

## Asymmetric Higgs sector and neutrino mass in an $SU(2)_R$ model

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The asymmetric Higgs sector of one  $SU(2)_L \times SU(2)_R$  bidoublet ( $\phi_1^0, \phi_1^-; \phi_2^+, \phi_2^0$ ) and one  $SU(2)_R$  doublet [but no  $SU(2)_L$  doublet] is considered in a nonsupersymmetric left-right extension of the standard model of particle interactions. The inverse seesaw mechanism for neutrino mass is naturally implemented with the addition of fermion singlets, allowing thereby the possibility of breaking  $SU(2)_R$  at the TeV scale. Flavor-changing neutral Higgs couplings to quarks are studied in two scenarios, where the  $SU(2)_R$  charged-current mixing matrix is given either by the Cabibbo-Kobayashi-Maskawa matrix  $V_R = V_{CKM}$  (scenario I) or  $V_R = 1$  (scenario II). We consider the bounds on these scalar particle masses from  $K - \bar{K}$  and  $B - \bar{B}$  mixing, as well as  $b \rightarrow s\gamma$ . We find that, whereas in scenario I, they are of order 10 TeV, as in other left-right models, they may be well below 1 TeV in scenario II, thus allowing them to be within reach of detection at the forthcoming Large Hadron Collider.

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### I. INTRODUCTION

In the nonsupersymmetric  $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  extension of the standard  $SU(3)_C \times SU(2)_L \times U(1)_Y$  model (SM) of particle interactions, the Higgs sector must be enlarged from the one  $SU(2)_L$  scalar doublet of the SM. There are several ways to do this, as discussed comprehensively in Ref. [1]. In the canonical approach, a Higgs triplet is used to break  $SU(2)_R$  at a large scale, and  $\nu_R$  gets a large Majorana mass. A Higgs bidoublet is then added to break  $SU(2)_L$ , and all fermions obtain Dirac masses, with  $\nu_L$  getting a small seesaw mass. In this scenario, the  $SU(2)_R$  breaking scale is presumably beyond the reach of present accelerators, such as the Large Hadron Collider (LHC). Even if we try to lower this scale, the canonical model has severe difficulties with flavor-changing neutral currents, in contradiction with what is experimentally observed.

The purpose of this paper is to elaborate on a simple alternative [1], where the  $SU(2)_R$  breaking scale may be lowered to 1 TeV, using the inverse seesaw mechanism for neutrino mass [2–5]. We choose a Higgs sector which contains only one  $SU(2)_L \times SU(2)_R$  bidoublet and one  $SU(2)_R$  doublet [but no  $SU(2)_L$  doublet]. Of course, flavor-changing neutral Higgs couplings are still unavoidable. However, as we show in this paper, a scenario exists where they are sufficiently suppressed. Since the  $SU(2)_R$  charged-current mixing matrix is unknown, we consider two scenarios, where it is given either by the Cabibbo-Kobayashi-Maskawa matrix  $V_R = V_{CKM}$  (scenario I) or  $V_R = 1$  (scenario II). We consider the bounds on the corresponding scalar particle masses from  $K - \bar{K}$  and  $B - \bar{B}$  mixing, as well as  $b \rightarrow s\gamma$ . We find that, whereas in scenario I, they are of order 10 TeV, as in other left-right

models, they may be well below 1 TeV in scenario II, thus allowing them to be within reach of detection at the forthcoming Large Hadron Collider.

### II. ASYMMETRIC LEFT-RIGHT MODEL

#### A. Particle content and neutrino mass

The fermion content of the minimal  $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  gauge model is well known, i.e.,

$$\psi_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \sim (1, 2, 1, -1/2), \quad (1)$$

$$\psi_R = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_R \sim (1, 1, 2, -1/2),$$

$$Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \sim (3, 2, 1, 1/6), \quad (2)$$

$$Q_R = \begin{pmatrix} u \\ d \end{pmatrix}_R \sim (3, 1, 2, 1/6),$$

where the  $U(1)$  charge is normalized to  $(B - L)/2$  so that the electric charge is given by  $Q = I_{3L} + I_{3R} + (B - L)/2$ . Here a neutral fermion singlet

$$S_L \sim (1, 1, 1, 0) \quad (3)$$

is also added per family, which will have important implications for the neutrino masses, as shown below.

To obtain masses for the quarks and leptons, a Higgs bidoublet

$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix} \sim (1, 2, 2, 0) \quad (4)$$

is needed. In a nonsupersymmetric model, which is being considered here, the dual of  $\Phi$ , i.e.,

$$\tilde{\Phi} = \sigma_2 \Phi^* \sigma_2 = \begin{pmatrix} \bar{\phi}_2^0 & -\phi_1^+ \\ -\phi_2^- & \bar{\phi}_1^0 \end{pmatrix} \sim (1, 2, 2, 0) \quad (5)$$

must also be used. To break  $SU(2)_R \times U(1)_{B-L}$  to  $U(1)_Y$ , an  $SU(2)_R$  Higgs doublet

$$\Phi_R = \begin{pmatrix} \phi_R^+ \\ \phi_R^0 \end{pmatrix} \quad (6)$$

is added, which also links  $\bar{\nu}_R$  with  $S_L$  to form a Dirac mass  $m_R$ . Since  $S_L$  is a gauge singlet, it is also allowed to have a Majorana mass  $m_S$ ; hence the  $3 \times 3$  neutrino mass matrix spanning  $(\bar{\nu}_L, \nu_R, \bar{S}_L)$  is of the form

$$\mathcal{M}_{\nu,S} = \begin{pmatrix} 0 & m_D & 0 \\ m_D & 0 & m_R \\ 0 & m_R & m_S \end{pmatrix}, \quad (7)$$

where  $m_D$  is the usual Dirac mass linking  $\bar{\nu}_L$  to  $\nu_R$  through  $\langle \phi_1^0 \rangle$  and  $\langle \bar{\phi}_2^0 \rangle$ . A quick look at the above shows clearly that if  $m_S = 0$ , then the lepton number is conserved with a linear combination of  $\nu_L$  and  $S_L$  forming a Dirac fermion with  $\nu_R$ , and the orthogonal combination is exactly massless. This means that it is natural for  $m_S$  to be small, thereby triggering the inverse seesaw mechanism, resulting in

