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$\mathbf{SU(5)_{flip} \times SU(5)'}$ from $\mathbf{Z_{12-I}}$

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Based on the $\mathbf{Z}_{12-I}$ orbifold compactification of heterotic string theory, we construct a flipped-SU(5) model with three families of standard model matter and ingredients for dynamical supersymmetry breaking through the hidden sector matter $\overline{\mathbf{10}}'$ and $\mathbf{5}'$ of SU(5)$'$, which are neutral under the visible sector flipped-SU(5). The appearance of one chiral set $\overline{\mathbf{10}}'$ and $\mathbf{5}'$ is the new feature of the present flipped-SU(5) string model with $\sin^2\theta_W^0 = \frac{3}{8}$. As required, all the exotic states are shown to decouple from low energy physics. Above the compactification scale, the flipped-SU(5) gauge symmetry is enhanced to SO(10) gauge symmetry by including the Kaluza-Klein modes. The threshold correction is calculated by counting Kaluza-Klein modes, and we show that the model allows a very wide range for the hidden-sector confining scale (10^{11} GeV–10^{16} GeV).

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I. INTRODUCTION

The flipped-SU(5) model [$\equiv$ SU(5) $\times$ U(1)$_X$], called SU(5)$_{\rm flip}$, was contrived for the alternative embedding of the standard model (SM) SU(2) singlets in the irreducible representations of SU(5) [1–3], in contrast to the well-known Georgi-Glashow SU(5)$_{\rm GG}$ grand unified theory (GUT) [4]. As a result, the doublet/triplet splitting in the Higgs representations, $\mathbf{5}$ and $\bar{\mathbf{5}}$, is so easy in SU(5)$_{\rm flip}$ [3]. A distinctive feature of the SU(5)$_{\rm flip}$ model is the GUT breaking mechanism by the Higgs representations $\mathbf{10}$ and $\overline{\mathbf{10}}$ of SU(5) rather than the adjoint $\mathbf{24}$, reducing the rank of the SU(5)$_{\rm flip}$ by one unit. With the advent of string constructions of the supersymmetric (SUSY) GUT models, and, particularly, with the difficulty in obtaining an adjoint Higgs for GUT breaking in the heterotic string, the GUT breaking by $\mathbf{10}_1$ and $\overline{\mathbf{10}}_{-1}$ in the SU(5)$_{\rm flip}$ model becomes a great advantage. Earlier string construction obtaining 4D SU(5)$_{\rm flip}$ was done in the fermionic construction [5]. Recently, a realistic SU(5)$_{\rm flip}$ model was obtained in a $\mathbf{Z}_{12-I}$ orbifold construction [6].

SUSY breaking is one of the important issues that a realistic model should address. Dynamical SUSY breaking (DSB) in the hidden sector can be realized simply by the representations $\overline{\mathbf{10}}'$ plus $\mathbf{5}'$ of SU(5)$'$ [7], and $\overline{\mathbf{16}}'$ of SO(10)$'$ [8]. Other hidden-sector gauge groups may be possible. Recent DSB models at unstable vacua are known to be possible with vectorlike representations in the hidden sector [9]. However, we will concentrate on the simple SU(5)$'$ model with only one *chiral set*, i.e. $\overline{\mathbf{10}}'$ plus $\mathbf{5}'$, because it would be relatively easy to realize SU(5)$'$ compared to SO(10)$'$, in the compactification of the heterotic string. In this spirit, DSB at an unstable vacuum and at a stable vacuum has been discussed already [10,11]. We note that here the minimal supersymmetric standard model (MSSM) with the SU(5)$'$ gauge group with one chiral set ($\overline{\mathbf{10}}' \oplus \mathbf{5}'$) in the hidden sector was obtained, but the bare value of the weak mixing angle was not $\frac{3}{8}$ [11]. In principle, the weak mixing angle could fit the observed one with the power-law–type threshold effects from the Kaluza-Klein (KK) towers, if relatively large extra dimensions are assumed [12].

Even if SU(5)$'$ is a prominent example to clearly include dynamical SUSY breaking (even in global SUSY), so far the SU(5)$'$ hidden sector gauge group has not attracted much attention. One reason may be that SU(5)$'$ is relatively hard to obtain compared to the rank 4 gauge groups such as SU(4)$'$ or SO(8)$'$ [13]. Moreover, the requirement that one family, ($\overline{\mathbf{10}}' \oplus \mathbf{5}'$), in the hidden sector must be neutral under the SM gauge group is much harder to satisfy compared to obtaining just ($\mathbf{5}' \oplus \bar{\mathbf{5}}'$)'s.

If the fundamental scale is 10^{16} GeV, where the visible- and hidden-sector gauge couplings are unified, $\alpha_v = \alpha_h \approx 1/25$, the SU(5)$'$ confining scale with one family ($\overline{\mathbf{10}}'$ and $\mathbf{5}'$) would be 10^{11} GeV, which may be useful for SUSY breaking by gauge mediation. But it is quite lower than the conventional hidden-sector scale of order 10^{13} GeV in gravity mediation. In this paper, we will point out that the confinement scale can be extremely sensitive to the threshold correction by KK modes, and so including just a few KK towers can lift the confinement scale up to 10^{16} GeV. Thus, if KK modes are considered, a very large class of hidden-sector gauge groups that we did not yet explore could be phenomenologically viable, allowing for the possibility of the gauge coupling unification at the fundamental scale.

The items discussed above from string compactifications are obtained piecemeal: (1) the SM gauge group with three families [14], (2) gauge coupling unification at $M_{\rm GUT}$ [15], (3) DSB with one set of $\overline{\mathbf{10}}'$ and $\mathbf{5}'$ [11], (4) the contribution of the KK spectrum to the threshold correction [12], and (5) all the exotics being vectorlike under SM gauge symmetry. In particular, the specific model presented in

Ref. [11] has the Lee-Weinberg electroweak gauge group with $\sin^2\theta_W^0 = \frac{1}{4}$, which needs two scales of the gauge group breaking by the Higgs mechanism.

In fact, the string models of $\mathrm{SU}(5)_{\mathrm{flip}}$ satisfying conditions (1), (2), and (5) are not known, except for those of Refs. [5,6]. Actually, it is nontrivial to obtain $\mathbf{10}_H$ and $\overline{\mathbf{10}}_H$ from string theory, which are essential for breaking the $\mathrm{SU}(5)_{\mathrm{flip}}$ down to the SM gauge group.[1] In this paper, we present an interesting model which satisfies not only (1), (2), and (5) but also all the above features, in particular, $\sin^2\theta_W^0 = \frac{3}{8}$ at the string or GUT scale. The model is based on the $\mathbf{Z}_{12-I}$ orbifold compactification of the heterotic string [6,11,12,16]. So, we give most of the construction technique in a succinct form in the Appendix.

This paper is organized as follows. In Secs. II and III, we will construct a SUSY GUT model $\mathrm{SU}(5)_{\mathrm{flip}} \times \mathrm{SU}(5)'$ and present the massless spectra from the untwisted and twisted sectors. In Sec. IV, we will discuss the Yukawa couplings needed for the realization of the MSSM. Section V is devoted to the derivation of the KK spectrum and the discussion of its effect on the gauge couplings of the visible and hidden sectors. Section VI gives our conclusion. In the Appendix, we sketch the technique for obtaining the massless spectrum and Yukawa couplings.

II. $\mathbf{Z}_{12-I}$ ORBIFOLD MODEL AND U SECTOR FIELDS

We employ the $\mathbf{Z}_{12-I}$ orbifold compactification scheme for the extra 6D space, which preserves $N = 1$ SUSY in the noncompact 4D spacetime [17,18]. $\mathbf{Z}_{12-I}$ orbifolds are known to give phenomenologically interesting MSSMs [6,12,16,18].

The $\mathbf{Z}_{12-I}$ orbifold is an $\mathrm{SO}(8) \times \mathrm{SU}(3)$ lattice, and the Wilson lines W_3 and W_4 ($= W_3$) can be introduced in the 2D SU(3) lattice [17,18]. We take the following shift vector V and the Wilson line W_3,

$$\begin{aligned} V &= (0\ 0\ 0\ 0\ 0; \tfrac{-1}{6}\ \tfrac{-1}{6}\ \tfrac{-1}{6})(0\ 0\ 0\ 0\ 0\ \tfrac{1}{4}\ \tfrac{1}{4}\ \tfrac{-2}{4})', \\ W_3 = W_4 \equiv W &= (\tfrac{2}{3}\ \tfrac{2}{3}\ \tfrac{2}{3}\ \tfrac{2}{3}\ \tfrac{2}{3}; 0\ \tfrac{-2}{3}\ \tfrac{2}{3})(\tfrac{2}{3}\ \tfrac{2}{3}\ \tfrac{2}{3}\ \tfrac{2}{3}\ 0\ \tfrac{-2}{3}\ 0\ 0)', \end{aligned} \tag{1}$$

which are associated with the boundary conditions of the left-moving bosonic string. For modular invariance in $\mathbf{Z}_{12-I}$ orbifold compactification, V and W should be specially related to the twist vector $\phi = (\frac{5}{12}\ \frac{4}{12}\ \frac{1}{12})$. ϕ is associated with the boundary conditions of the right-moving superstrings, preserving only $N = 1$ SUSY in 4D. The twist vector $\phi = (\frac{5}{12}\ \frac{4}{12}\ \frac{1}{12})$ specifies the $\mathbf{Z}_{12-I}$ orbifold. This model gives

$$V^2 - \phi^2 = \frac{1}{6}, \qquad W^2 = \frac{16}{3}, \qquad V \cdot W = \frac{-1}{6}. \tag{2}$$

Hence, the modular invariance conditions in $\mathbf{Z}_{12-I}$ orbifold compactification are satisfied [18]: $12 \cdot (V^2 - \phi^2) =$ even integer, $12 \cdot W^2 =$ even integer, and $12 V \cdot W =$ integer.

