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Exclusive J/Ψ electroproduction in a dual model

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Exclusive J/Ψ electroproduction is studied in the framework of the analytic S-matrix theory. The differential and integrated elastic cross sections are calculated using the modified dual amplitude with Mandelstam analyticity model. The model is applied to the description of the available experimental data and proves to be valid in a wide region of the kinematical variables s, t, and Q^2. Our amplitude can be used also as a universal background parametrization for the extraction of tiny resonance signals.

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I. INTRODUCTION

High-energy exclusive J/Ψ electroproduction was intensively studied in recent years. The theoretical tools include the quark model, perturbative QCD and Regge-pole models (for relevant reviews see Ref. [1]). The main source of the data at high energies is the HERA collider at DESY [2–6], while at low energies experimental studies at JLab are promising, see for instance the recent paper, Ref. [7].

In Ref. [8] a model based on vector meson dominance (VMD) and a dual amplitude with Mandelstam analyticity (DAMA) [9] was proposed to describe J/Ψ photoproduction; it resulted in a very good description of the data in the region of s from the threshold to high energies, where pure Pomeron exchange dominates. J/Ψ photoproduction is an ideal tool to study the Pomeron exchange.

In the framework of the VMD model [10], the photoproduction scattering amplitude is proportional to the sum of the relevant hadronic amplitudes

$$A(\gamma p \to Vp) = \sum_V \frac{e}{f_V} A(Vp \to Vp), \qquad (1)$$

where $V = \rho, \omega, \phi, J/\Psi, \ldots, f_V$ is a decay constant of the corresponding vector meson, and e is the vector meson-photon coupling constant. Comparison with the experimental data [11] confirms the validity of VMD in photoproduction reactions. VMD in electroproduction becomes more complicated and less obvious because of the asymmetry between the photon virtuality and the produced vector meson mass. A "skewed symmetric" VMD model for vector meson electroproduction was developed and applied in Refs. [12–15], while explicit Regge-pole models were constructed recently in Refs. [16–19].

VMD reduces J/Ψ photo- (electro-)production $\gamma(\gamma^*)p \to J/\Psi(J/\Psi^\star)p$ to a purely hadronic process $J/\Psi p \to J/\Psi p$ [10]. Because of the Okubo-Zweig-Iizuka rule [20] and the two-component duality picture [21], the high-energy Regge behavior of this scattering amplitude is completely determined by a Pomeron or, equivalently, by a Reggeized two-gluon ladder exchange, while its low-energy behavior is that of a smooth background due to the exotic quantum numbers of the direct channel. To a lesser extent, this is true also for $\phi - p$ scattering; however, in the latter case ordinary meson exchange is present due to $\omega - \phi$ mixing. Heavier vector mesons are as good as $J/\Psi - p$, but relevant data are less abundant. So, we find $J/\Psi - p$ scattering to be an ideal testing field for diffraction, where it can be studied uncontaminated by secondary trajectories [8]. A detailed introduction to the Okubo-Zweig-Iizuka rule and the two-component duality, both relevant to the present discussion, can be found in Ref. [8].

In this paper we study electroproduction of J/Ψ meson in the process $\gamma^*(Q^2)p \to J/\Psi p$ that gives a unique opportunity to study the properties of the Pomeron at different scales Q^2. We extend our model to processes with virtual particles based on a modified version of DAMA. Modification of the amplitude is done in such a way that all properties and symmetries of on-mass shell amplitude are secured.

The paper is organized as follows: in Sec. II we introduce the model, while Sec. III is dedicated to J/Ψ electroproduction and a description of the available experimental data. Our conclusions are drawn in Sec. IV.

II. THE MODEL

With the advent of the HERA experiments, the Regge-pole model was applied to off-mass shell processes by making some of its parameters, e.g. the Pomeron intercept, Q^2 dependent. When going back to low energies, two problems should be solved. One is the construction of a relevant dual amplitude, interpolating between low and high energies, and the second is its off-mass shell continuation.

To solve the first problem, in Ref. [8] we applied DAMA to J/ψ photoproduction. The parameters of the model were fitted to the experimental data on the differential cross section, $d\sigma/dt$. With these parameters we calculated also the total J/Ψ photoproduction cross section, as a model prediction, and obtained good agreement with the experimental data.