$$m_\nu \simeq \frac{m_D^2 m_S}{m_R^2}. \quad (8)$$

Note that there is no entry in Eq. (7) linking  $\nu_L$  and  $S_L$  because the  $SU(2)_L$  Higgs doublet is absent. This is important for the validity of Eq. (8). Instead of the canonical seesaw formula  $m_\nu \simeq -m_D^2/m_R$ , which is small if  $m_R$  is large, Eq. (8) lets  $m_\nu$  be small if  $m_S$  is small, even if  $m_R$  is not too large. Thus the inverse seesaw mechanism is suitable for bringing down the scale of  $SU(2)_R$  breaking to 1 TeV, with verifiable phenomenology at the LHC. Note also that the mixing of  $\nu_L$  with  $S_L$  is of order  $m_D/m_R$  which may now be non-negligible and results in deviations from unitarity [6] of the neutrino mixing matrix.

## B. Higgs sector

The most general Higgs potential consisting of  $\Phi_R$ ,  $\Phi$ , and  $\tilde{\Phi}$  is given by

$$\begin{aligned} V = & m_R^2 \Phi_R^\dagger \Phi_R + m^2 \text{Tr}(\Phi^\dagger \Phi) + \frac{1}{2} \mu^2 \text{Tr}(\Phi^\dagger \tilde{\Phi} + \tilde{\Phi}^\dagger \Phi) \\ & + \frac{1}{2} \lambda_R (\Phi_R^\dagger \Phi_R)^2 + \frac{1}{2} \lambda_1 [\text{Tr}(\Phi^\dagger \Phi)]^2 \\ & + \frac{1}{2} \lambda_2 \text{Tr}(\Phi^\dagger \Phi \Phi^\dagger \Phi) + \frac{1}{8} \lambda_3 \{[\text{Tr}(\Phi^\dagger \tilde{\Phi})]^2 \\ & + [\text{Tr}(\tilde{\Phi}^\dagger \Phi)]^2\} + \frac{1}{2} \lambda_4 [\text{Tr}(\Phi^\dagger \Phi)][\text{Tr}(\Phi^\dagger \tilde{\Phi} + \tilde{\Phi}^\dagger \Phi)] \\ & + f_1 \Phi_R^\dagger (\tilde{\Phi}^\dagger \tilde{\Phi}) \Phi_R + f_2 \Phi_R^\dagger (\Phi^\dagger \Phi) \Phi_R + f_3 \Phi_R^\dagger (\Phi^\dagger \tilde{\Phi} \\ & + \tilde{\Phi}^\dagger \Phi) \Phi_R, \end{aligned} \quad (9)$$

where all parameters have been chosen real for simplicity. Let  $\langle \phi_R^0 \rangle = v_R$  and  $\langle \phi_{1,2}^0 \rangle = v_{1,2}$ , then the minimum of  $V$  is given by

$$\begin{aligned} V_0 = & m_R^2 v_R^2 + m^2 (v_1^2 + v_2^2) + 2\mu^2 v_1 v_2 + \frac{1}{2} \lambda_R v_R^4 \\ & + \frac{1}{2} \lambda_1 (v_1^2 + v_2^2)^2 + \frac{1}{2} \lambda_2 (v_1^4 + v_2^4) + \lambda_3 v_1^2 v_2^2 \\ & + 2\lambda_4 (v_1^2 + v_2^2) v_1 v_2 + f_1 v_1^2 v_R^2 + f_2 v_2^2 v_R^2 \\ & + 2f_3 v_1 v_2 v_R^2, \end{aligned} \quad (10)$$

where  $v_R$  and  $v_{1,2}$  satisfy

$$v_R (m_R^2 + \lambda_R v_R^2 + f_1 v_1^2 + f_2 v_2^2 + 2f_3 v_1 v_2) = 0, \quad (11)$$

$$\begin{aligned} v_1 [m^2 + f_1 v_R^2 + (\lambda_1 + \lambda_2) v_1^2 + (\lambda_1 + \lambda_3) v_2^2 + 3\lambda_4 v_1 v_2] \\ + v_2 (\mu^2 + f_3 v_R^2 + \lambda_4 v_2^2) = 0, \end{aligned} \quad (12)$$

$$\begin{aligned} v_2 [m^2 + f_2 v_R^2 + (\lambda_1 + \lambda_2) v_2^2 + (\lambda_1 + \lambda_3) v_1^2 + 3\lambda_4 v_1 v_2] \\ + v_1 (\mu^2 + f_3 v_R^2 + \lambda_4 v_1^2) = 0. \end{aligned} \quad (13)$$

A solution exists where  $v_2 \ll v_1$ , i.e.,

$$v_2 \simeq \frac{-(\mu^2 + f_3 v_R^2 + \lambda_4 v_1^2) v_1}{m^2 + f_2 v_R^2 + (\lambda_1 + \lambda_3) v_1^2}, \quad (14)$$

with

$$v_1^2 = \frac{m_R^2 f_1 - m^2 \lambda_R}{\lambda_R (\lambda_1 + \lambda_2) - f_1^2}, \quad v_R^2 = \frac{-m_R^2 - f_1 v_1^2}{\lambda_R}. \quad (15)$$

Fine-tuning is of course unavoidable. In the limit  $v_2 = 0$ , the physical Higgs bosons are  $\phi_2^\pm$  and  $\text{Im} \phi_2^0$  with masses squared

$$\begin{aligned} m^2(\phi_2^\pm) &= (f_2 - f_1) v_R^2, \\ m^2(\text{Im} \phi_2^0) &= (f_2 - f_1) v_R^2 - (\lambda_2 + \lambda_3) v_1^2, \end{aligned} \quad (16)$$

and three linear combinations of  $\text{Re} \phi_1^0$ ,  $\text{Re} \phi_R^0$ ,  $\text{Re} \phi_2^0$ , with the mass-squared matrix

$$\mathcal{M}^2 = \begin{pmatrix} 2(\lambda_1 + \lambda_2)v_1^2 & 2f_1v_1v_R & 2\lambda_4v_1^2 \\ 2f_1v_1v_R & 2\lambda_Rv_R^2 & 2f_3v_1v_R \\ 2\lambda_4v_1^2 & 2f_3v_1v_R & (f_2 - f_1)v_R^2 - (\lambda_2 - \lambda_3)v_1^2 \end{pmatrix}. \quad (17)$$

