodology and the data sets used. Finally, we use the recent value of the Hubble constant from the SHOES (Supernovae and H_0 for the Equation of State) program, $H_0 = 74.2 \pm 3.6\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ (1σ) [39], which updates the value obtained from the Hubble Key Project [40]. We also incorporate baryon density information from big bang nucleosynthesis $\Omega_b h^2 = 0.022 \pm 0.002$ (1σ) [41], as well as a top-hat prior on the age of the Universe, $10\ \mathrm{Gyr} < t_0 < 20\ \mathrm{Gyr}$.

III. RESULTS

In Table I we show the mean values and marginalized 68% confidence level limits for the cosmological parameters considered in this analysis for the WMAP + SNe(UNION) + BAO and WMAP + SNe(Constitution) + BAO data sets. We also consider a "global" data set: WMAP + SNe + BAO + CFHTLS + CMB + WL + ISW + LSS. The $w_i(z)$ $(i = 1, 2, \ldots 5)$ entries refer to the uncorrelated values of the dark energy EOS parameters. All values are compatible with a cosmological constant $(w = -1)$ at the 2σ level; Table II presents the best-fit parameters for the WMAP + UNION + BAO analysis, as well as for the global data set. The goodness of fit for each case can be expressed as the χ^2 per degree of freedom, $\chi^2_{\mathrm{red}} = \chi^2/\nu$; for WMAP + UNION + BAO, we have $\chi^2 = 2968.8$ for $\nu = 2864$, which translates into $\chi^2_{\mathrm{red}} = 1.04$; in the case

TABLE I. Mean values and marginalized 68% confidence levels for the cosmological parameters. The set of $w(z)_i$ represents the measured values of the dark energy EOS in uncorrelated redshift bins.

Parameter	WMAP + UNION + BAO	WMAP + Constitution + BAO	All data sets	Future data sets
$\Omega_b h^2$	0.02281 ± 0.00057	0.02278 ± 0.00058	0.02304 ± 0.00056	0.02270 ± 0.00015
$\Omega_c h^2$	0.1128 ± 0.0059	0.1144 ± 0.0060	0.1127 ± 0.0018	0.1100 ± 0.0012
Ω_Λ	0.728 ± 0.018	0.715 ± 0.017	0.728 ± 0.016	0.751 ± 0.008
n_s	0.964 ± 0.014	0.963 ± 0.014	0.971 ± 0.014	0.962 ± 0.004
τ	0.085 ± 0.017	0.084 ± 0.016	0.088 ± 0.017	0.084 ± 0.05
Δ_R^2	$(2.40 \pm 0.10) \times 10^{-9}$	$(2.40 \pm 0.10) \times 10^{-9}$	$(2.40 \pm 0.10) \times 10^{-9}$	$(2.40 \pm 0.10) \times 10^{-9}$
$w(z = 1.7)$	$\cdots$	$\cdots$	$\cdots$	$-1.55^{+0.46}_{-0.44}$
$w(z = 1)$	$-1.72^{+0.73}_{-0.81}$	$-1.68^{+0.73}_{-0.85}$	$-1.07^{+0.21}_{-0.20}$	-1.03 ± 0.10
$w(z = 0.75)$	$-0.71^{+0.44}_{-0.47}$	$-0.47^{+0.34}_{-0.33}$	$-0.86^{+0.025}_{-0.26}$	-0.98 ± 0.08
$w(z = 0.5)$	$-0.65^{+0.29}_{-0.30}$	$-1.06^{+0.41}_{-0.40}$	-0.86 ± 0.14	-1.00 ± 0.05
$w(z = 0.25)$	-1.05 ± 0.10	-1.04 ± 0.07	-1.00 ± 0.07	-1.00 ± 0.02
$w(z = 0)$	-0.97 ± 0.22	-0.86 ± 0.13	$-1.02^{+0.17}_{-0.18}$	-0.99 ± 0.05
σ_8	0.814 ± 0.055	0.815 ± 0.057	0.810 ± 0.024	0.811 ± 0.012
Ω_m	0.272 ± 0.018	0.285 ± 0.017	0.272 ± 0.016	0.249 ± 0.008
H_0	70.7 ± 2.0	69.4 ± 1.7	70.8 ± 2.0	73.1 ± 1.0
z_{reion}	10.8 ± 1.4	10.8 ± 1.4	11.0 ± 1.5	10.7 ± 0.4
t_0	13.65 ± 0.14	13.67 ± 0.15	13.67 ± 0.13	13.60 ± 0.06