As to the second problem, we extend our analysis to off-shell $J/\Psi p$ scattering (electroproduction). A modified-DAMA (M-DAMA) formalism, including Q^2 dependence was proposed in Ref. [22]. Our strategy is to keep fixed the parameters obtained in the on-shell case, Ref. [8], and to fit the data on J/Ψ electroproduction in the generalized model, varying only the new parameters connected with the off-shell continuation of the model [22,23].

Let us briefly review the main properties of our dual model.

A. Kinematics

The photoproduction scattering amplitude with the use of VMD is [8]

$$A_{\gamma p\to J/\Psi p}(s,t) = c_V A_{J/\Psi p\to J/\Psi p}(s,t), \tag{2}$$

where c_V is the $\gamma - J/\Psi$ coupling, and we use the following kinematic relation between Mandelstam variables

$$s + t + u = 2M_p^2 + 2M_{J/\Psi}^2, \tag{3}$$

corresponding to the VMD amplitude on the right-hand side of Eq. (2).

Similarly, for the case of electroproduction (i.e. photoproduction by virtual photons) we write our generalized scattering amplitude in the form

$$A_{\gamma^* p\to J/\Psi p}(s,t,Q^2) = c_V A_{J/\Psi^* p\to J/\Psi p}(s,t,Q^2), \tag{4}$$

where Q^2 is the photon virtuality. The kinematic relation is also modified to meet the right-hand side of Eq. (4):

$$s + t + u = 2M_p^2 + 2M_{J/\Psi}^2 - Q^2. \tag{5}$$

In the photoproduction limit ($Q^2 \to 0$) Eqs. (4) and (5) reproduce Eqs. (2) and (3).

B. Dual amplitude

The DAMA appeared as a generalization of narrow-resonance (e.g. Veneziano) dual models, intended to overcome the manifestly nonunitarity of the latter [9]. Contrary to narrow-resonance dual models, DAMA requires nonlinear, complex trajectories. The dual properties of DAMA were studied in Ref. [24].

The DAMA amplitude [9] is given by

$$D(s,t) = c\int_0^1 dz\left(\frac{z}{g}\right)^{-\alpha(s')-1}\left(\frac{1-z}{g}\right)^{-\alpha_t(t'')}, \tag{6}$$

where $\alpha(s)$ and $\alpha(t)$ are Regge trajectories in the s and t channel correspondingly; $x' = x(1-z)$, $x'' = xz$ ($x = s, t, u$); g and c are parameters, $g > 1$, $c > 0$.

For $s \to \infty$ and fixed t DAMA is Regge-behaved

$$D(s,t) \sim s^{\alpha_t(t)-1}. \tag{7}$$

The pole structure of DAMA is similar to that of the Veneziano model except that multiple poles appear on daughter levels [9]:

$$D(s,t) = \sum_{n=0}^{\infty} g^{n+1}\sum_{l=0}^{n}\frac{[-s\alpha'(s)]^l C_{n-l}(t)}{[n-\alpha(s)]^{l+1}}, \tag{8}$$

where $C_n(t)$ is the residue, whose form is fixed by the t-channel Regge trajectory (see [9]). The pole term in DAMA is a generalization of the Breit-Wigner formula, comprising a whole sequence of resonances lying on a complex trajectory $\alpha(s)$. Such a "Reggeized" Breit-Wigner formula has little practical use in the case of linear trajectories, resulting in an infinite sequence of poles, but it becomes a powerful tool if complex trajectories with a limited real part and hence a restricted number of resonances are used.

A simple model of trajectories satisfying the threshold and asymptotic constraints of DAMA is a sum of square roots [9] $\alpha(s) \sim \sum_i \gamma_i \sqrt{s_i - s}$. The number of thresholds included depends on the model; the simplest is with a single threshold

$$\alpha(s) = \alpha(0) + \alpha_1(\sqrt{s_0} - \sqrt{s_0 - s}). \tag{9}$$

Imposing an upper bound on the real part of this trajectory,

$$\mathrm{Re}\,\alpha(s) < 0 \Rightarrow \alpha(0) + \alpha_1\sqrt{s_0} < 0, \tag{10}$$

we get an amplitude that does not produce resonances [9,24,25]. The imaginary part of such a trajectory instead rises indefinitely, contributing to the total cross section with a smooth background. This ansatz for the exotic trajectory was suggested in Ref. [8].