### C. Gauge bosons

The structure of the scalar sector leads in general to both  $W_L - W_R$  and  $Z - Z'$  mixing. The former vanishes in the limit  $v_2 \rightarrow 0$  and so will be suppressed for the above choice of vacuum expectation values. In contrast, the  $Z - Z'$  mixing term is proportional to  $v_1^2$ , which is unacceptably large. To cancel this contribution, a simple possibility is to add a Higgs bidoublet  $X \sim (1, 2, 2, -1)$  with vacuum expectation value  $v_3$ . In that case, the choice  $v_3^2/v_1^2 = 1 - 2\sin^2\theta_W$  (for  $g_L = g_R$ ) will lead to zero mixing at tree level; details are given in the Appendix. Note that  $X$  will not affect the  $\rho$  parameter (at tree level) in precision electroweak measurements, nor will it contribute to quark or lepton masses. In particular, it does not link  $\nu_L$  with  $S_L$  in Eq. (7), otherwise the inverse seesaw mechanism would be invalidated. The present experimental limits on  $W_R$  and  $Z'$  are, respectively, 715 and 860 GeV.

## III. FLAVOR-CHANGING PROCESSES FROM NEUTRAL HIGGS COUPLINGS

### A. General structure

Since both  $\Phi$  and  $\bar{\Phi}$  couple to the quarks and leptons, flavor-changing interactions through the exchange of neutral Higgs scalars are unavoidable. The question is whether they can be suppressed [7]. Consider the Yukawa terms

$$(h_{ij}^u\phi_1^0 + h_{ij}^d\bar{\phi}_2^0)\bar{u}_{iL}u_{jR} + (h_{ij}^u\phi_2^0 + h_{ij}^d\bar{\phi}_1^0)\bar{d}_{iL}d_{jR}. \quad (18)$$

In the limit  $v_2 = 0$ , both *up* and *down* quark masses come from only  $v_1$ . Hence

$$\begin{aligned} h_{ij}^u v_1 &= U_L \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} U_R^\dagger, \\ h_{ij}^d v_1 &= D_L \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix} D_R^\dagger, \end{aligned} \quad (19)$$

where  $U_{L,R}$  and  $D_{L,R}$  are unitary matrices, with

$$U_L^\dagger D_L = V_{\text{CKM}}, \quad U_R^\dagger D_R = V_R, \quad (20)$$

being the quark mixing matrix for the known left-handed charged currents and that for their unknown right-handed counterparts. This means that in the basis of quark mass eigenstates, the structure of flavor-changing neutral currents through scalar exchange is determined, i.e.,

$$\frac{\text{Re}\phi_1^0}{v_1} \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} + \frac{\bar{\phi}_2^0}{v_1} V_{\text{CKM}} \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix} V_R^\dagger \quad (21)$$

for the *up* quarks, and

$$\frac{\text{Re}\phi_1^0}{v_1} \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix} + \frac{\phi_2^0}{v_1} V_{\text{CKM}}^\dagger \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} V_R \quad (22)$$

for the *down* quarks. Hence  $\text{Re}\phi_1^0$  behaves as the SM Higgs boson, and at tree level, all flavor-changing effects come from  $\phi_2^0$ , whereas in one loop, there are also contributions from  $(\phi_2^+, \phi_2^0)$ . Note that for  $v_1^2 \ll v_R^2$ , this electroweak doublet has the common mass of  $\sqrt{f_2 - f_1}v_R$ .

In the lepton sector, the analog of  $V_{\text{CKM}}$  is unknown because the neutrino mass matrix depends on  $m_D$ ,  $m_R$ , and  $m_S$ . In fact, we could choose  $m_D$  to be diagonal in the  $(e, \mu, \tau)$  basis and still have the freedom to obtain the observed neutrino mixing matrix from  $m_R$  and  $m_S$ . In that case,  $\phi_2^0$  would have no flavor-changing leptonic interactions.

### B. $K - \bar{K}$ and $B - \bar{B}$ mixing

We now apply Eq. (22) to  $K - \bar{K}$  and  $B - \bar{B}$  mixing. In the two scenarios (I and II) considered for the  $V_R$  matrix mentioned in the Introduction, the  $\phi_2^0$  couplings are of the form

$$(I) \quad V_R = V_{\text{CKM}}: \frac{\phi_2^0}{v_1} \bar{d}_{iL} d_{jR} \sum_k m_{u_k} V_{u_k d_i}^* V_{u_k d_j} + \text{H.c.} \quad (23)$$

$$(II) \quad V_R = 1: \frac{\phi_2^0}{v_1} \bar{d}_{iL} d_{jR} m_{u_j} V_{u_j d_i}^* + \text{H.c.} \quad (24)$$

We use the formulas presented in Ref. [8]. The mass difference of a neutral meson and its antiparticle is written in terms of its SM and other contributions:

$$\Delta M_X = (\Delta M)_{X,\text{SM}} + (\Delta M)_{X,\text{New}} \quad (25)$$

where  $\Delta M_X = \Delta M_K$ ,  $\Delta M_{B_d}$ ,  $\Delta M_{B_s}$ , and  $(\Delta M)_{X,\text{SM}}$  denotes the SM (one-loop) contribution, and  $(\Delta M)_{X,\text{New}}$  is everything else. In our case, the latter comes from the flavor-changing  $\phi_2^0$  couplings. The resulting expression for the mass difference is then given by

$$(\Delta M)_{X,\text{New}} = \frac{G_F^2 M_W^2}{6\pi^2} S_X [\bar{P}_2^{LR} C_2^{LR} + \bar{P}_1^{SLL} (C_1^{SLL} + C_1^{SRR})] \quad (26)$$

where the constant  $S_X$  includes strong-interaction effects, and the coefficients  $P$  include next-to-leading QCD corrections, while the functions  $C$  denote the Wilson coefficients of the operator-product expansion (OPE) expansion for the relevant hadronic matrix elements.