The massless gauge sector corresponds to the states satisfying $P \cdot V =$ integer and $P \cdot W =$ integer, where P is the $\mathrm{E}_8 \times \mathrm{E}_8'$ root vector. They are

$$\mathrm{SU}(5)\colon (\underline{1\ -1\ 0\ 0\ 0}; 0^3)(0^8)', \tag{3}$$

$$\mathrm{SU}(5)'\colon \begin{cases} (0^8) & (\underline{1\ -1\ 0\ 0\ 0}\ 0\ 0\ 0)' \\ \pm\ (0^8) & (\underline{+\ -\ -\ -\ -}\ +\ +\ +)', \end{cases} \tag{4}$$

$$\mathrm{SU}(2)'\colon \pm\ (0^8)(+\ +\ +\ +\ +\ +\ +\ +)', \tag{5}$$

where the underline means all possible permutations. Thus, the gauge group is

$$[\{\mathrm{SU}(5) \times \mathrm{U}(1)_X\} \times \mathrm{U}(1)^3] \times [\mathrm{SU}(5) \times \mathrm{SU}(2) \times \mathrm{U}(1)^3]', \tag{6}$$

where $\mathrm{SU}(5) \times \mathrm{U}(1)_X$ is identified with the flipped-SU(5). The $\mathrm{U}(1)_X$ charge operator of the flipped-SU(5) is [6]

$$X = \frac{1}{\sqrt{40}}(-2\ \ -2\ \ -2\ \ -2\ \ -2; 0^3)(0^8)'. \tag{7}$$

The normalization factor $\frac{1}{\sqrt{40}}$ is determined such that the norm of the X [in general, all U(1) charge operators in the level-one heterotic string theory [18]] is $\frac{1}{\sqrt{2}}$. This value is exactly the one given as the normalization required for the $\mathrm{SU}(5) \times \mathrm{U}(1)_X$ embedded in SO(10). Since the standard model hypercharge is defined as

$$Y = \sqrt{\tfrac{3}{5}}\,(\tfrac{1}{3}\ \tfrac{1}{3}\ \tfrac{1}{3}\ \tfrac{1}{2}\ \tfrac{1}{2}; 0^3)(0^8)', \tag{8}$$

the weak mixing angle at the string scale is $\sin^2\theta_W^0 = \frac{3}{8}$. From now on, we will drop the normalization factors "$\frac{1}{\sqrt{40}}$" and "$\sqrt{\frac{3}{5}}$" just for simplicity.

The massless states of chiral matter in the U sector (U) are the states satisfying $P \cdot V = \{\frac{-5}{12}$ or $\frac{4}{12}$ or $\frac{1}{12}\}$, and $P \cdot W =$ integer. In Table I, the chiral fields in the U sector are tabulated. Note that there does not appear to be any flipped-SU(5) singlets in U. From the U sector, we obtain one family of the MSSM matter,

TABLE I. The U sector chiral states. There are no hidden-sector chiral states and no flipped-SU(5) singlets.

Visible states	$P \cdot V$	χ	$\mathrm{SU}(5)_X$
$(\underline{+\ -\ -\ -\ -}; +\ -\ -)(0^8)'$	$\frac{1}{12}$	L	$\mathbf{5}_3$
$(\underline{+\ +\ +\ -\ -}; -\ -\ +)(0^8)'$	$\frac{1}{12}$	L	$\mathbf{10}_{-1}$
$(+\ +\ +\ +\ +; -\ +\ -)(0^8)'$	$\frac{1}{12}$	L	$\mathbf{1}_{-5}$

[1] In the flipped-SU(5), the R-parity violating operators do not appear at the renormalizable Lagrangian level. If an exact R parity is also required in the compactification, the number of phenomenologically viable string SM models becomes drastically reduced.

$$\overline{\mathbf{10}}_{-1} + \mathbf{5}_3 + \mathbf{1}_{-5} \qquad (\text{and their } \mathcal{CTP} \text{ conjugates}), \quad (9)$$

where $\overline{\mathbf{10}}_{-1}$, $\mathbf{5}_3$, $\mathbf{1}_{-5}$ contain $\{d_L^c, q_L, \nu_L^c\}$, $\{u_L^c, l_L\}$, and e_L^c, respectively. It is tempting to interpret this as the third (top quark) family, but the low dimensional Yukawa couplings prefer one family in the twisted sector as the third family.

III. TWISTED SECTOR FIELDS

There are 11 twisted sectors, T_k with $k = 1, 2, \ldots, 11$. The $\mathcal{CTP}$ conjugates of the chiral states in T_k are provided in T_{12-k}. Thus, it is sufficient to consider $k = 1, 2, \ldots, 6$. While the U and T_6 sectors contain both chiral states and their $\mathcal{CTP}$ conjugates, T_1, T_2, T_4, and T_7 (T_{11}, T_{10}, T_8, and T_5) sectors yield only the left-handed (right-handed) chiral states. The T_3 (and T_9) sector includes both left- and right-handed chiral states. So, to obtain the left-handed states we will take $\mathcal{CTP}$ conjugations for the right-handed states of the T_3 and T_5 sectors. All the twisted sector fields are listed in [19].

From untwisted and twisted sectors, altogether there appear six pairs of Higgs doublets from T_4, T_7, and T_6, among which the MSSM Higgs doublet pair is chosen. We will explain in Sec. IV that, except one pair of $\{\mathbf{5}_{-2}, \bar{\mathbf{5}}_2\}$, the other pairs of five-plets with $X = \pm 2$ in the T_4, T_7, and T_6 sectors achieve superheavy masses, when some singlets under $[\mathrm{SU}(5) \times \mathrm{U}(1)_X] \times [\mathrm{SU}(5) \times \mathrm{SU}(2)]'$ obtain vacuum expectation values (VEVs) of order the string scale. There, we will also explain how to decouple the color triplets from the electroweak doublets in the Higgs quintets.

A. The flipped-SU(5) spectrum

The visible sector chiral states of the twisted sectors are

$$T_4\colon 2(\overline{\mathbf{10}}_{-1} + \mathbf{5}_3 + \mathbf{1}_{-5}),\ 2(\mathbf{5}_{-2} + \bar{\mathbf{5}}_2), \qquad (10)$$

$$T_3, T_9\colon (\mathbf{10}_1 + \overline{\mathbf{10}}_{-1}), \qquad (11)$$

$$T_7\colon (\mathbf{5}_{-2} + \bar{\mathbf{5}}_2), \qquad (12)$$

$$T_6\colon 3(\mathbf{5}_{-2} + \bar{\mathbf{5}}_2). \qquad (13)$$

To get the left-handed states from the T_9 and T_7 sectors, we applied the $\mathcal{CTP}$ conjugations to the right-handed states of the T_3 and T_5 sectors. From Eq. (10), we note that two families of the MSSM matter fields appear from T_4. The additional family needed appears from the U sector we presented above.

To break the flipped-SU(5) down to the SM, we need $\mathbf{10}_1$ ($\equiv \mathbf{10}_H$) and $\overline{\mathbf{10}}_{-1}$ ($\equiv \overline{\mathbf{10}}_H$), which appear from T_3 and T_9. As explained later, they couple to the $\{\mathbf{5}_{-2}, \bar{\mathbf{5}}_2\}$ ($\equiv \{\mathbf{5}_h, \bar{\mathbf{5}}_h\}$) so that the pseudo-Goldstone mode $\{D, D^c\}$ included in $\{\mathbf{10}_H, \overline{\mathbf{10}}_H\}$ pairs up with the triplets contained in $\{\mathbf{5}_{-2}, \bar{\mathbf{5}}_2\}$ and becomes superheavy.

B. The hidden-sector $\mathrm{SU}(5)'$ spectrum

The hidden-sector fields appear from twisted sectors. The chiral multiplets under $\mathrm{SU}(5)' \times \mathrm{SU}(2)'$ are listed as follows.

$$T_4\colon 3(\mathbf{5}', \mathbf{1}')_{-5/3},\ 3(\bar{\mathbf{5}}', \mathbf{1}')_{5/3},\ 2(\mathbf{1}', \mathbf{2}')_{-5/3},\ 2(\mathbf{1}', \mathbf{2}')_{5/3}, \qquad (14)$$

$$T_2\colon (\mathbf{1}', \mathbf{2}')_{5/3},\ (\mathbf{1}', \mathbf{2}')_{-5/3}, \qquad (15)$$

$$T_1\colon (\overline{\mathbf{10}}', \mathbf{1}')_0,\ (\mathbf{5}', \mathbf{2}')_0,\ (\bar{\mathbf{5}}', \mathbf{1}')_0,\ (\mathbf{1}', \mathbf{2}')_0,\ (\bar{\mathbf{5}}', \mathbf{1}')_{-5/3},\ (\mathbf{1}', \mathbf{2}')_{-5/3},\ 2(\mathbf{1}', \mathbf{2}')_{5/3}, \qquad (16)$$

$$T_7\colon (\mathbf{5}', \mathbf{1}')_{5/3},\ 2(\mathbf{1}', \mathbf{2}')_{-5/3},\ (\mathbf{1}', \mathbf{2}')_{5/3}. \qquad (17)$$

Again, we replaced the right-handed states in the T_5 sector by the left-handed ones in T_7 by $\mathcal{CTP}$ conjugations. Yet we have not included non-Abelian group singlets. The vectorlike representations are assumed to obtain superheavy masses when the neutral singlet under the flipped-SU(5) develops VEVs of order the string scale. We will discuss this in Sec. IV.

Removing vectorlike representations from Eqs. (14)–(17), we still have

$$(\overline{\mathbf{10}}', \mathbf{1}')_0,\ (\mathbf{5}', \mathbf{2}')_0,\ (\bar{\mathbf{5}}', \mathbf{1}')_0,\ (\mathbf{1}', \mathbf{2}')_0. \qquad (18)$$

The hidden-sector $\mathrm{SU}(2)'$ is broken by a GUT scale VEV of $(\mathbf{1}', \mathbf{2}')_0$ of (18). Then, out of the representations of (18), one hidden-sector chiral set remains,

$$\overline{\mathbf{10}}'_0,\ \mathbf{5}'_0, \qquad (19)$$

which is the key toward the DSB with $\mathrm{SU}(5)'$ [7].

Representations in (19) do not carry any visible sector quantum numbers, and the flipped-SU(5) is not broken by the DSB in the hidden sector. Our construction of one hidden-sector chiral set $\overline{\mathbf{10}}' + \mathbf{5}'$ with additional vectorlike pairs of $\mathbf{5}'$ and $\bar{\mathbf{5}}'$ does not change the fate of DSB as noted in [20]. But inclusion of supergravity effects gives a runaway solution at large values of the dilaton field [21]. The barrier separation between the SUSY breaking minimum and the runaway point must be very high.