WMAP + Constitution + BAO we have $\chi^2 = 3131.4$ for $\nu = 2954$ so that $\chi^2_{\mathrm{red}} = 1.06$.[1]

As we can see from Table I and from Fig. 1, there is no discrepancy between the Union and Constitution data sets; moreover, the addition of cosmological probes of cosmic clustering noticeably reduces the uncertainty in the determination of the dark energy parameters, especially at high redshifts. We also checked that the use of a different number of bins does not sensibly affect our results; results from the Markov chain Monte Carlo runs with two, three, or four bins and using the combination of WMAP + UNION + BAO data sets are always compatible with a cosmological constant. When using only one bin, we get $w = -0.97 \pm 0.08$; using bins at $z = [1, 0]$ we have $w(z = 1) = -0.95^{+0.12}_{-0.11}$ and $w(z = 0) = -1.03 \pm 0.10$; with bins at $z = [1, 0.5, 0]$ we get $w(z = 1) = -1.27^{+0.77}_{-0.75}$, $w(z = 0.5) = -0.96 \pm 0.11$, and $w(z = 0) = -1.11^{+0.14}_{-0.17}$; and finally, for $z = [1, 0.5, 0.25, 0]$ we obtain $w(z = 1) = -1.42^{+0.50}_{-0.55}$, $w(z = 0.5) = -0.69^{+0.25}_{-0.23}$, $w(z = 0.25) = -1.06 \pm 0.11$, $w(z = 0) = -0.96 \pm 0.22$.

To reinforce our conclusions, we also created several mock data sets for upcoming and future SN, BAO, and CMB experiments. The quality of future data sets allows us to constrain the dark energy evolution beyond redshift $z = 1$. We thus consider an additional bin at $z = 1.7$, with a similar constraint: $w(z > 1.7) = w(z = 1.7)$. We consider a mock catalog of 2298 SNe, with 300 SNe uniformly distributed out to $z = 0.1$, as expected from ground-based low redshift samples, and an additional 1998 SNe binned in 32 redshift bins in the range $0.1 < z < 1.7$, as expected from JDEM or similar future surveys [42]. The error in the distance modulus for each SN is given by the intrinsic error, $\sigma_{\mathrm{int}} = 0.1$ mag. In generating the SN catalog, we do not include the effect of gravitational lensing, as this is expected to be small [43]. In addition, we use a mock catalog of 13 BAO estimates, including 2 BAO estimates at $z = 0.2$ and $z = 0.35$, with 6% and 4.7% uncertainties (in D_V), respectively, 4 BAO constraints at $z = [0.6, 0.8, 1.0, 1.2]$ with corresponding fiducial survey precisions (in D_V) of [1.9, 1.5, 1.0, 0.9]% (V5N5 from [44]), and 7 BAO estimates with precision [0.36, 0.33, 0.34, 0.33, 0.31, 0.33, 0.32]% from $z = 1.05$ to $z = 1.65$ in steps of 0.1 [45].

We simulate Planck data using a fiducial ΛCDM model, with the best-fit parameters from WMAP5, and noise properties consistent with a combination of the Planck 100–143–217 GHz channels of the high frequency instru-

TABLE II. Maximum likelihood values for the set of basic parameters considered in the analysis.

Parameter	WMAP + UNION + BAO	Global data set
$\Omega_b h^2$	0.02293	0.02307
$\Omega_c h^2$	0.1087	0.1131
Ω_Λ	0.732	0.732
n_s	0.964	0.979
τ	0.085	0.083
Δ_R^2	2.38×10^{-9}	2.31×10^{-9}
$w(z = 1)$	-1.26	-1.22
$w(z = 0.75)$	-0.21	-0.82
$w(z = 0.5)$	-0.63	-0.79
$w(z = 0.25)$	-1.07	-0.96
$w(z = 0)$	-0.94	-0.98

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[1] We note that in general any χ^2 based on supernovae measurements gives us only a rough idea of the goodness of fit, as the published supernovae errors are scaled to ensure an appropriate χ^2 in the fit.