Let us consider first case (I) of our model, i.e.,  $V_R = V_{\text{CKM}}$ . Here the Wilson coefficients  $C_1^{SLL}$ ,  $C_1^{SRR}$  are equal:

$$C_1^{SLL} = C_1^{SRR} = \frac{16\pi^2}{G_F^2 M_W^2} \left(\frac{r_X^{LL}}{v_1}\right)^2 \left[ \frac{1}{m_{\text{Re}\phi_2^0}^2} - \frac{1}{m_{\text{Im}\phi_2^0}^2} \right], \quad (27)$$

and suppressed because the mass difference between  $\text{Re}\phi_2^0$  and  $\text{Im}\phi_2^0$  is small compared to their sum, whereas  $C_2^{LR}$  is of the form

$$C_2^{LR} = \frac{16\pi^2}{G_F^2 M_W^2} \left(\frac{r_X^{LR}}{v_1}\right)^2 \left[ \frac{1}{m_{\text{Re}\phi_2^0}^2} + \frac{1}{m_{\text{Im}\phi_2^0}^2} \right], \quad (28)$$

which has no such suppression. In case (I), the various  $r$ 's in each system are also the same:  $r_X^{LR} = r_X^{LL} = r_X^{RR} = r_X$ , where

$$r_K = m_u V_{ud} V_{us} + m_c V_{cd} V_{cs} + m_t V_{td} V_{ts}, \quad (29)$$

$$r_{B_d} = m_u V_{ud} V_{ub} + m_c V_{cd} V_{cb} + m_t V_{td} V_{tb}, \quad (30)$$

$$r_{B_s} = m_u V_{us} V_{ub} + m_c V_{cs} V_{cb} + m_t V_{ts} V_{tb}. \quad (31)$$

We have also assumed for simplicity that all the  $V_{\text{CKM}}$  entries are real.

Obviously there are large contributions coming from those terms proportional to  $m_t$  or  $m_c$ . However, there is also a natural suppression for the  $C^{LL}$  and  $C^{RR}$  Wilson coefficients, because their contributions are proportional to the effective  $\langle \phi_2^0 \phi_2^0 \rangle$  propagator, i.e.,  $m^{-2}(\text{Re}\phi_2^0) - m^{-2}(\text{Im}\phi_2^0)$ . Whereas  $\text{Im}\phi_2^0$  is a mass eigenstate,  $\text{Re}\phi_2^0$  is not, but if  $f_3$  and  $\lambda_4$  are small in Eq. (17), then it is approximately so, and their combined contribution for  $v_1^2 \ll v_R^2$  is naturally suppressed, i.e.,

$$\begin{aligned} & \frac{1}{(f_2 - f_1)v_R^2 - (\lambda_2 - \lambda_3)v_1^2} \\ & - \frac{1}{(f_2 - f_1)v_R^2 - (\lambda_2 + \lambda_3)v_1^2} \\ & \simeq \frac{-2\lambda_3 v_1^2}{(f_2 - f_1)^2 v_R^4}. \end{aligned} \quad (32)$$

This suppression persists even if  $f_3$  and  $\lambda_4$  are not neglected. We simply replace  $\lambda_3$  by

$$\lambda_3 + \frac{2f_3 \lambda_4 f_1 - f_3^2 (\lambda_1 + \lambda_2) - \lambda_4^2 \lambda_R}{\lambda_R (\lambda_1 + \lambda_2) - f_1^2}. \quad (33)$$

This feature of our model would allow  $v_R$  to be at the TeV scale, without running into conflict with present data on  $K - \bar{K}$  and  $B - \bar{B}$  mixing as far as  $C^{LL}$  and  $C^{RR}$  are concerned. Unfortunately, this suppression does not work for  $C^{LR}$ , which is proportional to  $m^{-2}(\text{Re}\phi_2^0) + m^{-2}(\text{Im}\phi_2^0)$ . However, as we show below in case (II), the  $C^{LR}$  coefficients are further suppressed by light quark masses in the  $r$ 's, which allows  $\phi_2^0$  to be lighter than 1 TeV.

In case (II), i.e.,  $V_R = 1$ , the  $r$  values are related by  $(r_X^{LR})^2 = r_X^{LL} r_X^{RR}$ , with

$$r_K^{LL} = m_c V_{cd}, \quad r_K^{RR} = m_u V_{us}, \quad (34)$$

$$r_{B_d}^{LL} = m_t V_{td}, \quad r_{B_d}^{RR} = m_u V_{ub}, \quad (35)$$

$$r_{B_s}^{LL} = m_t V_{ts}, \quad r_{B_s}^{RR} = m_c V_{cb}. \quad (36)$$

From the above, it is clear that whereas  $C^{LR}$  is not suppressed by Eq. (32), it is much smaller than what it is in case (I), because of the smallness of  $r^{LR}$ .

As mentioned in Ref. [8], there are large theoretical uncertainties associated with these expressions. To make an estimate, we simply require the absolute value of the contribution of new physics to be less than the corresponding experimental value. In what follows, we shall obtain bounds for the combination of parameters:  $1/\Delta^2 = m^{-2}(\text{Re}\phi_2^0) - m^{-2}(\text{Im}\phi_2^0)$  and  $1/\Sigma^2 = m^{-2}(\text{Re}\phi_2^0) + m^{-2}(\text{Im}\phi_2^0)$ . Let us define  $\Sigma^2 = m_2^2/2$  and  $1/\Delta^2 = \delta^2/m_2^4$ , where  $m_2$  is the approximate mass of  $\text{Re}(\phi_2^0)$  or  $\text{Im}(\phi_2^0)$ , and  $\delta^2$  is a measure of the splitting between their squared masses.