C. Regge trajectories

For the t-channel Pomeron trajectory we use an expression with two square roots thresholds:

$$\alpha(t) = \alpha^P(t) = \alpha^P(0) + \alpha_1^P(\sqrt{t_1} - \sqrt{t_1 - t}) + 2\alpha_2^P(t_2 - \sqrt{(t_2 - t)t_2}) \tag{11}$$

with a light (lowest) threshold $t_1 = 4m_\pi^2$, while the value of the heavy one, t_2, were fitted to the photoproduction data together with the other parameters of the trajectory [8] (see Table I).

TABLE I. Fitted values of the adjustable parameters.

On-Mass Shell Ref. [8]	
$\alpha^E(0) = -1.83$	$\alpha_1^E(0) = 0.01$ (GeV^{-1})
$\alpha^P(0) = 1.2313$	$\alpha_1^P(0) = 0.13498$ (GeV^{-1})
$\alpha_2^P(0) = 0.04$ (GeV^{-2})	$t_2 = 36$ (GeV^2)
$g = 13629$	$c = 0.0025$ (GeV^{-2})
χ^2/d.o.f. $= 0.83$	
Off-Mass Shell	
$Q_0^2 = 3.464^2$ (GeV^2), $a = 2.164$, $n = 2.131$	
χ^2/d.o.f. $= 1.2$	

The direct-channel exotic trajectory is given by Eq. (9), with condition (10), where the relevant threshold value is $s_0 = (m_{J/\Psi} + m_P)^2$. The other parameters of the trajectory—$\alpha(0)$ and α_1—were fitted to the data in Ref. [8] (see Table I).

D. Off-mass shell dual model

To extend our model off-mass shell we need to construct the Q^2-dependent dual amplitude. To this aim we use the so-called M-DAMA formalism developed in Ref. [22]. The scattering amplitude is given by

$$D(s,t,Q^2) = c\int_0^1 dz\left(\frac{z}{g}\right)^{-\alpha(s')-\beta(Q^{2''})-1} \times\left(\frac{1-z}{g}\right)^{-\alpha_t(t'')-\beta(Q^{2'})}, \tag{12}$$

where $\beta(Q^2)$ is a monotonically decreasing dimensionless function of Q^2; $x' = x(1-z)$, $x'' = xz$, where $x \equiv s, Q^2, t$.

It has been shown in Ref. [22] that by choosing the β function in the form

$$\beta(Q^2) = -\frac{\alpha_t(0)}{\ln g}\ln\left(\frac{Q^2+Q_0^2}{Q_0^2}\right), \tag{13}$$

all asymptotics of the amplitude at large s remain valid and the Q^2 behavior of the amplitude is in qualitative agreement with the experiment. Clearly at $Q^2 = 0$ we have $\beta(0) = 0$ so that we reproduce the on-mass shell amplitude studied in Ref. [8].

Now we can calculate the amplitude $D(s,t,Q^2)$ using the same numerical method as in Ref. [8]. The replacement of DAMA by M-DAMA results only in one more parameter, namely, the characteristic virtuality scale Q_0^2.

E. Scattering amplitude and cross sections

From the amplitude $D(s,t,Q^2)$, Eq. (12) we can construct the full scattering amplitude. Following Refs. [8,22,23], in order to secure $(s-u)$ symmetry, the complete amplitude can be written as

$$A(s,t,u,Q^2) = (s-u)(D(s,t,Q^2) - D(u,t,Q^2)). \tag{14}$$

Although, in the off-shell case, this symmetry is progressively violated as Q^2 increases, we try this form as the first approximation [23].

For the exotic Regge trajectory without resonances, like the one given by Eq. (9) with condition (10), the scattering amplitude $A(s,t,u,Q^2)$, Eq. (14), is given by a convergent integral, and can be calculated for any s, t, and Q^2 without analytical continuation, needed otherwise, as discussed in [9,22].

