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$\Upsilon(1S) \to \gamma f_2(1270)$ decay

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Decay $\Upsilon(1S) \to \gamma f_2(1270)$ is studied by an approach in which the tensor meson, $f_2(1270)$, is strongly coupled to gluons. Besides the strong suppression of the amplitude $\Upsilon(1S) \to \gamma gg$, $gg \to f_2$ by the mass of the b-quark, d-wave dominance in $\Upsilon(1S) \to \gamma f_2(1270)$ is revealed from this approach, which provides a large enhancement. The combination of these two factors leads to larger $B(\Upsilon(1S) \to \gamma f_2(1270))$. The decay rate of $\Upsilon(1S) \to \gamma f_2(1270)$ and the ratios of the helicity amplitudes are obtained and they are in agreement with the data.

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The measurements

$$B(\Upsilon(1S) \to \gamma f_2(1270)) = (10.2 \pm 0.8 \pm 0.7) \times 10^{-5}, \quad (1)$$

$$B(\Upsilon(1S) \to \gamma f_2(1270)) = (10.5 \pm 1.6(\mathrm{stat})^{+1.9}_{-1.8}(\mathrm{syst})) \times 10^{-5} \quad (2)$$

have been reported by CLEO in the channel of $\Upsilon(1S) \to \gamma f_2(1270)$, $f_2(1270) \to \pi^+\pi^-$ [1], and $f_2 \to \pi^0\pi^0$ [2], respectively. It is known that

$$B(J/\psi \to \gamma f_2(1270) = (1.43 \pm 0.11) \times 10^{-3}. \quad (3)$$

[3] $B(\Upsilon(1S) \to \gamma f_2(1270))$ is about 1 order of magnitude smaller than $B(J/\psi \to \gamma f_2(1270))$. The CLEO Collaboration has reported the measurements of $B(\Upsilon(1S) \to \gamma\eta'(\eta))$ whose upper limits are smaller than $B(J/\psi \to \gamma\eta'(\eta))$ by almost 3 orders of magnitudes [4]. In Ref. [5] the dependences of $B(J/\psi, \Upsilon(1S) \to \gamma\eta'(\eta))$ on corresponding quark masses are found and an explanation of very small $B(\Upsilon \to \gamma(\eta', \eta))$ is presented. The question is why $B(\Upsilon \to \gamma f_2)$ is not too small in comparison with $B(J/\psi, \Upsilon(1S) \to \gamma\eta'(\eta))$. $B(\Upsilon \to \gamma f_2)$ has been studied by many authors. In Ref. [6] a QCD analysis for $B(\Upsilon(1S) \to \gamma f_2(1270))$ has been done. In Ref. [7] the ratio $\frac{B(\Upsilon\to\gamma f_2)}{B(J/\psi\to\gamma f_2)}$ has been studied by using soft-collinear theory and nonrelativistic QCD. In 1983 we studied the radiative decay $J/\psi \to \gamma f_2(1270)$ [8]. In this paper the same approach exploited in Ref. [8] is used to study $\Upsilon \to \gamma f_2$.

The study done in Ref. [8] is based on the arguments presented in Refs. [9] that the tensor meson $f_2(1270)$ contains glueball components

$$|f_2\rangle = \cos\phi|q\bar{q}\rangle + \sin\phi|gg\rangle. \quad (4)$$

Tensor glueballs have been studied by many authors [10]. The mass of 2^{++} glueball has been calculated by lattice QCD [11]. In Ref. [11(a)] $M(2^{++}) = 2390(30)(120)$ MeV is reported. On the other hand, besides $f_2(1270)$ and $f_2'(1525)$ other isoscalar 2^{++} mesons: $f_2(1565)$, $f_2(1640)$, $f_2(1810)$, $f_2(1910)$, $f_2(1950)$, $f_2(2010)$, $f_2(2150)\ldots$ have been reported [3]. $f_2(1270)$, $f_2'(1525)$, $f_2(1640)$, $f_2(1910)$, $f_2(1950)$ are observed in J/ψ radiative decays. $J/\psi \to \gamma f_2(1270)$, $\gamma f_2(1950)$ have greater branching ratios. Some of them are possible radial excitations of $f_2(1270)$ and $f_2'(1525)$. It is also possible that one of them is a 2^{++} glueball.