Using Eqs. (27) and (28), we obtain the following general expression:

$$\begin{aligned} & \left(\frac{r_X^{LR}}{v_1}\right)^2 \frac{2P_2^{LR}}{m_2^2} + \left[ \left(\frac{r_X^{LL}}{v_1}\right)^2 + \left(\frac{r_X^{RR}}{v_1}\right)^2 \right] \frac{P_1^{SLL} \delta^2}{m_2^4} \\ & = \frac{3}{8S_X} \Delta M_X^{\text{exp}}. \end{aligned} \quad (37)$$

For the  $K - \bar{K}$  system,  $S_K = m_K F_K^2 \eta_2 \hat{B}_K$ , with  $F_K = 160$  MeV,  $m_K = 498$  MeV,  $\eta_2 = 0.57$ , and  $\hat{B}_K = 0.85$ . At the scale  $\mu = 2$  GeV,  $\bar{P}_2^{LR} = 30.6$ ,  $\bar{P}_1^{SLL} = -9.3$ ,  $\Delta M_K^{\text{exp}} = 3.48 \times 10^{-12}$  MeV. Notice that in case (I), both  $P_2^{LR}$  and  $C_2^{LR}$  dominate over the  $LL$  and  $RR$  contributions. Therefore the resulting bound is not sensitive to the parameter  $\delta$ , and the bound on  $m_2$  is given by

$$m_2 \geq 25 \text{ TeV}. \quad (38)$$

For the  $B - \bar{B}$  systems, we take the corresponding parameters from the Particle Data Group [9], so that for  $(B_d, B_s)$

$$m_2 \geq 12(11) \text{ TeV}. \quad (39)$$

These results are in agreement with [7].

In case (II), if we take  $\delta = 0$  (i.e., only the  $LR$  contribution), we obtain a much smaller bound for the  $K$  system, i.e.,  $m_2 \geq 1.1$  TeV. However, for the  $B_d$ , and  $B_s$  systems,

the same procedure yields the bounds  $m_2 \geq 60(900)$  GeV, respectively. Thus for the  $B_d$  system, it seems more appropriate to consider  $\delta \neq 0$ , in which case the bound becomes  $m_2^2/\delta \geq 3.7$  TeV.

### C. $b \rightarrow s\gamma$

To evaluate the contribution of  $(\phi_2^+, \phi_2^0)$  to  $b \rightarrow s\gamma$ , we consider the relevant terms in Eq. (22). For case (I), i.e.,  $V_R = V_{\text{CKM}}$ , the important ones are

$$\begin{aligned} & \frac{m_t}{v_1} |V_{tb}|^2 (\phi_2^0 \bar{b}_L + \phi_2^+ \bar{t}_L) b_R + \frac{m_t}{v_1} V_{ts}^* V_{tb} (\phi_2^0 \bar{s}_L + \phi_2^+ \bar{c}_L) b_R \\ & + \frac{m_t}{v_1} V_{tb}^* V_{ts} (\phi_2^0 \bar{b}_L + \phi_2^+ \bar{t}_L) s_R + \text{H.c.} \end{aligned} \quad (40)$$

For case (II), i.e.,  $V_R = 1$ , they are

$$\begin{aligned} & \frac{m_t}{v_1} V_{tb}^* (\phi_2^0 \bar{b}_L + \phi_2^+ \bar{t}_L) b_R \\ & + \frac{m_t}{v_1} V_{ts}^* (\phi_2^0 \bar{s}_L + \phi_2^+ \bar{c}_L) b_R + \text{H.c.} \end{aligned} \quad (41)$$

The SM contribution (from the  $W$  exchange) is of the form  $\bar{s}_L \sigma_{\mu\nu} b_R$  which is classified [10] as  $O_7$ . Using the above interactions, there is only one such contribution coming from the  $\phi_2^0$  exchange, i.e.,  $\bar{s}_L b_R$  and  $\bar{b}_R b_L$ , which is proportional to  $V_{ts}^* m_t^2/v_1^2$  in both cases (I) and (II), assuming that  $V_{tb} = 1$ , which is of course a very good approximation. In contrast to the usual two-Higgs-doublet model, the  $\phi_2^+$  contribution is suppressed here because it is proportional to  $m_b$ . As for  $O_7'$ , i.e.,  $\bar{s}_R \sigma_{\mu\nu} b_L$ , both  $\phi_2^0$  and  $\phi_2^+$  have contributions proportional to  $V_{ts}^* m_t^2/v_1^2$ , but since the  $b \rightarrow s\gamma$  rate is proportional to

$$|C_7|^2 + |C_7'|^2 = |A_{\text{SM}} + A_{\phi_2^0}|^2 + |A'_{\phi_2^0} + A'_{\phi_2^+}|^2, \quad (42)$$

the latter can be safely ignored. Using Ref. [10], we find

$$A_{\text{SM}} \sim \frac{3m_t^2}{m_W^2} \left[ \frac{2}{3} F_1(x_t) + F_2(x_t) \right] \quad (43)$$

$$A_{\phi_2^0} \sim \frac{m_t^2}{m_{\phi_2^0}^2} \left[ -\frac{1}{3} F_1(x_b) \right] \quad (44)$$

where  $x_t = m_t^2/m_W^2$ ,  $x_b = m_b^2/m_{\phi_2^0}^2$ , and the functions  $F_{1,2}$  are given by

$$F_1(x) = \frac{1}{12(x-1)^4} (x^3 - 6x^2 + 3x + 2 + 6x \ln x), \quad (45)$$

$$F_2(x) = \frac{1}{12(x-1)^4} (2x^3 + 3x^2 - 6x + 1 - 6x^2 \ln x). \quad (46)$$

We now require the amplitude ratio  $|A_{\phi_2^0}/A_{\text{SM}}|$  to be less than 10%, so that it is well within the experimental accuracy. This translates to an estimated lower bound for  $m_{\phi_2^0}$  of about 200 GeV, as shown in Fig. 1.