C. The other vectorlike exotic states

The remaining charged states under the flipped-SU(5) are the singlets of $\mathrm{SU}(5) \times \mathrm{SU}(5)' \times \mathrm{SU}(2)'$. They are listed as follows.

$$T_4\colon 4 \cdot \mathbf{1}_{-5/3},\ 4 \cdot \mathbf{1}_{5/3}, \qquad (20)$$

$$T_2\colon \mathbf{1}_{-10/3},\ 2 \cdot \mathbf{1}_{5/3},\ \mathbf{1}_{10/3},\ 2 \cdot \mathbf{1}_{-5/3}, \qquad (21)$$

$$T_1\colon \mathbf{1}_{10/3},\ 3 \cdot \mathbf{1}_{-5/3},\ \mathbf{1}_{-10/3},\ 2 \cdot \mathbf{1}_{5/3}, \qquad (22)$$

$$T_7\colon \mathbf{1}_{10/3},\ 2 \cdot \mathbf{1}_{-5/3},\ \mathbf{1}_{-10/3},\ 3 \cdot \mathbf{1}_{5/3}. \qquad (23)$$

These are singlet exotics. Since they are also vectorlike under the flipped-SU(5), they could obtain superheavy masses if the needed neutral singlets develop VEVs of order the string scale. Hence, we can get the same low energy field spectrum as that of the MSSM. Such vectorlike superheavy exotics could be utilized [22] to explain the recently reported high energy cosmic positron excess [23–26].

IV. SINGLETS AND YUKAWA COUPLINGS

It is necessary to make exotics vectorlike and heavy. For this purpose, many singlets are required to develop large VEVs. In Table II, we list singlet fields. At least the following fields are given large VEVs at the string scale,

$$S_2,\ S_3,\ S_4,\ S_5,\ S_7,\ S_{11},\ S_{12},\ S_{15},\ S_{16},\ S_{17},\ S_{18},\ S_{21},\ S_{22}. \tag{24}$$

These VEVs are possible through higher dimensional terms in the superpotential. The selection rules for the allowed superpotential are summarized in the Appendix.

Our strategy is to construct composite singlets (CSs) which have H-momenta, $(1\ 0\ 0)$, $(-1\ 0\ 0)$, $(0\ 1\ 0)$, $(0\ -1\ 0)$, $(0\ 0\ 1)$, $(0\ 0\ -1)$. Then, using only singlets developing VEVs at the string scale $M_{\rm string}$ with any integer set $(l\ m\ n)$, we can attach an appropriate number of CSs such that they make the total H-momentum $(-1\ 1\ 1)$. Since their VEVs are of order $M_{\rm string}$, generically the Yukawa couplings multiplied by them are not suppressed.

A. Composite singlets

Specifically, let us consider a CS composed of S_2 with $(N^L)_j = 1_{\bar{1}}$, S_{21} with $(N^L)_j = \{2_3, 1_2\}$, and S_{22} with $(N^L)_j = 2_{\bar{1}}$ from T_4^0, T_1^0, and T_7^0, respectively. The CS "$S_2S_{21}S_{22}$" fulfills selection rules (a) and (c) of the Appendix, and its H-momentum is calculated as $[(\frac{-1}{3}\ \frac{1}{3}\ \frac{1}{3}) + (1\ 0\ 0)] + [(\frac{-7}{12}\ \frac{4}{12}\ \frac{1}{12}) + (0\ -1\ -2)] + [(\frac{-1}{12}\ \frac{4}{12}\ \frac{7}{12}) + (2\ 0\ 0)] = (2\ 0\ -1)$. S_3 with $(N^L)_j = 1_{\bar{1}}$, 1_2, or 1_3 from T_4^0, S_5 $[(N^L)_j = 0]$ from T_6, and S_{17} $[(N^L)_j = 2_3]$ from T_2^0 form another useful set. "$S_3S_5S_{17}$" also fulfills (a) and (c), and its H-momentum is given by $(0\ 1\ -1)$, $(-1\ 0\ -1)$, or $(-1\ 1\ -2)$. Similarly, "S_5S_7" satisfies (a) and (c) and gives the H-momentum of $(-1\ 0\ 1)$. By properly multiplying $S_2S_{21}S_{22}$, $S_3S_5S_{17}$, S_5S_7 (and their higher powers), one can indeed construct CSs whose H-momenta are $(1\ 0\ 0)$, $(-1\ 0\ 0)$, $(0\ 1\ 0)$, $(0\ -1\ 0)$, $(0\ 0\ 1)$, $(0\ 0\ -1)$. For instance, $(1, 0, 0)$ can be obtained from $(2\ 0\ -1) + (-1\ 0\ 1)$, namely, $(S_2S_{21}S_{22})(S_5S_7)$, and $(0\ 0\ 1)$ is achieved from $(S_2S_{21}S_{22})(S_5S_7)^2$.

One can easily see that all the states in T_4^+ and T_4^- achieve string scale masses via $\langle S_4\rangle$. The states in $\{T_2^+, T_2^-\}$, $\{T_1^+, T_7^-\}$, and $\{T_1^-, T_7^+\}$ pair up to be superheavy

TABLE II. Left-handed $SU(5) \times U(1)_X \times SU(5)' \times SU(2)'$ singlet states. The right-handed states in T_3 and T_5 are converted to the left-handed ones of T_9 and T_7, respectively.

Sectors	Singlet states	χ	$(N^L)_j$	$\mathcal{P}(f_0)$	Label
T_4^0	$(0\ 0\ 0\ 0\ 0; \frac{-2}{3}\ \frac{-2}{3}\ \frac{-2}{3})(0^8)'$	L	0	3	S_1
T_4^0	$(0\ 0\ 0\ 0\ 0; \frac{-2}{3}\ \frac{1}{3}\ \frac{1}{3})(0^8)'$	L	$1_{\bar{1}}, 1_2, 1_3$	2, 3, 2	S_2
T_4^0	$(0\ 0\ 0\ 0\ 0; \frac{1}{3}\ \frac{-2}{3}\ \frac{1}{3})(0^8)'$	L	$1_{\bar{1}}, 1_2, 1_3$	2, 3, 2	S_3
T_4^0	$(0\ 0\ 0\ 0\ 0; \frac{1}{3}\ \frac{1}{3}\ \frac{-2}{3})(0^8)'$	L	$1_{\bar{1}}, 1_2, 1_3$	2, 3, 2	S_4
T_6	$(0\ 0\ 0\ 0\ 0; 0\ 1\ 0)(0\ 0\ 0\ 0\ 0\ \frac{1}{2}\ \frac{-1}{2}\ 0)'$	L	0	2	S_5
T_6	$(0\ 0\ 0\ 0\ 0; 0\ 0\ 1)(0\ 0\ 0\ 0\ 0\ \frac{-1}{2}\ \frac{1}{2}\ 0)'$	L	0	2	S_6
T_6	$(0\ 0\ 0\ 0\ 0; 0\ -1\ 0)(0\ 0\ 0\ 0\ 0\ \frac{-1}{2}\ \frac{1}{2}\ 0)'$	L	0	2	S_7
T_6	$(0\ 0\ 0\ 0\ 0; 0\ 0\ -1)(0\ 0\ 0\ 0\ 0\ \frac{1}{2}\ \frac{-1}{2}\ 0)'$	L	0	2	S_8
T_3	$(0\ 0\ 0\ 0\ 0; \frac{-1}{2}\ \frac{-1}{2}\ \frac{-1}{2})(0\ 0\ 0\ 0\ 0\ \frac{3}{4}\ \frac{-1}{4}\ \frac{-1}{2})'$	L	0	1	S_9
T_3	$(0\ 0\ 0\ 0\ 0; \frac{-1}{2}\ \frac{1}{2}\ \frac{1}{2})(0\ 0\ 0\ 0\ 0\ \frac{3}{4}\ \frac{-1}{4}\ \frac{-1}{2})'$	L	0	1	S_{10}
T_3	$(0\ 0\ 0\ 0\ 0; \frac{1}{2}\ \frac{1}{2}\ \frac{-1}{2})(0\ 0\ 0\ 0\ 0\ \frac{-1}{4}\ \frac{3}{4}\ \frac{-1}{2})'$	L	0	1	S_{11}
T_3	$(0\ 0\ 0\ 0\ 0; \frac{1}{2}\ \frac{1}{2}\ \frac{-1}{2})(0\ 0\ 0\ 0\ 0\ \frac{-1}{4}\ \frac{-1}{4}\ \frac{1}{2})'$	L	$1_1, 1_3$	2, 1	S_{12}
T_9	$(0\ 0\ 0\ 0\ 0; \frac{1}{2}\ \frac{1}{2}\ \frac{1}{2})(0\ 0\ 0\ 0\ 0\ \frac{-3}{4}\ \frac{1}{4}\ \frac{1}{2})'$	L	0	1	S_{13}
T_9	$(0\ 0\ 0\ 0\ 0; \frac{1}{2}\ \frac{-1}{2}\ \frac{-1}{2})(0\ 0\ 0\ 0\ 0\ \frac{-3}{4}\ \frac{1}{4}\ \frac{1}{2})'$	L	0	2	S_{14}
T_9	$(0\ 0\ 0\ 0\ 0; \frac{-1}{2}\ \frac{-1}{2}\ \frac{1}{2})(0\ 0\ 0\ 0\ 0\ \frac{1}{4}\ \frac{-3}{4}\ \frac{1}{2})'$	L	0	2	S_{15}
T_9	$(0\ 0\ 0\ 0\ 0; \frac{-1}{2}\ \frac{-1}{2}\ \frac{1}{2})(0\ 0\ 0\ 0\ 0\ \frac{1}{4}\ \frac{1}{4}\ \frac{-1}{2})'$	L	$1_{\bar{1}}, 1_{\bar{3}}$	1, 1	S_{16}
T_2^0	$(0\ 0\ 0\ 0\ 0; \frac{-1}{3}\ \frac{-1}{3}\ \frac{-1}{3})(0\ 0\ 0\ 0\ 0\ \frac{-1}{2}\ \frac{1}{2}\ 0)'$	L	$2_{\bar{1}}, 2_3$	1, 1	S_{17}
T_2^0	$(0\ 0\ 0\ 0\ 0; \frac{-1}{3}\ \frac{-1}{3}\ \frac{-1}{3})(0\ 0\ 0\ 0\ 0\ \frac{1}{2}\ \frac{-1}{2}\ 0)'$	L	$2_{\bar{1}}, 2_3$	1, 1	S_{18}
T_1^0	$(0\ 0\ 0\ 0\ 0; \frac{-1}{6}\ \frac{-1}{6}\ \frac{-1}{6})(0\ 0\ 0\ 0\ 0\ \frac{-3}{4}\ \frac{1}{4}\ \frac{1}{2})'$	L	3_3	1	S_{19}
T_1^0	$(0\ 0\ 0\ 0\ 0; \frac{-1}{6}\ \frac{-1}{6}\ \frac{-1}{6})(0\ 0\ 0\ 0\ 0\ \frac{1}{4}\ \frac{-3}{4}\ \frac{1}{2})'$	L	3_3	1	S_{20}
T_1^0	$(0\ 0\ 0\ 0\ 0; \frac{-1}{6}\ \frac{-1}{6}\ \frac{-1}{6})(0\ 0\ 0\ 0\ 0\ \frac{1}{4}\ \frac{1}{4}\ \frac{-1}{2})'$	L	$\{1_1, 1_3\}, \{2_3, 1_2\}, 6_3$	1, 1, 1	S_{21}
T_7^0	$(0\ 0\ 0\ 0\ 0; \frac{5}{6}\ \frac{-1}{6}\ \frac{-1}{6})(0\ 0\ 0\ 0\ 0\ \frac{-1}{4}\ \frac{-1}{4}\ \frac{1}{2})'$	L	2	1	S_{22}
T_7^0	$(0\ 0\ 0\ 0\ 0; \frac{-1}{6}\ \frac{5}{6}\ \frac{-1}{6})(0\ 0\ 0\ 0\ 0\ \frac{-1}{4}\ \frac{-1}{4}\ \frac{1}{2})'$	L	2	1	S_{23}
T_7^0	$(0\ 0\ 0\ 0\ 0; \frac{-1}{6}\ \frac{-1}{6}\ \frac{5}{6})(0\ 0\ 0\ 0\ 0\ \frac{-1}{4}\ \frac{-1}{4}\ \frac{1}{2})'$	L	$2_{\bar{1}}$	1	S_{24}