According to Ref. [3], $f_2(1270)$ is closer to a flavor singlet. Therefore, it is natural that there is a mixing between $f_2(1270)$ and the 2^{++} glueball (4).

According to Ref. [3], the mixing angle of $f_2'(1525)$ and $f_2(1270)$ is $\theta_{\mathrm{quad}} = 29.6°$. The decays of f_2, $f_2' \to \gamma\gamma$ can be used to estimate the mixing angle ϕ (4)

$$R^{\mathrm{th}} = \frac{\Gamma(f_2(1270) \to 2\gamma)}{\Gamma(f_2'(1525) \to 2\gamma)} = \cos^2\phi \frac{(5\sin\alpha + \sqrt{2}\cos\alpha)^2}{(5\cos\alpha - \sqrt{2}\sin\alpha)^2} \frac{m_{f_2'(1525)}}{m_{f_2(1270)}} = 37.87\cos^2\phi,$$

where $\alpha = 84.3°$ [3]. The experimental value of this ratio is $32.18(1 \pm 0.22)$ [3]. Therefore, the mixing angle is estimated to be $\phi \sim 22.81°$. If the linear mixing angle of $f_2'(1525)$ and $f_2(1270)$ $\theta_{\mathrm{lin}} = 28°$ is taken [3], $R^{\mathrm{th}} = 29.04\cos^2\phi$. Therefore, only the lower values of the experimental ratio $32.18(1 \pm 0.22)$ [3] can be used to estimate the angle ϕ. The contribution of the glueball component (4) to the two photon decay is suppressed by $O(\alpha_s^2)$ and is ignored.

In the radiative decay $J/\psi \to \gamma f_2$ the $q\bar{q}$ component of $f_2(1270)$ is suppressed by $O(\alpha_s^2(m_c))$ [9]

$$\frac{\Gamma(J/\psi \to \gamma + (q\bar{q}))}{\Gamma(J/\psi \to \gamma + (gg))} \sim \alpha_s^2(m_c). \quad (5)$$

Therefore, the glueball component of f_2 is dominant in the decay $J/\psi \to \gamma f_2$. Similarly, the glueball component of f_2 is expected to be dominant in the decay $\Upsilon(1S) \to \gamma f_2$. In QCD the radiative decays J/ψ, $\Upsilon \to \gamma f_2$ are described as J/ψ, $\Upsilon \to \gamma gg$, $gg \to f_2$. The coupling between f_2 and two gluons is written as [8]

$$G^{ab}_{\alpha\beta,\lambda_2}(x_1,x_2) = \langle f_{gg\lambda_2}|T\{A^a_\alpha(x_1)A^b_\beta(x_2)\}|0\rangle = \delta_{ab}e^{(i/2)p_f(x_1x_2)}G(0)\sum_{m_1m_2}c^{2\lambda_2}_{1m_11m_2}e^{*m_1}_\alpha e^{*m_2}_\beta, \quad (6)$$

where $G(0)$ is taken as a parameter and $c^{2\lambda_2}_{1m_11m_2}$ is a Clebsch-Gordan coefficient. Using Eq. (6), the helicity amplitudes of $J/\psi \to \gamma f_2$ are presented in Ref. [8]. Replacing m_c by m_b in Eqs. (3, 4, 11) of Ref. [8], the helicity amplitudes of $\Upsilon(1S) \to \gamma f_2$ are obtained

$$\begin{aligned} T_0 &= -\frac{2}{\sqrt{6}}(A_2 + p^2A_1), \\ T_1 &= -\frac{\sqrt{2}}{m_\Upsilon}(EA_2 + m_f p^2 A_3), \\ T_2 &= -2A_2, \end{aligned} \quad (7)$$