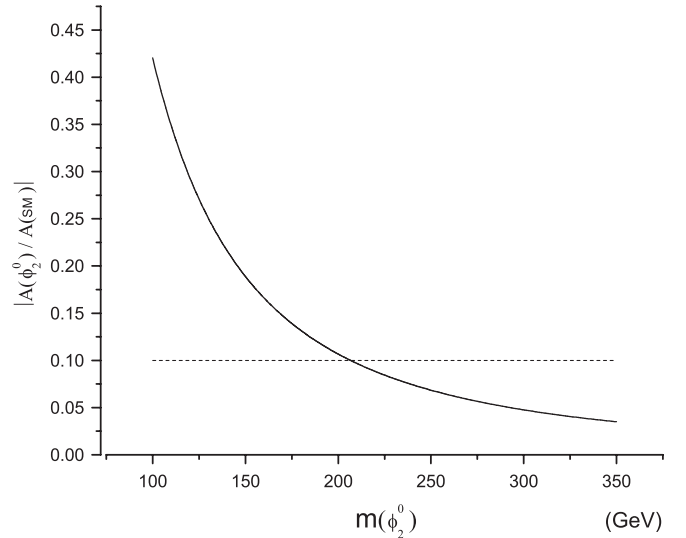


FIG. 1. Plot of  $|A_{\phi_2^0}/A_{\text{SM}}|$  vs  $m_{\phi_2^0}$ .

## IV. CONCLUSION

We have studied in this paper a simple nonsupersymmetric left-right extension of the standard model. The asymmetric Higgs sector of this model consists of one  $SU(2)_L \times SU(2)_R$  bidoublet and one  $SU(2)_R$  doublet [but no  $SU(2)_L$  doublet]. With the addition of neutral fermion singlets, the inverse seesaw mechanism for neutrino mass is naturally implemented, suggesting that the  $SU(2)_R$  breaking scale may be lowered to 1 TeV. We then analyzed the unavoidable problem of flavor-changing couplings of the neutral Higgs bosons of this model and showed that in the limit of  $v_2 = \langle \phi_2^0 \rangle = 0$ , these effects are naturally suppressed in case (II) ( $V_R = 1$ ) [but not in case (I) ( $V_R = V_{\text{CKM}}$ ), which has the same constraint as other left-right models that the  $SU(2)_R$  breaking scale is above 10 TeV.] From  $K - \bar{K}$  and  $B - \bar{B}$  mixing, we find  $v_R = \langle \phi_R^0 \rangle$  to be consistent with less than about 1 TeV in case (II). From  $b \rightarrow s\gamma$ , we find  $m_{\phi_2^0}$  to be above 200 GeV. The new particles of this model, i.e.,  $W_R^\pm$ ,  $Z'$ , the heavy pseudo-Dirac neutral fermion of mass  $m_R$  from the pairing  $S_L$  with  $\nu_R$ , and the heavy Higgs particles  $\text{Re}\phi_R^0$  and  $(\phi_2^+, \phi_2^0)$ , are all consistent with having masses below 1 TeV in case (II) and are potentially observable at the LHC.

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## APPENDIX

With only the bidoublet  $\Phi$ , our model exhibits an unavoidable  $Z - Z'$  mixing term proportional to  $v_1^2$ , implying

thus a very large value of  $v_R$ . This can be remedied by enlarging the scalar sector through the addition of another bidoublet  $X$  of  $(B - L)/2 = -1$ ,

$$X = \begin{pmatrix} \chi_1^- & \chi_2^0 \\ \chi_1^- & \chi_2^- \end{pmatrix} \sim (1, 2, 2, -1), \quad (\text{A1})$$

and its corresponding dual  $\tilde{X} = \sigma_2 X^* \sigma_2$ . We list in this Appendix the modifications resulting from its addition.

*Vector-boson masses:* Let the neutral components of  $\Phi$ ,  $\Phi_R$ , and  $X$  acquire vacuum expectation values

$$\langle \phi_{1,2}^0 \rangle = v_{1,2}, \quad \langle \phi_R^0 \rangle = v_R, \quad \langle \chi_2^0 \rangle = v_3, \quad (\text{A2})$$

and denote the neutral gauge bosons associated with  $SU(2)_{L,R}$  by  $W_{L,R}^0$  and the  $U(1)$  gauge boson by  $B$ , with  $g_{L,R}$  and  $g'$  the gauge couplings for  $SU(2)_{L,R}$  and  $U(1)$ , respectively. The resulting mass-squared matrix in the  $(W_R^0, W_L^0, B)$  basis is then given by

$$\mathcal{M}^2 = 2 \begin{pmatrix} g_R^2(v_1^2 + v_2^2 + v_3^2 + v_R^2) & -g_L g_R(v_1^2 + v_2^2 - v_3^2) & -g' g_R(v_R^2 + 2v_3^2) \\ -g_L g_R(v_1^2 + v_2^2 - v_3^2) & g_L^2(v_1^2 + v_2^2 + v_3^2) & -2g' g_L v_3^2 \\ -g' g_R(v_R^2 + 2v_3^2) & -2g' g_L v_3^2 & -g' g_R(v_R^2 + 4v_3^2) \end{pmatrix}. \quad (\text{A3})$$

The photon  $A$ , the neutral gauge boson  $Z$  of the SM, and the new  $Z'$  are then linear combinations, determined according to

$$\begin{pmatrix} W_R^0 \\ W_L^0 \\ B \end{pmatrix} = \mathcal{R} \begin{pmatrix} A \\ Z \\ Z' \end{pmatrix}; \quad \mathcal{R} = e \begin{pmatrix} 1/g_R & t_W/g_R & -1/(g' c_W) \\ 1/g_L & -1/(t_W g_L) & 0 \\ 1/g' & t_W/g' & 1/(g_R c_W) \end{pmatrix}, \quad (\text{A4})$$

where  $c_W = \cos\theta_W$ ,  $t_W = \tan\theta_W$  and the weak-mixing angle  $\theta_W$  and the proton charge  $e$  are defined by

$$\tan\theta_W = \frac{g' g_R / g_L}{\sqrt{g'^2 + g_R^2}}, \quad \frac{1}{e^2} = \frac{1}{g_R^2} + \frac{1}{g_L^2} + \frac{1}{g'^2}. \quad (\text{A5})$$