via $\langle S_2\rangle$, $\langle S_3\rangle$, and $\langle S_4\rangle$. Similarly, the singlet states in $\{T_1^+, T_7^-\}$ and $\{T_1^-, T_7^+\}$ pair up to be superheavy.

In order to break the flipped-SU(5) to the SM gauge group, we need GUT scale ($\approx$ string scale in our case) VEVs for $\overline{\mathbf{10}}_H$ and $\mathbf{10}_H$, which we have from T_3 and T_9, respectively. The μ-type term $\mathbf{10}_H\overline{\mathbf{10}}_H$ and terms with its higher powers are allowed. Thus, SUSY vacua where $\langle\overline{\mathbf{10}}_H\rangle = \langle\mathbf{10}_H\rangle \approx M_{\mathrm{string}} \approx M_{\mathrm{GUT}}$ can, in principle, exist.

We regard a pair of $\mathbf{5}_h$ and $\bar{\mathbf{5}}_h$ in T_4^0 as the Higgs fields containing two Higgs doublets of the MSSM. For the missing partner mechanism, we need the couplings $\overline{\mathbf{10}}_H\overline{\mathbf{10}}_H\bar{\mathbf{5}}_h$ and $\mathbf{10}_H\mathbf{10}_H\mathbf{5}_h$. These couplings are allowed in the superpotential by multiplying CSs, $S_{18}S_{11}S_{16}$ and $S_{17}S_{12}S_{15}$, respectively.

The vectorlike $\mathbf{5}$'s and $\bar{\mathbf{5}}$'s appearing in the T_6 sector obtain string scale masses. Via $\langle S_{21}\rangle$ one pair of five-plets in T_7^0 can pair up with one pair of five-plets in T_4^0 to be superheavy. The remaining pair of $\mathbf{5}$ and $\bar{\mathbf{5}}$ in T_4^0, i.e. $\{\mathbf{5}_h, \bar{\mathbf{5}}_h\}$, can get a mass term (or μ term) via $S_{17}S_{18}$ and S_1. Note that while a VEV $S_{17}S_{18}$ has been assumed, a VEV S_1 is not yet assumed. But it can be determined by soft terms such that $\mu \equiv \langle S_{17}S_{18} + S_1\rangle \approx m_{3/2}$, as in the MSSM supplied by singlets (NMSSM).

The MSSM matter states in the T_4^0 sector couple to the Higgs pair, $\mathbf{5}_h$ and $\bar{\mathbf{5}}_h$, in the same sector. Additionally, $\langle S_2S_3S_4\rangle$ can be multiplied to suppress the size of the Yukawa couplings. The matter states in the untwisted sector can couple to them via S_2, S_3, and S_4: $\overline{\mathbf{10}}_{-1}\overline{\mathbf{10}}_{-1}\bar{\mathbf{5}}_h \times \langle S_4^2\rangle$, $\overline{\mathbf{10}}_{-1}\mathbf{5}_3\mathbf{5}_h \times \langle S_2S_4\rangle$, and $\mathbf{1}_{-5}\mathbf{5}_3\bar{\mathbf{5}}_h \times \langle S_2S_3\rangle$. Since there are in total 21 $[=(2+3+2)\times 3]$ states in S_2, S_3, and S_4, they can be utilized to suppress the size of the Yukawa couplings.

B. White dwarf axions and one pair of Higgsino doublets

In this subsection, we comment on how the needed horizontal symmetry can arise from our heterotic string compactification. But, we will not endeavor to discuss accidental global symmetries arising at some specific vacua [27–29]. In Ref. [25], a variant very light axion has been introduced to enhance the axion-electron coupling. This enhancement was motivated from the unexpected extra energy loss from the white dwarf evolution [30]. It is needed to distinguish families by the quantum numbers of an Abelian horizontal gauge symmetry $\mathrm{U}(1)_H$ so that the mixing angles are of $\mathcal{O}(10^{-1})$–$\mathcal{O}(10^{-3})$. The Peccei-Quinn symmetry broken at $\sim 10^{11}$ GeV cannot achieve this goal due to the small mixing $F_a/M_P \sim 10^{-7}$. Let us choose the H direction as

$$H = \tfrac{1}{2}(1\ 1\ 1\ 1\ 1\ 3\ \ -1\ 1)(0\ 0\ 0\ 0\ 0\ a\ b\ c)', \tag{25}$$

where

$$b = 2a - 20, \qquad c = \tfrac{3}{2}a - 7. \tag{26}$$

The H quantum numbers of the visible sector quark and Higgs fields are shown below in the square brackets:

$$\begin{gathered} U\colon \overline{\mathbf{10}}_{-1}\ [0], \qquad T_4\colon 2\ \overline{\mathbf{10}}_{-1}\ [0], \qquad U\colon \mathbf{5}_{+3}\ [0], \\ T_4\colon 2\ \mathbf{5}_{+3}\ [-1], \qquad U\colon \mathbf{1}_{-5}\ [0], \qquad T_4\colon 2\ \mathbf{1}_{-5}\ [+1], \\ T_4\colon 2\ \mathbf{5}_{-2}\ [1], \qquad T_4\colon 2\ \bar{\mathbf{5}}_2\ [0], \qquad T_7\colon \mathbf{5}_{-2}\ [2],\ \bar{\mathbf{5}}_2\ [1] \end{gathered} \tag{27}$$

which have a $U(1)_H - SU(5)^2$ anomaly. But this anomaly is canceled by the Green-Schwarz mechanism [31]. The H quantum numbers of (27) are minus those anticipated in Ref. [25], and hence can act as the needed horizontal gauge symmetry. To realize the scenario of [25], we need $\mathbf{5}_{-2}$ [1] and $\bar{\mathbf{5}}_{+2}$ [1], which appear in T_4 and T_7, respectively.

As seen in the previous subsection, one quintet-antiquintet pair in T_7 is coupled to one quintet-antiquintet pair in T_4 via $\langle S_{21}\rangle$, and we assume that the other remaining pair in T_4 contains the MSSM Higgs. In this subsection, we will assume that $\langle S_{21}\rangle$ and $\langle S_4S_{16}\rangle$ are fine-tuned to be zero. This is possible because the quantum numbers of S_{21} and S_4S_{16} are the same. Instead, we need the following singlet VEVs to remove two Higgs quintet-antiquintet pairs,

$$T_4\colon S_1\ [-1], \qquad T_1\colon S_{19}\ [-2],$$

which break the $\mathrm{U}(1)_H$ gauge symmetry. Note that the VEVs of Eq. (24) break $\mathrm{U}(1)_H$. Here, we assume that $\langle S_1\rangle$ and $\langle S_{19}\rangle$ are the dominant $\mathrm{U}(1)_H$ breaking sources.

The $\mathrm{U}(1)_H$ invariant couplings of the form $T_4T_4T_4$ remove two pairs of Higgs quintets and antiquintets of T_4. Note that in the previous subsection $\langle S_1\rangle$ was adjusted to give a light mass term ("μ term") of one quintet-antiquintet pair in T_4. The $\mathrm{U}(1)_H$ invariant coupling of the form $T_1T_4T_7$ removes one Higgs quintet-antiquintet pair from T_4 and T_7. Thus, the 3×3 Higgsino mass matrix takes the form

$$\begin{array}{ccc|l} S_1[-1] & S_1[-1] & 0 & \mathbf{5}^a_{-2}[1](T_4) \\ S_1[-1] & S_1[-1] & 0 & \mathbf{5}^b_{-2}[1](T_4) \\ S_{19}[-2] & S_{19}[-2] & 0 & \mathbf{5}^c_{-2}[2](T_7) \\ \hline \bar{\mathbf{5}}_2[0](T_4) & \bar{\mathbf{5}}_2[0](T_4) & \bar{\mathbf{5}}_2[1](T_7) & \end{array} \tag{28}$$

It is obvious that $\bar{\mathbf{5}}_2[1](T_7) \equiv \bar{\mathbf{5}}^{\mathrm{EW}}_{-2}$ is massless at this level. If $\langle S_1\rangle = V_1$ and $\langle S_{19}\rangle = V_2$ and the Yukawa couplings are set to 1, the matching massless $\mathbf{5}^{\mathrm{EW}}_{-2}$ is a linear combination of $\mathbf{5}$'s from T_4 and T_7,

$$\mathbf{5}^{\mathrm{EW}}_{-2} = \frac{-V_2(\mathbf{5}^a + \mathbf{5}^b) + 2V_1\mathbf{5}^c}{\sqrt{4V_1^2 + 2V_2^2}}, \tag{29}$$

where the superscripts a, b, and c denote their origins from T_4 and T_7 as indicated in Eq. (28).

V. KALUZA-KLEIN SPECTRUM

The relatively light KK modes ($M_{\rm KK} < 1/\sqrt{\alpha'}$) associated with the relatively large extra dimensions can arise only in the nonprime orbifolds such as $\mathbf{Z}_{12-I}$. This is because KK excitations are possible only under a trivial (untwisted) boundary condition, which leads to $N = 2$ (or $N = 4$) SUSY spectra. In the $\mathbf{Z}_{12-I}$ orbifold, for instance, the boundary conditions associated with the SU(3) sublattice of the 6D compact space in the U, T_3, T_6, and T_9 sectors become trivial and allow $N = 2$ SUSY sectors [12].