III. J/Ψ ELECTROPRODUCTION

The transverse differential cross section is given by

$$\frac{d\sigma_T}{dt}(s,t,Q^2) = \frac{1}{16\pi\lambda(s, m_{J/\Psi}^2, m_P^2)}|A(s,t,u,Q^2)|^2, \tag{15}$$

where $\lambda(x,y,z) = x^2+y^2+z^2-2xy-2yz-2xz$.

The total transverse cross section reads

$$\sigma_T(s,Q^2) = \int_{-t_{\max}=s/2}^{t_{\rm thr.}\approx 0} dt\frac{d\sigma_T}{dt}(s,t,Q^2). \tag{16}$$

When calculating the total elastic cross section it is important to take into account also the longitudinal component of the cross section, which was negligible for the photoproduction case [8]. Thus, the total elastic cross section is given by the sum of longitudinal and transverse components

$$\sigma_{el}(s,Q^2) = (1+R(Q^2))\sigma_T(s,Q^2), \tag{17}$$

where $R = \sigma_L/\sigma_T$.

In Ref. [26] the following expression for $R_V(Q^2)$, universal for all studied vector mesons $V \equiv \rho_0, \phi, \omega, J/\Psi$, was proposed:

$$R_V(Q^2) = \left(\frac{am_V^2+Q^2}{am_V^2}\right)^n - 1, \tag{18}$$

where a and n are the adjustable parameters. For our particular case, we choose $R = R_{J/\Psi}(Q^2)$.

Thus, in order to describe J/Ψ electroproduction data we have three adjustable parameters: Q_0^2, a, and n. In the fitting procedure we include 76 data points on σ_{el}, 32 data points on $d\sigma/dt$ and 8 data points on $R = R_{J/\Psi}(Q^2)$ at $Q^2 \neq 0$ from Refs. [2–6]. The resulting values of fitted parameters are: $Q_0^2 = 3.464^2$ (GeV^2), $a = 2.164$, $n = 2.131$. All other parameters are fixed to the values obtained in Ref. [8]. The agreement is very good, χ^2/d.o.f. $= 1.2$ for 116 data points, as seen in Table I.

The results are shown in Figs. 1–3. In Fig. 1 the differential cross section as a function of t is shown for different values and Q^2 for two values of c.m. energy $W = \sqrt{s}$, 90 GeV, and 110 GeV. As one can see, the model correctly describes the data both in t and in Q^2. This striking