In terms of these fields, the above  $3 \times 3$  mass-squared matrix is reduced to a  $2 \times 2$  one, spanning only  $(Z, Z')$  with entries

$$m_Z^2 = \frac{e^2}{2} \frac{(1 + t_W^2)^2}{t_W^2} (v_1^2 + v_2^2 + v_3^2); \quad m_{Z'}^2 = \frac{e^2}{2} \frac{g_R^4 (v_1^2 + v_2^2 + v_3^2 + v_R^2) + 2g'^2 g_R^2 (v_R^2 + 2v_3^2) + g'^4 (v_R^2 + 4v_3^2)}{(c_W g' g_R)^2};$$

$$\Delta_Z = -\frac{e^2}{2} \frac{1 + t_W^2}{c_W t_W g' g_R} [g_R^2 (v_1^2 + v_2^2 - v_3^2) - 2g'^2 v_3^2], \quad (\text{A6})$$

where  $\Delta_Z$  is the  $Z - Z'$  mixing term. For the charged vector bosons, the analogous mass terms are

$$m_{W_L}^2 = \frac{1}{2} g_L^2 (v_1^2 + v_2^2 + v_3^2);$$

$$m_{W_R}^2 = \frac{1}{2} g_R^2 (v_1^2 + v_2^2 + v_3^2 + v_R^2); \quad (\text{A7})$$

$$\Delta_W = -\frac{1}{2} g_L g_R v_1 v_2.$$

Note that the  $\rho$  parameter is one at tree level, i.e.,  $m_{W_L}^2 = c_W^2 m_Z^2$ , in the absence of mixing, i.e.,  $\Delta_W = \Delta_Z = 0$ . This can be achieved by taking  $v_2 \ll v_1$  as already discussed in the text, and requiring

$$v_3^2 = \frac{v_1^2 + v_2^2}{1 + 2g'^2/g_R^2} \simeq \frac{v_1^2}{1 + 2g'^2/g_R^2}$$

$$\equiv u^2 v_1^{2 \xrightarrow{g_L = g_R}} (1 - 2\sin^2\theta_W) v_1^2. \quad (\text{A8})$$

Without this cancellation from  $X$ ,  $\Delta_Z$  would have been

unacceptably large. *Scalar potential:* With the addition of  $X$ , more terms occur in the Higgs potential:

$$V_X = m_X^2 \text{Tr} X^\dagger X + f'_1 |\det X|^2 + f'_2 |\text{Tr} \Phi^\dagger X|^2 + f'_3 |\text{Tr} \tilde{\Phi}^\dagger X|^2$$

$$+ f'_4 [(\text{Tr} \Phi^\dagger X)(\text{Tr} X^\dagger \tilde{\Phi}) + \text{H.c.}] + f'_5 (\text{Tr} X^\dagger X)^2$$

$$+ f'_6 (\text{Tr} \Phi^\dagger \Phi)(\text{Tr} X^\dagger X) + f'_7 [(\det \Phi)(\text{Tr} X^\dagger X) + \text{H.c.}]$$

$$+ f'_8 |\Phi_R|^2 (\text{Tr} X^\dagger X) + f'_9 \text{Tr}(\Phi^\dagger X X^\dagger \Phi)$$

$$+ f'_{10} [\text{Tr}(\Phi^\dagger X \tilde{\Phi}^\dagger X) + \text{H.c.}] + f'_{11} [\tilde{\Phi}_R^\dagger \Phi^\dagger X \Phi_R + \text{H.c.}]$$

$$+ f'_{12} \text{Tr}(\Phi^\dagger \Phi X^\dagger X) + f'_{13} \text{Tr}(X^\dagger X)^2 + f'_{14} \Phi_R^\dagger X^\dagger X \Phi_R, \quad (\text{A9})$$

where  $\tilde{\Phi}_R = i\sigma_2 \Phi_R^*$ . The full potential is then  $V \rightarrow V + V_X$ . The minimum value of  $V$ , which we denote by  $V_0$ , occurs when the various neutral fields are set equal to their corresponding vacuum expectation values:

$$\begin{aligned}
V_0 = & m_R^2 v_R^2 + m^2(v_1^2 + v_2^2) + 2\mu^2 v_1 v_2 + \frac{1}{2}\lambda_R v_R^4 + \frac{1}{2}\lambda_1(v_1^2 + v_2^2)^2 + \frac{1}{2}\lambda_2(v_1^4 + v_2^4) + \lambda_3 v_1^2 v_2^2 + 2\lambda_4(v_1^2 + v_2^2)v_1 v_2 \\
& + f_1 v_1^2 v_R^2 + f_2 v_2^2 v_R^2 + 2f_3 v_1 v_2 v_R^2 + m_X^2 v_3^2 + f_9' v_1^2 v_3^2 + 2f_7' v_1 v_2 v_3^2 + f_{12}' v_2^2 v_3^2 + f_6'(v_1^2 + v_2^2)v_3^2 \\
& + f_{13}' v_3^4 + f_5 v_3^4 + 2f_{11}' v_1 v_3 v_R^2 + f_{14}' v_3^2 v_R^2 + f_8' v_3^2 v_R^2,
\end{aligned} \tag{A10}$$

where  $v_{R,1,2,3}$  satisfy

$$\begin{aligned}
0 = & v_1[m^2 + f_1 v_R^2 + (\lambda_1 + \lambda_2)v_1^2 + (\lambda_1 + \lambda_3)v_2^2 + 3\lambda_4 v_1 v_2] + v_2(\mu^2 + f_3 v_R^2 + \lambda_4 v_2^2) \\
& + v_3[(f_6' + f_9')v_1 v_3 + f_7' v_2 v_3 + f_{11}' v_R^2]; \\
0 = & v_2[m^2 + f_2 v_R^2 + (\lambda_1 + \lambda_2)v_2^2 + (\lambda_1 + \lambda_3)v_1^2 + 3\lambda_4 v_1 v_2] + v_1(\mu^2 + f_3 v_R^2 + \lambda_4 v_1^2) \\
& + v_3[(f_6' + f_{12}')v_2 v_3 + f_7' v_1 v_3]; \\
0 = & v_R(m_R^2 + \lambda_R v_R^2 + f_1 v_1^2 + f_2 v_2^2 + 2f_3 v_1 v_2) + v_3[(f_8' + f_{14}')v_R v_3 + 2f_{11}' v_1 v_R]; \\
0 = & v_3[m_X^2 + (f_6' + f_9')v_1^2 + 2f_7' v_1 v_2 + (f_{12}' + f_6')v_2^2 + 2(f_{13}' + f_5')v_3^2 + (f_{14}' + f_8')v_R^2] + f_{11}' v_1 v_R^2,
\end{aligned} \tag{A11}$$