The KK modes associated with the relatively large extra dimensions R ($\equiv R_3 = R_4$) of the SU(3) sublattice, whose masses compose a KK tower of (integer)/R, should also satisfy the mass shell conditions [12], Eqs. (A1) and (A2) of the Appendix, for $M_{\rm KK} < 1/\sqrt{\alpha'}$. Hence, the KK modes in the U sector still arise from the $\mathrm{E}_8 \times \mathrm{E}_8'$ root vectors.[2] But $P \cdot W =$ integer is not necessary for the KK states in the decompactification limit. In addition, the Gliozzi-Scherk-Olive (GSO) projection condition in the U sector is relaxed from $P \cdot V =$ integer to $P \cdot 3V =$ integer [12]. The $\mathrm{E}_8 \times \mathrm{E}_8'$ roots satisfying these are

$$\mathrm{SO}(10)\colon\ (\underline{\pm 1\ \ \pm 1\ 0\ 0\ 0};0\ 0\ 0)(0^8)', \tag{30}$$

$$\mathrm{SO}(6)\colon\ (0\ 0\ 0\ 0\ 0;\underline{\pm 1\ \ \pm 1\ 0})(0^8)', \tag{31}$$

$$\mathrm{E}_6'\colon \begin{cases} (0^8) & (\underline{\pm 1\ \ \pm 1\ 0\ 0\ 0};0\ 0\ 0)' \\ \pm(0^8) & (\underline{+\ -\ -\ -\ -};+\ +\ +)' \\ \pm(0^8) & (\underline{+\ +\ +\ -\ -};+\ +\ +)' \\ \pm(0^8) & (+\ +\ +\ +\ +;+\ +\ +)', \end{cases} \tag{32}$$

$$\mathrm{SU}(2)_{\rm K}'\colon\ \pm(0^8)(0\ 0\ 0\ 0\ 0;1\ \ -1\ 0)'. \tag{33}$$

Thus, above the compactification scale the gauge group is enhanced to

$$[\mathrm{SO}(10) \times \mathrm{SO}(6)] \times [\mathrm{E}_6 \times \mathrm{SU}(2)_{\rm K} \times \mathrm{U}(1)]'. \tag{34}$$

In the visible sector, the flipped-SU(5) in the massless case is embedded in the simple group SO(10). Therefore, between the GUT scale ($\sim$ compactification scale $1/R$) and the string scale $1/\sqrt{\alpha'}$, the MSSM gauge couplings are unified, including the $\mathrm{U}(1)_X$ coupling. $\mathrm{SU}(5)'$ and $\mathrm{SU}(2)'$ of the hidden sector are embedded in E_6'. Note that the $\mathrm{SU}(2)_{\rm K}'$ emerging in 6D space is different from the $\mathrm{SU}(2)'$ gauge symmetry observed from the massless spectrum. The condition for KK matter states from the U sector is also relaxed from $P \cdot V = \{\frac{-5}{12}, \frac{4}{12}, \frac{1}{12}\}\ (\mathrm{mod}\ Z)$ to $P \cdot 3V = \pm\frac{1}{4}\ (\mathrm{mod}\ Z)$ [12]. They form $N = 2$ hypermultiplets. The KK matter states from the U sector are shown in Table III.

Among the twisted sectors of $\mathbf{Z}_{12-I}$, only T_3, T_6, and T_9 can provide KK states. The KK states from T_9 are all the $\mathcal{CTP}$ conjugates of the KK states from T_3. As in the U sector, the KK modes from T_3, T_6, and T_9 should also satisfy the mass shell conditions. However, the required GSO projection is also relaxed. Following the guide of Ref. [12], where the authors discussed how the KK states are consistent with the modular invariance, one can derive the KK spectrum from the twisted sectors T_3 and T_6. The results are presented in Table IV. One can check that the KK spectra in Tables III and IV cancel the 6D gauge anomalies. The beta function coefficients $b_{\mathcal{G}}^{N=2}$ of SO(10) and E_6' by KK modes with $N = 2$ SUSY are

$$b_{\mathrm{SO}(10)}^{N=2} = -2 \times 8 + 2 \times (2 \times 8 + 1 \times 10) = 36, \tag{35}$$

$$b_{\mathrm{E}_6'}^{N=2} = -2 \times 12 + 2 \times 3 \times 2 = -12. \tag{36}$$

The KK masses are nothing but the excited momenta ($= \vec{m}_3, \vec{m}_4$) in the SU(3)-dual lattice of Fig. 1(b) in $\mathbf{Z}_{12-I}$. The Wilson line W^I lifts some KK spectra and breaks the gauge symmetry, say $\mathcal{G}$ to $\mathcal{H}$. This is because the momentum vectors $\vec{m}_3$, $\vec{m}_4$ are shifted by $P^I W^I$, where P^I indicates the $\mathrm{E}_8 \times \mathrm{E}_8'$ weight vectors. This is clearly seen

TABLE III. The KK spectrum from the U sector. The $\overline{\mathbf{16}}$ collectively denotes $(\underline{+\ -\ -\ -\ -})$, $(\underline{+\ +\ +\ -\ -})$, and $(+\ +\ +\ +\ +)$, which are $\mathbf{5}$, $\overline{\mathbf{10}}$, and $\mathbf{1}$, respectively, in terms of SU(5). Here we drop the $\mathcal{CTP}$ conjugates so that the fields listed above are all we need.

Visible states	4D χ	$\mathrm{SO}(10) \times \mathrm{SO}(6)$
$(\overline{\mathbf{16}};\underline{+\ -\ -})(0^8)'$	L, R	$(\overline{\mathbf{16}}, \mathbf{4})$
$(\overline{\mathbf{16}};+\ +\ +)(0^8)'$	L, R	
Hidden states	**4D χ**	**$\mathrm{E}_6' \times \mathrm{SU}(2)_{\rm K}'$**
$(0^8)(\overline{\mathbf{16}};\underline{+\ -}\ -)'$	L, R	
$(0^8)(\underline{\pm 1\ 0\ 0\ 0\ 0};\underline{1\ 0}\ 0)'$	L, R	$(\overline{\mathbf{27}}, \mathbf{2})'$
$(0^8)(0\ 0\ 0\ 0\ 0;\underline{-1\ 0}\ \ -1)'$	L, R	
$(0^8)(0\ 0\ 0\ 0\ 0;\underline{-1\ 0}\ 1)'$	L, R	$(\mathbf{1}, \mathbf{2})'$

TABLE IV. The KK spectrum from the T_3 and T_6 sectors. All the states are singlets under E_6'. The $\overline{\mathbf{16}}$ in T_6 collectively denotes $(\underline{+\ -\ -\ -\ -})$, $(\underline{+\ +\ +\ -\ -})$, and $(+\ +\ +\ +\ +)$, which are $\mathbf{5}$, $\overline{\mathbf{10}}$, and $\mathbf{1}$, respectively, in terms of SU(5). In the T_6 sector, we drop the $\mathcal{CTP}$ conjugates.

$P + 3V$	T_k	$(N^L)_j$	4D χ	$\mathrm{SO}(10) \times \mathrm{SO}(6) \times \mathrm{SU}(2)_{\rm K}'$
$(0^5;\underline{+\ +\ -})(0^5;\underline{\tfrac{3}{4}\ \tfrac{-1}{4}}\ \tfrac{-1}{2})'$	T_3	0	L, R	$4 \times (\mathbf{1}, \mathbf{4}; \mathbf{2}')$
$(0^5;-\ -\ -)(0^5;\underline{\tfrac{3}{4}\ \tfrac{-1}{4}}\ \tfrac{-1}{2})'$	T_3	0	L, R	
$(0^5;\underline{+\ +\ -})(0^5;\tfrac{-1}{4}\ \tfrac{-1}{4}\ \tfrac{1}{2})'$	T_3	1_1, 1_3	L, R	$8 \times (\mathbf{1}, \mathbf{4}; \mathbf{1}')$
$(0^5;-\ -\ -)(0^5;\tfrac{-1}{4}\ \tfrac{-1}{4}\ \tfrac{1}{2})'$	T_3	1_1, 1_3	L, R	
$(\overline{\mathbf{16}};0\ 0\ 0)(0^5;\tfrac{-1}{4}\ \tfrac{-1}{4}\ \tfrac{1}{2})'$	T_3	0	L, R	$4 \times (\overline{\mathbf{16}}, \mathbf{1}; \mathbf{1}')$
$P + 6V$	**T_k**	**$(N^L)_j$**	**4D χ**	**$\mathrm{SO}(10) \times \mathrm{SO}(6) \times \mathrm{SU}(2)_{\rm K}'$**
$(0^5;\underline{\pm 1\ 0\ 0})(0^5;\underline{+\ -}\ 0)'$	T_6	0	L, R	$3 \times (\mathbf{1}, \mathbf{6}; \mathbf{2}')$
$(\underline{\pm 1\ 0^4};0^3)(0^5;\underline{+\ -}\ 0)'$	T_6	0	L, R	$5 \times (\mathbf{10}, \mathbf{1}; \mathbf{2}')$

[2] The states of $\mathrm{E}_8 \times \mathrm{E}_8'$ *weights*, not satisfying $P^2 = 2$, are the string excited states with masses of (integer)/$\sqrt{\alpha'}$.