$$E = \frac{1}{2m_f}(m^2_\Upsilon + m^2_f), \qquad p = \frac{1}{2m_f}(m^2_\Upsilon - m^2_f), \quad (8)$$

where

$$\begin{aligned} A_1 &= -a\frac{2m^2_f - m_\Upsilon(m_\Upsilon - 2m_b)}{m_b m_\Upsilon[m^2_b + \frac{1}{4}(m^2_\Upsilon - 2m^2_f)]}, \\ A_2 &= -a\frac{1}{m_b}\left\{\frac{m^2_f}{m_\Upsilon} - m_\Upsilon + 2m_b\right\}, \\ A_3 &= -a\frac{m^2_f - \frac{1}{2}(m_\Upsilon - 2m_b)^2}{m_b m_\Upsilon[m^2_b + \frac{1}{4}(m^2_\Upsilon - 2m^2_f)]}, \\ a &= \frac{16\pi}{3\sqrt{3}}\alpha_s(m_b)G(0)\psi_\Upsilon(0)\frac{\sqrt{m_\Upsilon}}{m^2_b}, \end{aligned} \quad (9)$$

where $\psi_\Upsilon(0)$ is the wave functions of Υ at origin. The decay width of $\Upsilon \to \gamma f_2$ is derived as

$$\Gamma(\Upsilon \to \gamma f_2) = \frac{32\pi\alpha}{81}\sin^2\phi\,\alpha^2_s(m_b)G^2(0)\psi^2_\Upsilon(0) \times \frac{1}{m^4_b}\left(1 - \frac{m^2_f}{m^2_\Upsilon}\right)\{T^2_0 + T^2_1 + T^2_2\}. \quad (10)$$

The ratios of the helicity amplitudes are defined as

$$x = \frac{T_1}{T_0}, \qquad y = \frac{T_2}{T_0}. \quad (11)$$

The expressions for these quantities for $J/\psi \to \gamma f_2$ can be found in Ref. [8].

The wave functions of Υ or J/ψ at the origin are related to their leptonic decay rates:

$$\frac{\psi^2_\Upsilon(0)}{\psi^2_J(0)} = 4\frac{\Gamma_{\Upsilon\to e^+e^-}}{\Gamma_{J/\psi\to e^+e^-}}\frac{m^2_\Upsilon}{m^2_{J/\psi}}. \quad (12)$$

The parameters $\sin^2\phi G^2(0)$ are canceled in the ratio

$$R = \frac{B(\Upsilon \to \gamma f_2)}{B(J/\psi \to \gamma f_2)}.$$

Taking $\alpha_s(m_c) = 0.3$, $\alpha_s(m_b) = 0.18$ [6], and $m_c = 1.29$ GeV (the experimental value is $m_c = 1.27^{+0.07}_{-0.11}$ GeV [3]), $m_b = (5.04 \pm 0.075 \pm 0.04)$ GeV [3]. It is obtained

$$R = 0.071(1 \pm 0.17) \quad (13)$$

which agrees with the experimental data [see, Eqs. (1)–(3)].

The ratios of the helicity amplitudes are found to be

$$x^2 = 0.058, \qquad y^2 = 5.9 \times 10^{-3}. \quad (14)$$

They are consistent with the experimental values [1]

$$x^2 = 0.00^{+0.02+0.01}_{-0.00-0.00}, \qquad y^2 = 0.09^{+0.08+0.04}_{-0.07-0.03}. \quad (15)$$