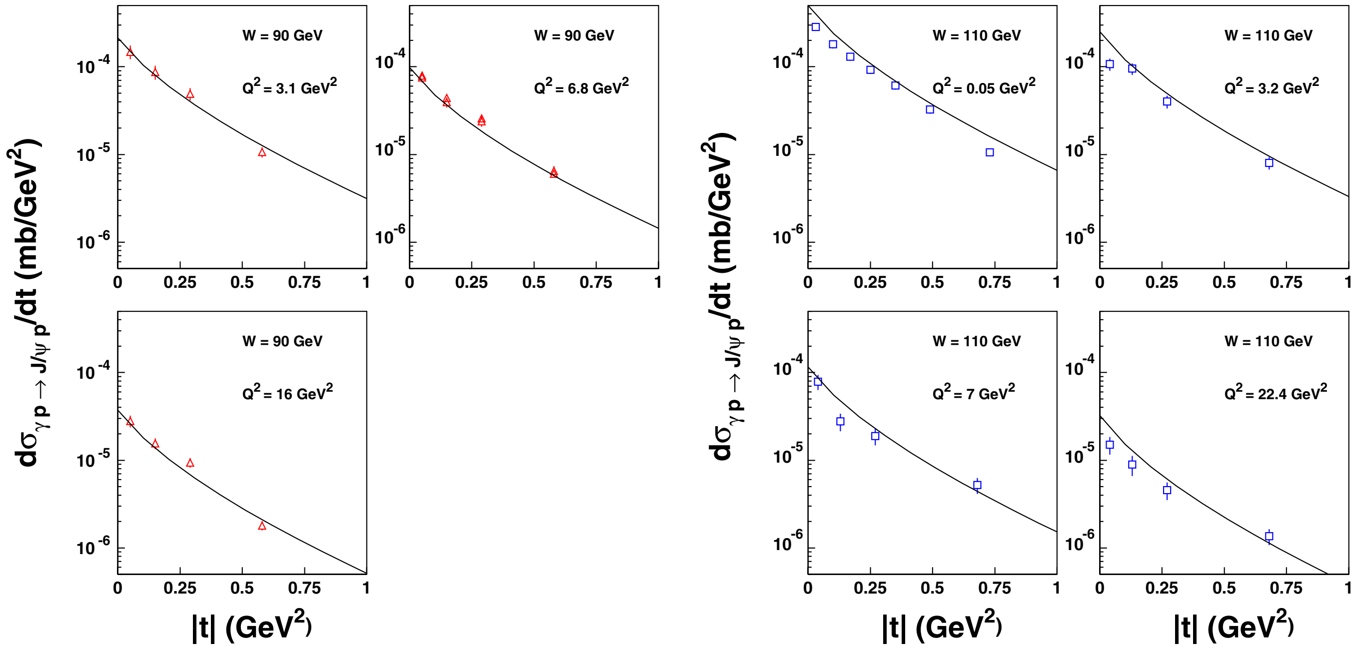

FIG. 1 (color online). J/Ψ differential cross sections as a function of t for different Q^2, for left panel—$W = 90$ GeV, experimental points are from Ref. [5]; right panel—$W = 110$ GeV, experimental points are from Ref. [6].

agreement shows that the off-mass-shell generalization of dual amplitude, proposed in Ref. [22], works.

The Q^2 dependence of J/Ψ electroproduction integrated elastic cross section at $W = 90$ GeV is presented in Fig. 2, left panel. The agreement is rather good.

On the right panel of Fig. 2 we study the behavior of the integrated elastic cross section of J/Ψ electroproduction as a function of W for different values of Q^2. The agreement with data is also fairly good; however, one can notice that the fits deteriorate progressively as Q^2 increases.