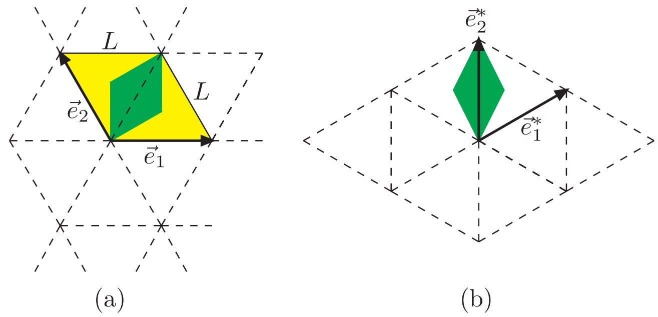

FIG. 1 (color online). The SU(3) lattice (a) and its dual lattice (b). In panel (a) the torus is inside the light grey parallelogram and the fundamental region is inside the dark grey parallelogram.

from the expression for KK masses [12]:

$$M_{\mathrm{KK}}^2 = \sum_{m_a, m_b} \frac{2\tilde{g}^{ab}}{3R^2}(m_a - P\cdot W)(m_b - P\cdot W), \qquad (37)$$

where R is the radius of the SU(3) torus, m_a, m_b ($a, b = 3, 4$) are integers, and $\tilde{g}^{ab}$ of the SU(3)-dual lattice is defined as

$$\tilde{g}^{\,ab} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}. \qquad (38)$$

We list the masses of the first two excited KK states for $P \cdot W =$ integer:

$$M_{\mathrm{KK}}^2 = \begin{cases} \frac{4}{3R^2} & \text{for } (m_3, m_4) = \pm(1,0), \pm(0,1), \pm(1,-1) \\ \frac{4}{R^2} & \text{for } (m_3, m_4) = \pm(1,1), \pm(2,-1), \pm(1,-2). \end{cases} \qquad (39)$$

See Fig. 2. For $P \cdot W = \frac{1}{3} +$ integer, we have

$$M_{\mathrm{KK}}^2 = \begin{cases} \frac{4}{9R^2} & \text{for } (m_3, m_4) = (0,0), (1,0), (0,1) \\ \frac{16}{9R^2} & \text{for } (m_3, m_4) = (1,1), (1,-1), (-1,1). \end{cases} \qquad (40)$$

In the next excited level, there are six KK states, whose

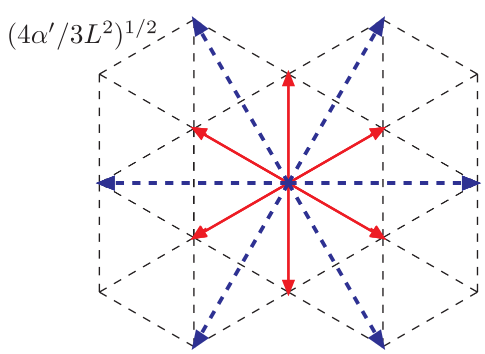

FIG. 2 (color online). The KK modes with $P \cdot W =$ integer. The length of the solid arrows is $(4\alpha'/3L^2)^{1/2}$ and that of the thick dashed arrows is $2(\alpha'/L^2)^{1/2}$.

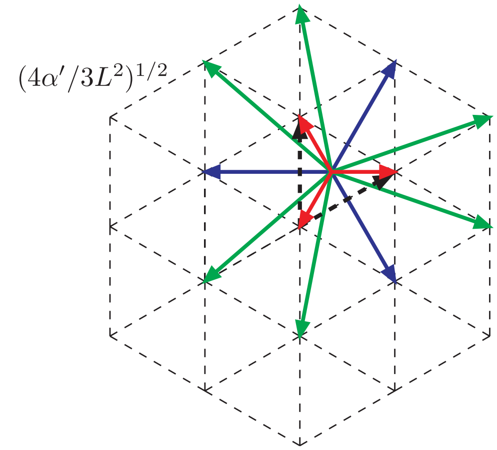

FIG. 3 (color). The KK modes with $P \cdot W = \frac{1}{3}$ mod integer. The lengths of the red (short), blue (medium), and green (long) arrows are $\frac{2}{3}(\alpha'/L^2)^{1/2}$, $\frac{4}{3}(\alpha'/L^2)^{1/2}$, and $\frac{2}{3}(7\alpha'/L^2)^{1/2}$, respectively.

mass squared is $\frac{28}{9R^2}$. See Fig. 3. As seen from Eq. (37), the KK mass squared for the states with $P \cdot W = -\frac{1}{3} +$ integer and $(-m_3, -m_4)$ is the same as that for the states with $P \cdot W = \frac{1}{3} +$ integer and (m_3, m_4). Nonvanishing vectors (m_3, m_4) do not affect the GSO projection conditions [12]. In Table V, we display the KK states satisfying $P \cdot W =$ integer. Thus the KK states in Tables III and IV also contain the states of $P \cdot W = \pm\frac{1}{3}$ plus an integer.

By the constraint $P \cdot W =$ integer, in the visible sector 6D SO(10) is broken to the flipped-SU(5), and SO(6) to $\mathrm{SU}(2) \times \mathrm{U}(1)^2$. The 6D hidden-sector gauge group E_6' is also broken to $\mathrm{SU}(5)' \times \mathrm{SU}(2)' \times \mathrm{U}(1)'$. But $P \cdot W =$ integer still leaves $N = 2$ SUSY intact. While the root vectors of the flipped-SU(5) and $\mathrm{SU}(5)' \times \mathrm{SU}(2)'$ are those of Eqs. (3)–(5), the roots of the 6D SU(2) in the visible sector are $\pm(0^5; 0\ 1\ 1)(0^8)'$. They are broken to U(1) below the compactification scale. The beta function coefficients $b_{\mathcal{H}}^{N=2}$ via the states with $P \cdot W =$ integer are

$$b_{\mathrm{SU}(5)}^{N=2} = -2 \times 5 + 2 \times (\tfrac{1}{2} \times 12 + \tfrac{3}{2} \times 5) = 17, \qquad (41)$$

$$b_{\mathrm{U}(1)_X}^{N=2} = \frac{1}{40} \times 2 \times (3^2 \times 10 + 1^2 \times 10 + 1^2 \times 40 + 2^2 \times 50) = 17, \qquad (42)$$

$$b_{\mathrm{SU}(5)'}^{N=2} = -2 \times 5 + 2 \times \tfrac{1}{2} \times 3 = -7. \qquad (43)$$

The beta function coefficients $b_{\mathcal{G}/\mathcal{H}}^{N=2}$ via the "matter" states with $P \cdot W = \pm\frac{1}{3} +$ integer are $b_{\mathcal{G}}^{N=2} - b_{\mathcal{H}}^{N=2}$. Since $b_{\mathrm{SU}(5)}^{N=2}$ is the same as $b_{\mathrm{U}(1)_X}^{N=2}$, and both are included in $b_{\mathrm{SO}(10)}^{N=2}$ in Eq. (35), the KK modes in this model, which respect the $N = 2$ SUSY, do not affect the gauge coupling unification of SU(5) and $\mathrm{U}(1)_X$. Accordingly, only the

TABLE V. The KK spectrum satisfying $P \cdot W =$ integer. Here we drop the $\mathcal{CTP}$ conjugates.

$P + kV$	T_k	4D χ	(SU(5), SU(2))(SU(5), SU(2))′
$(\underline{+\ -\ -\ -\ -};+\ +\ +)(0^8)'$	U	L, R	$(\mathbf{5},\mathbf{2})(\mathbf{1},\mathbf{1})'$
$(\underline{+\ -\ -\ -\ -};+\ -\ -)(0^8)'$	U	L, R	
$(\underline{+\ +\ +\ -\ -};-\ -\ +)(0^8)'$	U	L, R	$(\overline{\mathbf{10}},\mathbf{1})(\mathbf{1},\mathbf{1})'$
$(+\ +\ +\ +\ +;-\ +\ -)(0^8)'$	U	L, R	$(\mathbf{1},\mathbf{1})(\mathbf{1},\mathbf{1})'$
$(0^8)(\underline{+\ -\ -\ -}\ -;+\ -\ -)'$	U	L, R	
$(0^8)(+\ +\ +\ +\ +;+\ -\ -)'$	U	L, R	$(\mathbf{1},\mathbf{1})(\mathbf{5},\mathbf{2})'$
$(0^8)(\underline{1\ 0\ 0\ 0}\ 0;1\ 0\ 0)'$	U	L, R	
$(0^8)(0^5;0\ -1\ -1)'$	U	L, R	
$(0^8)(\underline{+\ +\ +\ -}\ -;-\ +\ -)'$	U	L, R	$(\mathbf{1},\mathbf{1})(\bar{\mathbf{5}},\mathbf{1})'$
$(0^8)(0\ 0\ 0\ 0\ -1;0\ 1\ 0)'$	U	L, R	
$(0^8)(-\ -\ -\ -\ +;-\ +\ -)'$	U	L, R	$(\mathbf{1},\mathbf{1})(\mathbf{1},\mathbf{2})'$
$(0^8)(0\ 0\ 0\ 0\ 1;0\ 1\ 0)'$	U	L, R	
$(0^8)(0\ 0\ 0\ 0\ 0;0\ -1\ 1)'$	U	L, R	$(\mathbf{1},\mathbf{1})(\mathbf{1},\mathbf{1})'$
$(0^5;-\ +\ +)(0^5;\frac{3}{4}\ \frac{-1}{4}\ \frac{-1}{2})'$	T_3	L, R	$4\times(\mathbf{1},\mathbf{2})(\mathbf{1},\mathbf{1})'$
$(0^5;-\ -\ -)(0^5;\frac{3}{4}\ \frac{-1}{4}\ \frac{-1}{2})'$	T_3	L, R	
$(0^5;+\ +\ -)(0^5;\frac{-1}{4}\ \frac{3}{4}\ \frac{-1}{2})'$	T_3	L, R	$4\times(\mathbf{1},\mathbf{1})$
$(0^5;+\ +\ -)(0^5;\frac{-1}{4}\ \frac{-1}{4}\ \frac{1}{2})'$	T_3	L, R	$8\times(\mathbf{1},\mathbf{1})(\mathbf{1},\mathbf{1})'$
$(\underline{+\ +\ +\ -\ -};0\ 0\ 0)(0^5;\frac{-1}{4}\ \frac{-1}{4}\ \frac{1}{2})'$	T_3	L, R	$4\times(\overline{\mathbf{10}},\mathbf{1})(\mathbf{1},\mathbf{1})'$
$(0^5;0\ 1\ 0)(0^5;+\ -\ 0)'$	T_6	L, R	$6\times(\mathbf{1},\mathbf{2})(\mathbf{1},\mathbf{1})'$
$(0^5;0\ 0\ -1)(0^5;+\ -\ 0)'$	T_6	L, R	
$(\underline{10}^4;0^3)(0^5;-\ +\ 0)'$	T_6	L, R	$10\times(\mathbf{5},\mathbf{1})(\mathbf{1},\mathbf{1})'$

fields in the $N = 1$ SUSY sector, i.e. the massless states, affect the unification.

From the beta function coefficients, we can expect that the MSSM gauge couplings rapidly increase in the ultraviolet region. On the other hand, the hidden-sector gauge coupling is asymptotically free. Therefore, a large disparity between couplings of the visible and hidden sectors at the compactification scale can be understood by going up above the compactification scale. In other words, starting with a unified coupling at the string scale, the hidden-sector SU(5)′ coupling can be of order 1 near the GUT scale.