Equation (14) shows that the helicity amplitudes $T_{1,2}$ are small and the amplitude T_0 makes the dominant contribution to the decay rate of $\Upsilon \to \gamma f_2$. On the other hand, Eq. (13) indicates that the $B(\Upsilon \to \gamma f_2)$ is not too small in comparison with $B(\Upsilon \to \gamma\eta')$. These two results (13) and (14) are obtained by the approach studied in Ref. [8]. In this approach there are two processes for the decay $\Upsilon \to \gamma f_2$: $\Upsilon \to \gamma + gg$ and the two gluons are coupled to the f_2 meson. The process $\Upsilon \to \gamma + gg$ leads to very strong suppression by the mass of the b quark in the amplitudes (9). The same suppression appears in $\Upsilon \to \gamma\eta'$ [5] too. On the other hand, the coupling between the two gluons and the f_2 meson leads to enhancements in the helicity amplitudes (7) and (8). From Eq. (8) $E = 35.9$ GeV and $p = 34.6$ GeV are determined. The E and the p with large values appear in the helicity amplitudes $T_{0,1}$ and not in T_2. That is why the ratio y is so small (14). In T_1 (7) there is a factor $\frac{1}{m_\Upsilon}$ which makes T_1 smaller. Therefore, the helicity amplitude T_0 makes the dominant contribution. In the helicity amplitude T_0 p^2A_1 is much greater than A_2, therefore,

$$T_0 \cong -\frac{2}{\sqrt{6}}p^2A_1. \quad (16)$$

The combination of the T_0 dominance and Eq. (16) leads to

$$\Gamma(\Upsilon \to \gamma f_2) \propto p^4. \quad (17)$$

The T_0 dominance has been found in Ref. [6] and $R \sim 0.059$ is obtained. $m_c = 1.5$ GeV is taken in Ref. [6]. In this paper, a not too small $B(\Upsilon \to \gamma f_2)$ in comparison with $B(\Upsilon \to \gamma\eta')$ results in the competition between the suppression and the enhancement in the decay $\Upsilon \to \gamma f_2$.

In QCD J/ψ, $\Upsilon \to$ light hadrons are described as J/ψ, $\Upsilon \to 3g$ whose decay width is proportional to $\alpha^3_s m_V$, where m_V is the mass of J/ψ, Υ, respectively. Putting these factors together, the ratio is expressed as

$$
\begin{aligned}
R &= \frac{B(\Upsilon \to \gamma f_2)}{B(J/\psi \to \gamma f_2)} \\
&= \frac{\Gamma(\Upsilon \to \gamma f_2)}{\Gamma(J/\psi \to \gamma f_2)} \frac{\Gamma(J/\psi \to lh)}{\Gamma(\Upsilon \to lh)} \frac{B(\Upsilon \to lh)}{B(J/\psi \to lh)} \\
&= 1.06 \frac{\alpha_s(m_c)}{\alpha_s(m_b)} \frac{p_\Upsilon^4}{p_J^4} \frac{m_J m_c^6}{m_\Upsilon m_b^6} \frac{[m_c^2 + \frac{1}{4}(m_J^2 - 2m_f^2)]^2}{[m_b^2 + \frac{1}{4}(m_\Upsilon^2 - 2m_f^2)]^2} \frac{(1 - \frac{m_f^2}{m_\Upsilon^2})}{(1 - \frac{m_f^2}{m_\Upsilon^2})} \\
&\quad \times \frac{\{2m_f^2 - m_\Upsilon(m_\Upsilon - 2m_b)\}^2}{\{2m_f^2 - m_J(m_J - 2m_c)\}^2 + 6\frac{m_f^2}{m_J^2}\{m_f^2 - \frac{1}{2}(m_J - 2m_c)^2\}^2},
\end{aligned} \tag{18}
$$

where

$$
p_J = \frac{m_J^2}{2m_f}\left(1 - \frac{m_f^2}{m_J^2}\right). \tag{19}
$$

The competition between the suppression and the enhancement in the decay $\Upsilon \to \gamma f_2$ makes the dependence of $\frac{B(\Upsilon \to \gamma f_2)}{B(J/\psi \to \gamma f_2)}$ on quark masses much weaker than the ratio $\frac{B(\Upsilon \to \gamma \eta'(\eta))}{B(J/\psi \to \gamma \eta'(\eta))}$ [5].

In summary, very small ratios of the helicity amplitudes and the dominance of the enhanced amplitude T_0 are obtained by the approach [8] in which the $f_2(1270)$ is strongly coupled to two gluons. These results agree with the data.

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