Let us define

$$\begin{aligned}
z_1 = & f_1 + u f_{11}', \\
z_2 = & f_{11}' + u(f_8' + f_{14}'), \\
z_3 = & f_6' + f_9' + 2(f_5' + f_{13}')u^2, \\
z_4 = & \lambda_1 + \lambda_2 + u^2(f_6' + f_9'), \\
z_5 = & f_{12}' - f_9' - (2f_{13}' - f_1')u^2, \\
z_6 = & u f_{14}' + f_{11}'.
\end{aligned} \tag{A12}$$

Using this notation, the vacuum expectation values have the following solution with  $v_2 \ll v_1$ :

$$\begin{aligned}
v_2 \simeq & \frac{-[\mu^2 + f_3 v_R^2 + (\lambda_4 + u^2 f_7')v_1^2]v_1}{m^2 + f_2 v_R^2 + [\lambda_1 + \lambda_3 + u^2(f_6' + f_{12}')v_1^2]}, \\
v_1^2 = & \frac{m_R^2 z_1 - \lambda_R m^2}{\lambda_R z_4 - z_1(z_1 + u z_2)}, \\
v_R^2 = & \frac{-m^2 z_4 + m^2(z_1 + u z_2)}{\lambda_R z_4 - z_1(z_1 + u z_2)},
\end{aligned} \tag{A13}$$

where  $u = g_R/\sqrt{g_R^2 + 2g'^2}$  was introduced in (A8), and  $v_3$  is determined by that same equation. In order for (A11) to be consistent with (A8), the parameters in the potential

must also satisfy

$$\begin{aligned}
u m_X^2 [z_1(z_1 + u z_2) - z_4 \lambda_R] = & m^2 [z_2(z_1 + u z_2) - u z_3 \lambda_R] \\
& + m_R^2 (u z_1 z_3 - z_2 z_4).
\end{aligned} \tag{A14}$$

There are many ways to obtain the desired hierarchy,

$$v_R \gg v_1 \sim v_3 \gg v_2. \tag{A15}$$

For example, let  $m_R \gg m, \mu$  and  $|f_3|, |z_1| \ll 1$ ; then  $v_R^2 \simeq m_R^2/\lambda_R$ ,  $v_1^2 \simeq (z_1/z_4)v_R^2$ , and  $v_2 \simeq -(f_3/f_2)v_1$ .

In the limit  $v_2 = 0$ , the (un-normalized) would-be Goldstone fields associated with the  $Z, Z', W_R^+$ , and  $W_L^+$  vector bosons are, respectively,

$$\begin{aligned}
G = & \text{Im}(c_\alpha \phi_1^0 + s_\alpha \chi_2^0); \\
G' = & \text{Im}[\phi_R^0 - s_{2\alpha} \epsilon(u \phi_1^0 - \chi_2^0)]; \\
G_R^+ = & \phi_R^+ + \epsilon(u \chi_1^+ - \phi_2^+); \\
G_L^+ = & c_\alpha \phi_1^+ + s_\alpha \chi_2^+;
\end{aligned} \tag{A16}$$

where

TABLE I. Physical mass eigenstates and their corresponding masses in the model containing  $X$ . We have ignored corrections of order  $v_1/v_R$  and  $v_2/v_1$ ; the various parameters are constrained by the requirement that all masses squared must be positive.

Field	(mass) <sup>2</sup>
$\chi_1^{++}$	$-\mathbf{v}_R^2 z_6/u$
$\chi_1^+$	$-\mathbf{v}_R^2 z_6/u$
$\phi_2^+$	$\mathbf{v}_R^2 (f_2 - z_1)$
$(-s_\alpha \phi_1^+ + c_\alpha \chi_2^+)$	$-2\mathbf{v}_R^2 f_{11}'/s_{2\alpha}$
$\text{Re} \phi_R^0$	$2\lambda_R \mathbf{v}_R^2$
$\text{Im} \phi_2^0, \text{Re} \phi_2^0$	$\mathbf{v}_R^2 (f_2 - z_1)$
$\text{Im}(-s_\alpha \phi_1^0 + c_\alpha \chi_2^0), \text{Re}(-s_\alpha \phi_1^0 + c_\alpha \chi_2^0)$	$-2\mathbf{v}_R^2 f_{11}'/s_{2\alpha}$
$\text{Re}(c_\alpha \phi_1^0 + s_\alpha \chi_2^0)$	$-2\mathbf{v}_1^2 [(c_\alpha z_1 + s_\alpha z_2)^2/\lambda_R - (s_\alpha^2 z_3 + c_\alpha^2 z_4)]$

$$\begin{aligned} \epsilon &= \frac{v_1}{v_R}; & s_\alpha &= \sin\alpha; & c_\alpha &= \cos\alpha; \\ s_{2\alpha} &= \sin 2\alpha; & \tan\alpha &= u. \end{aligned} \quad (\text{A17})$$

The physical scalars and their corresponding masses can be obtained from the potential in a straightforward manner:

there is a single doubly-charged field, 3 singly charged fields, and 6 (real) neutral fields. In obtaining the various expressions we have assumed (A15). The results are presented in Table I: they indicate that the field  $\text{Re}(c_\alpha \phi_1^0 + s_\alpha \chi_2^0)$  has a mass  $O(v_1)$  and plays the role of the SM Higgs boson; the other physical scalars have masses of order  $v_R$ .

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