When a gauge group $\mathcal{G}$ is broken to a subgroup $\mathcal{H}$ by the Wilson line and further broken to $\mathcal{H}_0$ by orbifolding,[3] the renormalization group (RG) evolution of the gauge coupling of $\mathcal{H}_0$, including the effects by KK modes, is described at low energies by

$$\frac{4\pi}{\alpha_{\mathcal{H}_0}(\mu)} = \frac{4\pi}{\alpha_*} + b_{\mathcal{H}_0}^{N=1}\log\frac{M_*^2}{\mu^2} + b_{\mathcal{H}}^{N=2}\Delta^0 + b_{\mathcal{G}/\mathcal{H}}^{N=2}\Delta^{\pm}. \tag{44}$$

We assume that the dilaton has been stabilized by a nonperturbative effect [32]. This can be discussed also in the context of SUSY breaking as in Ref. [21]. In Eq. (44), $b_{\mathcal{H}}^{N=2}\Delta^0$ ($b_{\mathcal{G}/\mathcal{H}}^{N=2}\Delta^{\pm}$) denotes the threshold correction by the KK modes of $P \cdot W = 0$ ($\pm\frac{1}{3}$) mod integer, respecting the $N = 2$ SUSY. $b_{\mathcal{H}_0}^{0}$ in Eq. (44) is the beta function coefficient contributed by the $N = 1$ SUSY sector states. As discussed above, the KK mass towers by the states with $P \cdot W = \frac{1}{3} +$ integer and with $P \cdot W = -\frac{1}{3} +$ integer are the same. $b_{\mathcal{G}/\mathcal{H}}^{N=2}$ is given by $b_{\mathcal{G}}^{N=2} - b_{\mathcal{H}}^{N=2}$.

As seen in Eqs. (35), (36), and (41)–(43), the beta function coefficients by KK modes are quite large. Accordingly, only the KK states residing in the lowest few layers of the KK mass tower would be involved in the RG evolution of the visible SU(5) gauge coupling, before it reaches $\mathcal{O}(1)$. So the field theory analysis, keeping only such relatively light KK modes for the RG analysis of the gauge couplings, would give a good approximation. Reference [12] presents a full stringy analysis on the threshold correction in $\mathbf{Z}_{12-I}$.

If $16/9R^2 < M_*^2 < 28/9R^2$, Δ^0 includes the contributions from six KK modes with mass squared $4/3R^2$, while Δ^+ (and also Δ^-) includes contributions from three KK modes of $4/9R^2$ and three KK modes of $16/9R^2$. Thus, the threshold corrections by such KK modes are given by

$$b_{\mathcal{H}}^{N=2}\Delta^0 = 17\cdot 6\cdot \log\left(\frac{3R^2M_*^2}{4}\right), \tag{45}$$

$$b_{\mathcal{G}/\mathcal{H}}^{N=2}\Delta^{\pm} = 19\cdot 3\cdot 2\left[\log\left(\frac{9R^2M_*^2}{4}\right) + \log\left(\frac{9R^2M_*^2}{16}\right)\right], \tag{46}$$

[3]In our model, $\mathcal{G} = [\mathrm{SO}(10)\times\mathrm{SO}(6)]\times[\mathrm{E}_6\times\mathrm{SU}(2)_{\mathrm{K}}\times\mathrm{U}(1)]'$, $\mathcal{H} = [\mathrm{SU}(5)\times\mathrm{U}(1)_X\times\mathrm{SU}(2)\times\mathrm{U}(1)^2]\times[\mathrm{SU}(5)\times\mathrm{SU}(2)\times\mathrm{U}(1)^3]'$, and $\mathcal{H}_0 = [\mathrm{SU}(5)\times\mathrm{U}(1)_X\times\mathrm{U}(1)^3]\times[\mathrm{SU}(5)\times\mathrm{SU}(2)\times\mathrm{U}(1)^3]'$.

where $\mathcal{H} = \mathrm{SU}(5)$ and $\mathcal{G} = \mathrm{SO}(10)$. We assume $1/R \approx M_{\mathrm{GUT}}$ and $\alpha_* = 1$. With $\alpha_{\mathrm{SU}(5)} = \frac{1}{25}$, we estimate $R^2 M_*^2 \approx 2.5$,[4] which is consistent with our assumption $16/9R^2 < M_*^2 < 28/9R^2$. With $R^2 M_*^2 \approx 1.9$, and

$$b^{N=1}_{\mathrm{SU}(5)'} = -3 \times 5 + \tfrac{1}{2} \times 3 + \tfrac{3}{2} = -12 \tag{47}$$

by $(\overline{\mathbf{10}}', \mathbf{1}')_0$, $(\mathbf{5}', \mathbf{2}')_0$, and $(\bar{\mathbf{5}}', \mathbf{1}')_0$ in Eq. (19), one can also estimate the confining scale of the hidden $\mathrm{SU}(5)'$. In the beta function coefficient of (47), we also included $\mathrm{U}(1)_X$-neutral $(\mathbf{5}' \oplus \bar{\mathbf{5}}')$ in addition to (19) to obtain the lowest possible $\mathrm{SU}(5)'$ confining scale. It is just below $\mu \approx 4/3R \approx 0.8 M_*$. Therefore, e.g., if $M_* = 2 \times 10^{16}$ GeV, the confining scale of the hidden sector is 1.6×10^{16} GeV. Indeed, the string scale can be much lower than 10^{18} GeV in strongly coupled heterotic string theory (or heterotic M theory), if the compactified 11th dimension is relatively large [33].

However, the hidden-sector confining scale is very sensitive to $R^2 M_*^2$. If $M_*^2 \lesssim 4/9R^2$, all the KK modes do not contribute to the RG evolution of the gauge couplings up to the string scale M_*, and so we should adopt only the usual 4D RG equation. If $M_* = 2 \times 10^{16}$ GeV and so $\alpha^{-1}_{\mathrm{SU}(5)'} = 25$ at that scale, the confining scale can be much lower, down to 10^{11} GeV. Here, we assume $\mathrm{SU}(2)'$ is broken at the compactification scale and only $\overline{\mathbf{10}}'$ and $\mathbf{5}'$ draw down the confining scale.

Below the confinement energy scale, the order parameters are composite fields rather than $\mathrm{SU}(5)'$ gauginos and quarks. As noticed in Ref. [21], the gaugino condensation scale or $N = 1$ SUSY breaking scale can be much lower than the confinement scale, as discussed in Refs. [21,34,35].

The threshold correction by the KK modes allows a very wide range of the $\mathrm{SU}(5)'$ confinement scale, from 10^{11} GeV to 10^{16} GeV. Moreover, as noticed in Ref. [21], the gaugino condensation scale can be quite low compared to the confinement scale of $\mathrm{SU}(5)'$. Thus, even in the case where the confinement scale is above 10^{13} GeV, one can obtain $N = 1$ SUSY breaking effects in the visible sector of order 10^{2-3} GeV via gravity mediation. If the condensation scale is below 10^{13} GeV, SUSY breaking effects in the visible sector by gauge mediation can dominate over those by gravity mediation, and here one may resort to the gauge mediation scenario [11].

[4]Considering that the first excited KK mass squared is $4/9R^2$, one could define the effective compactification scale, $R_{\mathrm{eff}} \equiv \frac{3}{2} R$. Then, $R_{\mathrm{eff}}^2 M_*^2 = 5.6$. So at $\mu = M_*/\sqrt{5.6} = 0.4 \times M_*$, the first excited KK modes appear.

VI. CONCLUSION

We have constructed a phenomenologically viable flipped-$\mathrm{SU}(5) \times$ hidden-$\mathrm{SU}(5)'$ model, based on the $\mathbf{Z}_{12-I}$ orbifold compactification of heterotic string theory. The flipped-SU(5) breaks down to the SM gauge group by nonzero VEVs of $\mathbf{10}_H$ and $\overline{\mathbf{10}}_H$. The doublet/triplet splitting problem is very easily resolved, because the missing partner mechanism simply works in the flipped-SU(5). In this model, we could obtain $\sin^2\theta_W = \frac{3}{8}$ at the string (or GUT) scale as desired. We have shown that all the extra states beyond the MSSM field spectrum are vectorlike under the flipped-SU(5) and obtain superheavy masses by VEVs of some neutral singlets.

In this model, the KK modes do not affect the gauge coupling unification in the visible sector, because the flipped-SU(5) gauge symmetry is enhanced to the SO(10) gauge symmetry above the compactification scale. On the other hand, they could cause a big difference between the visible and hidden gauge couplings at the compactification scale. Depending on the size of such disparity between the visible and hidden gauge couplings at the compactification scale, a wide range of the confining scale of $\mathrm{SU}(5)'$ is possible: 10^{11} GeV–10^{16} GeV. With the hidden matter $\overline{\mathbf{10}}'$ and $\mathbf{5}'$, the gaugino condensation scale or the $N = 1$ SUSY breaking scale can be a few orders lower than the hidden-sector $\mathrm{SU}(5)'$ confinement scale.

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APPENDIX

The string excited states are irrelevant to low energy physics. The masslessness conditions for the left- and right-moving strings are

$$\text{left-moving string}: \frac{(P + kV_f)^2}{2} + \sum_i N_i^L \tilde{\phi}_i - \tilde{c}_k = 0, \tag{A1}$$

$$\text{right-moving string}: \frac{(s + k\phi)^2}{2} + \sum_i N_i^R \tilde{\phi}_i - c_k = 0, \tag{A2}$$

where $k = 0, 1, 2, \ldots, 11$; $V_f = (V + m_f W)$ with $m_f = 0, +1, -1$; and i runs over $\{1, 2, 3, \bar{1}, \bar{2}, \bar{3}\}$. Here $\tilde{\phi}_j \equiv k\phi_j$ mod Z such that $0 < \tilde{\phi}_j \leq 1$, and $\tilde{\phi}_{\bar{j}} \equiv -k\phi_j$ mod Z such that $0 < \tilde{\phi}_{\bar{j}} \leq 1$. N_i^L and N_i^R indicate oscillating numbers for the left and right movers. P and s $[\equiv (s_0, \tilde{s})]$ are the $E_8 \times E_8'$ and SO(8) weight vectors, respectively. The values of $\tilde{c}_k$, c_k are listed as follows [17,18]:

$$2\times\tilde{c}_k=\begin{cases}\frac{210}{144}, & k=1; \quad \frac{192}{144}, \quad k=4\\ \frac{216}{144}, & k=2; \quad \frac{210}{144}, \quad k=5\\ \frac{234}{144}, & k=3; \quad \frac{216}{144}, \quad k=6\end{cases} \tag{A3}$$

and

$$2\times c_k=\begin{cases}\frac{11}{24}, & k=1; \quad \frac{1}{3}, \quad k=4\\ \frac{1}{2}, & k=2; \quad \frac{11}{24}, \quad k=5\\ \frac{5}{8}, & k=3; \quad \frac{1}{2}, \quad k=6.\end{cases} \tag{A4}$$