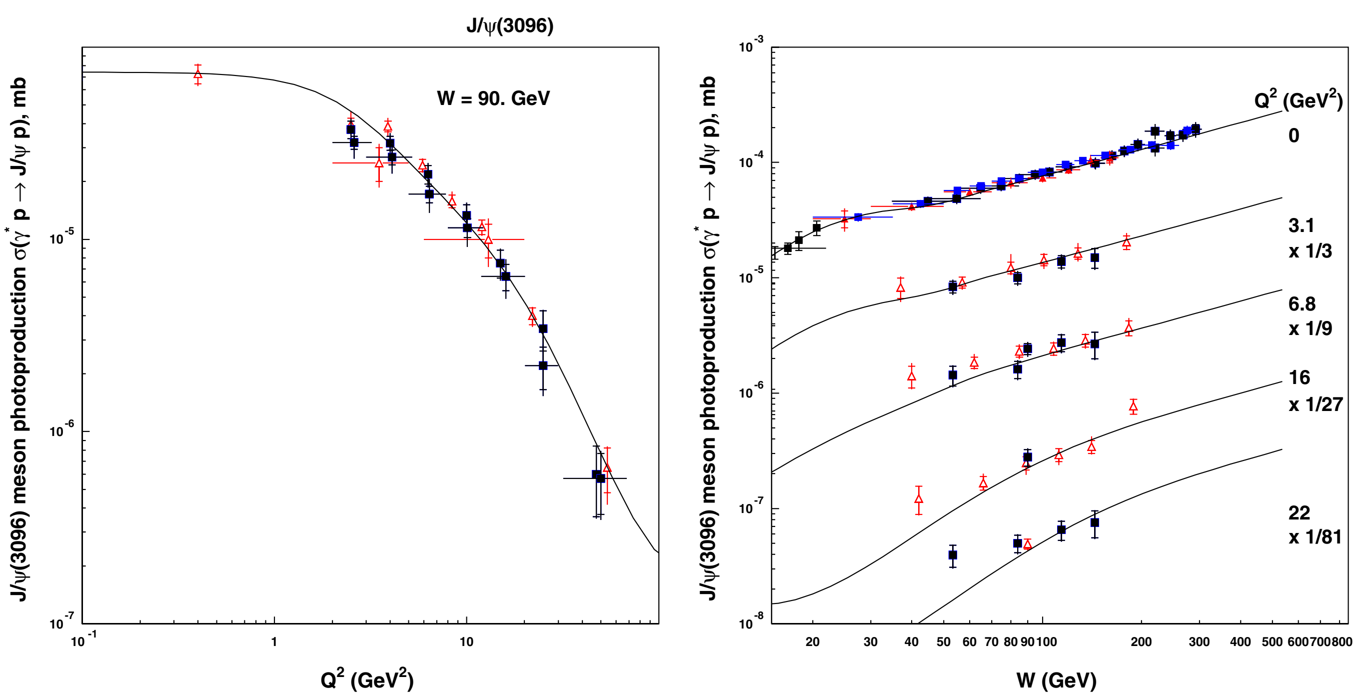

FIG. 2 (color online). J/Ψ elastic cross sections as a function of Q^2 for $W = 90$ GeV (left panel), and as a function of energy W for different Q^2 (right panel). Experimental points are from Refs. [2–6].

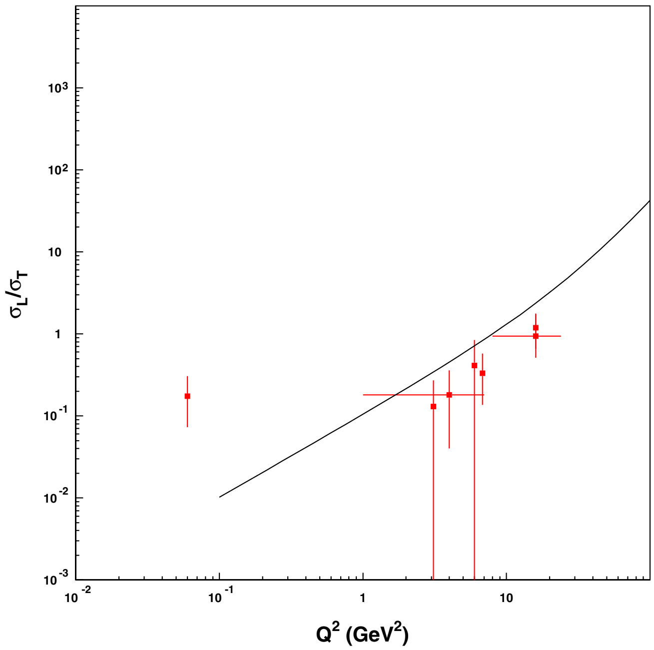

FIG. 3 (color online). The ratio $\sigma_L/\sigma_T = R(Q^2)$ as a function of Q^2. The experimental points are from Refs. [2,3,5].

One reason for this is the violation of the crossing symmetry in the model. Also, the ratio $R(Q^2)$, Eq. (18), taken from [26], with the parameters fitted in that paper, may be too restrictive. The calculated $R(Q^2)$ is compared to existing experimental data in Fig. 3.

The agreement is reasonable; however it may be that the parametrization of $R(Q^2)$ of Ref. [26], assumed to be universal for different vector mesons: ρ_0, ϕ, ω, J/Ψ, should be modified and adjusted to the particular case of J/Ψ.

IV. CONCLUSION

In this paper we describe J/Ψ electroproduction in the framework of M-DAMA, Refs. [22,23]. The model has one free parameter Q_0^2 in the scattering amplitude and the ratio σ_L/σ_T is parametrized as in Ref. [26], with two additional free parameters. It gives good description of the data in all available regions of the Mandelstam variables and photon virtualities. The model can be used to predict cross sections for J/Ψ photo- and electroproduction for any value of the Mandelstamian variable, and in a wide range of photon virtualities Q^2.

The fits deteriorate at highest values of Q^2. Two reasons for this limitation of the model can be two-fold. One can stem from the progressive violation of the $s - u$ symmetry, implemented in Eq. (14). This can be cured by adding interference terms, which however would make the model much more complicated and less practical in applications. The second reason seems obvious: with increasing Q^2, effects from QCD evolution, ignored in the model, may come into play. We plan to study both effects in a future paper.

We would like to stress that our results are not restricted to $J/\Psi p$ scattering alone. With a suitable readjustment of the fitted parameters, the model can be applied to any diffractive process.

Note that the model produces a smooth universal background in the near-to-threshold region of the reaction thus it can be used for the extraction of tiny resonance signals in the experiments with the energies close to the threshold (e.g. at the JLab [27]).

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