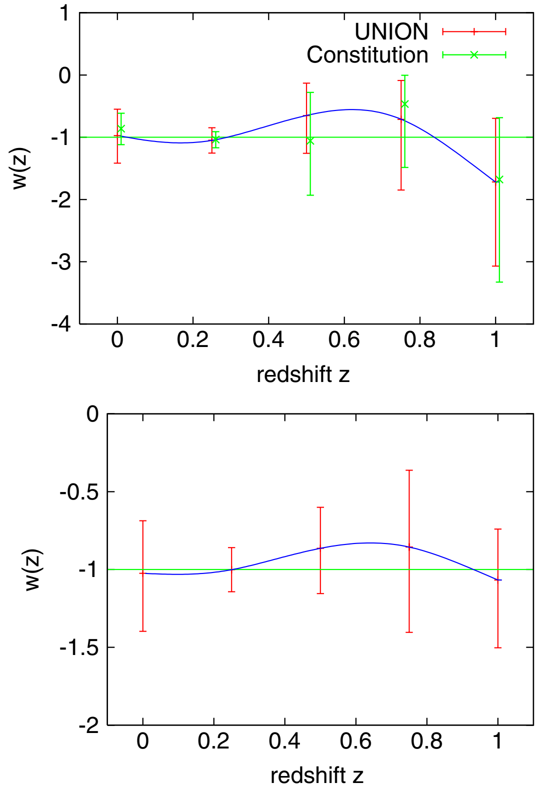

FIG. 1 (color online). The upper panel shows uncorrelated constraints on the dark energy EOS parameters using both WMAP + UNION + BAO and WMAP + Constitution + BAO; the points for the Constitution data set have been slightly shifted to facilitate comparison between the two cases: we find no significant difference between UNION and Constitution. The dark (blue) line is the reconstructed $w(z)$ using a cubic spline interpolation between the nodes and it is related to WMAP + UNION + BAO; the lower panel shows results from a "global" data set given by WMAP + UNION + BAO + WL + ISW + LSS; error bars are always at 2σ.

ment [46], and fitting for temperature and polarization using the full-sky likelihood function given in [47]. In addition, we use the same priors on the Hubble parameter and on the baryon density as considered above. As can be seen from Table I and Fig. 2, future data will reduce the uncertainties in w_i by a factor of at least 2, with the relative uncertainty below 10% in all but the last bin (at $z = 1.7$).

IV. CONCLUSIONS

One of the main tasks for present and future dark energy surveys is to determine whether or not the dark energy density is evolving with time. We have performed a global analysis of the latest cosmological data sets, and have constrained the dark energy equation of state using a very flexible and almost model independent parametrization. We determine the equation of state $w(z)$ in five independent redshift bins, incorporating the effects of dark energy perturbations. We find no evidence for a temporal evolution of dark energy—the data are completely consistent with a cosmological constant. Our result agrees with previous findings [48–51], while significantly improving the overall constraints on dark energy EOS [12–15].

Bayesian evidence models strongly suggest that the dark energy is a cosmological constant, given that the cosmological constant remains a very good fit to the data as the number of dark energy parameters increases (see e.g. [52] and references therein). We show that future experiments, such as Planck or JDEM, will be able to reduce the uncertainty on $w(z)$ to less than 10% in multiple redshift bins, thereby mapping any temporal evolution of dark energy with high precision. With these data it will be possible to measure the temporal derivative of the equation of state parameters, dw/dz, useful in discriminating between two broad classes of "thawing" and "freezing" models [7].

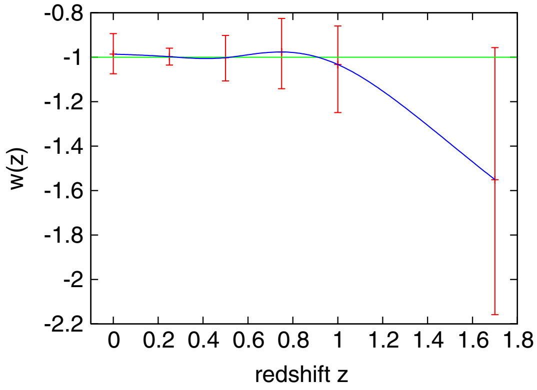

FIG. 2 (color online). Uncorrelated constraints on the dark energy equation of state parameters, for mock data sets from Planck and JDEM; error bars are at 2σ.

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Note added.—As we were completing this paper we became aware of the work reported in [53], which considers a similar analysis of cosmological data to constrain $w(z)$. While the authors of that work find weak evidence for evolution of the EOS, we find no such evidence. The two analyses differ in the way $w(z)$ is interpolated (we use a spline, while they employ a tanh function), as well as by the different calculations of the effects of dark energy perturbations. Furthermore, we analyze different data sets; in this paper we have utilized both the latest BAO measurements [30,31] and the latest value of the Hubble constant from the SHOES program [39].

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