The multiplicity for a given massless state is calculated with the generalized GSO projector in the $\mathbf{Z}_{12-I}$ orbifold,

$$\mathcal{P}_k(f)=\frac{1}{12\cdot 3}\sum_{l=0}^{11}\tilde{\chi}(\theta^k,\theta^l)e^{2\pi i l\Theta_k},$$

where $f\ (=\{f_0,f_+,f_-\})$ denotes twist sectors associated with $kV_f=kV,\ k(V+W),\ k(V-W)$. The phase Θ_k is given by

$$\Theta_k=\sum_i(N_i^L-N_i^R)\hat{\phi}_i+\left(P+\frac{k}{2}V_f\right)V_f-\left(\tilde{s}+\frac{k}{2}\phi\right)\phi,$$

where $\hat{\phi}_j=\phi_j$ and $\hat{\phi}_{\bar{j}}=-\phi_j$. Here, $\tilde{\chi}(\theta^k,\theta^l)$ denotes the degeneracy factors, which are summarized in Table VI [17,18]. Note that $\mathcal{P}_k(f_0)=\mathcal{P}_k(f_+)=\mathcal{P}_k(f_-)$ for $k=0$, 3, 6, 9. In addition, the left-moving states should satisfy

$$P\cdot W=0 \bmod Z \text{ in the } T_3, T_6, T_9 \text{ sectors.}$$

Neglecting the oscillator numbers, H-momenta of states in various sectors, $H_{\mathrm{mom},0}\ [\equiv(\tilde{s}+k\phi+\tilde{r}_-)]$, are assigned as [6,17,18]

$$\begin{aligned}&U_1\colon(-1,0,0), \qquad U_2\colon(0,1,0), \qquad U_3\colon(0,0,1),\\ &T_1\colon\left(\frac{-7}{12},\frac{4}{12},\frac{1}{12}\right), \qquad T_2\colon\left(\frac{-1}{6},\frac{4}{6},\frac{1}{6}\right),\\ &T_3\colon\left(\frac{-3}{4},0,\frac{1}{4}\right), \qquad T_4\colon\left(\frac{-1}{3},\frac{1}{3},\frac{1}{3}\right),\\ &\left\{T_5\colon\left(\frac{1}{12},\frac{-4}{12},\frac{-7}{12}\right)\right\}, \qquad T_6\colon\left(\frac{-1}{2},0,\frac{1}{2}\right),\\ &T_7\colon\left(\frac{-1}{12},\frac{4}{12},\frac{7}{12}\right), \qquad T_9\colon\left(\frac{-1}{4},0,\frac{3}{4}\right),\end{aligned} \tag{A5}$$

from which T_5 will not be used since the chiral fields there are right-handed while the other fields are represented as left-handed. Including oscillators, the H-momenta $[\equiv(R_1,R_2,R_3)]$ are

$$(H_{\mathrm{mom}})_j=(H_{\mathrm{mom},0})_j-(N^L)_j+(N^L)_{\bar{j}}, \qquad j=1,2,3. \tag{A6}$$

The superpotential terms by vertex operators should respect the following selection rules [18]:

(a) Gauge invariance;

(b) H-momentum conservation with $\phi=(\frac{5}{12},\frac{4}{12},\frac{1}{12})$,

$$\sum_z R_1(z)=-1 \bmod 12, \qquad \sum_z R_2(z)=1 \bmod 3,$$
$$\sum_z R_3(z)=1 \bmod 12, \tag{A7}$$

where $z(\equiv A,B,C,\ldots)$ denotes the index of states participating in a vertex operator;

(c) Space group selection rules:

$$\sum_z k(z)=0 \bmod 12, \tag{A8}$$

$$\sum_z [km_f](z)=0 \bmod 3. \tag{A9}$$

If some singlets obtain string scale VEVs, however, condition (b) can be merged into Eq. (A8) in (c). Then, it is sufficient to consider (a) and (c) only.

There are 11 twisted sectors. For each twisted sector of T_1-T_6, the chiral fields are listed in the tables of Ref. [19].

TABLE VI. The degeneracy factor $\tilde{\chi}(\theta^k,\theta^l)$ in the $\mathbf{Z}_{12-I}$ orbifold.

$k\backslash l$	0	1	2	3	4	5	6	7	8	9	10	11
1	3	3	3	3	3	3	3	3	3	3	3	3
2	3	3	3	3	3	3	3	3	3	3	3	3
3	4	1	1	4	1	1	4	1	1	4	1	1
4	27	3	3	3	27	3	3	3	27	3	3	3
5	3	3	3	3	3	3	3	3	3	3	3	3
6	16	1	1	4	1	1	16	1	1	4	1	1

[1] S. M. Barr, Phys. Lett. **112B**, 219 (1982).
[2] J.-P. Derendinger, J. E. Kim, and D. V. Nanopoulos, Phys. Lett. **139B**, 170 (1984).
[3] I. Antoniadis, J. R. Ellis, J. S. Hagelin, and D. V. Nanopoulos, Phys. Lett. B **194**, 231 (1987).
[4] H. Georgi and S. L. Glashow, Phys. Rev. Lett. **32**, 438 (1974).
[5] I. Antoniadis, J. R. Ellis, J. S. Haglein, and D. V. Nanopoulos, Phys. Lett. B **208**, 209 (1988); **231**, 65 (1989); K. Dienes and A. Faraggi, Nucl. Phys. **B457**,

409 (1995); A. E. Faraggi, R. S. Garavuso, and J. M. Isidro, Nucl. Phys. **B641**, 111 (2002).
[6] J. E. Kim and B. Kyae, Nucl. Phys. **B770**, 47 (2007).
[7] G. Veneziano, Phys. Lett. **128B**, 199 (1983); I. Affleck, M. Dine, and N. Seiberg, Nucl. Phys. **B256**, 557 (1985); E. Poppitz and S. P. Trivedi, Phys. Lett. B **365**, 125 (1996).
[8] I. Affleck, M. Dine, and N. Seiberg, Nucl. Phys. **B256**, 557 (1985); H. Murayama, Phys. Lett. B **355**, 187 (1995).
[9] K. A. Intrilligator, N. Seiberg, and D. Shih, J. High Energy Phys. 04 (2006) 021.
[10] J. E. Kim, Phys. Lett. B **651**, 407 (2007).
[11] J. E. Kim, Phys. Lett. B **656**, 207 (2007).
[12] J. E. Kim and B. Kyae, Phys. Rev. D **77**, 106008 (2008).
[13] O. Lebedev, H. P. Nilles, S. Raby, S. Ramos-Sanchez, M. Ratz, P. K. S. Vaudrevange, and A. Wingerter, Phys. Rev. Lett. **98**, 181602 (2007).
[14] L. E. Ibanèz, J. E. Kim, H. P. Nilles, and F. Quevedo, Phys. Lett. B **191**, 282 (1987).
[15] S. Dimopoulos, S. Raby, and F. Wilczek, Phys. Rev. D **24**, 1681 (1981); C. W. Kim and U. Lee, Mod. Phys. Lett. A **6**, 1745 (1991); U. Amaldi, W. de Boer, and H. Furstenau, Phys. Lett. B **260**, 447 (1991); P. Langacker and M. Luo, Phys. Rev. D **44**, 817 (1991); J. R. Ellis, D. V. Nanopoulos, and J. Walker, Phys. Lett. B **550**, 99 (2002).
[16] J. E. Kim, J. H. Kim, and B. Kyae, J. High Energy Phys. 06 (2007) 034.
[17] Y. Katsuki, Y. Kawamura, T. Kobayashi, N. Ohtsubo, Y. Ono, and K. Tanioka, Nucl. Phys. **B341**, 611 (1990).
[18] K.-S. Choi and J. E. Kim, *Quarks and Leptons from Orbifolded Superstring*, Lecture Notes in Physics Vol. 696 (Springer-Verlag, Berlin, 2006).
[19] J.-H. Huh, J. E. Kim, and B. Kyae, arXiv:0904.1108.
[20] E. Poppitz and S. P. Trivedi, Phys. Lett. B **365**, 125 (1996); P. Pouliot, Phys. Lett. B **367**, 151 (1996).
[21] J. E. Kim, Phys. Lett. B **678**, 197 (2009).
[22] K. J. Bae and B. Kyae, J. High Energy Phys. 05 (2009) 102.
[23] O. Adriani *et al.* (PAMELA Collaboration), Nature (London) **458**, 607 (2009); Phys. Rev. Lett. **102**, 051101 (2009); J. Chang *et al.* (ATIC Collaboration), Nature (London) **456**, 362 (2008); S. Torii *et al.* (PPB-BETS Collaboration), arXiv:0809.0760.
[24] J.-H. Huh, J. E. Kim, and B. Kyae, Phys. Rev. D **79**, 063529 (2009); J.-H. Huh and J. E. Kim, Phys. Rev. D **80**, 075012 (2009).
[25] K. J. Bae, J. H. Huh, J. E. Kim, B. Kyae, and R. D. Viollier, Nucl. Phys. **B817**, 58 (2009).
[26] B. Kyae, J. Cosmol. Astropart. Phys. 07 (2009) 028.
[27] K.-S. Choi, I.-W. Kim, and J. E. Kim, J. High Energy Phys. 03 (2007) 116.
[28] K. S. Choi, H. P. Nilles, S. Ramos-Sanchez, and P. K. S. Vaudrevange, Phys. Lett. B **675**, 381 (2009).
[29] I.-W. Kim and J. E. Kim, Phys. Lett. B **639**, 342 (2006).
[30] J. Isern, E. García-Berro, S. Torres, and S. Catalán, Astrophys. J. **682**, L109 (2008).
[31] M. Green and J. Schwarz, Phys. Lett. **149B**, 117 (1984).
[32] V. Lowen and H. P. Nilles, Phys. Rev. D **77**, 106007 (2008); K. Choi, H. B. Kim, and H. D. Kim, Mod. Phys. Lett. A **14**, 125 (1999).
[33] E. Witten, Nucl. Phys. **B471**, 135 (1996); H. P. Nilles, M. Olechowski, and M. Yamaguchi, Nucl. Phys. **B530**, 43 (1998).
[34] Y. Meurice and G. Veneziano, Phys. Lett. **141B**, 69 (1984).
[35] J. E. Kim and H. P. Nilles, J. Cosmol. Astropart. Phys. 05 (2009) 010.